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Modelowanie zjawisk zmiany trasy przejazdu w sytuacjach nietypowych

Rerouting phenomena modelling of unexpected events in Dynamic Traffic Assignment

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It can be said, in fact, that research, by exploring the greatest and the smallest, contributes to the glory of God which is reflected in every part of the universe.

Pope John Paul II, Karol Wojtyła

Jestem Polakiem – więc mam obowiązki polskie: są one tym większe i tym silniej się do nich poczuwam, im wyższy przedstawiam typ człowieka.

I am the Pole – so my duties are Polish: they are even stronger and oblige me more when my humanity evolves.

Roman Dmowski, Myśli Nowoczesnego Polaka, Warszawa, 1903

Contents

.

List of figures:

List of tables:

.

1. Summary

1.1. Highlights

- The problem of unexpected events and how they affect the dynamic traffic networks is addressed.
- Drivers react to the unexpected events by changing the route while travelling – they reroute and change the traffic flow pattern.
- Dynamic traffic assignment models shall represent the above rerouting phenomena to provide realistic outcomes.
- Two new alternative rerouting macroscopic DTA models are proposed: Information Comply Model (ICM) for detailed phenomena representation, and Rolling Horizon for the real-time ITS applications.
- ICM models the probabilistic information spreading processes and handles the rerouting decision with a discrete choice model based on the rerouting utility.
- The proposed awareness model of ICM is capable to handle various information sources: radio, VMS, on-line information, application.
- The proposed rerouting utility of ICM measures both the expected travel gains and losses.
- Rolling-Horizon is a modification of a DTA problem, which can be applied sequentially to adjust the traffic pattern to the actual conditions.
- Models are implemented in the state-of-the-art macroscopic DTA framework (TRE by SISTeMA) and are applicable for the real-size networks with the acceptable computation time.
- Theoretical estimation framework is introduced for the proposed models using the two identified datasets: observed *od* paths and observed flows.

1.2. Scope

Main scope and motivation of the thesis is to quantify the impact of unexpected events on the traffic networks. Namely to estimate the state-of-the-network resulting from the phenomena taking place in the traffic networks in cases of unexpected events (mainly the rerouting phenomena). This is achieved by proposing the two original extensions of the DTA model capable to represent the drivers routing behavior in case of unexpected events.

1.3. Abstract

The thesis addresses problem of modelling the state of the traffic network in cases of unexpected events by proposing the two alternative macroscopic dynamic traffic assignment models. Indeed, both the traffic flows and the travel times change due to the events. Significant change of traffic flow pattern observed on the Warsaw bridges in case of the traffic event justified the need for the proposed models. Forecasting and quantifying how the events affect the traffic pattern, is of particular importance, since the proper traffic management actions rely on it.

Thesis identifies that the representation of the, so-called, rerouting phenomena is critical to properly represent the impact of unexpected events. The rerouting takes place when drivers traversing the network realize that the situation is atypical and try to avoid the negative consequences (delay) by changing their routes. This way the demand pattern of vehicle flows traversing the network is different from what was expected and what was computed with the classic traffic assignment methods. Thesis identifies reasons why the classical equilibrium

traffic assignment models are incapable to represent the atypical situation and its impact on the traffic network. Therefore a main contributions of the thesis are the new rerouting models for the macroscopic Dynamic Traffic Assignment: 1) the Information Comply Model (ICM) with a detailed behavioral representation of the rerouting phenomena and 2) the Rolling-Horizon (RH-DTA) applicable in real-time ITS systems. Models are implemented by author in the state-of-the-art macroscopic DTA framework (TRE by SISTeMA) and are applicable for the real-size networks with the acceptable computation time.

ICM redefines the DTA demand model to cover the rerouting phenomena. It models the phenomena through calculating probability of the two fundamental transitions between cognitive states of the drivers. Initially, the unaware drivers become aware about the event due to numerous sources of awareness. Aware drivers can, in turn, comply with the event and decide to reroute due to utility they see in the rerouting. Utility is expressed through a difference in travel costs. The above process is represented with the awareness and compliance model respectively. The awareness model is designed to represent the way the information is spread to the drivers in time and space, including the spreading profiles for the radio news, VMS and information available on-line. The compliance model calculates the utility as a function of the expected travel time saving while rerouting and expected delay while not rerouting. Only the drivers who are aware and perceive a high utility of rerouting will change their routes. The two above components are integrated within the Markov chain, which is a part of the modified Network Flow Propagation procedure of the DTA. This way the full picture of rerouting is obtained. The resulting ICM DTA model relies on variables available at hand within the DTA and avoids complex calculations so that it is computable for real-size networks (ca. 25% longer computation times than in the classic DTA). Unfortunately, the presented version of the model can handle only a single event.

The second model (Rolling-Horizon) is proposed to overcome the limitation of the ICM model and be applicable in the real-time applications. The DTA is applied as a rolling horizon (RH) sequence, a common decision making practice for stochastic dynamic environment usually applied for planning purposes (i.e. production management and logistics). Every new update (roll) of a horizon means updating expected realization of a stochastic process, which was unknown before. So that the previous decisions can be revised with a new information. By analogy, it is assumed that drivers update their routing decisions whenever they receive a new information, i.e. about an unexpected traffic event. To this end, the RH-DTA is proposed - a modified DTA problem that can be applied sequentially in RH. Each new horizon comes with a new DTA problem, starting with a traffic flows from a previous horizon, utilizing a new information to update the routing decisions and propagate the flows up to the new horizon when the sequence is repeated. RH sequence of consecutive DTA problems is proposed as a flexible solution suitable for various use-cases: immediate and delayed information, informed and uninformed drivers, single and multiple events. The resulting RH-DTA is, in fact, an extension of the model proposed by (Peeta and Mahmassani, 1995) suited for needs of contemporary traffic management centers and real-time macroscopic DTA.

In the final part of the thesis, a theoretical estimation framework for the proposed rerouting models is introduced. Two valuable datasets for the estimation are identified: *od* paths observed during an unexpected event; and traffic flows of the entire cut-set of the network observed over the long period including atypical situation. Formal analysis is proposed for both of them to reveal the rerouting phenomena. Both of them form an input for the proposed maximum likelihood estimation problem of the ICM model. Traffic flows crossing Warsaw bridges measured over a several consecutive days including day of the event allowed to formulate the following conclusions about the rerouting behavior: a) about 20% of affected traffic flow rerouted, b) rerouting flows are increasing in time, c) drivers show strategic capabilities, d) and maximize their utility while rerouting.

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1.4. Outline

Thesis begins with an chapter 2 containing the definitions, notation and abbreviations lists.

Chapter 3 defines the urban traffic dynamics followed by the discussion on the relevance of rerouting phenomena for traffic management. The detailed background for the problem, including general definitions and discussion about rerouting phenomena precedes an extensive literature review.

Before introducing the rerouting models the broad theoretical background in is presented in chapter 4. The Theoretical background provides insight on the methods, algorithms and concepts used in the thesis. Since the proposed rerouting models are implemented within the DTA, the emphasis is on the formal introduction to the theory; including notation, algorithms and equations. Further extended with description of the hybrid route-choice model, which is the theoretical foundation of the proposed models.

The central part of the thesis is chapter 0 where the problem of the thesis is solved through the two macroscopic DTA rerouting models. Thesis proposes there two alternative models, both designed as an extension of a classic DTA algorithm. Starting from Information Comply Model (ICM), which yields realistic representation of the rerouting behavior of drivers. Followed by the Rolling-Horizon DTA (RH-DTA) suited for real-time applications. Models are defined through the idea, followed with the theoretical formulations, and concluded with the implementation in the DTA framework. Both models are illustrated with the numerical examples. Finally, the proposed models are summarized and compared.

Chapter 6 shows methods to estimate and validate the proposed models with direct and indirect estimation methods shown. The indirect estimation method is illustrated with the case study of Warsaw bridges, where initial assumptions of the rerouting phenomena are supported.

Thesis is concluded with chapter 7 with the summary and the future research directions.

1.5. Acknowledgements

I would definitely not be able to make this thesis without the support; therefore, I truly thank everybody who supported me with this. My wife, Anna, who was patiently asking over last few years if "the accident at link number seven (of the toy-network I was working with) is already over?" and now I can say: "Yes it’s over!”. Prof. Andrzej Szarata, supervisor of the thesis, supported the research, provided a great workplace, helped to solve the bureaucracy and reviewed the ideas several times. Special thanks should go to prof. Guido Gentile and his whole Roman group at La Sapienza University and in SISTeMA srl. Prof. Gentile has spent hours to teach me what the DTA really is and, when I finally got it, he discussed the rerouting problems and ideas in depth. Long, productive, yet exhausting discussions overlooking Colloseo in Rome founded this thesis and definitely they will fund more productive research in future. Dr Lorenzo Meschini supported and reviewed the ideas, gave a valuable hints on the applicability for Real-Time DTA and ITS. I wish I had even half of patience which Dr Daniele Tiddi had teaching me how TRE (DTA framework by SISTeMA used in the thesis) works, spending hours, often working with me until the La Sapienza was closing. The other (non-roman) part of international DTA community should be also thanked. Most importantly Klaus Noekel from PTV Group who gave the idea of this thesis and pushed it forward several times, but also: Thomas Otterstatter (TU Stuttgart), who commented the papers; Willem Himpe (KU Leuven) who compared my solutions with theirs; Valentina Trozzi (ICL), with whom I worked on her hyperpath algorithm, which inspired part of the thesis; and many more. The special thanks should go to all the members of the Department of Transportation Systems, where I found a productive and creative atmosphere.

The models proposed in the thesis were implemented inside the mature transport modelling and DTA solutions, namely PTV Visum and Traffic Real-time Equilibrium. I had access to the core of both of them and was free to develop ideas. Thanks to Klaus Noekel for the PTV software (PTV Visum) and to Guido Gentile for the TRE Software by SISTeMA.

Other thanks go to Warsaw Municipality, specifically Mr. Daniel Gajewski from the department Zarząd Dróg Miejskich who not only gave us access to the data, but explained it in deep, pointing important characteristics. Finally, I acknowledge that during the work on my thesis I have been a scholarship fellow of the "**Doctus – Małopolski fundusz stypendialny dla doktorantów**" project co-funded by EU funds within European Social Fund.

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2. Notation, definitions and abbreviations

2.1. Definitions

unexpected event any event taking place in urban traffic network, not known by at least some percentage of drivers and significantly changing the network characteristics, i.e. capacity, or speed.

rerouting changing the currently chosen route due to the traffic event while travelling.

demand following most of the DTA researchers, the phenomena taking place in the transportation systems are divided into two parts: demand and supply. Demand refers to the drivers and their travel choices, most importantly route choices, i.e. their decisions about how to travel from origin to destination.

supply refers to the physical phenomena taking place at the transportation network (i.e. congestion, queuing, propagation, etc.)

demand pattern result of the route choice model used to propagate flows through the network in time and space. In the thesis the implicit (local) route choices constitute the demand pattern instead of the full *od* paths. Demand pattern can be either *typical* (if typical costs are used to make routing decisions) or *actual* (when actual situation on the network influences the choices). Demand pattern aggregates the individual routing decisions to the total probabilities applied on the total demand flows for propagation.

travel time time needed to traverse the given network element (arc, node, or turn depending on the network topology used) or to get to the destination. In general, the travel times are objective and result from a physical traffic flow processes, while travel cost are subjective and user-dependent.

travel cost generic disutility perceived by driver while travelling. In general, routes are chosen by users to minimize travel costs, which can be understood as any additive variable of the route, including: travel time, monetary costs (tolls), safety, comfort, etc.

temporal profile representation of the discretized time-varying variable. Formalized as a vector of variable values evaluated at each discrete time instant of the simulation, i.e. the queue length for each minute of the simulation is the temporal profile of the queue.

typical (also: expected, usual) the conditions observed in case when no event is present in the network and when all the drivers has made their routing decisions as if nothing would happen. It can refer to typical travel costs and times, i.e. the travel times observed during the typical

	day (with no event) and the DTA results on the typical network. Typical demand pattern is the set of route choices made as if nothing has happened. Denoted in the thesis with superscript ^ - for going straight and not rerouting.
actual	conditions observed in the network while travelling. In case of an unexpected event they differ from the typical. Travel times and costs observable in case of the event and resulting from the event. They are taken into account while making rerouting decisions. Denoted in the thesis with superscript ~ - for rerouting, trying to avoid the event.
user, driver	user of the transportation system; an individual travelling from origin to the destination, making the route choices, propagating through the network, summing up to the macroscopic flow. User of the traffic network is a driver. Terms user and driver are used interchangeably in the thesis.
state-of-the-network	result of the DTA containing meaningful representation of the phenomena both at the demand and supply parts. State-of-the-network is the set of temporal profiles of travel times $\boldsymbol{t}$, travel costs $\boldsymbol{c}$ and directed traffic flows $\boldsymbol{q}$ on the network elements.
performance pattern	output of the supply model of DTA, temporal profile of travel times over all the network elements (i.e. arcs).
perspective	temporal profile of travel costs expected by the user when making a route choice decisions. The perspective can be typical if no information is available; or actual, when drivers in making their decisions include information about the event impact. User can have a wrong perspective when he does not know the actual state of the network.
decision point	place where a rerouting decision can be made, in the graph G rerouting can take place at the nodes $n \in N$.
rerouting decision	the moment at which a single driver reroutes, technically identified as the moment when a driver starts using the actual costs for the routing.
awareness	knowing about the event and considering it while making routing decisions. The awareness can be both due to being informed (notified) about the event and due to guessing it by observation of the atypical situation.
market penetration	is a share of a drivers which have access (and use) given information source .
compliance	making a rerouting decision resulting from complying with the information, observation that made user aware of the event.

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topological order specific order of graph nodes sorted from the closest to the furthest to a specific destination.

shortest path path connecting given origins with a given destination at a minimal possible cost (or time), obtained on a graph with i.e. with Dijkstra algorithm (Dijkstra, 1959).

shortest tree is a subgraph of graph *G* containing the shortest paths from all the nodes to a given destination *d.*

2.2. Abbreviations

ATIS	Advanced Traveler Information Systems
CDF	Cumulative Distribution Function
DNL	Dynamic Network Loading
DTA	Dynamic Traffic Assignment
DUE	Dynamic User Equilibrium
GLTM	General Link Transmission Model
ICM	Information Comply Model
LWR	Lighthill, Whitham Richards' dynamic traffic flow model
NFP	Network Flow Propagation
PDF	Probability Density Function
RCM	Route Choice Model
RH	Rolling-Horizon
RH-DTA	Rolling-Horizon DTA
STKW	Simplified Theory of Kinematic Waves
TMC	Traffic Management Centre
UE	User Equilibrium
VMS	Variable Message Sign

2.3. Notation

$G(N, A)$	connected, oriented graph of arcs $a \in A$, nodes $n \in N$, and centroids of origin/destination zones $o, d \in N$;
$a \in A$	generic arc of the graph *G;*
$n \in N$	generic node of the graph *G;*
a^-, a^+	tail and head of a generic arc *a.* $a^-, a^+ \in N$;
i^-, i^+	backward and forward star of a generic node *i.* $i^+ = \{a : a \in A \wedge a^- = i\}$;
i_d^-, i_d^+	efficient backward and forward star of a generic node *i* towards destination *d,* i.e. subset of forward star which brings closer to destination *d*;
k	generic path (route), ordered set of nodes connected by arcs *a*, optionally defined with the arrival time instants at consecutive arcs t_a – to form a trajectory;
τ	time instant when referred to a temporal layer;
θ	arrival time at destination *d*, or simplification of $h_i^d(\theta)$;
$\delta_a(\tau)$	parameters of arc *a* at time τ (speed, capacity, etc.);
$q_a^d(\tau)$	flow at arc *a* at time τ, towards destination *d*;
$f_a(\tau)$	inflow of arc *a* at time τ;
$e_a(\tau)$	outflow of arc *a* at time τ;
$d_a^d(\tau)$	originating flow at arc *a* at time τ, towards destination *d*. Either *od* demand flow or flow loaded from previous run (in RH-DTA);
$p_a^d(\tau)$	arc conditional probability, probability of using arc *a* towards destination *d* conditional on being at a^- at time τ;
$r_a(\tau)$	splitting rate, portion of flow directed from node a^- to arc *a* at time τ regardless the destination *d*;
$t_a(\tau)$	travel time of arc *a* for users entering it at time τ;
$c_a(\tau)$	travel cost of arc *a* for users entering it at time τ;
$h_a^d(\theta)$	minimum exit time of arc $a \in A$ to reach $d \in D$ at time θ along the shortest tree;
$g_a^d(\theta)$	minimum cost from the head of arc $a \in A$ to reach destination at time θ along the shortest tree;
$w_i^d(\theta)$	node satisfaction, the expected cost to get to the destination *d* from node *i* at time θ;
$TO_i^d(\tau)$	position of the node *i* in the topological order of nodes. Topological order sorts the nodes subject to travel time needed to get to destination *d* at time τ and is computed within the shortest path calculation from minimum exit times *h* or costs *g*;
$Q_a^d(\tau)$	number of vehicles on arc *a* at time τ towards destination *d*;
$\hat{c}, \hat{t}, \hat{p}, \ldots$	typical characteristics of the network;

$\tilde{\ }$ $\tilde{\ }$ $\ldots$	actual characteristics of the network;
$\iota_i^d(\tau)$	share of flow at node *i* at time *τ* that becomes aware about the event;
$\kappa_i^d(\tau)$	share of aware flow that comply and reroute from node *i* towards *d* at time *τ*;
$\hat{q}_i^d(\tau)$	unaware flow at node *i*;
$a_i^d(\tau)$	aware flow at node *i*;
$\tilde{\ }$	rerouted flow at node *i*;
$\alpha_i^d(\tau)$	share of the total flow which reroutes at node *i* towards *d* at time *τ*;
$M(\tau)$	network delay due to the event at time *τ*;
$\Delta p_i^d(\tau)$	difference between typical and actual arc probabilities vectors of forward star from node *i* towards destination *d* at time τ, computed with cosine similarity;
$\Delta w_i^d(\tau)$	difference between typical and actual node satisfactions vectors of forward star from node *i* towards destination *d* at time *τ*;
S	source of awareness;
P^S	penetration rate of a given source of awareness;
$\iota_a^S(\tau),\ \iota_a(\tau)$	probability of becoming aware through a generic information source *S* while traversing arc *a* at time *τ*. Calculated with the probability density function (PDF) for spreading of a generic information source *S* and a joint probability of all the present sources;
$I_a^S,\ I_a$	cumulative density function for spreading of a generic information source *S* and a joint probability of all the present sources;
$\hat{k}$	typical (reference) realization of path *k* (recurrently observed on a typical day);
r	rerouting point, node at route *k* at which the rerouting takes place;
ε	generic event with the corresponding start time t_ε, communication time i_ε and network modifications (in the hybrid route-choice model and in *RH-DTA* the event coincides with the horizon and perspective, also denoted with *ε*). In chapter 6 ε denotes the extra flow related to that event;
$\mathbf{c}^\varepsilon$, $\mathbf{p}^\varepsilon$, $\ldots^\varepsilon$	network cost, travel times, etc. perceived in the perspective ε;
s_ε	share of drivers having access to the information about the event.

3. Introduction

3.1. Motivation

Typical conditions are, paradoxically, infrequent in the congested urban traffic networks where whole range of events happens every day. The events are one of the major causes for urban traffic delays (Lomax et al., 2012), as they are usually unexpected and drivers are not aware about them when making routing decisions. Accurate forecast of traffic flows and travel times is crucial to manage the traffic in urban areas and minimize the delays. Yet, both travel times and traffic flows can vary significantly due to the unexpected events as they affect both the supply side (network) and demand side (users) of the traffic network. Events impact supply side at not only the place of event i.e. through reducing capacity, but also the disruption propagates upstream causing delays and spillback at the remainder of the network. However, modelling the effects on the supply side is relatively straightforward subject to a fixed demand pattern and can be done within a macroscopic traffic flow model, i.e. Link Transmission Model (Corthout et al., 2009). Much more challenging is to represent the phenomena taking place at the demand side due to unexpected events, namely how drivers react to unexpected events. Indeed, when the actual travel costs (defined in chapter 2.1), known or only perceived, vary significantly from the expected ones; the driver may react by shifting his current route to a better one. Such phenomena are further referred to as rerouting and are the key phenomena to be defined and modelled in the thesis. Its representation is particularly challenging if the information reaches a driver who is already travelling toward the destination. As shown in the case study of Warsaw bridges in chapter 6, drivers react to the unexpected events and change their routes which, in turn, significantly change the traffic pattern of the network. The traffic forecast need to take into account the drivers rerouting behavior in cases of unexpected events to remain efficient for traffic managers.

The traffic phenomena observable in urban areas are strictly dynamic in their nature. Both traffic flows and the corresponding travel times fluctuate throughout the day. That is why a dynamic representation of traffic is becoming widely applied to define traffic in the urban areas. In contrast to static methods, the dynamic ones are capable of representing dynamics of the traffic, providing results in terms of travel times and traffic flows temporal profiles. Dynamic Traffic Assignment (DTA) model handles the traffic dynamics both on the supply side (through time-varying speeds, queues, spillbacks, etc.) and in the demand (i.e. peak hours).

Dynamic Traffic Assignment (DTA) for road networks has been, so far, mainly applied off-line for the strategic planning and for the design purposes to improve the static models with a realistic representation of traffic phenomena (including congestion, queue formation and spillback) and time-varying travel demand (AM/PM peak hours). Nowadays, thanks to the recent software and algorithmic advancements (Gentile, 2015), DTA models can work also in real-time environment to estimate the actual state-of-the-network and forecast its short-term evolution. Such DTA models are applied in the traffic management centers (TMC) to provide the actual and forecasted traffic flows and travel times. They operate in real-time on real-size networks, face a number of situations and seek for a realistic representation of any occurring traffic situation. TMC performs control actions appropriate for the actual state-of-the-network by taking counter-measures, such as changing signal settings, pointing driving directions on VMS, disseminating traffic information and forecast. The action needs to be quick and well-aimed to be efficient. Efficiency of those actions is measured on its impact on the travel costs, which are a function of the traffic flows. Thus, to evaluate properly the impact of undertaken actions TMC needs an accurate estimation and forecast of traffic flows.

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Yet reproducing the actual demand pattern for real-time applications raises number of issues that need to be solved both on the theoretical and on the practical side. Traditionally, the demand in DTA is modelled through the Dynamic User Equilibrium (DUE), which proved to be a good representation of a typical users' behavior. DUE is plausible for off-line environment yet not for real-time applications, where the actual situation can be far from typical, i.e. due to unexpected events and related rerouting phenomena. Traffic flows in case of real-time events are not simply the result of the iterative interactions between supply and demand leading to a Dynamic User Equilibrium. They are more the result of an adaptation process, where an initial, typical, demand pattern is adjusted en-route when information about the traffic event becomes available.

Main scope and motivation of the thesis is to quantify the impact of unexpected events on the traffic networks. Namely to estimate the state-of-the-network resulting from the phenomena taking place in the traffic networks in cases of unexpected events (mainly the rerouting phenomena).

3.2. Rerouting phenomena in dynamic traffic assignment

Thesis identifies that the rerouting phenomena, defined as changing the currently chosen route due to the unexpected traffic event, is crucial to model drivers behavior in cases of unexpected events. Rerouting is one of many possible reactions to an unexpected event. Reactions to the events can take place both pre-trip and en-route. Before departing (pre-trip) the driver can modify his departure time, destination, route, or even resign from the trip at all. During the trip (en-route) changing the departure time is not possible anymore, yet the driver still can change the destination, route or resign (come back to the origin). (Dobler et al., 2013) provides an extensive background on all possible reactions to unexpected events, while in this thesis the origin, destination and departure time are assumed to be fixed and the reaction is limited to the route choices.

Figure 3.1 illustrates the rerouting phenomena in the urban context, the driver travels from the origin (bottom right) to the destination (top left). A driver decided pre-trip to travel along his typical route (bold), yet due to the atypical situation in the network (some unexpected event marked with the cross ×) he changes his route to a different one (dashed) when he receives the information about the event. This is the essence of the rerouting phenomena addressed in the thesis.

Representing rerouting phenomena is particularly challenging if the information reaches a driver who is already travelling toward the destination. Respectively the central problem of the proposed models is to realistically represent the en-route route choices (rerouting). The rerouting is called by other authors en-route rerouting (Snowdon et al., 2012), adaptation (Gao et al., 2010), or strategy within the hyperpath (Trozzi et al., 2009) and if the phenomena is limited to the pre-trip route changing it is referred to as route-swapping and traditionally handled within day-to-day models (Watling et al., 1999). The models proposed in the thesis simultaneously handle the pre-trip and en-route rerouting.

The rerouting can take place due to number of traffic events. In thesis, they are all further called: unexpected events, defined as any event that is not known by at least some percentage of drivers and significantly changes the network characteristics, i.e. capacity, or speed. They are ranging from incidents, road closures, new signal plans, sport events, demonstrations to planned road works, etc. Rerouting phenomena originates in the rational process when some reason leads an individual to alternate his plans. The reason to reroute is to avoid the expected negative consequences of the event (i.e. additional delay, longer trip duration, etc.). The rerouting decision is preceded with becoming aware about the event: receiving information, being notified, or observing atypical situation in the traffic network.

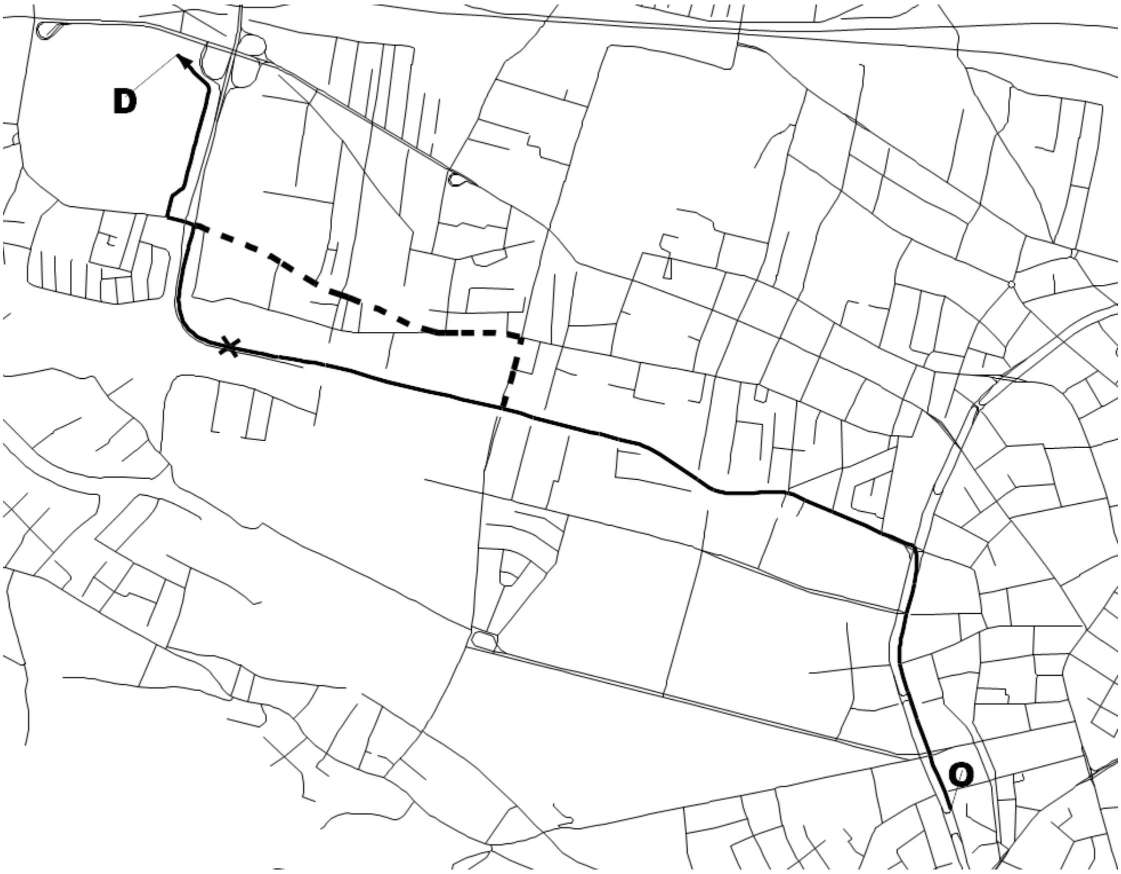

Figure 3.1 A typical route (bold) connecting origin O with the destination D. Due to the event (×) driver reroutes to the dashed route.

3.3. State-of-the-art and the problem statement

In general, DTA determines the traffic flows on the network satisfying the demand (Cascetta, 2009). It is done through the assignment methods, typically following the 'user-equilibrium' (UE) concept, being state at which no user finds it convenient to change his/her route and reduce the expected travel costs (Nash, 1951), (Wardrop, 1952). In the dynamic context 'user-equilibrium' becomes a dynamic user equilibrium (DUE), defined as a traffic pattern at which no driver finds it convenient to (unilaterally) change his/her route and departure time (see i.e. Friesz et al., 2000). DUE is obtained through the iterative process where at each iteration the route choices are adjusted based on the outcomes of decisions made in previous iterations. The process is converging to a fixed-point (Banach, 1922), where demand and supply are stabilized. The DTA results in the so-called state-of-the-network, representing both the supply (temporal profiles of travel costs and times) and the demand (traffic flows).

The state-of-the-practice provides, currently, two ways to handle unexpected events with DTA, both of them intrinsically wrong. One is to solve a DTA problem assuming that the event is known and included at the supply level. Such *predictive* DUE assumes that the information about unknown event travels back in time and is known at the origin before the event actually took place. Consequently, an unexpected event becomes, in fact, expected and known by users. Alternatively, event can be included in the supply side (network characteristics) and not included at the demand side (without effect on the route choices). In this case, instead of DTA, the DNL problem (defined in chapter 4.1.4) is solved to propagate the fixed demand pattern on the network with the event. This will result in no reaction to the incident; drivers will follow the previously chosen routes and drive straight into the place of event to form a queue. Such simulation can show the upper bound of the event impact at the supply level (i.e. maximal queue length), yet is not realistic as the rational drivers will try to minimize their cost and adapt to the unexpected situation (i.e. by rerouting). Both of the cases are illustrated on the toy-network in chapter 5.2.3 and become clearer with the in-depth theoretical background of DTA from chapter 4.1.

Based on the above brief introduction to the DTA, let's list the problems of modeling the rerouting phenomena in the macroscopic DTA assignment. The following issues arise while applying the DTA to model the rerouting phenomena:

1) issue of the iterative character of the DTA algorithm;
2) issue of the dynamic shortest path search applied for the unexpected events;
3) issue of the pre-trip route-choice disjointed from the flow propagation;
4) issue of the drivers' predictive capabilities in cases of unexpected events.

Let's start with the consequences of the iterative DTA algorithms and their convergence process. The day-to-day interpretation of DTA (i.e. Watling and Hazelton, 2003) is a good way to highlight this issue. In day-to-day interpretation, iterations are understood as days and adjusting is seen as learning from the previous days. Equilibrium is reached when drivers has learned their routing and do not need to seek for better routes, which implicitly means that the outcomes of their decisions are what they expected when they were deciding. Yet the learning is based on the critical assumption that next day will be the same as previous both in terms of network parameters and the demand (OD flows). Each iteration of DUE takes as an input the same network and the same origin demand; the routing decisions are the only independent variables of the DTA. This way DTA is suitable to reproduce the demand pattern (route choices) for any recurrent day for which the drivers had opportunity to learn the behavior, i.e. typical working day, typical Friday, holiday etc. Yet an unexpected event is not recurrent, event changes the network parameters so that demand pattern of DTA solution happens to be incorrect and a new demand pattern has to be calculated. However, the new arising pattern cannot be obtained with the iterative DTA, first due to not recurrent character of event – thesis addresses unexpected events that cannot be neither predicted, nor expected. Secondly, due to change of the initial conditions: the supply is different from what was in the previous iterations as the capacity or speed of the affected arc is different. Therefore, the iterative character of DTA needs to be revised.

The second group of issues arises when the dynamic shortest path algorithms are applied for unexpected changes in the network costs. Route choices in DTA rely on dynamic shortest path search, where the shortest path is computed to minimize the costs along the trajectory based on costs of arcs at the moment of traversing them (Dehne et al., 2010). Which implicitly means that drivers use for routing the future expected costs of arcs on the way to destination. In case of unexpected events not only the costs may be different from expected, but also this change can be still unknown. Correspondingly, the shortest path can be computed using either the actual costs (including the event) or the typical costs (of the usual day), both of them leading to wrong results. Using the actual costs within the route choice will result in a full knowledge about the future states of the network. This way users will make routing decisions to avoid the event even prior the event is known. Which will result in no effect of the event at the supply side (all users avoided the event, because in fact it was not unexpected, it was well known), being improbable. On the other hand using typical costs (learnt though the previous iterations) will results in no effect of the event at the decisions, the drivers will not reroute. Consequently, the dynamic shortest path search shall be revised as well.

The third issue, dual to the above, is the users making their route choices pre-trip based on expected costs taken from previous iterations. If the costs experienced en-route (while travelling) happens to be different from what was expected, they will influence only the subsequent iteration. The current iteration is completed using the demand pattern calculated pre-trip regardless of the actual travel costs. In other words routing within the single iteration is independent of the travel costs of this iteration. This is mainly due to the strict distinction between supply and demand models in the DTA where the current conditions simulated at the network do not affect the demand patterns of the flows propagating through the network.

The second issue implies the fourth: typically, for routing it is assumed that drivers are aware of costs of the whole network (which they utilize while searching for a shortest path). Within the supply model, the travel times are function of flows, i.e. the flows on the network affects the travel times. Therefore, if driver knows the costs, he implicitly needs to be aware of the routing decisions of others. This is perfectly consistent with the learning process for the recurrent situations, where drivers take into account also the others' decisions. Yet for the cases of unexpected events this would mean that drivers fully predict the rerouting decisions of others, which is doubtful. Even if driver gets the information about the event, he does not know who else received it and how others will react to it. He cannot fully predict the consequences of others' rerouting decisions.

The above issues are further elaborated and concluded into two limitations of the DTA models in chapter 4.2, after the theoretical background is formally introduced. The objective of the thesis is to overcome the above issues and propose the rerouting macroscopic DTA model.

3.4. Literature review

The formalization of traffic assignment in terms of econometric equilibrium (Wardrop, 1952) is commonly applied by most of researchers and the proposed models are not an exception. Rerouting models proposed in the thesis follow the fundamental traffic assignment assumptions, most importantly expected-utility concept (Bernoulli, 1738) as an explanation of routing behavior. As the rerouting phenomenon is strictly dynamic, the dynamic user equilibrium - DUE (Merchant and Nemhauser, 1978a,b) is considered. Specifically, the models proposed in the thesis originate from stochastic macroscopic DTA model proposed in (Bellei et al., 2005) with sequential route choices (Gentile et al., 2006) at the demand side and General Link Transmission Model (Gentile, 2010) on the supply side. The recent implementation of this framework can be found in (Gentile et al., 2013); however, chapter 4 of this thesis provides a thorough theoretical background. Broad review of DTA models by (Carey and Ge, 2003) is given as the reference, which can be further extended with (Gentile, 2010a).

The importance of the proposed model is justified within the definition of real-time DTA (Milkovits et al., 2010). Recent developments in theory and practice of DTA made it applicable for real size networks with acceptable computation times (Gentile, 2015). Thanks to that DTA can be applied not only in planning (off-line) context, but also for the real-time traffic management, i.e. Optima (Meschini and Gentile, 2010), Dynasmart (Mahmassani, 1997) (Peeta and Mahmassani, 1995), DynaMIT (Ben-Akiva et al., 1998), AIMSUN (Barcelo et al., 1998), (Olszewski et. al., 1995) etc. DTA in real-time is used to estimate and forecast the actual state-of-the-network (traffic flows and performance pattern) using the field data collected in real-time from numerous sources. The reference demand (OD matrix) and supply (network characteristics) are adjusted in real-time to reproduce the actual state-of-the-network. The real-time DTA results are used in TMC to perform control actions and to disseminate information. Yet applying DTA for real-time applications raises number of issues that need to be solved both on the theoretical and practical side. (Peeta and Mahmassani, 1995) and (Dobler, et. al., 2013) in clear way point why UE fails to represent rerouting phenomena which was used to identify the DTA issues from chapter 3.3.

Thesis focuses on the issue of extending the demand representation so that it covers not only a typical day, but also anomalous, non-recurrent cases commonly taking place in real

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urban networks. Therefore, a comprehensive literature review on route-choice models with real-time information, also en-route by (Gao et al., 2008) is referred. In this work, the Route-Choice Model (RCM) maximizing expected-utility through discrete-choice (logit) model (Gentile et al., 2006) is used. While actually, for the proposed models the RCM is transparent as long as it can be stored in local arc conditional choices and the node potentials with an implicit path enumeration. Recently number researches extended RCM to cover impact of real-time information on choices (Mahmassani and Liu, 1999; Peeta and Yu, 2005; Lee et al., 2010; Xu et al., 2011).

Stream of research analyzing day-to-day evolution of traffic network provides an excellent insight on the machinery of DTA and allows further extension for more general demand models. In fact, rerouting phenomena can be defined as: 'within-day adaptation of typical (equilibrium) route choices to the actual conditions due to unexpected events', which is similar to day-to-day interpretation. (Cantarella and Cascetta, 1995) gave a good theoretical background of the problem, further extended by (Watling and Hazelton, 2003). The question raised by (Watling and Hazelton, 2003) is not only what is the equilibrium and if it's unique and stable, but also if a new equilibrium is reached when network parameters and demand change, concluding that system is in endless evolution towards an equilibrium and hardly achieves it. Day-to-day research brought numerous concepts valuable for theoretical definitions of rerouting, namely: (Cantarella and Cascetta, 1995) handled day-to-day adaptation through two matrices: route switching probability matrix, and route transition matrix. (Watling, 1999) introduced cost updating rule to describe route-choice elasticity and proposed measure of "disequilibrium" being difference between costs of routes. (Watling and Hazelton, 2003) further developed his ideas and introduced multiple user classes and split demand into two parts (informed and uninformed) each utilizing different cost pattern for route choices, yet loaded on the network jointly within a single fixed-point problem. (Han et al., 2011) analyzed how the equilibrating process is affected by varying market penetration of Advanced Traveler Information Systems (ATIS). Further examined in (Bifulco, et al., 2013) showing that system can become instable for certain combination of equipped drivers and information they receive. ATIS can provide to the driver insight on the current (instantaneous) or forecasted state of the network. The information can be either the location of the event on which driver autonomously builds his route, or a full forecast arising from the event (Bifulco et al., 2009). (Bifulco et al., 2011) show that ATIS actions are most efficient when they accurately predict the performance pattern arising from the event both on the supply side (queues) and at the demand side (rerouting). The latter forms a circular dependency between predictive information and predicted travel costs addressed in the thesis in chapter 4.4.5.

Notable stream of papers by Gao, Frejinger and Ben-Akiva addressed rerouting phenomena through adaptive routing policies. (Gao et al., 2008) opposed routing policies (result of classic RCM) to the adaptive route choices and defined it as the mapping of the network 'support-points' (realization of travel times) to the routing decisions. The realization of policy is a single route, which can be adapted subject to the actual network conditions. The general concept of routing policy is somehow similar to the hyperpath (Trozzi et al., 2009) and the routing strategy (Hamdouch and Lawphongpanich, 2008). Routing policy is built on assumption that drivers can forecast actual travel times by observing realization of travel times while traveling. When driver observes actual state of the network (realization of travel times), he updates probability of downstream arcs travel times and, if he finds a better one than currently chosen, he reroutes. Which is somehow contradictory to the theory of the limited cognitive capacity pointed by (Gao et al., 2011). In the routing policy concept it is assumed that drivers use their experience to update policies, while thesis addresses mainly the unexpected, non-recurrent events. (Gao et al., 2010) further extended the above and proposed routing policy basing on Cumulative Prospect Theory (Kahneman and Tversky, 1979) which is more appropriate framework to model choice under risk than expected utility. The driver deciding to reroute is defined as making choice under risk and the asymmetrical, nonlinear utility allows evaluating the possible losses differently than the gains. In this thesis, the Prospect Theory was the motivation for the dual representation of the utility of ICM. Thanks to representing utility in terms of gains and in terms of losses, reformulation of the current discrete choice to the Prospect Theory in the future research is possible.

(Snowdon et al., 2012) followed similar approach like the routing policy, yet in the agent-based environment, where rerouting is driven by the experienced delay while traversing the network. He proposed an agent-based simulation in which agents everyday update knowledge on how perturbation on a single arc influences the remainder of the network. This knowledge is then stored in the correlation matrices. It is assumed that experiencing delay while travelling can lead to assumption that downstream arcs are also perturbed, which can, in turn, lead to the rerouting decision. In contrast (Mahmassani and Hu, 1997) assumed that user have access to the perfect knowledge about the network and rerouting takes place if the possible gains of changing the route are big enough. Moreover, it takes place only if the difference both relative and absolute is greater than so-called 'indifference-band'. Mahmassani propose term 'schedule-day' for what is called here typical.

Both Snowden and Mahmassani implicitly follow one of most valuable concepts for rerouting, namely the hybrid-model (Mahmassani et al., 1991) (Pel et al., 2009) (Qian et. al., 2012). Hybrid-model originates from a strong distinction between pre-trip and en-route route choice and ascertainment that rerouting takes place as a mixture of them, namely 'hybrid-routing': following pre-trip chosen route until there is a good reason to deviate from it and follow new route up to destination. Hybrid-model addresses rerouting with a sequential procedure executed at each decision point using utility of rerouting. Rerouting is calculated strictly subject to destination. The result of hybrid model is the new, recalculated route-choice probability at each node. Furthermore (Pel et. al, 2009) provide valuable considerations about information and define it as a function which transform actual costs into perceived costs used by user for routing. While (Qian et. al., 2012) defines the rerouting part of the hybrid route-choice model on the instantaneous travel times calculated with heuristics based on the Simplified Theory of Kinematic Waves (Lighthill and Whitham, 1955), which in general shall be avoided in ATIS environment. (Corthout and Immers, 2011) applied a hybrid model to focus on the event consequences at the supply side. Namely, they used the Link Transmission Model (Yperman, 2010) to assess the effect of the event in terms of queue length, spillback, gridlocks, etc. subject to the fixed demand.

The Rolling Horizon (RH) framework of (Sethi, et. al, 1991) is a commonly used in decision making theory for dynamic stochastic environment. Typically, a decision made in the stochastic environment is a solution of optimization problem for a given estimate of the stochastic process based on the actual data and its expected evolution. In RH, the decision is revised sequentially with every new *roll* of the *horizon*. New horizon provides an updated estimate of unknown stochastic process based on the actual data. Modifying initial conditions of the problem can lead to a different solution if the actual realization was not expected before. Instead of seeking for a single global optimum, the dynamic problem in RH is solved sequentially and local optima are found for each horizon, subject to currently available information. This way the decision made in stochastic environment can be adapted to actual realization of the stochastic process to find a new (possibly better) optimum as it is cyclically adapted to the actual data. The final decision is a concatenation of decisions made in the successive horizons. The hybrid-route choice model enables to apply DTA in the rolling-horizon, which was proposed earlier by (Peeta and Mahmassani, 1995). They identified DTA limitations and proposed to overcome some of them with Rolling Horizon approach, following (Gartner, 1982) who applied it for the signal control. (Peeta and Mahmassani, 1995) divide ATIS information into two types: *near-term* a 5-15 minute forecast based on the reliable data and *long-term* up to one hour, which will be overwritten when new *near-term* data will be available for that period. Each *stage* of the rolling horizon forms an assignment problem, which is solved through stage specific objective function minimizing travel times. RH-DTA sequence is called quasi-real time assignment problem, where the *quasi* is measured with the horizon duration. (Peeta and Mahmassani, 1995) introduced multiple user classes, including the switching (rerouting) class, on which the route switching formula is applied. Route switching formula enables rerouting only above a given travel time savings ('indifference-band'). The common-point between two consecutive horizons is obtained at the path flows level by defining the intermediate *node* for each path, which represent the new origins. Surprisingly the user classes which are not rerouting are treated as a static 'initial condition' and do not influence the temporal propagation of the total flow, which contradicts the actual phenomena.

The stated- and revealed-preference studies of rerouting are valuable to support estimation and validation of the model. (Emmerink et al., 1996) conducted a survey to see the impact of VMS and radio broadcasts on route choice. Sixty percent respondents claimed that their route-choice would be influenced by radio broadcasts, and forty percent by VMS signs. (Schlaich, 2010) got much less optimistic results with the revealed-preference data from floating mobile data. His research on how the VMS information affects route-choices showed ~30% of compliance. (Tracz et al., 1996) (Tracz et al. 1999) proposes thorough blackspot identification for traffic events, which allows to understand where the events are likely to take place.

An excellent statement of both the rerouting problem in assignment methods and definition of unexpected events of (Dobler, 2013) structured the problem of the thesis. The alternative definition of rerouting can be found in activity-based agent simulation (see Balmer, 2007). (Dobler, 2013) analyses the problem of within-day re-planning of agents' activity plans due to the events, which can include i.e. altering destination, later departure time or even resigning from the non-obligatory trips. Which links to the research stream on induced and surpressed traffic (Szarata, 2013). Within-day re-planning of Dobler is based on a strong distinction between iterative process adequate for recurrent conditions and single-shot simulation for special cases (i.e. re-planning, rerouting, evacuation), explaining why the classic assignment methods fail in modelling reaction of drivers to exceptional events. The agent structure proposed by Dobler uses the BDI structure – beliefs, desires, intentions, as defined in (Wooldridge, 2000), which is a good way to define rerouting where beliefs are perception of the network states (information and observation), desires are to avoid negative impact and finally intentions are to reroute or not. The BDI structure was an origin of the ICM structure of awareness and compliance models. Dobler proposes the Rayleigh distribution (Papoulis, 1984) to represent the information spreading process.

Problem of evacuation in case of unexpected events shows number of similarities with the thesis problem. Number of studies applied DTA and traffic models in general to simulate the evacuation scenarios of hurricanes, earthquakes, floods, etc. The same issues that made the iterative assignment unsuitable for cases of unexpected events made it unsuitable to simulate evacuation. The researches on the evacuation pointed how fully rational behavior changes when knowledge and time are limited (Drabek, 1969; Rosengren et al., 1975; Houts et al., 1988; Frey et al., 2011). Such considerations on random route-choice models led to conclusion that more detailed behavioral theories need to be covered to accommodate the rerouting phenomena. (Ben-Elia and Shiftan, 2010) gave an excellent comparison between the most popular route-choice models (see Ramming, 2002 for extensive route-choice models review) and cognitive theories, pointing works of (Tversky and Kahneman, 1992) where systematic violations of expected utility theory were revealed empirically. (Ben-Elia and

Shiftan, 2010) identified three alternative approaches to describe behavior under conditions of uncertainty: 'Hot Stove' (Denrell et al., 2001) which emphasizes asymmetry between bad and good outcomes of decisions. A bad experience lasts longer than good and decreases the tendency to take risk. Prospect Theory (Kahneman and Tversky, 1979) describes one-shot decisions and stresses that utilities are relative to 'point of reference' rather than to actual outcomes of choices (Razo and Gao, 2010). Decision makers in Prospect Theory will usually reveal risk-averse behavior in the case of gains, and risk-seeking behavior in the case of losses. Finally, the reinforced learning process (Busemeyer and Townsend, 1993) describes what happens when the decision maker receives no specific information describing the possible outcomes of choice and has to rely on feedback from past experiences, which leads to random choice if there is no previous experience. (Ben-Elia and Shiftan, 2010) raises an important issue of cognitive capacity which is often assumed to be infinite, e.g. drivers know current and future cost of each arc and, or even; like in (Gao et al., 2011), where they know correlation between travel times at each pair of arcs. Experiments by (Hölscher et al., 2011) show that our cognitive capacity is very limited and infinitive knowledge shall be avoided.

The binomial logit model (Ben-Akiva and Lerman, 1985) was used to model discrete choice in the compliance model of ICM. Whereas, the information spread model originated from evacuation models (Tweedie et al., 1986) where information about event spreads in time reaching audience. Thanks to the data from Twitter (Milstein et. al 2008) which became available lately, the information spread processes could have been understood in-depth. Numerous researches were conducted using Twitter data (Earle et al., 2012) (Leskovec et al., 2007) (Sakaki et al., 2010) (Sreenivasan et al., 2011) (Zaman et al., 2010) providing valuable information on the aspects of dynamics and range of the information spread process.

The dynamics are observable through the 'tweets' posted after emergencies: earthquake, hurricane, riots, etc. (Earle et al., 2012) proposed algorithm effectively detecting earthquake in few minutes by analyzing tweets (75% of earthquakes identified in below two minutes). (Procter et al., 2013) made an outstanding research on how fast the information dissipates though the communication network of Twitter. They investigated the Twitter traffic related to the false news (i.e. "Rioters released wild animals from London ZOO") and showed how the society reacts, believe and deny the false news. The analysis was made time dependent so that speed of the information is observable. This is probably the most evident and precise feedback on what is the process of informing in time through the internet. Several examples analyzed by Procter showed similar properties. The shape of the information spread curves resembles what researchers usually adopt while talking about information spread (Tweedie et al., 1986) (Cova and Johnson, 2002)(Lindell, 2008) – Rayleigh-like distribution (or any of similar shape). With important features: a) distribution is asymmetrical – the growing is much faster than decreasing, b) it is a convex function in the growing part (as opposed to concave Weibull distribution) i.e. the speed of spreading is growing. The study showed two different processes jointly: a) spreading the information, b) spreading the dementi. Both with completely different behavior: the information spreads rapidly, while dementi diffuses slowly. In traffic case that can be seen as news about a huge traffic event, the drivers are being informed according the diffusion model with given speed and impedance. While within the same time, the new information becomes available (i.e. "traffic back to normal") but those two alerts will be spread with various speeds due to their different importance. Thus, the actual number of drivers aware of the event is the product of those two phenomena.

Twitter research revealed other important phenomena of information, its virality (Leskovec et al., 2007) (Ghosh et al., 2011). (Ghosh and Lerman, 2012) showed that information in the communication network is either completely negligible and forgotten very fast or completely opposite: spreads like viruses through communication network, reposted forward with exponential probability (Sakaki et. al, 2010). The observations on virality suggest to parameterizing both the spread speed and penetration rate with significance of the information (measured with the total delay it causes).

3.5. Solutions

Based on the above literature review and problem statement the solutions can be proposed. To this aim, the two complementary models are proposed and broadly defined in chapter 5. Both of them become possible thanks to the concept of hybrid-route choice model (Peeta and Mahmassani, 1995), (Pel, et. al., 2009) elaborated within theoretical definitions of chapter 0.

Hybrid route choice model allowed formulating the two complementary models capable to represent rerouting phenomena. First one: Information Comply Model (ICM) aims to realistically represent the decision making process with a meaningful behavioral model that can be estimated with the available data. Second one: Rolling Horizon DTA (RH-DTA) is designed for the real-time environment, which is achieved with a flexible model applicable for any situation observable in the actual traffic networks. RH-DTA can work with any number of events and provide meaningful results in real-time, yet it simplifies the phenomena. Contrary to ICM, which provides reach representation of the phenomena, yet can work only for a single event.

Information Comply Model originates from the thorough definition of the driver's behavioral decision process. ICM divides the rerouting process into the two steps: becoming aware and deciding to reroute. Since in the urban traffic networks driver can become aware via number of information sources (radio, internet apps, VMS, etc.), all of them are handled jointly inside a proposed probabilistic awareness model quantifying who and where becomes aware of the event. Aware drivers are then making a rerouting decisions, driven by the rerouting utility quantified through the possible travel time gains ("How much faster can I get to destination if I reroute?") and possible travel time losses ("How much longer will I go to the destination if I do not reroute?"). Decision process is modelled with the proposed compliance model, being a binomial logit model with two possible decisions: reroute or not. To apply the above ICM machinery in the DTA framework, the Network Flow Propagation of the DTA is redefined. It handles the three states of the flow: unaware, aware and rerouted. Which form the three-state Markov chain with transitions computed with the awareness and compliance models. Such extended NFP can be directly integrated with the DTA model so that the rerouting process is represented.

The second proposed model, Rolling Horizon, can be applied in any real traffic situation. The Rolling Horizon concept, originating from the production planning, coupled with the hybrid route-choice model allowed to extend a classic DTA into RH-DTA model. Proposed RH-DTA is a modified DTA model, which can be applied sequentially, so that a DTA traffic pattern is adjusted every given time step (i.e. 10 minutes) based on the actual traffic conditions. Consistency between consecutive horizons is guaranteed through saving the snapshot of the directed traffic flows – number of vehicles with their destinations. This way RH-DTA overcomes the fundamental limitations of the equilibrium algorithms. Proposed generic RH-DTA framework is a flexible starting point to handle various real traffic situations, starting from the trivial ones up to the Real-time Rolling Horizon, being a fully operative engine of the TMC providing valuable estimates of state-of-the-network in real-time based on the actual traffic situation. The DTA issues pointed in chapter 3.3 are solved in the ICM by modifying the NFP model and utilizing two separate cost patterns: typical and actual. While, the RH-DTA addresses them by applying DTA sequentially in consecutive horizons, consistent through memorizing the directed traffic flows.

4. Theoretical background

This chapter formally presents the DTA algorithm within which the rerouting models from chapter 0 will be introduced. Aim of this chapter is to provide the notation framework, algorithmic structure, and solutions of the DTA model. Such theoretical background is valuable to understand the technical aspect of modelling the rerouting phenomena and to propose the rerouting models. The notation introduced here is used throughout the thesis, most importantly to propose the rerouting models. Readers who are already familiar with the macroscopic DTA can focus on the notation, DTA limitations and the hybrid model parts, skipping the remainder.

In fact the thesis extends the well-established DTA model, successfully applied worldwide, namely Traffic Real-time Equilibrium (TRE) originating from La Sapienza University of Rome (copyright by Guido Gentile @ SIAE 19.03.2013 n.8762). In the thesis TRE is utilized as much as possible to address the rerouting problem with minimum possible modifications. Therefore, only selected elements are modified while the whole algorithm idea and data structures remains intact.

This chapter introduces the macroscopic DTA framework concluded by pointing issues arising while modelling rerouting phenomena. Correspondingly, the two central limitations of a DTA model are formulated. As a remedy for the limitations, the hybrid demand model is introduced, being the theoretical foundation enabling to consistently propose the rerouting models.

4.1. Dynamic traffic assignment

This section formally introduces the macroscopic Dynamic Traffic Assignment (DTA) model. Rerouting models proposed in this thesis originate and were implemented within DTA as defined below.

Thesis follows and extends the DTA as defined by (Bellei et al., 2005), where the macroscopic dynamic assignment model was defined as a fixed-point problem (Banach, 1922). DTA, in general, is a set of two functionals: supply being a function of the demand and demand being a function of the supply. The demand side calculates the demand pattern for a given performance pattern, while the supply side calculates the performance pattern resulting from a given demand pattern. This forms a sequence solved iteratively, which (thanks to contraction of a consecutive steps) converges at a stable fixed-point further called equilibrium as proved in i.e. (Powell and Sheffi, 1982), (Mounce and Carrey, 2014).

The supply model aims to represent the traffic flow, with emphasis on its temporal dimension (evolution in time) in a dynamic setting. Main objective for a supply model is to accurately represent how traffic flows propagate in time through the network. This propagation is modelled strictly subject to the relation between the traffic flows and the road network. Road network is defined through set of arcs and nodes with their characteristics (capacity, number of lanes, free flow speed, signal setting, etc.). Crucial traffic flow phenomena affecting the traffic flow represented with the supply model are: hypocritical congestion, hypercritical congestion (queue), storage capacity (spillback) and junction delay (priority, signals, geometry) which all impact the performance.

In the thesis the supply is modelled with a General Link Transmission Model (GLTM) – first order macroscopic traffic flow model capable to represent the crucial traffic flow phenomena (Gentile, 2010). GLTM is founded on a simplified theory of kinematic waves (STKW) by (Lighthill and Whitham, 1955), further developed by (Daganzo, 1994) to a cell transmission model. In fact, GLTM is the generalization of a Link Transmission Model (Yperman, 2010) extended for a parabolic fundamental diagram and detailed junction

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representation, i.e. with signal representation (Tiddi, 2012) and the concept of conflict areas (Tiddi et al., 2013). GLTM version that is used in the thesis represents: congestion, spillback, queue formation and dissipation. The output of the supply model is the performance pattern in terms of traffic state at each arc: queue length, speed, travel time, number of vehicles, capacity, etc.

The demand model, on the other hand, aims to represent the drivers travel choices, most importantly the routes they choose to travel through the network. Drivers are assumed to minimize travel costs to get from the origin to the destination. The route choices are modelled as a function of a performance pattern, i.e. the route chosen from origin to destination is a function of the cost expected along the trajectory. Where the cost results from the phenomena taking place at the supply, i.e. how traffic flows through the network. The route-choices, in general, are driven by the travel costs that the driver expects to encounter along his *od* trajectory.

The demand model utilized in this thesis is computed with the sequential route choice model (RCM) defined in (Gentile, 2006), where the (Dial, 1971) idea of a sequential model with node weights is applied along the trajectories in dynamic setting. RCM works with an implicit path enumeration instead of calculating explicit paths and their probabilities. Routes are calculated implicitly through the local arc conditional probabilities, i.e. probability of choosing an arc subject to being at its tail node and travelling to a given destination. Final product of RCM is the demand pattern, i.e. time profile of arc conditional probabilities. However, the input for the supply model (GLTM), are the splitting rates, the aggregation of the arc conditional probabilities over all destinations. In this thesis RCM of (Gentile, et al. 2006) is applied; however, the proposed rerouting models can work with other similar models (i.e. Fosgerau et al., 2014). Moreover, results of any explicit path RCM (i.e. Ramming, 2002) can be transformed to arc conditional probabilities (Bellei et al., 2005) and become compatible with the proposed rerouting models.

The definition of the demand model is given more attention in the subsequent sections, as the proposed rerouting models refer mainly to the demand model of DTA. While, the supply model is introduced briefly without going into details. The GLTM is used in the thesis just as an example out of many supply models for which the rerouting models can be applied. In fact, the proposed rerouting models are transparent to the supply model, so that they can be applied also for the microscopic simulators, mesoscopic solutions, etc.

4.1.1. DTA notation

The thesis follows a classical definition of a road network, represented through an oriented graph $G(N, A)$, where N is the set of a nodes and $A \subseteq N \times N$ is the set of arcs. Each arc $a \in A$ is described through a vector of characteristics $\delta_a(\tau)$ that allows to represent its performances, i.e. speed, length, capacity, number of lanes, etc. The initial node of the generic arc $a \in A$ is referred to as tail and denoted $a^- \in N$, while the final node is referred to as head and denoted $a^+ \in N$. The set of arcs exiting the generic node $i \in N$ is referred to as forward star and denoted $i^+ = \{a \in A\colon a^- = i\}$. Symmetrically, the set of arcs entering node $i \in N$ is referred to as backward star and denoted $i^- = \{a \in A\colon a^+ = i\}$. Finally, let $Z \subseteq N$ be the subset of nodes, called zone centroids, where trips can start and end $o,d \in Z$.

Subscripts are used in the notation to denote the network element: i for node and a for arc. Superscript d denotes destination and optionally (for the RH-DTA) ε denotes perception specific variables as explained in the respective sections. Since the analysis is carried out in a dynamic context, all model variables are represented as the temporal profiles of the time variable $\tau \in T$ denoted in bracket. Practically, the continuous time profiles of time τ are discretized (integrated or averaged) with a given precision over a modelled time period T, yet in the thesis the time discretization is not address and the analysis is carried out in the continuous time domain.

The bold, vector, notation is used to represent the whole temporal profile. If sub- and superscripts are further omitted it denotes a vector over all network elements. For example **q** is a matrix $|A|\times|T|$ of traffic flow profiles for each arc in the network $\mathbf{q}=\{\mathbf{q}_a : a\in A\}$, while $\mathbf{q}_a=\{q_a(\tau):\tau\in T\}$ is a vector $|T|\times|1|$ representing time profile of a single network element.

Due to the trajectory based calculation in the demand model, two different time notations needs to be used in the equations: τ for time instant and θ for arrival time along trajectory. τ is used to evaluate the variable at particular time instant, i.e. $t_a(\tau)$ denotes travel time experienced by drivers entering arc a at time instant τ. Confusingly, second time variable θ needs to be introduced to denote the time instant subject to arriving at a given destination along the trajectory at time θ. Demand model is defined with θ because the flows propagate along the trajectories to arrive at destination d at time θ, therefore travel costs and times are evaluated at time instant $h_i^d(\theta)$ – time at which arc $a\in A$ is exited to reach $d\in D$ at time θ along the trajectory, h results from the shortest tree calculation. To simplify the thesis notation, θ is used as the time variable to represent $h_i^d(\theta)$ - the time at which variable is evaluated to reach destination at time θ so that $\theta\rightarrow h_i^d(\theta)$. This way one can use $c_a(\theta)$ to express the travel cost experienced by users entering arc a at time instant $h_i^d(\theta)$, such that they will arrive at destination at time θ; as well as $c_a(\tau)$ to express travel cost experienced by drivers entering arc a at time instant τ.

In thesis the algorithms are depicted with the block schemas (Schema 1 – 9) where arrows denote the sequence of calculation and input/output of the respective functionals. Sub-models (functionals) are denoted with the grey rounded boxes. White sharp boxes denote variables. Green background of the variable denotes an input variable. The blue background denotes the additional input and red denotes the additional output of the proposed rerouting models. All of the schemas shall be interpreted as an illustration of a single iteration of the iterative DTA algorithm, as will be discussed below the algorithm can start and end from any point of schema. So in fact the schemas shall fully describe the ideas of the respective algorithms.

4.1.2. DTA formulation

Using the introduced notation the preliminary definition of DTA can be formalized through both formal equations and algorithm schemas. First, let's define the result of DTA that contains a meaningful representation of the phenomena both at the demand and at supply side. Performance and demand patterns are enough to represent the actual phenomena-taking place in the network; they are further denoted **S** and called the state-of-the-network**. S** contains the temporal profiles of travel times **t**, travel costs **c** and directed traffic flows **q** on the network (4.1).

In general, DTA calculates the state-of-the-network as the function of supply $\boldsymbol{\delta}$ and demand **d**, which can be formally expressed with (4.2) so that the state-of-the-network (times, costs and flows) comes as a result from the interaction of supply **δ** and demand **d** solved inside the DTA. Such iterative interaction between supply and demand can be defined with a fixed-point problem by introducing two main parts:

a) demand (being function of supply) in which a demand pattern **q** is calculated for a given initial the time-varying demand $\mathbf{d}=\{d_{od}(\tau):\tau\in T;o,d\in Z\}$ and a performance pattern (travel costs **c** and travel times **t)** (4.3).

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b) supply (being function of demand) in which performance pattern (travel costs **c** and travel times **t**) is calculated for a given demand pattern of traffic flows **q** and the network characteristics **δ** (4.4)

$$\mathbf{S} = \{\mathbf{c}, \mathbf{t}, \mathbf{q}\} \quad (4.1)$$

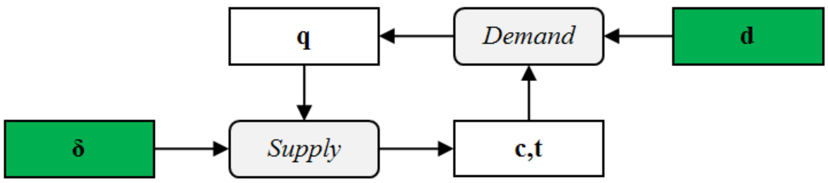
$$\mathbf{S} = DTA(\boldsymbol{\delta}, \mathbf{d}) \quad (4.2)$$

$$\mathbf{q} = Demand(\mathbf{c}, \mathbf{t}, \mathbf{d}) \quad (4.3)$$

$$\{\mathbf{c}, \mathbf{t}\} = Supply(\mathbf{q}, \boldsymbol{\delta}) \quad (4.4)$$

$$\mathbf{q} = Demand(Supply(\mathbf{q}, \boldsymbol{\delta}), \mathbf{d}) \quad (4.5)$$

$$\{\mathbf{c}, \mathbf{t}\} = Supply(Demand(\mathbf{c}, \mathbf{t}, \mathbf{d}), \boldsymbol{\delta}) \quad (4.6)$$



Schema 1 Simplified traffic assignment.

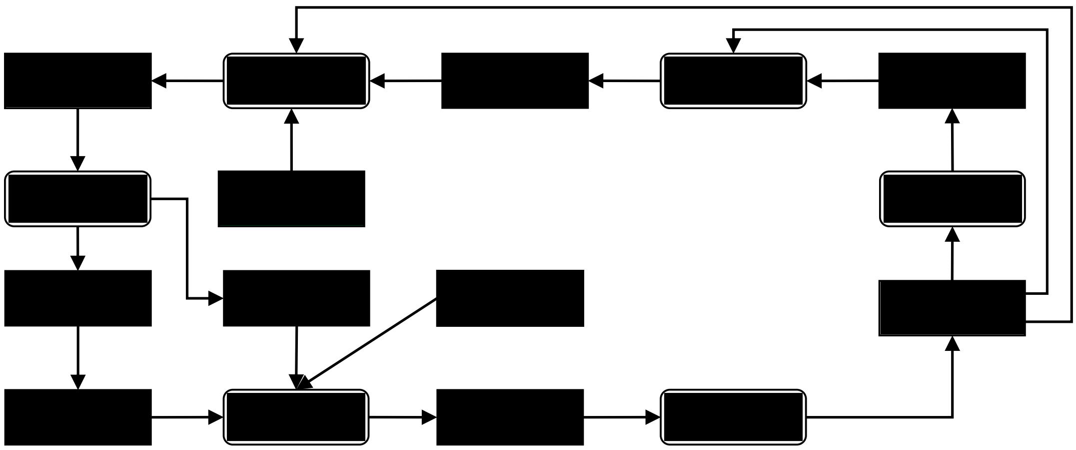
The interaction of the two above forms the DTA fixed-point iterative problem which can be equivalently expressed in terms of flows (4.5) and in terms of costs and times (4.6), as depicted in the form of an iterative Schema 1. Yet for better understanding of a DTA machinery, the above schema is further extended by introducing the following sub-models:

a) General Link Transmission Model (GLTM) – is the core of the supply model which reproduces performance pattern arising from a given demand pattern. It can be any kind of model that takes as an input: splitting rates **r**, initial demand flows **d** (both departing at origins and pre-loaded on the network) and network characteristics **δ** to calculate the network performances. In the thesis the General Link Transmission Model (Gentile, 2010) is used, however it can be easily substituted with any other traffic model, i.e. micro-simulation. GLTM is executed chronologically over time instants τ. Formally represented with (4.7).
b) First-In-First-Out Travel Time (FTT) – is a complementary model of GLTM to precisely calculate travel times **t** which are not seen directly as the GLTM calculates only inflows **f** and outflows **e** from which travel times can be recomputed (Gentile et al., 2007). Formally represented with (4.8).
c) Arc Cost Function (ACF) – calculates a generalized travel cost **c** of a network arcs from the network characteristics $\boldsymbol{\delta}$ and travel times **t.** Cost is utilized while making routing decisions and can depend on both objective parameters, i.e. travel time, as well as their subjective values perceived by user, i.e. arising from user preferences or lack of information about actual (objective) values. Cost is computed from the results of the supply model (i.e. travel time, queue length etc.) and other arc-additive attributes considered by the user while making a route-choice (i.e. safety, toll, scenery, curvature, etc., see i.e. (Ramming, 2002) for reference). Formally represented with (4.9).
d) Route Choice Model (RCM) – calculates the demand pattern in form of an arc conditional probabilities $\mathbf{p}^\mathbf{d}$ to destination d for given performance pattern of travel costs **c** and times **t**. RCM is executed destination by destination, chronologically over time instants τ. Formally represented with (4.10) and further elaborated in section 4.1.3.
e) Network Flow Propagation (NFP) – loads the demand $\mathbf{d}^\mathbf{d}$ towards a given destination using arc probabilities $\mathbf{p}^\mathbf{d}$ from the RCM and travel times **t** from the supply model. NFP

results in arc flows $\mathbf{q^d}$ destination by destination. Technically computed simultaneously with RCM. Formally represented with (4.11) and further elaborated in section 4.1.3.

f) Aggregation (SUM) – sums the destination specific flows $\mathbf{q^d}$ from NFP over destinations to obtain the demand flows **d** and the splitting rates **r** needed at the supply side. GLTM propagates the total flows without their destinations due to computational efficiency, so that destination specific flows $\boldsymbol{q}$ from the NFP need to be aggregated to the splitting rates. Optionally, Method of Successive Averages (MSA) can be integrated within this formula to guarantee contraction in Banach sense, needed to ensure existence of fixed-point of the DTA problem. Formally represented with (4.12).

$$\{\mathbf{f},\mathbf{e}\} = GLTM(\mathbf{r},\mathbf{d}) \tag{4.7}$$
$$\mathbf{t} = FTT(\mathbf{f},\mathbf{e}) \tag{4.8}$$
$$\mathbf{c} = ACF(\mathbf{t}) \tag{4.9}$$
$$\mathbf{p^d} = RCM(\mathbf{c},\mathbf{t}) \tag{4.10}$$
$$\mathbf{q^d} = NFP(\mathbf{p},\mathbf{t},\mathbf{d^d}) \tag{4.11}$$
$$\{\mathbf{r},\mathbf{d}\} = SUM(\mathbf{q^d}) \tag{4.12}$$
$$\mathbf{q}^n = Demand(\mathbf{c}^{n-1},\mathbf{t}^{n-1},\mathbf{d}) = Demand(Supply(\mathbf{q}^{n-1},\boldsymbol{\delta}),\mathbf{d}) \tag{4.13}$$



Schema 2 Dynamic Traffic Assignment.

Schema 2 represents a single iteration of a fixed-point problem of (4.5) through sequence of sub-models (4.7) to (4.12) which are executed in an iterative sequence. The fixed-point, coinciding with equilibrium, is reached at the convergence of iteratively solving DTA problem of schema 2. Formally, the state-of-the-network **S** is obtained after reaching the fixed-point of the problem (4.5), i.e. when the flows do not vary from iteration of iteration. The single (n-th) iteration of the algorithm, interpreted as a single cycle of block process from Schema 2, can be formally denoted as (4.13) where superscript n denotes values obtained at the n-th iteration. The relative gap of the above process can be measured i.e. with $\left\|\mathbf{q}^n - \mathbf{q}^{n-1}\right\|$ – arc flow difference between the successive iterations (Dial, 2006). For the DTA problem, the relative gap is minimized at the fixed-point (equilibrium). The DTA algorithm can, in general, be started from any of the sub-models (4.7) to (4.12) and exits with the state-of-the-network obtained from the last iteration (4.1). (Gentile, 2007) proposes to start from the free-flow travel times and compute the demand, while for the real-time environment (Gentile et al.,

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2013) the convergence is much faster while starting with the demand pattern and recomputing the supply part.

At the intermediate iterations route-choices **p** will yield different costs **c** than expected from the previous iteration, which is the inconsistency solved through the fixed point. Fortunately, at each iteration there is a consistency between the demand flows and travel times, i.e. the travel costs and times are calculated exactly for the flows that are actually propagated through the network in GLTM (Gentile et al., 2007). Such property of the fixed-point problem is highly beneficial for the real-time management and to model rerouting phenomena as will be shown in the further sections.

4.1.3. DTA demand model

Since the rerouting phenomena takes place at the demand side of DTA, the following section defines the demand model in detail. This chapter describes how the demand model is implemented in TRE, the broader definition and original version of the formulas used in this chapter can be found in (Gentile, et al. 2006). Generally, the demand pattern is calculated with the two sub-models: RCM in which the arc conditional probabilities are computed and NFP where the demand flows are propagated along the arc conditional probabilities computed in RCM.

Computing the demand pattern starts with the shortest tree computation, which can be obtained from any applicable dynamic shortest path search algorithm, i.e. label-setting, or label-updating algorithms. Detailing the shortest tree algorithms is out of the scope of the thesis, and the shortest tree is assumed to be an external input (for reference see Dehne et al., 2010). For the following considerations the shortest tree can be generically defined as a procedure computing a set of minimal costs $g_i^d(\theta)$ and exit times $h_i^d(\theta)$ to arrive at the destination *d* at time *θ*. Based on the minimal costs *g*, the topological order can be obtained with (4.14).

$$TO_i^d(\theta) > TO_j^d(\theta) \Leftrightarrow g_i^d(\theta) < g_j^d(\theta) \tag{4.14}$$

To compute the demand pattern the shortest tree is processed twice. First processing is in the topological order (4.14) from the destination to the furthest node. During this processing the node satisfactions (expected costs to get to destination) $w_i^d(\theta)$ are recursively computed through the Bellman equation (Bellman, 1957) as in (4.15). Subsequently, the node satisfactions are used in the second processing, executed in a reverse topological order (from the furthest node to destination) to recursively compute the arc conditional probabilities $p_i^d(\theta)$ using logit formula of (4.16). In fact the (4.15) and (4.16) form the multinomial logit model (Ben-Akiva and Lerman, 1985), where node weight (4.15) is a denominator – sum of utilities for all efficient alternatives available at node *i*. The utility of a single alternative is a function of travel costs *c* recursively updated to form a node satisfaction *w,* i.e. cost to get to destination *d* from current arc *a*. The resulting logit probabilities $p_a^d(\tau)$ of taking arc *a* to get to destination *d* being at the tail a^- at time *τ* is the resulting demand pattern used further in the thesis.

The second process of the shortest tree (during which arc conditional probabilities are calculated with (4.15)) can be directly integrated with NFP. In NFP the demand flow from the origin at node *i* to the destination *d* is propagated along the trajectories using the arc conditional probabilities *p*, calculated in RCM with (4.16).

The flow propagation is executed sequentially in the reverse topological order. It computes the arc flows with (4.17), cumulated the node flows from the backward star of the node with (4.18) and propagates it to the forward star with the respective arc conditional probabilities p (4.17). Such sequence is repeated until the flows reach their destination d. This sequence can be easily and effectively integrated with the RCM calculations and executed simultaneously, i.e. the flows are propagated with (4.17) as soon as the arc conditional probabilities p are computed with (4.16). NFP and RCM are shown separately at the algorithms to keep the algorithm structure clean. To compute the whole demand pattern, the RCM and NFP are calculated for each destination d for each arrival time instant θ of the simulation period T. Flows along the trajectory are finally transformed from the trajectory-related values of θ into the temporal profiles of τ. To this end trajectory flow values $q_i^d(\theta)$ are mapped onto the time instants using $\theta \to h_i^d(\theta)$ with (4.19).

$$w_i^d(\theta) = \begin{cases} \sum_{a \in i_d^+} \exp\left(\frac{-c_a(\theta)}{\eta}\right) w_{a^+}^d(\theta)^{\sqrt{\frac{g_{a^+}^d(\theta)}{g_{a^-}^d(\theta)}}} & \text{if } i \neq d \\ 1 & \text{if } i = d \end{cases} \tag{4.15}$$

$$p_a^d(\theta) = \begin{cases} \exp\left(\frac{-c_a(\theta)}{\eta}\right) w_{a^+}^d(\theta)^{\sqrt{\frac{g_{a^+}^d(\theta)}{g_{a^-}^d(\theta)}}} \Big/ w_{a^-}^d(\theta) & \text{if } a \in (a^-)_d^+ \\ 0 & \text{if } a \notin (a^-)_d^+ \end{cases} \tag{4.16}$$

$$q_a^d(\theta) = \left(d_{a^-}^d(\theta) + q_{a^-}^d(\theta)\right) p_a^d(\theta) \tag{4.17}$$

$$q_i^d(\theta) = \sum_{a \in i^-} q_a^d(\theta) \tag{4.18}$$

$$q_a^d(\tau) = \sum_{\theta \in T} q_a^d\left(h_{a^-}^d(\theta)\right) \tag{4.19}$$

In fact, a DTA problem as formulated in (2) can be solved with any implicit sequential RCM where the demand pattern is stored as a local arc conditional probabilities $p_a^d(\tau)$, computed as a function of travel times and costs. Finally, it should be noted that the above model is valid only if the topological order coincides with the increasing order of costs, i.e. (4.14) holds true. This feature is guaranteed only if costs are proportional to times, otherwise the circularity issue arises.

4.1.4. Dynamic network loading (DNL)

The Dynamic Network Loading (DNL) is a sub-problem of DTA, which consists of seeking, for a given route choices, an arc flow pattern consistent with the travel times through the supply model. DNL can be seen as a simplified DTA, without RCM. Although it is a simplified version of a DTA, it still has a circular dependency which needs to be solved iteratively. The iterations are needed to guarantee a temporal consistency between the traffic flows and travel times – not more than few iterations in practice as shown in (Gentile et al., 2007). The importance of DNL for rerouting comes from its ability to propagate the flows with the fixed demand pattern (arc conditional probabilities) yet for the network where travel times have changed. This way one can simulate how the flows are propagated along the typical demand pattern even though the network is not typical anymore, i.e. the unexpected event causes blockage on an arc, yet the information about the event is still unavailable. DNL solves this case by running a specific version of a DTA problem, where the expected travel times **t** and costs **c** are fixed, while the actual travel times differ. Consequently the arc

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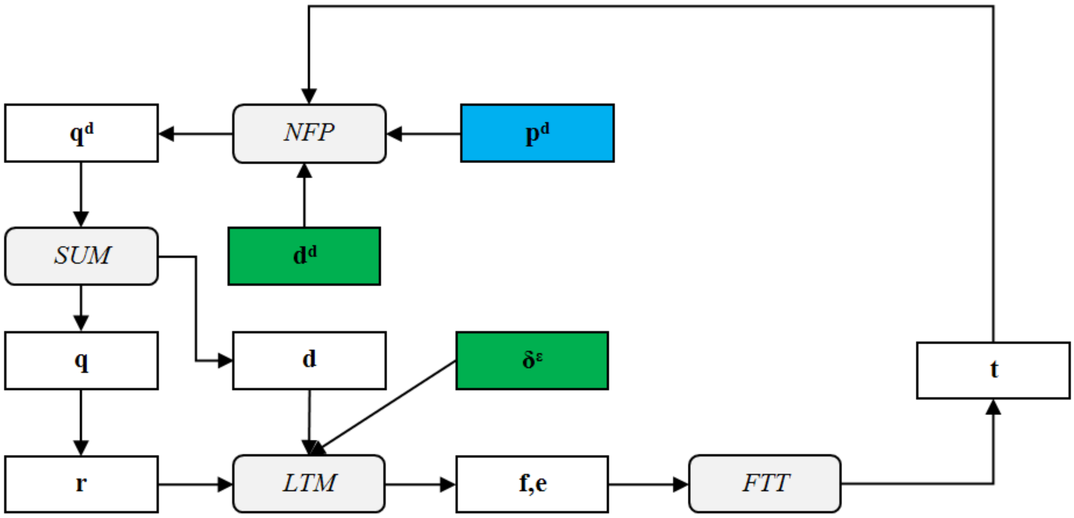
conditional probabilities **p** are shifted in time to match the actual travel times of the network and gain temporal consistency between demand and supply. This way the typical demand pattern can be applied for the actual traffic conditions (including events) as will be elaborated in further sections. Schema 3 depicts the DNL problem formally expressed with (4.20).

$$\{\mathbf{t},\mathbf{q}\} = DNL(\mathbf{p},\mathbf{d},\boldsymbol{\delta}) \tag{4.20}$$



Schema 3 Dynamic Network Loading.

4.2. *DTA limitations to model rerouting phenomena*

In this section, the two fundamental limitations of the above DTA algorithm (Schema 2) to model rerouting phenomena are identified. They are based on the DTA issues pointed in chapter 3.3, yet they point straight into theoretical structure of the DTA algorithm.

1. **Limit of *pre-trip routing based on costs from previous iterations.*** In a destination based dynamic RCM, the shortest path trees are calculated for the given arrival time at destination. That implies that route is chosen based on future state of the network (taken from the previous DTA iteration). Such formulation of Dynamic User Equilibrium , called by other authors *Predictive User Equilibrium*, is opposed to the so-called *Bostonian User Equilibrium* which assumes that choices are made upon instantaneous travel times (Friesz, 1993). Predictive route-choices mean implicitly that the driver departing from origin knows the state-of-the-network up to the time when he arrives at the destination. This is somehow not realistic as it assumes that the information travels back in time, i.e. the future state of the network is foreseen in advance. This property allows to formulate DTA as a day-to-day learning process where decisions are updated using previous experience (Watling and Hazelton, 2003). For a recurrent state of the network this is perfectly consistent with the routing behavior of the next day. Unfortunately, such paradigm is completely wrong for modelling unexpected traffic events, which, by definition, are not known in advance nor recurrent, but change significantly the performance pattern for that day. Using the actual network characteristics (including the event) to solve DTA problem (4.2) will result in a full knowledge about the future states of the network. Which, in turn, will lead to routing decisions to avoid the event even prior it is known. This way the event will have no effect at the supply side (all users successfully avoided the event, because, in fact, it was well known rather than unexpected), which is an unrealistic outcome.
2. **Limit of *pre-trip decision based on the expected future costs.*** Second limitation arises from the fixed demand pattern used in the flow propagation procedure. Flows are

propagated in the NFP (4.17) using routing decisions made pre-trip based on expected costs. If the costs experienced en-route (while travelling) happens to be different from what was expected, they will be taken into account for routing only in the next iteration. The current iteration will be completed using demand pattern calculated pre-trip regardless the actual travel costs. In other words routing within the single iteration is independent of the travel costs of this iteration, so that no adaptation, en-route rerouting, can be represented.

The two above features of the DTA: pre-trip routing based on costs from previous iterations and pre-trip decisions based on expected future costs are the most evident reasons why it is not an appropriate tool to model rerouting.

4.3. Hybrid route-choice model in implicit path enumeration

To overcome the aforementioned limitations of DTA as defined at Schema 2, the concept of the hybrid route-choice model (Mahmassani et al., 1991), (Pel et al., 2009), (Qian et. al., 2012) is utilized. In this chapter author formalizes the previous hybrid route-choice model concepts, and proposes an original Lemma proving that it can be used in a DTA models. The hybrid model will become a theoretical foundation for the rerouting models proposed in chapter 5.

Hybrid route-choice model is designed to handle the case when drivers change the *perspective* of the travel costs at a given time instant, i.e. when a new information updates and changes the expected travel costs. Let's denote a perspective with $\boldsymbol{\varepsilon}$, like the notation of the event, since the perspective will always be linked with the event which changes the perception of travel costs, therefore the two can share the same variable name without confusion. A generic perspective $\boldsymbol{\varepsilon}$ is defined through the specific set of travel costs used by drivers while making route-choices. The change of the perspective is defined as the change in the user's expectation of travel costs.

In general, drivers make routing decisions based on the perceived costs $\mathbf{c}$, yet they propagate through the network strictly according to the travel times $\mathbf{t}$, which constitutes the difference between RCM (where decisions are made upon costs) and NFP (where flows are propagated along trajectories chosen in RCM consistently with travel times). Generally, the travel times $\mathbf{t}$ are objective and result from a physical traffic flow processes, while travel costs are subjective and user-dependent. Such distinction allows to distinguish the demand patterns $\mathbf{p}$ of various perspectives of costs, which is typically applied for multiclass assignment where heterogeneous users choose routes differently, i.e. due to different value-of-time (Boyce and Ber-Gera, 2004). Hybrid model exploits it in a different way, by assuming that a user can change his perspective while travelling. Let's assume a generic perspective $\boldsymbol{\varepsilon}$ that updates a typical travel costs $\mathbf{c}$ to the perspective-specific costs $\mathbf{c}^{\varepsilon}$. Updated costs yield a new demand pattern $\mathbf{p}^{\varepsilon}$ according to (4.10), which can be redefined to the perspective specific RCM of (4.21). RCM subject to a given perspective (4.21) is calculated with the intact machinery of RCM (4.15) and (4.16) having as input perspective specific costs $\mathbf{c}^{\varepsilon}$ and objective travel times $\mathbf{t}$. Resulting perspective specific demand pattern $\mathbf{p}^{\varepsilon}$, can be applied in NFP (4.17) to propagate the flows.

To make hybrid-model operative the routing behavior needs to be *rolled* to perception ε at a given *roll time* J_{ε}, that is the time at which the information associated with a new perspective is delivered. This is done directly at the propagation level, where the sequential NFP procedure of (4.17) takes a conditional form of hybrid NFP. In the hybrid NFP (4.22), flows are propagated according to the typical demand pattern $p_a^d(\theta)$ up to time J_{ε} and further propagated according to the perspective specific demand pattern $p_a^{d|\varepsilon}(\theta)$.

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$$\mathbf{p}^{\varepsilon} = RCM(\mathbf{c}^{\varepsilon}, \mathbf{t}) \tag{4.21}$$

$$q_a^{d|\varepsilon}(\theta) = \begin{cases} \left(d_{a^-}^d(\theta) + q_{a^-}^d(\theta)\right) p_a^d(\theta) & \text{for } a : h_{a^-}^d(\theta) < J_\varepsilon \\ \left(d_{a^-}^d(\theta) + q_{a^-}^d(\theta)\right) p_a^{d|\varepsilon}(\theta) & \text{for } a : h_{a^-}^d(\theta) \geq J_\varepsilon \end{cases} \tag{4.22}$$

Before substituting a classic NFP of (4.17) with a hybrid NFP of (4.22) in DTA algorithm (Schema 2) let's demonstrate that the following author's lemma holds true:

Lemma. The hybrid NFP of (4.22) propagates flows consistently with the demand pattern *d*, i.e. the total flow arriving at destination $q_d^{d|\varepsilon}(\theta)$ is equal to the flows originating towards that destination $\sum_{i\in N} d_i^d(\theta)$.

Proof. To prove that (4.22) conserves the demand, let's first refer to the flow conservation rule of a sequential RCM, guaranteed through (4.23). (4.23) means that the total flow arriving at node *i* will be propagated towards the arcs of forward star, it can be derived straight from the logit model of (4.15) and (4.16). Moreover, by definition, the topological order calculated with the shortest tree algorithm coincides with the chronological order of propagation along the trajectories (since the NFP is executed in the reverse topological order, which at the same time is the chronological order due to (4.14)). If so, the recursive character of NFP arising from the sequential execution of (4.17) and (4.18) in topological order, guarantees that the total demand flow at origins will be propagated towards the destination ($q_d^d(\theta) = \sum_{i\in N} d_i^d(\theta)$).

To show that the demand is conserved also at the roll time J_ε, let's refer to a shortest tree to arrive at destination at θ. Such tree contains a unique cut-set $\mathbf{CS}(\varepsilon)$ such that $\mathbf{CS}(\varepsilon) = \{a \in A : h_{a^-}^d(\theta) < J_\varepsilon \leq h_{a^+}^d(\theta)\}$, i.e. set of the arcs that are traversed at roll time i_ε. From (4.22) it can be seen that $\mathbf{CS}(\varepsilon)$ is the set of the arcs at which the propagation of that tree needs to be rolled to the new perspective. As the topological order coincides with chronological order of trajectories (4.14), it is guaranteed that the flow propagated from origins is conserved at the cut-set $\mathbf{CS}(\varepsilon)$, as denoted with (4.24). The latter part of propagation, after the roll time, can be then redefined to fit the typical NFP where the flows at $\mathbf{CS}(\varepsilon)$ (which sums up to the demand through (4.24)) become the origins and are further propagated conserving the demand, as (4.23) holds true also for $\mathbf{p}^{\varepsilon}$. So that $q_d^{d|\varepsilon}(\theta) = \sum_{i\in N} d_i^d(\theta)$ which justifies the hybrid RCM and NFP of (4.22) and proves the Lemma.

$$\sum_{a\in i^+} p_a^d(\theta) = \sum_{a\in i_d^+} p_a^d(\theta) \equiv 1 \tag{4.23}$$

$$\sum_{i\in N: h_i^d(\theta) < t_\varepsilon} d_i^d(\theta) = \sum_{a\in \mathbf{CS}(\varepsilon)} q_a^d(\theta) \tag{4.24}$$

4.4. DTA issues in the rerouting context

This section brings the attention to features of the DTA algorithm (Schema 2) worth noting in context of rerouting phenomena. They are presented here, as they refer directly to the DTA algorithms of this chapter, but they were raised while designing and applying the algorithms for rerouting of the next chapter. Therefore, the proper significance of those remarks comes along with introducing the rerouting models.

4.4.1. Typical state-of-the-network and typical demand pattern

DTA (4.2) calculates the typical state-of-the-network with the typical demand pattern, understood as a stable trade-off between demand and supply in a full Wardrop sense (Wardrop, 1952). Which was argued by (Mahmassani, 1997) to be the state expected by the users to take place. In other words, drivers were expecting the conditions that they experience, i.e. route was chosen based on expectation of cost **c** that actually persist in the network along his trajectory. It is assumed in the proposed rerouting models, that driver will propagate through the network in line with a typical demand pattern **p** as long as he does not receive an information about atypical event in the network, which would change his perception of travel costs **c** in the network. Respectively, even for an atypical day with an unexpected event, drivers will act as if the day was typical as long as they do not receive the information about the event.

4.4.2. Implicit versus explicit path enumeration

In general, the paths in DTA can be handled twofold. With explicit paths, when the full paths and their traffic flows are memorized, connecting origin and destination through a sequence of arcs. Or implicitly, through local choices being arc conditional probability of choosing the next arc to get to the destination. Demand pattern stored implicitly with arc conditional probabilities is more flexible the one stored through the explicit the paths. Rerouting takes place in the network, when driver decides to update his routing decision. Therefore, he needs a routing guidance from the place where he is to the destination. This requires providing demand patterns not only for each origin-destination pair but also for each node-destination pair. If paths are stored implicitly, the flows can propagated not only along pre-computed paths connecting origin with destination, but from any node in the network to any origin. For explicit path enumeration it would significantly increase the memory consumption while it is anyhow calculated within implicit path enumeration. Therefore, implicit paths are better suited for the rerouting case than working with full paths.

4.4.3. Flows per destination versus splitting rates

Typically, the demand pattern in a DTA model can be stored as a splitting rates **r**, where the information about destination is aggregated. The typical state-of-the-network can be recomputed with supply model using only origin demand **d** and splitting rates **r**. Yet such recomputing is valid only as long as the travel times do not change. If some unexpected event affects the traffic flow (i.e. reduces capacity of a given arc) the splitting rates are not valid anymore and need to be recomputed. To regain consistency between travel times obtained in GLTM and traffic flows obtained in NFP, the DNL problem (see 4.1.4) needs to be solved. However solving the DNL problem starting from pre-computed splitting rates **r** is not satisfactory for the rerouting. The rerouting behavior is strictly subject to destination, thus destination index cannot be aggregated in the *SUM* sub-model of DTA (4.12). Therefore rerouting shall be handled at arc conditional probabilities **p** rather than splitting rates **r**.

4.4.4. Recomputing state-of-the-network from the performance pattern

In the rerouting models it is assumed that the drivers make a typical routing decisions if they do not receive any information about atypical conditions. Therefore, it is crucial to have access to the typical demand pattern while running rerouting models. Yet storing the demand pattern as arc conditional pattern **p** in the memory can be a serious data storage issue, for the real size networks it can easily reach gigabytes. Currently, it is more reasonable to re-compute **p** on the fly starting from the expected costs **c** and times **t** with RCM (4.10).

Moreover, in the typical machinery of DTA, saving directed flows **q** can stem a problem as the final result of DTA is averaged through iterations, usually with MSA procedure that loses the information about the destination. Therefore, the probabilistic route choice model (with logit as in (4.15)) is utilized for the rerouting, as it provides multipath solution every iteration. Thanks to this the traffic flows can be precisely recomputed from the travel times and costs with a single DTA iteration. The input needed to for the re-computation are the costs **c** and times **t** as in (4.3), which will in turn yield themselves in the supply model (4.4).

4.4.5. Iterative assignment and rerouting phenomena

For DTA the typical conditions should be obtained at the equilibrium, i.e. the convergence point of fixed-point problem (4.2) as the drivers are assumed familiar with the typical state and their behavior is equilibrated in Wardrop sense (Wardrop, 1952).

Significantly different behavior is represented in case when drivers reroute and do not act typically anymore. Based on what was stated in the literature review it may seem inappropriate to model rerouting with the classic Wardrop-like iterative algorithm resembling learning process. It seems counterintuitive, yet also has some support. Other researchers (Dobler et al., 2013) propose single-shot game where outcomes of decision are completely unknown and only forecasted based on experience. (Watling and Hazelton, 2003) argues that a traffic system is not reaching equilibrium in case of rerouting, and the relative gap is inevitable in the non-recurrent cases. Similar can be obtained within the fixed-point problem iterations: Drivers are assumed, to some extent, to be capable of forecasting consequences of the event considering also decisions of other users. Probably they are not able to intercept the exact outcomes but they are definitely not acting as if they were the only ones to reroute. Usually when drivers get informed about the event a decision process is as follows: "Bridge A is blocked, I need to take other route, the closest alternative is bridge B, but probably most of other drivers will try to take it, so it's better for me to take the further bridge C". In the study, being chapter 6.4 of this thesis, of traffic flow on bridges in Warsaw such behavior was observed in case of the traffic event – the rerouting flows were observed not only at the closest alternative bridge, but also spread on number of other bridges resembling the equilibration process. To cover forecasting capabilities, the fixed-point iterations are run to model rerouting. The behavior obtained during first iteration is based on costs as if no one else except the single driver would reroute. In the following iterations the costs and associated decisions are adjusted when outcomes of rerouting decisions are available. Each additional iteration improves the forecasting capability while making rerouting decision. Up to optimal fixed point, where rerouting decision can be seen as a specific type of equilibrium; equilibrium at which the rerouting decisions are optimal in sense that their outcomes are in-line with expectations.

Instead of following the common interpretation of DTA iterations in terms of day-to-day adaptations, in the rerouting model consecutive iterations can be understood in terms of a single-shot game where outcomes of decision made by other is unknown, yet can be predicted to some extent. Therefore, for the rerouting models (both RH-DTA and ICM) number of iterations is a parameter and should be further estimated to match actual behavior and forecasting capabilities of drivers. Yet the hypothesis that the rerouting strategic behavior can be modelled with the DTA iterations shall be further verified as well as the number of iterations (see 6.4).

4.4.6. Different topological order in case of the event

Algorithmic issue raises within Network Flow Propagation Model (NFP) that is executed in topological order. It is correctly assumed for the algorithm that the routing is restricted only to the efficient arcs (leading closer to the destination) and it is perfectly consistent if the routing is made based on exactly the same travel times as the propagation. Unfortunately, the different travel costs at the network can lead to a change in the topological order of nodes. Such situation results in 'losing' the flows at arc which are outside of topology, i.e. the flow conservation rule in (4.19) is not conserved. This issue is overcome with the hybrid model defined above, which guarantees the flow conservation.

Alternatively, it is proposed to execute NFP twice. First for full origin demand and using topological order of the typical costs but actual travel times. The rerouting flows (i.e. the flow of users that decided to reroute) are memorized in this NFP. Subsequently, another propagation is executed, yet using actual topological order and propagating only rerouted flows. Such solution solves also issue of 'multiple rerouting' preventing the driver from making a rerouting decision several times during trip (which is unlikely for the case of a single event). Such approach of two NFP is used in the Information Comply Model to conserve the total demand.

4.5. Summary of the theoretical background

This chapter provided prerequisites needed to introduce the rerouting models in the subsequent chapter. Both of the proposed models are applied in the DTA elaborated above, they follow the same concepts and use the same notations, in fact both of them can be seen as the extensions of the above DTA algorithms. The following chapter introduces the new concepts only when it is necessary to represent the rerouting behavior. One of the designing objectives for the proposed models was to modify the above DTA algorithms as little as possible.

Apart from providing a theoretical insight to understand the rerouting models, this chapter pointed the key issues arising while applying the DTA algorithms for the rerouting cases. The DTA elements that need to be changed are identified: the Route-Choice Model and the Network Flow Propagation. The key limitations of DTA arise in those two elements and they will be addressed in the following chapters. To overcome those limitations the hybrid-route choice model was proposed as the starting point for the design of the algorithms proposed in the subsequent chapter. Finally, number of remarks were introduced showing the issues arising in the DTA algorithm while applying the rerouting models.

5. Rerouting models for Dynamic Traffic Assignment

In this, central, part of the thesis the rerouting models are proposed. Using the DTA framework from previous section, the problem statement elaborated in first chapter is solved in the demand model of DTA. Two alternative models are proposed addressing two different motivations to model rerouting phenomena. One originates in ITS, another in the strategic, offline planning. One simplifies the phenomena, another gives it a broad behavioral context.

Both of them are consistent with the DTA (Schema 2) and are designed as its extensions. Both of them compute state-of-the-network in acceptable time for real-size networks. Both of them are an original author's contribution.

Information Comply Model is described first, with its broad definition of the phenomena and the intuitive structure of the model. The Rolling-Horizon model presented in the second part simplifies the phenomena, yet provides a framework for the real-time environment providing reasonable traffic forecasts for the ITS systems.

Models are introduced through defining the idea for the model, then the algorithms are proposed, followed with the implementation in the DTA framework. They are concluded with numerical examples illustrating the models. Finally they are summarized and compared.

5.1. Information Comply Model (ICM)

This chapter introduces the way of representing the rerouting phenomena in dynamic traffic networks with the Information Comply Model (ICM). ICM represents the driver's behavior in cases of unexpected events in form of a closed-form model applicable to the DTA algorithm from the previous chapter. The ICM, as defined in this chapter, was successfully implemented by author in the TRE software by SISTeMA.

5.1.1. Idea

Modelling rerouting phenomena, in general, can be understood as answering the question: *How do drivers adjust their routing decisions due to unexpected events?* Which can be seen as a decision making process of an individual (a single driver) who decides if and how to react. ICM answers this by identifying three parts of the underlying process: *when*, *why* and *where* do drivers reroute? Based on the general consideration of the rerouting phenomena from chapter 1.5, they can be answered as follows:

1. When? Rerouting can take place only when driver is aware of the event, i.e. when he received the information about the event, or experienced enough delay to consider rerouting.
2. Why? The reason for rerouting is to avoid negative consequences and minimize travel time to the destination.
3. Where? The target of rerouting is the optimal route subject to driver's expectations of the actual travel costs.

Such partition allows covering the cognitive process behind making rerouting decisions. Based on the above, the rerouting drivers can be defined as the ones who decided to change their route due to either observation or information about an event. This leads to identification of the three cognitive states in which the driver can be: unaware, aware and rerouted. The objective of ICM is to quantify number of drivers in each of those states for a given time, space and destination. Which is obtained through a Markov process with the three states of the flow and transition probabilities between them. At each node in the Network Flow Propagation, the transition probability between the three states is obtained through the:

1. Awareness model, answering when and how the unaware drivers receive the information, or experience enough delay to become aware about the rerouting.
2. Compliance model, answering when and why the aware drivers decide to reroute.

Such structure of ICM allows overcoming the fundamental limits of DTA identified in chapter 4.2. The limits can be paraphrased in the ICM context as follows:

1. The limit of the pre-trip routing based on costs from previous iterations is overcame two fold. First, it is assumed that the information precedes the reaction, i.e. only the aware drivers can reroute. Actual travel costs, by definition, cannot be utilized by unaware drivers. Becoming aware is possible only when either the information of observation was available, so that rerouting in ICM cannot take place prior the event is known or prior it affected the travel costs. But apart from this, it is assumed that drivers do not have a perfect knowledge about efficiency of their rerouting decisions, which are limited by their forecasting capabilities. Assumption about the imperfect knowledge is handled within iterative structure of DTA algorithm controlling the costs used for rerouting decisions, as described if further sections.
2. The limit of the pre-trip decision, based the on expected future costs is overcame by allowing drivers to utilize the actual costs while traversing the network, i.e. they can react to the received information, or observations while traveling. This is achieved with the compliance model, which allows rerouting flow during the Network Flow Propagation, i.e. to start utilizing the actual arc conditional probabilities while traversing the network. This way the concept of the hybrid route choice model from section 4.3 is used in ICM.

ICM is designed to satisfactorily represent the rerouting phenomena and allow a straightforward integration with the DTA (2). Emphasize was to provide a realistic model which could be estimated and validated to match the actual phenomena. In chapter 6.4 the estimation schema based on the real data is proposed. To ensure the realistic representation the following key properties are included within the ICM model:

1. For the awareness model:
   1.1. The information progressively spreads in time gradually reaching drivers, i.e. the number of drivers who has already received the information is the increasing function of time.
   1.2. Important information spreads faster and reach bigger audience, i.e. the greater significance of the event, the more drivers will know about it.
   1.3. Drivers can become aware of the event if they observe its consequences empirically, i.e. if they are stuck in the atypical, unexpected queue.
   1.4. There are numerous sources of awareness simultaneously notifying drivers about the traffic.
2. For the compliance model:
   2.1. Drivers reroute to minimize the possible losses, namely to avoid negative impact of additional travel costs to get to the destination.
   2.2. Dually to the above, drivers reroute to maximize their possible gains, namely to minimize their travel cost to get to the destination.
3. For the route chosen by the rerouting drivers:
   3.1. Drivers reroute to minimize their expected time to get to the destination.
   3.2. Drivers act strategically and consider that other drivers will also make a rerouting decisions.

The above properties are handled within the respective machinery of ICM, resembling various aspects of the rerouting phenomena. ICM can be seen as a multi-dimension function

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conserving the above properties through the algorithm structure and formulation of the sub-models, as shown in the subsequent sections.

To make the ICM operative and applicable as an extension of the DTA Algorithm (Schema 2) the above initial representation of the rerouting through the decision process of a single driver needs to be translated into the language of the macroscopic DTA. Hence, the following technical assumptions are made. First of all, not everyone has access to the information and not everyone complies with it, so that the rerouting is handled as the probability that individual reroutes (see Cantarella and Cascetta, 1995). Moreover, since the macroscopic DTA model operating on flows, not single drivers, is used in the thesis; it is convenient to operate on the share of the flow, rather than probability. The probability is handled internally with awareness and compliance models executed as the Markov process within the NFP, which operates on the macroscopic traffic flows. The rerouting decision in ICM model is restricted in space to the so-called *decision points* i.e. the points at which the rerouting can take place (there's an opportunity to change route). In DTA the decision points are all nodes $n \in N$. Placing decision points at nodes is especially convenient as it fits the logic of the sequential NFP used in the DTA (4.17).

In conclusion, the rerouting defined above as the decision process of a single driver, can be handled macroscopically by propagating the flows in three states (unaware, aware and rerouted) and computing transition probabilities between states. Technically, the idea of ICM is to operate on the two cost patterns: typical (see chapter 4.4.1, denoted with ^) and actual (observed and/or expected during the event, denoted ~). The rerouting decision, i.e. transition of the flow state from aware to rerouted, can be then defined as the moment at which driver starts using the actual costs for routing instead the typical. This way the concept of the hybrid route choice model from section 4.3 is used in ICM. Like in the hybrid route choice model, the propagation through the network is partitioned into two parts: prior rerouting and after. Both the unaware and aware flows act as if it was a typical day and follow the typical demand pattern. The drivers in those states do not follow the actual demand pattern, as they did not decide to reroute yet. The actual demand pattern is utilized only by the rerouted flow, which has already decided to reroute.

5.1.2. Methodology

In this section, the technical and formal aspects of the ICM model are presented. In the first part of this section the awareness model is elaborated, including a thorough considerations on the possible sources of information in the traffic networks and the way they can be handled jointly. It is followed by the definition of the compliance model with emphasis on how the utility of the rerouting can be represented in the DTA. Then the input needed for the ICM is described as well as the parameterization and the meaning of the respective parameters. Finally the route choice process made by rerouting drivers is discussed.

5.1.2.1. *Awareness model*

The awareness model answers who and when becomes aware of the event. The term aware is used to merge the two major sources: information and observation. In general, it is assumed that driver can become aware of the event due to the received information and/or experiencing atypical situation being consequence of the event, i.e. atypical delay. The following formal part introduces the awareness model for the ICM model, which allows to compute the probability of becoming aware in traffic network, where number of awareness sources are available. This is obtained by introducing the so-called spreading profiles for each of independent sources of awareness as defined below.

Awareness is computed at the arc level, with the probability of becoming aware while traversing the arc, further denoted as $\iota_a(\tau)$. As the traversing takes place in time and space, a driver is exposed to both the information and observation that make him become aware (Bifulco, et al 2013). Drivers in the traffic networks can become aware by means of numerous sources *S*, yet to model rerouting the important is fact of being aware in general, regardless of what and how many sources.

Let's first consider the probability $\iota_a^S(\tau)$ of becoming aware by means of a given source *S* out of available sources **S**. The generic source of awareness is defined with its market penetration P^S and the, so-called, spreading profile ι^S. The market penetration is the share of the drivers having access and utilizing source *S,* while the spreading profile tells how the information is spread to the recipients, with number of possible shapes of the spreading profile discussed below. In general, the spreading profile ι^S is a function of the explanatory variables (i.e. space, time, etc.) and is defined as a probability density function (PDF) of becoming aware by means of a given source *S*. The spreading profile provides the continuous representation of probability of becoming aware for a given time and space (for simplicity denoted as a $\iota^S(\tau)-$ function of time only). The probability of becoming aware while traversing an arc can be obtained by integrating the spreading profile $\iota^S(\tau)$ from the tail and the entry time to the head and the exit time in the space of explanatory variables of the spreading profile. Practically, as the spreading profile is a PDF, the integration can be substituted with the cumulative density function (CDF) being an integral of PDF (which is assumed to be integrable). Thanks to this, the probability of becoming aware by means of a source *S* can be calculated with (5.1), obtained either through the integration of PDF ($I=\int \iota$) or, equivalently, as a difference between CDF at head and at tail by using the travel time of arc t_a as a time during which driver can become aware. The second term of (5.1) is much more efficient as the CDF usually have a closed form and can be evaluated directly. In (5.1) the integration domain is not specified as the spreading profiles can be functions of various variables as will be shown below (i.e. it is a function of time for a radio news and a function of space for VMS). The resulting probability ι^S shall be further multiplied with the market penetration P^S. Market penetration for each source can be obtained through the stated preference study in which drivers would state what sources do they use, see (Emmerink et al., 1996), (Bifulco et al., 2011) for reference.

$$\iota_a^S(\tau)=\int_{a^-}^{a^+} P^S\cdot\iota^s(\tau)=P^S\cdot\left(I_{a^+}^S(\tau)-I_{a^-}^S(\tau-t_a)\right) \tag{5.1}$$

In the thesis any number of independent sources of awareness $S\in\mathbf{S}$ in the traffic network is considered. In the remainder, it is assumed that the sources of awareness are mutually independent; as well as the observation is assumed to be independent from the information. Thanks to this the simplified formula for the joint probability can be used to determine the total awareness probability $\iota_a(\tau)$ with (5.2), where, due to mutual independence, $P(S')=P(S'|S'')$ and $P(S'\cap S'')=P(S')\cdot P(S'')$. Actually, the desired output for the ICM is the probability of becoming aware from at least one source while traversing the given arc, denoted $\iota_a(\tau)$. It can be computed with the probability of the complementary event, namely that the driver did not become aware by any source, given with a product of complementary probabilities $(1-\iota_a^S(\tau))$ over all possible sources $S\in\mathbf{S}$. The complement of this event is the desired probability of becoming aware (from at least one source) computed with (5.2) and being the output of the awareness model.

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$$\iota_a(\tau) = 1 - \prod_{S\in\mathbf{S}} \left(1 - \iota_a^S(\tau)\right) \tag{5.2}$$

Below, the general definition of the spreading profile is applied for the sources of awareness popular in the contemporary traffic networks. Mind that the awareness model computes the probability of becoming aware while traversing the arc, not the probability of being aware. In general, this allows more consistent probabilistic results along the trajectories in NFP, where flows from various origins are merged and information about origin is lost. The general probabilistic machinery is handled with the Markov chain of three flow states described below. Finally it is worth noting that for consistency the $\iota_a(\tau)$ is evaluated at the exit time of the arc a and integrated since the entry time $\tau - t_a$. Mind that the proposed awareness model apart from covering the en-route rerouting, can be also applied to evaluate the pre-trip rerouting, which takes place at the origins, when drivers are aware of the event before departing. This can be achieved in the straightforward way, by using the event communication time in (5.1) evaluated at the origin. This way the drivers at the origin can become aware and reroute, which is usually referred to as the pre-trip rerouting.

Sources of awareness and their spreading profiles

In general, the following paradigms of becoming aware while travelling through the traffic network are identified:

1. fixed time information, i.e. news broadcasted through the radio;
2. fixed place information, i.e. message shown at a road-side VMS;
3. observing atypical situation, i.e. queuing in the unexpected queue;
4. information available on-line, i.e. published at a webpage, or to the mobile application.

Each of them defined with a different spreading profile $\iota^S(\tau)$ and specific penetration rate P^S. Becoming aware on-line is the most challenging and complicated process, therefore it is addressed in the separate section, while the description of the first three sources is compact enough to be listed below:

1. The traffic information broadcasted at a fixed time (i.e. the radio news) reaches the recipients wherever they are. It can be assumed that everyone who is listening to the news (equal to the radio market penetration P^{NEWS}) at the time of broadcast will become aware, regardless the location. Thus, the spreading profile of news can be defined as a function of time, being null when the news are not broadcasted and constant at the time of broadcasting (which for consistency of PDF is limited to a single time instant τ_{NEWS}). This way a spreading profile can be expressed with the functional form of the Dirac delta (5.3), with positive value at τ_{NEWS}, such that the CDF after the broadcasting is over is equal to the market penetration. Thanks to the Dirac delta formulation, the integral to CDF can be handled directly with (5.4) where the probability of becoming aware due to the news is positive only during the broadcasting $\tau_{NEWS} \in \left(\tau - t_a, \tau\right)$.

$$\iota_a^{NEWS}(\tau) = \begin{cases} 1 \cdot P^{NEWS} & \tau = \tau_{NEWS} \\ 0 & \text{otherwise} \end{cases} \tag{5.3}$$

$$I_a^{NEWS}(\tau) = \begin{cases} 1 \cdot P^{NEWS} & \tau \in \left(\tau_{NEWS}, t_a + \tau_{NEWS}\right) \\ 0 & \text{otherwise} \end{cases} \tag{5.4}$$

2. The information broadcasted at a given location, i.e. through a Variable Message Sign (VMS) can be reached only by those who were traversing the arc with VMS while it broadcasted the information. Therefore, the spreading profile for VMS forms the similar functional shape as above, yet being the function of not only time, but most importantly of the space. Namely, the probability of becoming aware by means of this source is positive only for the drivers crossing a VMS-equipped arc during its broadcast time and is null otherwise (for the other arcs and when the information is not broadcasted). $I_a^{VMS}(\tau)$ is given, likewise the $I_a^{NEWS}(\tau)$ with the Dirac delta of (5.5). Where P^{VMS} can be understood as the probability of noticing the VMS while passing by, rather than market penetration, as VMS is visible for every driver crossing it. For consistency, the positive value of PDF is assumed only at the tail of arc *a*, so that the $I^{VMS}(\infty) = P^{VMS}$ can be guaranteed through CDF of (5.6).

$$\iota_a^{VMS}(\tau) = \begin{cases} 1 \cdot P^{VMS} & \text{at VMS arc and during the broadcast} \\ 0 & \text{otherwise} \end{cases} \tag{5.5}$$

$$I_a^{VMS}(\tau) = \begin{cases} 1 \cdot P^{VMS} & \text{if } a = a_{VMS} \text{ and } \left[\tau - t_a, \tau\right) \cap \tau_{BROADCAST} \neq \varnothing \\ 0 & \text{otherwise} \end{cases} \tag{5.6}$$

3. Becoming aware from the observation is handled by measuring the experienced delay while traversing the arc. The motivation for such source of awareness is twofold. First of all, it is assumed that if driver experiences the atypical delay he is curious of the reason and uses the available information sources to understand the reason of the actual situation, i.e. on-line. Moreover, urban drivers are usually experienced and they have a routing policies. (Gao et al., 2008) and (Snowdon et al., 2012) argue that an experienced driver can correctly approximate the state of the whole network from experiencing the atypical travel times. The spreading profile of observing can be represented as a function of the ratio between the actual arc travel time $\tilde{}$ and the typical one $\hat{t}_a(\tau)$. In the thesis the naïve formulation of the spreading is proposed with (5.7), where the probability is proportional to the delay with a given constant of proportionality *λ*. Although the observation of the delay is evaluated locally at the current arc, it is somehow cumulated within the aware flow, so that experiencing more and more delay on the several arcs will magnify the awareness. Mind that the proposed mathematical formulation of the model is just hypothetical and not supported with any empirical observations, it shall be further verified within the estimation procedure (see chapter 6).

$$I_a^{OBSERVATION}(\tau) = \lambda \cdot \Delta t_a = \lambda \cdot \left(1 - \frac{\tilde{}}{\hat{t}_a(\tau)} \right) \tag{5.7}$$

Information spread model

The most interesting and challenging is to represent the information available online, which can be checked by the driver at any time. Such information is presumably the most popular nowadays and includes: mobile applications, social networks, online services, etc. To model such cases the following original *information spread model* is proposed.

The model originated from the *Twitter API* data, where numerous studies were conducted to data-mine the information spread processes. The recent research (Procter et. al,

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2013) gave answer on how the information spreads through the *Twitter* network. Twitter data supported the assumptions of the early researchers (Tweedie et al., 1986) that the Rayleigh distribution well explains the probability of receiving the information in time. This can be seen at the empirically observer profile of *Twitter* data at the Figure 5.1 and the theoretical Rayleigh distribution at Figure 5.2 showing the probability of becoming aware as an empirical and theoretical function of time. The number of twitter users receiving the information is given with a PDF of Rayleigh distribution (5.8), being function of time past the event was published on-line: $\tau - \tau_0^{ON-LINE}$. (5.8) is parameterized with the single parameter: σ having a straightforward interpretation as the time when the spreading is fastest.

Based on the above the probability of receiving information online while traversing a given arc $\iota_a^{ON-LINE}(\tau)$ is defined with the Rayleigh distribution and computed through the CDF (5.9). The spreading profile for the on-line sources can be then expressed with (5.11).

$$PDF(\tau) = \frac{\tau - \tau_0^{ON-LINE}}{\sigma^2} e^{-\frac{\left(\tau - \tau_0^{ON-LINE}\right)^2}{2\sigma^2}} \tag{5.8}$$

$$CDF(\tau) = \int_{-\infty}^{\tau} PDF(t)dt = 1 - e^{-\frac{\left(\tau - \tau_0^{ON-LINE}\right)^2}{2\sigma^2}} \tag{5.9}$$

$$i_a^{ON-LINE}(\tau) = P^{ON-LINE} \int_{\tau - t_a}^{\tau} PDF(t)dt = P^{ON-LINE}\left(CDF(\tau) - CDF(\tau - t_a)\right) \tag{5.10}$$

$$i_a^{ON-LINE}(\tau) = P^{ON-LINE}\left(e^{-\frac{\left((\tau - t_a) - \tau_0^{ON-LINE}\right)^2}{2\sigma^2}} - e^{-\frac{\left(\tau - \tau_0^{ON-LINE}\right)^2}{2\sigma^2}} \right) \tag{5.11}$$

Moreover, for the sake of realism, let's extend the above model to cover the, so-called, *virality* of the information (Leskovec et al., 2007). In general, the significant information is viral and spreads like a virus. For the awareness model it means that the viral information a) reaches more audience and b) reaches it faster. Traffic information can become viral if it is significant, meaning that the event is being communicated faster if it significantly affects the network. The significance of the event can be approximated with $M(\tau)$ – relative increase in travel times over the whole network, computed with (5.12). To compute *M* efficiently in practice it is convenient to define a so-called time-window, i.e. time period $(\tau - \Delta\tau^-; \tau + \Delta\tau^+)$ from which the total delay is computed in (5.12).

To include the virality effect in the information spread model both the market penetration $P^{ON-LINE}$ and the spreading speed σ can be expressed as the functions of the event significance $M(\tau)$. The default Rayleigh spreading process can be then accelerated for the viral events, so that σ parameter decreases with the increasing virality. Furthermore, the viral information reaches more drivers as it is repeatedly reposted on the social networks and somehow increases the default market penetration $P^{ON-LINE}$. In the thesis, the naïve assumption is followed that the virality is proportional to the $M(\tau)$ both at the spreading speed and a penetration rate. This way the default Rayleigh PDF of (5.8) can be now redefined to (5.13) including $M(\tau)$. Finally the (5.11) can be reformulated to (5.14) which includes the virality effect through *M.* Figure 5.3 and Figure 5.4 show how the base parameterization of Rayleigh CDF can change with varying *M* modifying the scale parameter, which speeds up the spreading (Figure 5.3) and adjusting market penetration, which increases the total share of aware drivers (Figure 5.4).

$$M(\tau) = \sum_{a \in A} \int_{\tau-\Delta\tau^-}^{\tau+\Delta\tau^+} \big(\tilde{\ } \qquad \vartheta)\big) d\theta \Big/ \sum_{a \in A} \int_{\tau-\Delta\tau^-}^{\tau+\Delta\tau^+} \big(\hat{t}_a(\theta) \cdot q_a(\theta)\big) d\theta \tag{5.12}$$

$$PDF(\tau) = M(\tau) \cdot \frac{\tau - \tau_0^{ON-LINE}}{\sigma^2} e^{-\frac{\left(\tau - \tau_0^{ON-LINE}\right)^2}{2(\sigma / M(\tau))^2}} \tag{5.13}$$

$$t_a^{ON-LINE}(\tau) = P^{ON-LINE} \cdot \left(M(\tau - t_a) \cdot e^{-\frac{\left((\tau - t_a) - \tau_0^{ON-LINE}\right)^2}{2(\sigma / M(\tau - t_a))^2}} - M(\tau) \cdot e^{-\frac{\left(\tau - \tau_0^{ON-LINE}\right)^2}{2(\sigma / M(\tau))^2}} \right) \tag{5.14}$$

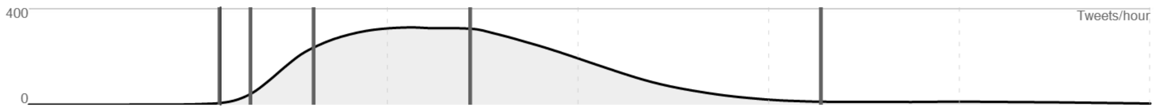

Figure 5.1 Twitter posts frequency after the event – empirically observed for the fake news on London riots (Procter, 2013).

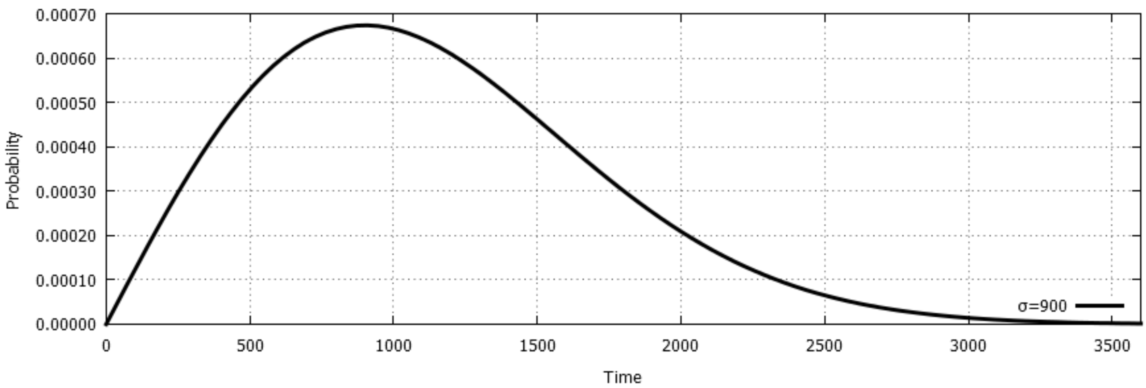

Figure 5.2 Probability density function (PDF) of the Rayleigh distribution with σ = 900 seconds.

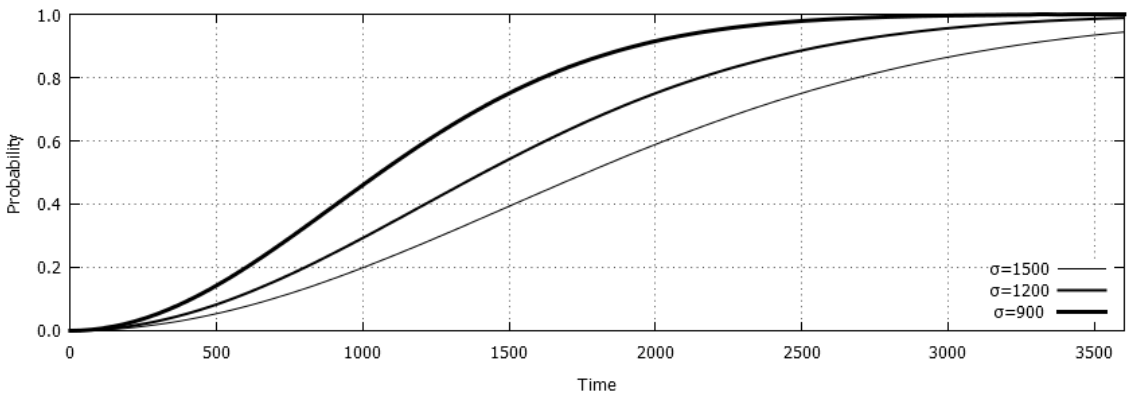

Figure 5.3 Cumulative density function (CDF) of the Rayleigh distribution where the basic Rayleigh parameter σ of 1500 seconds changes to 1200 and 900 seconds respectively, due to the total network delay M (5.14).

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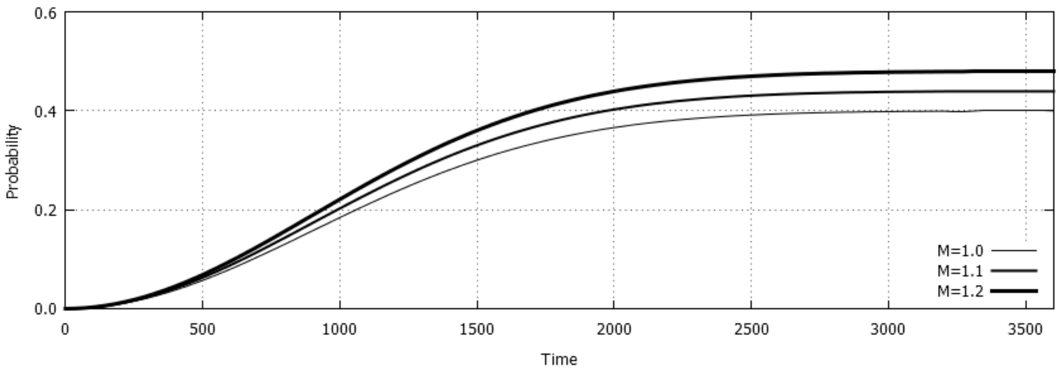

Figure 5.4 Cumulative density function (CDF) of the Rayleigh distribution, where penetration rate $P^{ON\text{-}LINE}$ is multiplied with the network delay *M* of 1.1 and 1.2 respectively.

To summarize the awareness model the Figure 5.5 is presented. It depicts the cumulated probability of becoming aware *I* for a generic driver traversing the network in the following cases:

- In the default setting, the information is available only on-line (thick, black line) and it reaches 40% of drivers ($P^{ON-LINE} = 40\%$).
- If the information is broadcasted also at the radio news (thin black line), the joint probability of being aware rises up to 60%, as the two independent sources amplify the awareness due to (5.2).
- If driver passes also the VMS sign with the information broadcasted (at 11:12), the joint probability (5.2) reaches 90%.
- If the information spreads virally (dotted line) it spreads fast and reaches more drivers (up to 80%) due to increasing *M* in (5.14).
- If driver experiences also the atypical delay (dashed line) while traveling the joint probability rises to 55% due to (5.7).

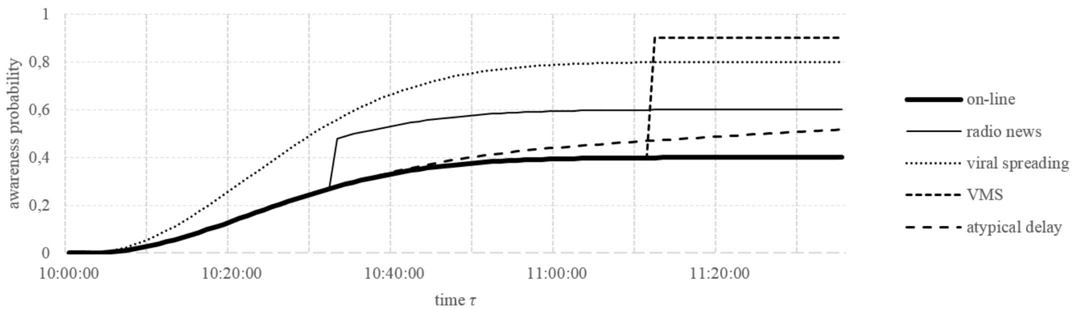

Figure 5.5 Illustration of the cumulated awareness probability as a function of time in various cases of on-line information coupled with: radio news, VMS, atypical delay and viral spreading.

5.1.2.2. *Compliance model*

The above awareness model provides information on how the drivers become aware while travelling through the network and being exposed to number of awareness sources. In the subsequent step, the aware drivers are making the decision whether they actually reroute or not. This is achieved with a compliance model telling what share of aware users will actually reroute in a given situation: where they are, where the travel to and what is the traffic situation. Compliance model is designed to realistically represent the actual drivers decisions to reroute or not in the actual traffic situation. A discrete choice model is chosen as an appropriate tool to represent such choice process, namely the binomial logit model with two alternatives: do and do not reroute. The decision is based on the utility of rerouting perceived by drivers, expressed as functional of the explanatory variables representing the expected consequences of the rerouting decision. The outcome of the compliance model is the $\kappa_i^d(\tau)$ – share of the aware drivers making rerouting decision at a given decision point (node) at a given time, subject to their destination. The utility is formed with the two elements, Δp to measure possible travel time gains, and Δw to measure potential time losses, computed as follows.

The possible gains are handled, through the $\Delta p_i^d(\tau)$, basing on arc conditional probabilities, to assess how much time (or cost) can be saved by making a rerouting decision at given node. It is achieved by telling how does the actual arc conditional probabilities differ from the typical ones. To this end the vector of probabilities of a forward star of the current decision point (node) $p_i^d(\tau) = \left\{p_a^d(\tau) : a^- \in i\right\}$ is used. Two such vectors are compared: typical $\hat{p}$ (computed with the typical costs) and actual $\tilde{p}$ (computed with costs present in case of the event) . The actual arc conditional probabilities $\tilde{p}$ are computed within the RCM (4.15) from the actual costs $\tilde{c}$. They differ from the typical ones $\hat{p}$ if the cost pattern changed significantly, i.e. if the minimal cost to get to the destination g_a^d is not obtained along the $\hat{p}$ anymore. The cost difference to get to the destination, measured directly with g_a^d, can be indirectly expressed as the function of p as they are dependent through (4.15). Yet to handle the difference at the node level the two vectors of the forward star, typical $\hat{p}$ and actual $\tilde{p}$, need to be compared. It can be achieved with the cosine similarity (Singhal, 2001) and computed with (5.15). As the arc conditional probabilities vectors are normalized (due to (4.23)), the cosine similarity takes values from 0 to 1. Cosine similarity is used in (5.15) with negative sign so that Δp increases with the decreasing similarity between the two vectors.

$$\Delta p_i^d(\tau) = 1 - \frac{\sum_{a\in i^+} \tilde{p} \qquad (\tau)}{\sqrt{\sum_{a\in i^+} \tilde{p} \qquad}\sqrt{\sum_{a\in i^+} \hat{p}_a^d(\tau)^2}} \tag{5.15}$$

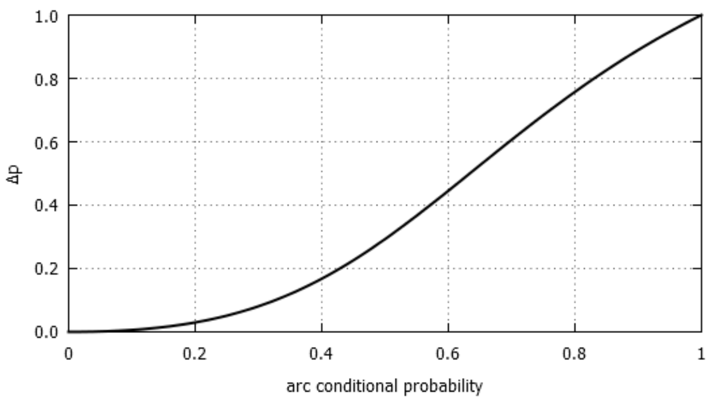

Figure 5.6 Cosine similarity Δp of vector $\{1,0\}$ and $\{x,1-x\}$ computed with (5.15).

Figure 5.6 shows the example of the Δp computation for the two vectors: $\{1,0\}$ of the typical arc conditional probabilities $\hat{p}$ and $\{x,1-x\}$ of the actual arc conditional probabilities $\tilde{p}$. Δp is expressed at Figure 5.6 as the function of x – probability of alternative arc that has null probability in the typical case. For the boundary cases $\Delta p = 0$ for identical vectors , i.e.

$\{1,0\}$ and $\{1,0\}$, and $\Delta p = 1$ for orthogonal vectors, i.e. $\{1,0\}$ and $\{0,1\}$. The shape of the function seen at Figure 5.6 proves that the cosine-similarity is an efficient measure the cost difference to get to the destination from a given node and Δp computed with (5.15) can be a meaningful part of the rerouting utility. Interestingly the resulting shape of cosine similarity implicitly handles the so-called *indifference band* of the rerouting (Mahmassani, 1997) i.e. the rerouting takes place only if the possible gains are high and can be skipped below some small difference. The small values of cosine similarity for the small differences between $\hat{p}$ and $\tilde{\imath}$ handle this internally.

The second measure, $\Delta w_i^d(\tau)$, is defined through the node potentials *w,* calculated within RCM (4.15). It quantifies the dual measure to the above Δp, namely a negative consequence of not rerouting at the current decision point. It is achieved through measuring the actual node potentials $\tilde{\imath}$ which can be reached from the current node in two cases: rerouting and not rerouting. The actual node potential reached at the forward star if a driver is not rerouting and follows the typical arc conditional probabilities $\hat{p}$ can be expressed with (5.16). Correspondingly, the actual node potential reached at the forward star if a driver reroutes and follows the actual arc conditional probabilities $\tilde{\imath}$ can be expressed with (5.17). The ratio between the two above (5.18) is the $\Delta w_i^d(\tau)$ which can be seen as the possible loss if driver will not follow the actual arc conditional probabilities from the current node.

$$\sum_{a \in i^+} \hat{p}_a^d(\tau)\tilde{\imath} \qquad (5.16)$$

$$\sum_{a \in i^+} \tilde{\imath} \qquad \tilde{} \qquad (5.17)$$

$$\Delta w_i^d(\tau) = \frac{\sum_{a \in i^+} \hat{p}_a^d(\tau)\tilde{\imath}}{\sum_{a \in i^+} \tilde{\imath} \qquad \tilde{}} \qquad (5.18)$$

There is a subtle distinction between Δp and Δw, which, prima facie, may seem a duplicates. The Δp will increase as soon as the costs become different, i.e. it will suggest the driver to optimize his route as soon as his typical choices are not optimal anymore. For example, if the motorway is blocked Δp will tell to escape at the first junction for the alternative route. Yet at the same time Δw can remain low, as there are still opportunities to avoid the event downstream. Δw increases significantly for the points with the *final call* for the rerouting decision, where there is the last chance to reroute and delay will be unavoidable further. Such decision points are not identified by Δp. Moreover, the Prospect Theory (Kahneman and Tversky, 1979), broadly applied in modelling the decision process, relies on fact that possible gains are perceived differently than possible losses. This way, in the future research, the proposed compliance model can be reformulated with the Prospect Theory instead of the discrete choice model.

Introduction of the two above variables allows to propose the meaningful form of the rerouting utility used in the following discrete-choice model. The idea is that the probability of rerouting while being aware is the function of the rerouting utility, which is, in turn, computed with the Δp and Δw variables. The greater the utility, the higher probability of rerouting. For the boundary cases: if the utility is null, the rerouting probability is infinitesimal; while for the infinite utility the probability reaches one. The utility is assumed to represent the actual factors leading to the rerouting decision, to ensure the proper representation of the decision process, the binomial discrete-choice model is applied as follows.

Let's define utility of rerouting at node *i* at time *τ* while travelling towards destination *d* as $\tilde{U}$ $\tilde{}$, with μ being random term and $\tilde{V}$ being the linear in-parameters function of the two explanatory variables Δp, Δw and the constant (so-called Alternative Specific Constant – *ASC*). The systematic part of utility takes form of (5.19) with two coefficients $\beta_{\Delta p}$, $\beta_{\Delta w}$. In (5.19) both Δp and Δw are normalized to the $(-1,1)$ range to make the model consistent. The codomain of Δp is (0,1) range, while Δw takes values from one up (the maximum is network- and event-dependent and can even reach infinity for the nodes with a dead-end situations, i.e. the last escape junction at the motorway). In practice let's use some fixed upper bound $\max(\Delta w)$ as the maximum value of Δw and use it in the normalization. This way the domain of V is $(-1,1)$ and can be further adjusted with model coefficients $\beta_{\Delta p}$, $\beta_{\Delta w}$ and the constant.

Such defined utility can be applied to the standard binomial logit model (5.20) with two alternatives: do and do not reroute. The utility of not rerouting in binomial model is by definition assumed to be zero. The probability of rerouting comes as the result of comparing the utility of rerouting with the null utility of not rerouting within the logit model. The rerouting behavior can be further parameterized through the *ASC* and the logit shape parameter η.

$$\tilde{V} \qquad SC + \beta_{\Delta p}\left(\Delta p_i^d(\tau) - 0,5\right) + \beta_{\Delta w}\left(\Delta w_i^d(\tau) / \max(\Delta w) - 0,5\right) \tag{5.19}$$

$$\kappa_i^d(\tau) = \frac{e^0}{e^0 + e^{-\tilde{V}}} \qquad \frac{1}{1+e} \tag{5.20}$$

The output of the compliance model is $\kappa_i^d(\tau)$, the share of the aware drivers that decide to reroute at a given node subject to a given destination at time *τ*, depicted as the function of utility at Figure 5.7.

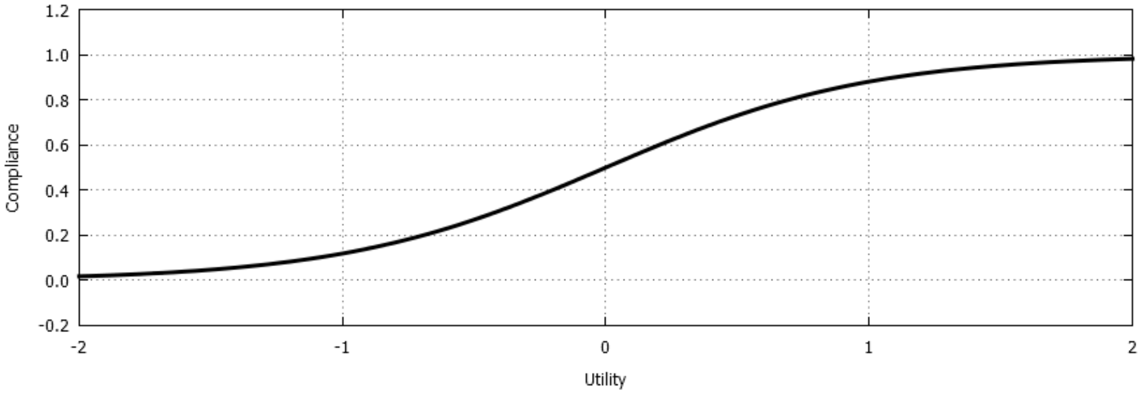

Figure 5.7 Share of complying drivers over utility.

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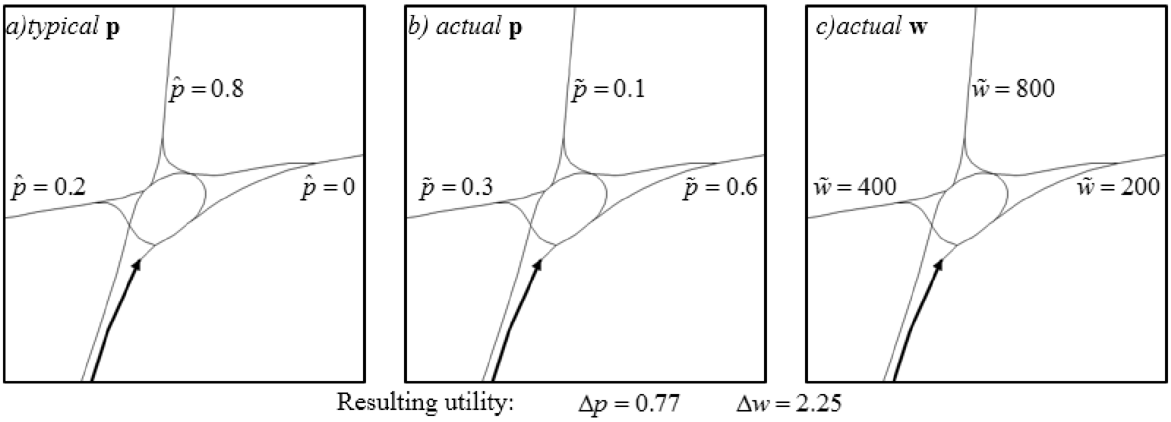


Figure 5.8 Compliance model illustrated for the case of a single decision point (node) – a junction of the traffic network. Driver approaches from the bottom and faces the rerouting decision with the following explanatory variables of the forward star, as shown: a) typical demand pattern, b) actual demand pattern, c) actual node potentials at the three arc of the forward star (left, straight, right). The resulting compliance (5.20) is computed with on Δp (5.15) and Δw (5.18).

5.1.2.3. *Input and parameterization*

As shown above the ICM can be expressed as a function evaluated on the explanatory variables at a given decision point, time and subject to a given destination. It outputs the share of drivers becoming aware (within the awareness model) and complying (within the compliance model). The explanatory variables needed to compute ICM result from a compromise between computation times and completeness of the model. Focus was on the variables that are anyhow calculated while solving a DTA problem. This was mainly due to computation time issues, as the ICM is executed numerous times in real-size applications. Namely for the network with *N* nodes, *D* destinations, *T* time intervals and *I* iterations there is $N \times D \times T \times I$ executions, which results in up to billion calculations for the real-size networks. Therefore, emphasis was to conserve the key properties of the model with variables available within the DTA Algorithm (Schema 2). Fortunately, the utility of the rerouting is well explained with the variables available at hand during RCM computation (arc conditional probabilities *p* and node potentials *w*). So that it can be obtained locally while computing the DTA demand pattern, this way computation time can be significantly reduced (in the final implementation the calculation times rose only by 25% compared to the classic DTA algorithm). Explanatory variables of the awareness model are even easier to obtain. The total delay *M* (5.12) is computed for the whole network and can be seamlessly joined with other DTA procedures executed over whole network (i.e. Arc Cost Function). The spreading profiles are proposed either through the Rayleigh distributions (on-line information), where integral can be quickly computed with closed form exponential formula (5.14), or through piecewise linear conditional functions (5.4), (5.6). The observation is handled locally and requires only relative delay of the respective arc, computed with (5.7). In conclusion, the lightweight algorithm is obtained allowing for quick computations. The following DTA variables are used as the input for the ICM:

- typical state-of-the-network in terms of travel costs $\hat{c}_a(\tau)$ and times $\hat{t}_a(\tau)$
- actual state-of-the-network in terms of travel costs $\tilde{}$ and times $\tilde{}$ (obtained on the fly while computing the ICM as discussed in part 4.4.1).
- event ε, with its communication time i_ε and impact on the network $\boldsymbol{\delta}^{\varepsilon}$.

This way the fast computation is guaranteed, yet for the practical applicability it has to be guaranteed that ICM provides the realistic results. This is obtained through the parameterization, allowing on one hand a flexible calibration to the actual phenomena and feasible estimation based on the available data on the other as elaborated in chapter 6. To represent the actual phenomena the parameters represent the identified properties from the section 5.1.1. Which led to the following ICM parameterization:

1. For the awareness model:
   1.1. The information progressively spreads in time reaching drivers at different time instants, which is parameterized through the spreading profiles ι^S of the respective sources of awareness with respective market penetration P^S.
   1.2. Important information spreads faster and reaches bigger audience, which is handled with the virality of the event. Currently defined as linear function of the network delay M through formulas (5.12) and (5.14), which shall be further verified.
   1.3. Drivers can become aware of the event if they observe its consequences empirically, which is handled with the experienced delay. Currently expressed with a linear formulation of (5.7), which shall be further parameterized and verified.

   Parameters of the awareness model are jointly denoted as $\mathbf{a}$.
2. For the compliance model:
   2.1. Drivers reroute to maximize the expected gains, which is parameterized with the $\beta_{\Delta p}$ and used as the weight in the utility.
   2.2. Dually to the above, drivers reroute to minimize their expected losses, which is parameterized with the $\beta_{\Delta w}$, used as the weight in the utility for the logit model.
   2.3. The compliance model is further parameterized with the logit parameter η, reflecting the sensitivity of drivers decisions, along with the Alternative Specific Constant capable to reproduce the inclination of drivers to reroute.
3. For the rerouting path, the resulting route chosen by the rerouting driver is parameterized with the n_{DTA}, the number of iterations of the DTA problem as elaborated in section 4.4.5.

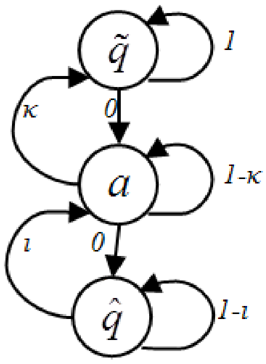
To sum up, the ICM model can be parameterized through the set of parameters $\{\mathbf{a}, \beta_{\Delta p_{id}}, \beta_{\Delta w_{id}}, \eta\}$. The estimation and verification of the ICM parameters is further elaborated in chapter 6, showing how the rerouting phenomena can be observed and how an ICM model can be estimated from the observed flows and paths.

5.1.3. Network Flow Propagation for the Information Comply Model

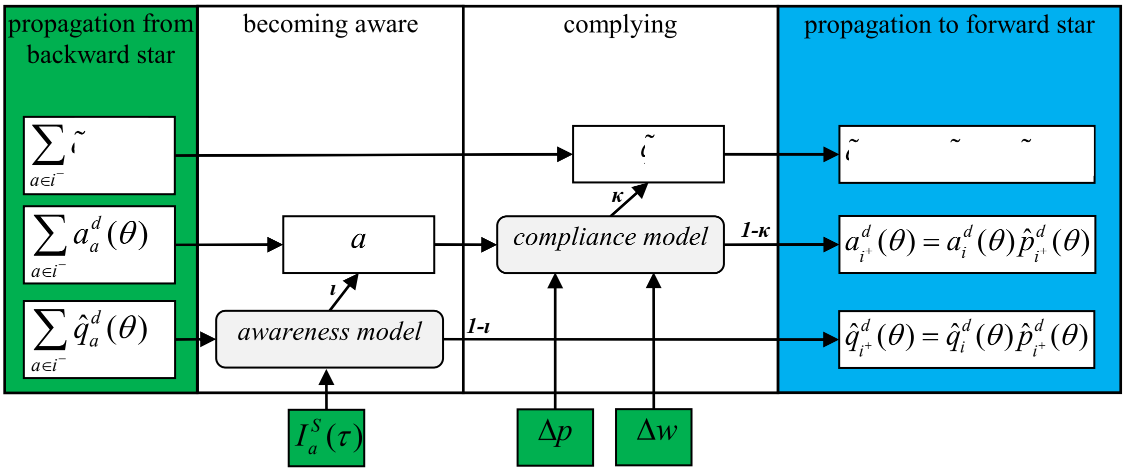
The way to apply the above awareness and compliance models inside the DTA is the following modification of the DTA problem (Schema 2), mainly the network flow propagation (NFP) model as described in section 4.1.3. In general, it is possible thanks to the partition of the propagated flow into the three states: unaware $\hat{q}$, aware a, rerouted $\tilde{r}$. All of them are propagated simultaneously in the NFP procedure, adapted for the ICM as follows. The total demand flows **d** is conserved, yet the propagation behavior differs across the states. Initially the flow is unaware, it becomes aware while being propagated and, finally, the aware flow can reroute. Both unaware $\hat{q}$ and aware flows a are propagated with the typical arc conditional probabilities $\hat{p}$ yet with the actual travel times $\tilde{t}$ (which can be different from typical due to the event). While the rerouted flows $\tilde{r}$ are propagated with the actual probabilities $\tilde{p}$ (including the event) and with the actual travel times. This way the idea of the hybrid-route choice model (4.21) is utilized in ICM, namely the rerouting driver changes the perspective of costs from typical to actual at the time when he complies to the event.

To compute the flow in each state and transitions between them, the Schema 5 is proposed to extend the NFP (4.17) at a generic node. It takes as an input the flow in each of the three state propagated from backward star. Then the respective transitions are computed with the awareness and compliance models of ICM to update the flow in each state and propagate it forward. The proposed schema can be seen as the Markov chain with the three possible states depicted at Schema 4. The only allowed transitions are: a) becoming aware and b) complying (rerouting while already being aware). Transition probabilities are computed directly within the ICM. The probability of becoming aware ι is computed within the awareness model (5.2), while the probability of complying κ is computed with the compliance model (5.20). The awareness model outputs the probability of becoming aware while propagating from the previous to the current node, computed with (5.2). This way the part of flow that becomes aware ι is separated from the part that is still unaware $(1-\iota)$. Subsequently, the aware flow a is transitioned to the rerouted flow $\tilde{r}$ with the compliance model (5.20). This way the part of the aware flow that decided to reroute κ is separated from the part that remains aware (and stick to the typical probabilities) $(1-\kappa)$.

To fit this idea into the NFP, the general formulas (4.17) and (4.18) need to be redefined for each of the states as follows. In each state the flow is propagated differently, with various demand patterns used and various flow conservation. The unaware flow $\hat{q}$ is reduced with the part which became aware ι (5.21) and is propagated forward using the typical demand pattern $\hat{p}$ with (5.22) – which includes the demand flows d. Respectively the aware flow is first increased with the flow which became aware and then reduced with the flow which complied (5.23). The aware flow is still propagated with a typical demand pattern $\hat{p}$, yet the demand flows are not included anymore (5.24). Finally, the rerouting flow is increased with the flow that complied (5.25) and is propagated forward with the actual demand pattern $\tilde{p}$ (5.26). Such schema allows to consistently apply the ICM model inside the NFP computed along the trajectories towards a single destination. Moreover, it allows to apply the awareness model (through (5.2)) and compliance (through (5.20)).



Schema 4 Three states of the flow in ICM represented with the simplified Markov chain.



Schema 5 ICM inside NFP for a generic node *i*.

$$\hat{q}_i^d(\theta) = \sum_{a\in i^-} \hat{q}_a^d(\theta)\cdot\left(1-\iota_i^d(\theta)\right) \quad (5.21)$$

$$\hat{q}_a^d(\theta) = \left(d_{a^-}^d(\theta) + \hat{q}_{a^-}^d(\theta)\right)\hat{p}_a^d(\theta) \quad (5.22)$$

$$a_i^d(\theta) = \left(\sum_{a\in i^-} a_a^d(\theta) + \sum_{a\in i^-} \hat{q}_i^d(\theta)\cdot\iota_i^d(\theta)\right)\cdot\left(1-\kappa_i^d(\theta)\right) \quad (5.23)$$

$$a_a^d(\theta) = a_{a^-}^d(\theta)\,\hat{p}_a^d(\theta) \quad (5.24)$$

$$\ldots {}_a^d(\theta) + \sum_{a\in i^-} \hat{q}_i^d(\theta)\cdot\iota_i^d(\theta)\Bigg)\cdot\kappa_i^d(\theta) \quad (5.25)$$

$$(5.26)$$

$$\alpha_i^d(\tau) = \ldots \quad (5.27)$$

Algorithmic issue raises within the NFP formulas proposed above, which are executed in the topological order (4.14). It is correctly assumed for the algorithm that the routing is restricted to the efficient arcs only (leading closer to the destination) and it is perfectly consistent if the route choice is computed with the same travel times as the propagation. Unfortunately, the topological order of typical situation can differ from the actual one, as the event can significantly impact the travel times. Such situation results in the *lost* flows at arcs that are outside of topology, i.e. the flow conservation rule in (4.19) is not conserved. This

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issue is overcame in ICM by executing the NFP twice. The first propagation is executed in the typical topological order (based on the typical costs $\hat{c}$). During the first propagation, the unaware and aware flows are propagated, while the rerouted flows are only memorized and stored at the nodes. Then, the second NFP is executed, now with the actual topological order and propagating only the rerouted flows with the actual arc conditional probabilities. Otherwise it could happen that: a) the same driver makes rerouting decision several times during trip, b) flows which travel against the actual topological order (are going further from the destination in term of actual travel times) are lost and do not reach the destination. Therefore, the above NFP is executed as a two separate processes executed in two various topological orders. Namely, the propagation with the equations (5.21) to (5.25) is computed during the first run with memorizing the rerouted flows $\tilde{q}$ (5.25). Then the second NFP propagates the rerouted flows along the actual topological order with (5.26).

Finally, for the further analysis let's define the share of the total flow that reroutes at the current node as α expressed with (5.27), which will be main input for the estimation and verification in chapter 6. Rerouting share α is the most meaningful output of the ICM model, it tells where and when the rerouting takes place, as well as what is the share of rerouting drivers.

5.1.4. Rerouting path

The significant aspect of the rerouting phenomena is handled implicitly within the ICM. Namely the route chosen by the drivers who reroute, which in macroscopic DTA would be expressed as the demand pattern $\tilde{p}$ used by the rerouted flows in (5.26). ICM assumes that the rerouted flow will make the rerouting decision using the actual arc conditional probabilities as supported i.e. by (Mahmassani, 1997). Actual arc conditional probabilities $\tilde{p}$ based on the actual costs $\tilde{c}$ assume that drivers have full access to the perfect knowledge on the actual and predicted travel costs. This may seem at the first sight that the rerouting flows are following the optimal route to get to the destination, likewise the solution of the optimal route guidance problem. In fact, the actual arc conditional probabilities $\tilde{p}$ are computed using the probabilistic RCM of (4.15). Which is, in fact, the probabilistic, logit model with parameter η, so that not only optimal routes are chosen. Theoretically, all the efficient arcs shall have non-zero probabilities in the logit model. Nevertheless, the assumption that the rerouting drivers are making their routing decisions based on the full knowledge of actual costs shall be further verified (see chapter 6.).

5.1.5. DTA with Information Comply Model – ICM DTA

The ICM is designed to work within the classic DTA framework and this section provides insight on how the two are integrated and how the results are obtained. The DTA problem of (4.2) is redefined with the ICM to form (5.28) and referred to as the ICM DTA problem depicted at schema 6. Likewise the default DTA, the ICM DTA provides the results in terms of state-of-the-network (traffic flows, travel costs and times), yet including rerouting phenomena.

The ICM DTA problem of (5.28) retains the structure and logic of the default DTA algorithm (schema 2) and modifies only the following parts. The fundamental difference lays in the way the demand part of DTA is handled. To apply the hybrid route-choice and apply the ICM machinery, the demand pattern (arc conditional probabilities) needs to be computed twice: for the typical costs $\hat{\mathbf{c}}$ and then for the actual costs $\tilde{\mathbf{c}}$, expressed with (5.29) and (5.30) respectively. To compute the typical demand the typical travel costs $\hat{\mathbf{c}}$ need to be known, which is the additional input of the ICM DTA. Furthermore, the flow is propagated in the three states, so that schema 5 need to be integrated with the ICM DTA, so that the default NFP, expressed with formulas (4.17) and (4.18), need to be replaced with the modified NFP defined above through formulas (5.21) to (5.26). Finally, due to issue of varying topological order mentioned above, NFP needs to be executed twice.

ICM DTA (schema 6) can be executed with the default DTA input of: the demand **d** and the network $\boldsymbol{\delta}^{\varepsilon}$. The network used in ICM DTA $\boldsymbol{\delta}^{\varepsilon}$ includes the event and the way it changes the network parameters, i.e. reduces capacity, speed, number of lanes, or blocks the arc (the typical network parameters are not needed as they are memorized through the typical costs $\hat{\mathbf{c}}$). The additional input needed to execute the ICM DTA is the typical state-of-the-network (travel times $\hat{\mathbf{t}}$ and costs $\hat{\mathbf{c}}$). The explanatory variables of ICM are computed on the fly during executing the DTA algorithm. The way the input variables and internal DTA variables are used inside the new Network Flow Propagation model is depicted at Schema 7. Schema 7 is the extended version of Schema 5 with the DTA input and output shown. Schema 7 can be directly used inside DTA algorithm as shown in schema 6, denoted there as the ICM block. The logic and structure of the resulting DTA (Schema 6) remains almost the same as in the default DTA (schema 2).

ICM requires longer computation times then the default DTA mainly due to executing the RCM twice (for the typical and then for the actual costs) and the NFP twice (for unaware and aware flows and then for rerouted flows). Red arrows at Schema 7 depicts the elements that need to be recomputed with the second NFP (reason for this is explained in sections 4.4.6 and 5.1.3). Explanatory variables of the compliance model (Δp and Δw) are computed within RCM using the typical and actual node weights and arc conditional probabilities with (5.15) and (5.18) respectively. They need to be computed simultaneously with RCM and NFP, as they are strictly related to the cost pattern from the current decision point to the destination. The input needed to compute awareness model is less related to the RCM machinery. $M(\tau)$ used in the information spread model can be computed globally for the whole network and expressed as the function of time. The spreading profile of the respective sources of awareness $I_a^S(\tau)$ needs to be evaluated over the backward star to obtain $\iota_a(\tau)$ with (5.2) based on time instant, location, etc. Computing time for the awareness and compliance models of ICM is acceptable and computation time is longer than the default DTA mainly due to running shortest path calculations twice (for typical and for actual costs). Fortunately, the number of iterations can be much lower than in typical DTA, as discussed in 4.4.5.

ICM DTA, is still an iterative sequence and the resulting state-of-the-network will fluctuate throughout the iterations. The input of actual travel times $\tilde{\mathbf{t}}$ and costs $\tilde{\mathbf{c}}$ will change from iteration to iteration, so will the output of the awareness and compliance model as well as the flow propagated in each state. The typical cost and times are fixed and do not change between iterations, yet the utility of the rerouting changes between the iterations. In fact, the circularity affects mainly the compliance model that is computed directly from the actual costs. In a nutshell: the utility of the rerouting is high when no one reroutes and then in the next iteration it becomes low as everybody reroutes. To a lesser extent it refers to the awareness model, where spreading profiles are fixed and only the total network delay M and observed delay Δt_a can alter the awareness process. The transitions between unaware and aware flow ι are expected to be more or less fixed between the iterations and the fixed point would be handled mostly at the compliance model, through revising the compliance transitions κ and actual arc conditional probabilities calculated with (5.30) based on the actual costs. This allows to express the fixed point for the ICM DTA in terms of the rerouting flows $\tilde{\iota}$, with the meaningful relative gap expressed through $\|\tilde{\iota}\ \ \tilde{}\ $. The actual times and costs computed on the supply (4.8) are the function of the total flows, including the rerouted

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flows. Consequently, the rerouted flows become dependent of the actual costs resulting from the previous rerouted flows, which constitutes the fixed-point problem. This way, the DTA fixed-point problem of (4.13) can be expressed for the ICM DTA as (5.31). As argued in section 4.4.5, the realistic solution is not obtained at the fixed-point of the DTA problem (5.28), as the network is far from equilibrium in the case of an unexpected event (Watling and Hazelton, 2003). However at least some iterations need to be executed to solve the internal DNL problem (chapter 4.1.4) so that the typical arc conditional probabilities $\hat{p}$ become consistent with the actual travel times $\tilde{}$. So that the given number of iterations n_{DTA} shall executed reflect the forecasting capabilities and actual disequilibrium of the network, further discussed in chapter 6.

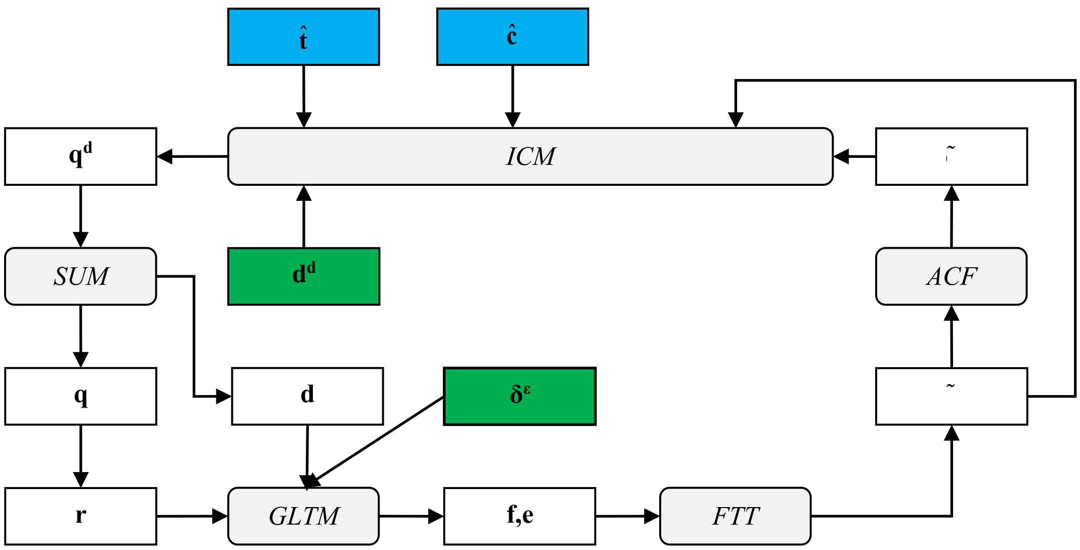
This way the new DTA model is proposed through the schema 6 and expressed formally with (5.28), referred to as the ICM DTA. It both retains the logic of the macroscopic DTA described in chapter 4.1 and includes the ICM machinery. The resulting algorithm remains efficient, the calculation time at the medium scale network (1300 links, 430 nodes, 30 zones) rose only by 25% compared with the classical TRE. The ICM DTA problem solves the thesis main problem – it calculates the state-of-the-network in cases of unexpected events, as shown with the following illustrative examples.

$$\mathbf{S} = DTA\_ICM(\boldsymbol{\delta}^{\varepsilon}, \mathbf{d}, \hat{\mathbf{c}}, \hat{\mathbf{t}}) \quad (5.28)$$

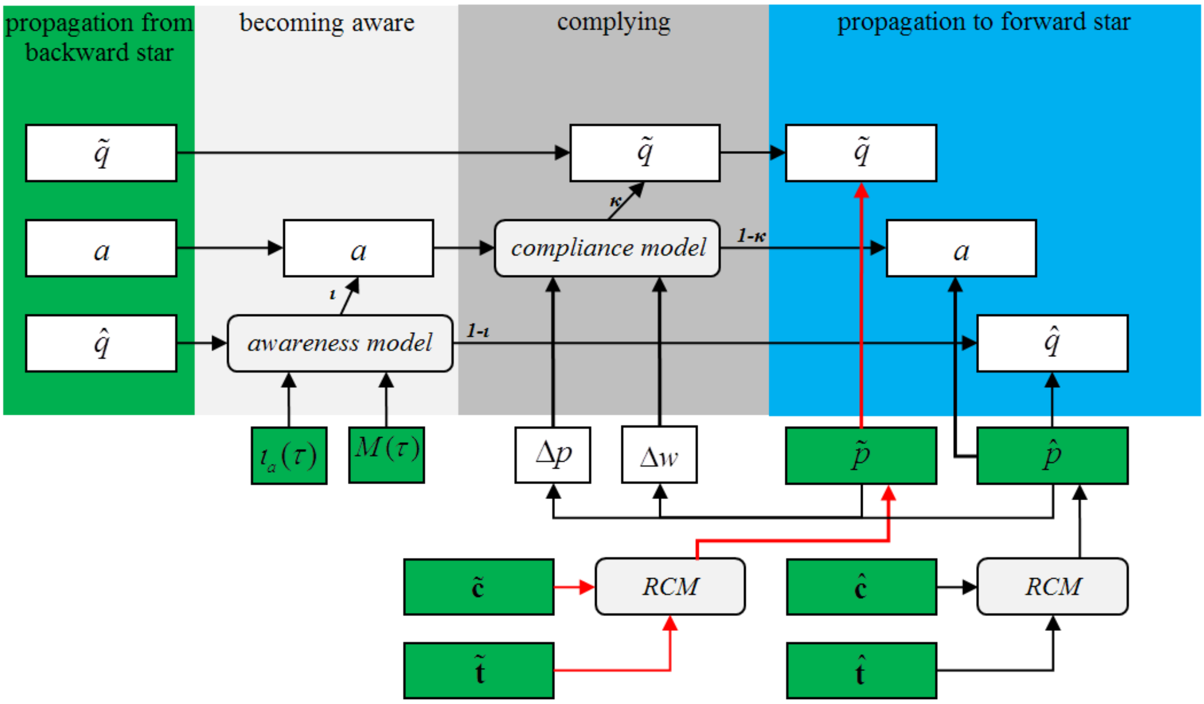
$$\hat{\mathbf{p}} = RCM(\hat{\mathbf{c}}, \hat{\mathbf{t}}) \quad (5.29)$$

$$\tilde{} \qquad \tilde{} \quad (5.30)$$

$$\{\hat{\mathbf{q}}^{n}, \mathbf{a}^{n}, \tilde{} \qquad \tilde{} \quad (5.31)$$



Schema 6 DTA problem with ICM.



Schema 7 Network Flow Propagation and Route Choice model of Information Comply Model.

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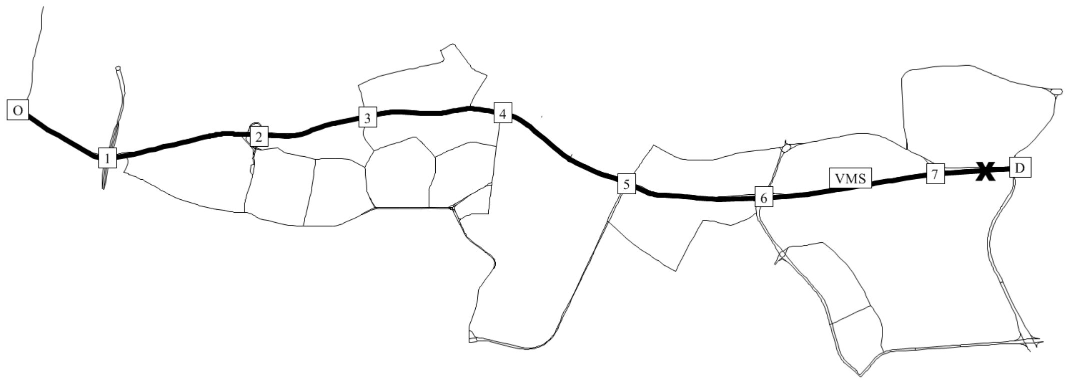
5.1.6. *Illustrative examples*

Let's consider a single route between a given origin and a destination traversed in time (Figure 5.9). The route consists of a seven decision points at which the rerouting decision can be made. Close to the destination, at 7$^{\underline{10}}$, there is an event that blocks the arc, so that the typical route (chosen at the origin, marked with bold at Figure 5.9) is no longer feasible and drivers start seeking for alternatives in the remainder of the traffic network from Figure 5.9.



A path (bold) connecting origin O with destination D, passing through the 7 descision points and one VMS sign; unexpected event marked with X.

Figure 5.9 Network of the illustrative example

In the illustrative example, a generic flow of 100 vehicles departs from the origin *O* at 7$^{\underline{00}}$ and propagates towards the destination *D.* While propagating through the network drivers receive various information from different sources, observe the actual situation and estimate their expected rerouting utility. Based on this they make a rerouting decisions, modelled with the ICM model. The tables below show the values of the ICM explanatory variables, resulting transition probabilities and traffic flows in the three states propagated forward from each of the decision points. Mind that to keep the examples demonstrative, the ICM parameters were intentionally magnified, especially the awareness model is defined so that almost everybody becomes aware.

5.1.6.1. *Multiple source of awareness*

Table 5.1 ICM results at 7 consecutive decision points traversed during propagation. Information broadcasted at the radio, on-line and at the VMS.

Decision point		Flow			Awareness model							Compliance model		
i	τ	$\hat{q}$	a	$\tilde{\iota}$	ι	$\iota_a^{NEWS}(\tau)$	$\iota_a^{VMS}(\tau)$	$\iota_a^{O-L}(\tau)$	$M(\tau)$	$\iota_a^{O}(\tau)$	Δt_a	κ	Δp	Δw
1	7:00	100	0	**0**	**0%**	-	-	0%	1	-	-	-	0.00	1.00
2	7:10	97	3	**0**	**3%**	-	-	3%	1	-	-	**1%**	0.01	1.01
3	7:15	93	7	**0**	**3%**	-	-	3%	1	-	-	**1%**	0.01	1.00
4	7:20	62	37	**0**	**33%**	30%	-	4%	1.01	-	-	**0%**	0.00	1.00
5	7:25	60	29	**11**	**4%**	-	-	4%	1.01	-	-	**28%**	0.61	1.47
6	7:40	54	8	**27**	**10%**	-	-	10%	1.02	-	-	**78%**	0.76	2.65
7	8:20	5	1	**55**	**91%**	-	80%	6%	1.04	50%	0.5	**98%**	1.00	4.00

i - decision point index
τ - time instant at which i-th decision point is reached
$\iota_a^{O-L}(\tau)$ - probability of receiving the information online while traversing arc a (5.14)

$\iota_a^O(\tau)$ - probability of becoming aware while traversing arc a due to observation (5.7)

The first example shows the case where the information is provided through the radio news, is available on-line and is broadcasted on the VMS. Table 5.1 illustrates how the rerouting phenomenon is modelled with ICM in case of full information. The results along the trajectory are examined. At the 1st decision point there is no information available, so the whole flow remains unaware $\hat{q}$. During propagation from 1st to 2nd decision point ($7^{\underline{00}}$ to $7^{\underline{10}}$), 3% of the drivers become aware of the event by checking the on-line sources, yet none of them decides to reroute as the utility remains low. At $7^{\underline{17}}$ the information about the event is broadcasted on the radio, with a penetration rate $P^{NEWS} = 30\%$. The drivers were propagating between the 3rd and 4th decision point at that time, thus the aware flow a significantly increases at 4th decision point, due to (5.4). At $7^{\underline{20}}$ the event starts impacting the whole network, causing the total delay up to the 4% at $8^{\underline{20}}$ ($M(8^{20}) = 1.04$). So that the on-line information is spread faster and reaches the bigger audience, as the default penetration rate $P^{ON-LINE} = 30\%$ is magnified due to (5.14). Between decision points 6th and 7th, 80% of the drivers notice the VMS informing about the event (5.6). Moreover, the actual travel time of that arc is 50% longer than typical, so that drives become aware also due to observation (5.7). Thanks to multiple sources of awareness ι reaches 91% at 7th decision point, due to (5.2).

The rerouted flow $\tilde{\iota}$ remains null up to 5th decision point, although the 40% of the flow is already aware up to that time. Due to low utility in the compliance model (5.20) drivers do not reroute prior the 5th decision point. The reason for this was both lack of reasonable alternatives at decision points 1st to 4th and the actual demand pattern $\tilde{\iota}$ being still in-line with the typical $\hat{p}$.

The actual and typical demand patterns stars being different from 5th decision point on, where the alternative routes (see Figure 5.9) are found. At the 5th decision point the compliance reaches 28%, both because of $\Delta p_5^D(07^{25})$ and $\Delta w_5^D(07^{25})$. The potential gains, Δp computed with (5.15), as well as the potential losses, Δw computed with (5.18), are significant at 5th, 6th and 7th decision points, leading to a high compliance κ. The potential losses Δw are highest at the last decision point, which is the last opportunity to avoid the blockage caused by the event. To avoid the infinite value of Δw at this decision point the upper bound of 4 is used as the $\max(\Delta w)$ (5.19).

For the above setting 95% of the drivers became aware and 93% has actually rerouted. Around 10 drivers remained unaware and propagated into the event, one aware driver decided to ride into the blocked arc. The rerouting decisions were made at 5th, 6th and 7th decision points. Mind that the propagation of the rerouted flows in not shown in the table as they will make new route choices based on the actual costs and possibly use different route. It is, however, possible that the actual and typical demand patterns overlap partially, so that the rerouted drivers will take the same path like the not rerouting for some downstream arcs, yet the compliance model is designed to model rerouting at the decision points where the demand patterns diverge.

5.1.6.2. *Information available at the origin*

To demonstrate the functionality of the ICM in the following examples the possible variations of the above case are presented. Let's first analyze the same trajectory, yet traversed an hour later, when information is available on-line before the departure shown in Table 5.2 In this case roughly $P^{ON-LINE} = 31\%$ of drivers are aware already at the origin. Since the compliance values did not change compared to the Table 5.1, the rerouting will take

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place only at decision points 5th to 7th. The drivers do not become aware from on-line sources anymore, as the spreading process is over. But the information is still provided by the radio news at $8^{\underline{17}}$ and through the VMS prior the 7th decision point. The results are somehow similar, yet the flows become aware earlier and reroute earlier, i.e. at 5th decision point 14% of drivers decide to reroute instead of 11%.

Table 5.2 ICM results at 7 consecutive decision points traversed during propagation. The event is already known by 30% of drivers at the departure time $8^{\underline{00}}$. Information broadcasted at the radio, on-line and at the VMS.

Decision point		Flow propagated forward			Awareness model							Compliance model		
i	τ	$\hat{q}$	a	$\tilde{\iota}$	ι	$t_a^{NEWS}(\tau)$	$t_a^{VMS}(\tau)$	$t_a^{O-L}(\tau)$	$M(\tau)$	$t_a^{O}(\tau)$	Δt_a	κ	Δp	Δw
1	8:00	69	31	**0**	**31%**	-	-	31%	1.04	-	-	-	0.00	1.00
2	8:10	69	31	**0**	**-**	-	-	-	1.04	-	-	**1%**	0.01	1.01
3	8:15	69	31	**0**	**-**	-	-	-	1.04	-	-	**1%**	0.01	1.00
4	8:20	48	51	**0**	**30%**	30%	-	-	1.04	-	-	**0%**	0.00	1.00
5	8:25	48	37	**14**	**-**	-	-	-	1.04	-	-	**28%**	0.61	1.47
6	8:40	48	8	**29**	**-**	-	-	-	1.04	-	-	**78%**	0.76	2.65
7	9:20	5	1	**51**	**91%**	-	80%	-	1.04	50%	0.5	**98%**	1.00	4.00

5.1.6.3. *VMS placed at the initial part of the route*

Table 5.3 shows the impact of placing the VMS between the 1st and 2nd decision point. The first example is now modified so that 80% of drivers become aware by the VMS. In this case some drivers decide to reroute early (at 2nd and 3rd decision point), yet most of them reroutes later, interestingly the rerouting flow is now greater at the 6th , not 7th decision point.

Table 5.3 ICM results at 7 consecutive decision points traversed during propagation. Information broadcasted at the radio, on-line and at the VMS placed at the initial part of the route.

Decision point		Flow propagated forward			Awareness model							Compliance model		
i	τ	$\hat{q}$	a	$\tilde{\iota}$	ι	$t_a^{NEWS}(\tau)$	$t_a^{VMS}(\tau)$	$t_a^{O-L}(\tau)$	$M(\tau)$	$t_a^{O}(\tau)$	Δt_a	κ	Δp	Δw
1	7:00	100	0	**0**	**0%**	-	-	0%	1	-	-	-	0.00	1.00
2	7:10	19	80	**1**	**81%**	-	80%	3%	1	-	-	**1%**	0.01	1.01
3	7:15	19	80	**1**	**4%**	-	-	3%	1	-	-	**1%**	0.01	1.00
4	7:20	12	86	**0**	**33%**	30%	-	4%	1.01	-	-	**0%**	0.00	1.00
5	7:25	12	62	**24**	**4%**	-	-	4%	1.01	-	-	**28%**	0.61	1.47
6	7:40	11	14	**49**	**10%**	-	-	10%	1.02	-	-	**78%**	0.76	2.65
7	8:20	5	0	**20**	**53%**	-	-	6%	1.04	50%	0.5	**98%**	1.00	4.00

5.1.6.4. *No information sources*

Table 5.4 present the case when no other information sources are available. The information about the event is not available on-line, is not broadcasted on the radio and is not shown at VMS. Now the only source of awareness is the experienced delay Δt_a , which is now greater as the event causes more impact (it is unknown, so the drivers do not avoid it). Drivers remain unaware until they observe the consequences between 5th and 6th decision point. Due to the lack of awareness, no one reroutes at the 5th decision point event though the κ is reasonably high. The rerouting takes place when the drivers experience the delay of 30% and 40% respectively, which leads to rerouting at 6th and 7th decision points. Yet still 42% of the flow remains unaware (did not guess from the atypical delay) and drives into the event.

Table 5.4 ICM results at 7 consecutive decision points traversed during propagation. No information about the event is available; drivers can only observe the atypical delay.

Decision point		Flow propagated forward			Awareness model							Compliance model		
i	τ	$\hat{q}$	a	$\tilde{\iota}$	ι	$\iota_a^{NEWS}(\tau)$	$\iota_a^{VMS}(\tau)$	$\iota_a^{O-L}(\tau)$	$M(\tau)$	$\iota_a^{O}(\tau)$	Δt_a	κ	Δp	Δw
1	7:00	100	0	**0**	**-**	-	-	-	1	-	1	-	0.00	1.00
2	7:10	100	0	**0**	**-**	-	-	-	1	-	1	**1%**	0.01	1.01
3	7:15	100	0	**0**	**-**	-	-	-	1	-	1	**1%**	0.01	1.00
4	7:20	100	0	**0**	**-**	-	-	-	1.01	-	1	**0%**	0.00	1.00
5	7:25	100	0	**0**	**-**	-	-	-	1.03	-	1	**28%**	0.61	1.47
6	7:40	70	7	**23**	**30%**	-	-	-	1.05	30%	1.3	**78%**	0.76	2.65
7	8:20	42	1	**34**	**40%**	-	-	-	1.10	40%	1.4	**98%**	1.00	4.00

5.1.6.5. *Event with no impact*

Let's assume the theoretical situation where an event is broadly communicated, yet it has no impact on the travel costs and times. For example there was an accident, yet it blocked only one out of three lanes and the capacity is enough to carry all the flows without additional delays. The base example is now modified so that the utility is null for all the decision points. The drivers become aware almost exactly like in the base example (the information spreading is the same, only it is not magnified with *M).* So that at the 7th decision point 95% of the drivers are aware, yet no one reroutes as there is no rerouting utility, the actual costs are exactly like the typical ones.

Table 5.5 ICM results at 7 consecutive decision points traversed during propagation. Information broadcasted at the radio, on-line and at the VMS. Event do not causes a delay, the utility of rerouting is null, no rerouting is modelled.

Decision point		Flow propagated forward			Awareness model							Compliance model		
i	τ	$\hat{q}$	a	$\tilde{\iota}$	ι	$\iota_a^{NEWS}(\tau)$	$\iota_a^{VMS}(\tau)$	$\iota_a^{O-L}(\tau)$	$M(\tau)$	$\iota_a^{O}(\tau)$	Δt_a	κ	Δp	Δw
1	7:00	100	0	**0**	**0%**	-	-	-	1	-	1	-	0.00	1.00
2	7:10	97	3	**0**	**3%**	-	-	3%	1	-	1	-	0.00	1.00
3	7:15	93	7	**0**	**3%**	-	-	3%	1	-	1	-	0.00	1.00
4	7:20	62	38	**0**	**33%**	30%	-	4%	1	-	1	-	0.00	1.00
5	7:25	60	40	**0**	**4%**	-	-	4%	1	-	1	-	0.00	1.00
6	7:40	54	46	**0**	**10%**	-	-	10%	1	-	1	-	0.00	1.00
7	8:20	5	95	**0**	**81%**	-	80%	5%	1	-	1	-	0.00	1.00

5.1.6.6. *Lost opportunity to reroute*

The ICM do not allow rerouting for the unaware drivers, i.e. the transition probability between unaware and rerouted flow is by definition null. To demonstrate this, the network of the base case is modified so that the only possible alternative to reroute is at the 2nd decision point, on points 3rd to 7th there are no rerouting alternatives. However while making rerouting decision at 2nd decision point at 7^{10} the event is known only by 3% of drivers, 99% of which will reroute due to very high utility. The remainder will make a rerouting decision, yet will not be able to actually reroute as there are no rerouting opportunities. Drivers need to drive straight to the event, as the rerouting opportunity is lost. The results are shown in Table 5.6.

Table 5.6 ICM results at 7 consecutive decision points traversed during propagation. Information broadcasted at the radio, on-line and at the VMS. Yet the last opportunity to reroute is gone and the rerouting drivers do not have any alternative route to reroute.

Decision point		Flow propagated forward			Awareness model							Compliance model		
i	τ	$\hat{q}$	a	$\tilde{\iota}$	ι	$\iota_a^{NEWS}(\tau)$	$\iota_a^{VMS}(\tau)$	$\iota_a^{O-L}(\tau)$	$M(\tau)$	$\iota_a^{O}(\tau)$	Δt_a	κ	Δp	Δw
1	7:00	100	0	**0**	**0%**	0%	0%	0%	1	0%	1	**0%**	0.00	1.00
2	7:10	97	3	**3**	**3%**	0%	0%	3%	1	0%	1	**99%**	1.00	4.00
3	7:15	84	13	**0**	**13%**	0%	0%	4%	1.01	10%	1.1	**0%**	0.00	4.00
4	7:20	45	52	**0**	**47%**	30%	0%	4%	1.04	20%	1.2	**0%**	0.00	4.00
5	7:25	30	77	**0**	**34%**	0%	0%	5%	1.1	30%	1.3	**0%**	0.00	4.00
6	7:40	16	81	**0**	**47%**	0%	0%	12%	1.15	40%	1.4	**0%**	0.00	4.00
7	8:20	1	96	**0**	**91%**	0%	80%	10%	1.3	50%	1.5	**0%**	0.00	4.00

5.1.7. Summary of the Information Comply Model

Information Comply Model originates from the thorough definition of the drivers behavior and their decision process. Thanks to this, it provides a valuable, meaningful results in-line with what can be observed (see chapter 6). Dividing the decision process into two steps: becoming aware and deciding to reroute, allows handling two processes separately. Thanks to this the number of information sources available in contemporary agglomerations can be handled consistently. Impact of VMS, online sources, radio broadcasts can be handled jointly and further extended with drivers guessing about atypical situation from an atypical delay. So that the number of aware drivers (knowing about the event) can be consistently estimated as a function of space and time, which is crucial for the ICM model, as only aware drivers will reroute. Their rerouting decision is driven by the utility they see in rerouting. Utility is computed based on two factors: possible travel time gains ("How much faster can I get to destination if I reroute?"), possible travel time losses ("How much longer will I go to the destination if I do not reroute?"). With such approach the rerouting is modelled only at places where it actually has a big utility (travel time differences) through a compliance model, being a binomial logit model with two possible decisions: reroute or not. To make the machinery consistent the two models are applied in the Network Flow Propagation model which now handles three states of flow: unaware, aware and rerouted. The transitions between the states are handled with the awareness and compliance models forming the Markov process. Such extended NFP is integrated with the DTA algorithm – ICM DTA, so that it provides the number of rerouting drivers for each time and space in the network and includes them in the state-of-the-network. Which is the scope of the thesis.

5.2. Rolling Horizon DTA

The second, alternative, solution for the problem of the thesis is the Rolling Horizon DTA model described in this chapter. ICM model, proposed in the previous chapter, provides a comprehensive, thorough representation of the phenomena, yet it cannot be applied for real-time environment where number of unexpected events take place in real-time and are unknown both to drivers and to traffic managers. That's the main motivation to propose a Rolling Horizon DTA (RH-DTA), where the behavioral representation of the rerouting phenomena is simplified, yet still present.

Rolling horizon (RH) sequence is a common decision making practice for stochastic dynamic environment. Every new roll of a horizon updates actual and expected realization of a stochastic process that was unknown before. So that the previous decisions can be revised with new information. By analogy, it is assumed that drivers update their routing decisions whenever they receive a new information, i.e. about an unexpected traffic event. In this chapter, a RH-DTA is proposed as a modified DTA problem that can be applied sequentially in RH. Each new horizon comes with a new DTA problem, starting with a traffic flows from a previous horizon, utilizing a new information to update the routing decisions and propagate the flows up to the new horizon when the sequence is repeated. The general RH sequence of the consecutive DTA problems is proposed as a flexible starting point to be extended to handle various use-cases: immediate and delayed information, informed and uninformed drivers, single and multiple events; as shown in the chapter.

5.2.1. Rolling Horizon framework

Rolling Horizon (RH) framework is commonly used in a decision-making theory for dynamic stochastic environment. Typically, a decision made in the stochastic environment is a solution of an optimization problem for a given estimate of the stochastic process based on the actual data and its expected evolution (forecast). In RH, the decision is revised sequentially with every *roll* of the *horizon* (Sethi, et al., 1991). New horizon provides an updated estimate of an unknown stochastic process based on the actual data. Modifying the initial conditions of the problem can lead to a different solution if the actual realization was not expected before. Instead of seeking for a single global optimum, the dynamic problem in RH is solved sequentially and local optima are found for each horizon (subject to currently available information). This way the decision made in stochastic environment can be adapted to the actual realization of the stochastic process to find a new (possibly better) optimum as it is cyclically adapted to the actual data. The final decision is a concatenation of the decisions made in the successive horizons.

In order to apply DTA in RH, drivers are treated as a decision makers updating their route-choice decisions en-route every time they receive a new information about actual travel costs. They are making decisions in a stochastic environment starting from the expected (typical) costs that can evolve far from initial expectations (i.e. due to events). Decisions are updated whenever the new information is available, if current routing decision based on the expected cost becomes inefficient it is updated to re-optimize the decision. Notably, drivers cannot utilize the actual costs of the network prior they know it, i.e. routing decision cannot be changed without receiving information. This way instead of solving the single global DTA problem for the expected travel costs a sequence of DTA problems is solved, one for each horizon, when the actual update of travel costs is available.

In the RH-DTA, the flows are propagated from the origin to the destination with several successive routing decisions, which can change with every roll of the horizon. Even though decisions along the route are discontinuous the consistency of the final solution is granted at the state-of-the-network level, i.e. temporal profiles of traffic flows and travel times are continuous and the demand is conserved. End of the previous horizon is the starting point for a new horizon, thanks to this, the continuity of state is guaranteed and results of successive horizons can be concatenated to arc flows consistent with *od* demand. To complete the picture two user-classes are introduced with respect to en-route behavior. Drivers who do not receive information, has made their decisions pre-trip based on their expectations and will not change it while travelling. They are a static background for the RH model. More important for the RH model are the rerouting drivers who will update their decisions with every new *roll* of the *horizon.* Multiclass version of RH that handles the two user-classes simultaneously is proposed in the thesis.

5.2.2. Rolling Horizon DTA model

To integrate the RH concept with DTA, the demand model (4.3) is extended with the concept of a hybrid-route choice model, described in detail in the chapter 4.3 originating from concepts of (Peeta and Mahmassani, 1995), (Pel, et. al., 2009). The hybrid route-choice model of (4.21) applied to NFP in (4.22) is capable to represent the adaptation process, where the initial demand pattern **p** is adjusted en-route to the new demand pattern $\mathbf{p}^{\varepsilon}$ when information about traffic event becomes available. Which is the phenomenon that is aimed to be covered in this thesis.

The straightforward way to implement the hybrid model inside DTA (schema 2) would be to simply replace the standard NFP formulas with a hybrid-modification of (4.21). However, due to efficiency and consistency of results, different approach is proposed here. Namely, the hybrid route-choice is applied through a sequence of DTA problems solved one after the another, further referred to as RH-DTA. The reason for that is not only the easier implementation, but most importantly handling the inconsistency between travel times at the roll-time. Hybrid route-choice (4.22) model works fine only when the shortest tree travel times do not change after changing the perspective, i.e. $h_i^d(\theta) = h_i^{d|\varepsilon}(\theta)$, which unfortunately cannot be guaranteed in the real-time environment when the expected travel times are never realized accurately. Particularly in the case when information about the event comes delayed, the horizon rolls forward at the time when an unexpected event has already impacted the travel times. This can lead to inconsistency of traffic flows between consecutive horizons, which is overcame in RH-DTA through memorizing the number of vehicles, as shown below.

A generic horizon is further denoted with ε, defined through its starting at time H_ε and identified with the event ε which changes the perspective. It finishes when the new horizon $\varepsilon+1$ arrives at time $H_{\varepsilon+1}$. Horizon is associated with a new information that it provides, usually consumed algorithmically through updating the network characteristics to $\boldsymbol{\delta}^{\varepsilon}$. The network $\boldsymbol{\delta}^{\varepsilon}$ includes the event and the way it changes the network parameters, i.e. reduces capacity, speed, number of lanes, or block the arcs. The horizon changes the expected costs form **c** to $\mathbf{c}^{\varepsilon}$ which in turn yield the new demand pattern $\mathbf{p}^{\varepsilon}$ (4.10).

Elemental component RH-DTA is the modified DTA problem of (4.2) computed for a single horizon and corresponding perspective of costs $\mathbf{c}^{\varepsilon}$ (5.32). Apart from calculating RCM consistent with the perspective of current horizon (4.21), additional input and output **Q** is introduced. **Q** is the number of the vehicles present in the network at the roll time H_ε. **Q** can be seen as the snapshot of the traffic flows in the network taken at the roll time instant and stored at the two resolutions: total flows obtained through (5.33) at the supply level (GLTM traffic flow model) and flows per destination obtained with (5.34) at the demand level (NFP). At the equilibrium the two coincide, i.e. the demand flows per destination sums up to the flows of the supply model. Demand conservation proved for the hybrid NFP (4.22) in Lemma of the chapter 4.3 is conserved in RH-DTA through the number of vehicles per destination

$Q_a^{d|\varepsilon}$ guaranteeing consistency between two successive DTA problems of RH-DTA. (5.34) stores the total number of vehicles that is then loaded back on the network through (5.35).

$$\{\mathbf{c},\mathbf{t},\mathbf{q},\mathbf{Q}^{\varepsilon}\} = DTA^{\varepsilon}(\boldsymbol{\delta}^{\varepsilon},\mathbf{d}^{\varepsilon},\mathbf{Q}^{\varepsilon-1}) \tag{5.32}$$

$$Q_a^{\varepsilon}(\tau) = \int_{H_\varepsilon - t_a(\tau)}^{H_\varepsilon} q_a(\theta)d\theta \tag{5.33}$$

$$Q_a^{d|\varepsilon} = \sum_{\theta\in T}\begin{cases} q_a^d(\theta) & \text{if } h_{a^-}^d(\theta) < H_\varepsilon \le h_{a^+}^d(\theta) \\ 0 & \text{otherwise}\end{cases} \tag{5.34}$$

$$q_a^{d|\varepsilon}(\theta) = Q_a^{d|\varepsilon}\frac{\left(h_{a^+}^d(\theta) - h_{a^-}^d(\theta)\right)}{t_a\left(h_{a^-}^d(\theta)\right)} \tag{5.35}$$

The fundamental sequence of RH-DTA (5.36) is the sequence of the two DTA problems applied one after another. First, for the typical perspective and stored through number of vehicles **Q** at the roll time. **Q** is then used as the input for the second DTA problem calculated for a new perspective ε, where memorized flows **Q** are used. Hybrid route-choice model is applied by using the memorized vehicles **Q** and using them as the demand flows **d** in (4.22). In fact each of nodes for which the flows **Q** were saved starts playing as the origin (5.35) for the second DTA problem.

The result of RH-DTA is a concatenation of the states-of-the-network calculated with the first DTA before H_ε and with the second after (5.36). The Schema 8 depicts DTA for the rolling-horizon. It differs from Schema 2 with a new input and output **Q** as well as with constrained demand $\mathbf{d}^{\mathbf{d}|\boldsymbol{\varepsilon}}$ cutting the demand flows prior the roll time H_ε (5.37). On the algorithm schema the blue boxes denote an additional input from the previous stages and red boxes denote output for the next stage. (5.36) can be applied recursively for a multiple horizons so that each new horizon brings a new DTA computed starting from the results of the previous DTA (5.32). The newly computed DTA overwrites the resulting state-of-the-network only for $\tau > H_\varepsilon$, which can be further overwritten for $\tau > H_{\varepsilon+1}$ when new horizon ε+1 arrives. In general, the DTA calculated for a given perspective is valid during the validity of the horizon of this perspective and is overwritten when a new horizon arrives.

$$\{\mathbf{c},\mathbf{t},\mathbf{q}\} = \begin{cases} DTA(\boldsymbol{\delta},\mathbf{d}) & \text{for } \tau \le H_\varepsilon \\ DTA^{\varepsilon}(\boldsymbol{\delta}^{\varepsilon},\mathbf{d}^{\varepsilon},\mathbf{Q}) & \text{for } \tau > H_\varepsilon \end{cases} \tag{5.36}$$

$$d_i^{d|\varepsilon}(\theta) = \begin{cases} d_i^d(\theta) & \text{for } \theta : h_i^d(\theta) > H_\varepsilon \\ 0 & \text{otherwise}\end{cases} \tag{5.37}$$

The Schema 8 is the central element of RH-DTA that formally allows to solve number of real-time cases in traffic network. Below illustrative examples shows how it can be applied for the practical situations and extended when needed.

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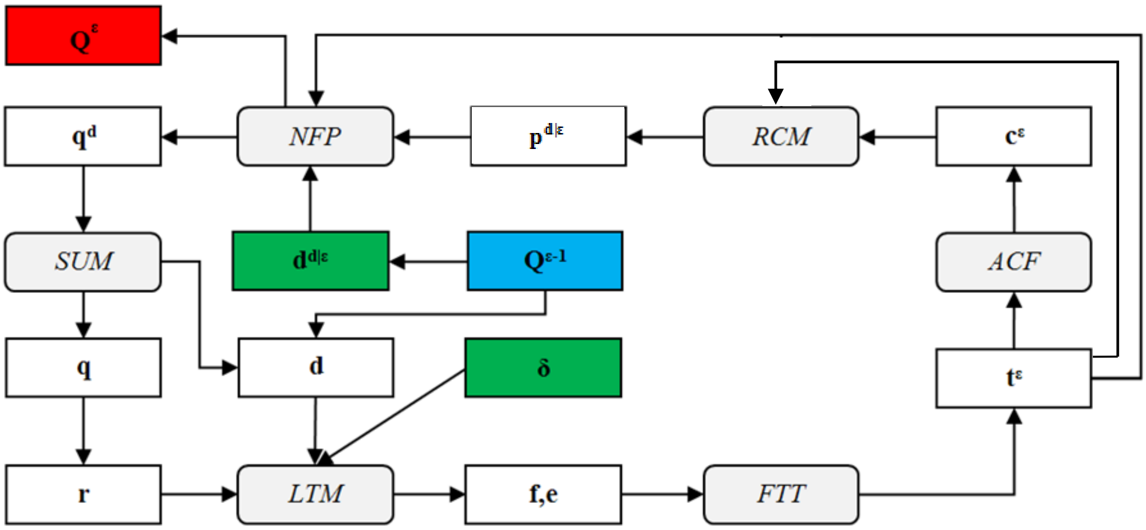

Schema 8 DTA for Rolling Horizon.

5.2.3. Illustrative examples

To refer the above theoretical considerations to the practical application number of possible use-cases for RH-DTA are shown below. The use-cases are illustrated with the results obtained on the illustrative toy-network. The RH was implemented by author in the Traffic Real-time Equilibrium (TRE) software by SISTeMA s.r.l. and was tested on the multiple real-size networks; however, in this paper objective is to point the phenomena and possible use-cases, which are evident on the proposed toy-network. The toy network (Figure 5.10) was designed for a single OD pair with the two connections: lower - fast and efficient, and upper - alternative used only when the lower connection is not efficient anymore, i.e. it is blocked by the unexpected event. 'Escape-arcs' between two routes are inefficient to travel from *O* to *D*, they can be useful only while making en-route rerouting to avoid delay on the lower route. The assignment lasts from $2^{\underline{00}}$ to $3^{\underline{00}}$ and the demand of 200 vehicles is propagated from the origin to the destination, the event happens at $02^{\underline{30}}$ at the lower route (× points the location) and causes drop of capacity to 10 veh/h (from 2000 veh/h).

Results of the RH DTA are presented through the snapshot of flows at the whole network and traffic flow profiles of the three selected arcs. Snapshot of traffic flows on the whole network is shown at the specific time instant – five minutes past the event (02:35), the thickness of the bars is proportional to number of vehicles. The temporal flow profiles is showed for the three arcs:

- #1 arc towards the event;
- #2 'escape arc';
- #3 'alternative arc'.

Additionally, the queue length at the event arc (**X**) is showed.

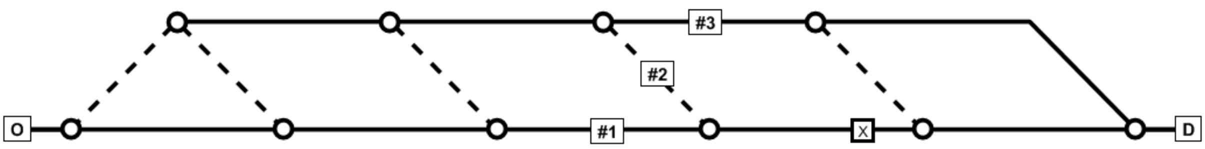

Figure 5.10 The toy network.

5.2.3.1. *Basic cases: no event, unknown event, known event*

The first three cases are provided as a background to highlight the rerouting phenomena.
In the case when no unexpected event takes place (at the equilibrium), only lower route is utilized in route-choice, alternative and escape arcs are empty and the flow on lower route is stable (200 veh/h), which is depicted at Figure 5.11 and Figure 5.12.

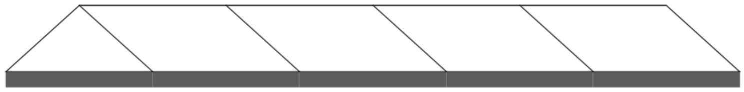

Figure 5.11 Flows (thickness of the bars) on the toy-network at $02^{\underline{35}}$.

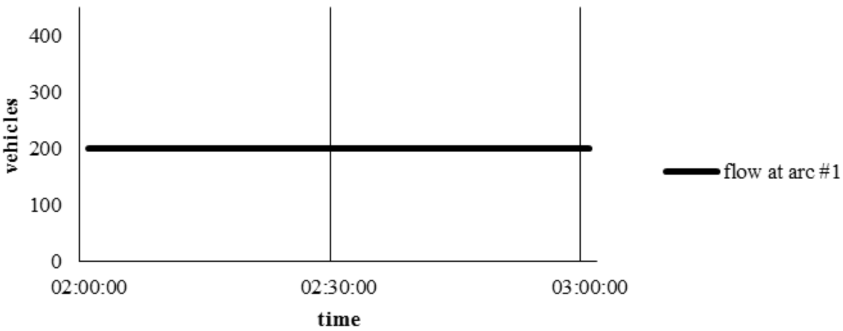

Figure 5.12 Temporal profiles of flow and queue at the toy network – typical case.

In the case of unexpected event, which is unknown and no information is provided; the demand pattern remains typical, as a new perspective is unavailable. *RH* does not need to be applied in this case. The flows are propagated only along the lower route; alternative and escape arcs are empty. Due to capacity drop the queue builds up at the place of the event, the outflow from the event arc is reduced to 10 veh/h. Formally the case of no information is solved with DNL problem (as defined in 4.1.4). This way the flows are propagated using a the typical demand pattern **p**, computed with a typical DTA (Schema 2), but on the network with the event $\boldsymbol{\delta}^{\varepsilon}$. Rerouting phenomena is not observed as the information about the new perspective is not available, yet the impact of the event is modelled adequately. Figure 5.14 shows the observed temporal profiles of flows and time of the event (red vertical line) without pointing the communication time as the event is not communicated at all, so that the queue builds up at the event arc.

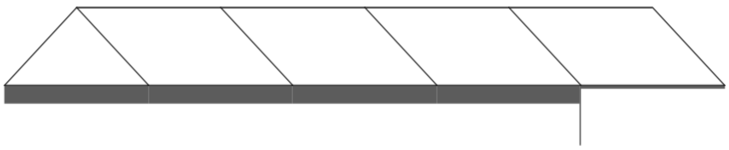

Figure 5.13 Flows (thickness of the bars) and queue (vertical line) on the toy-network at $02^{\underline{35}}$.

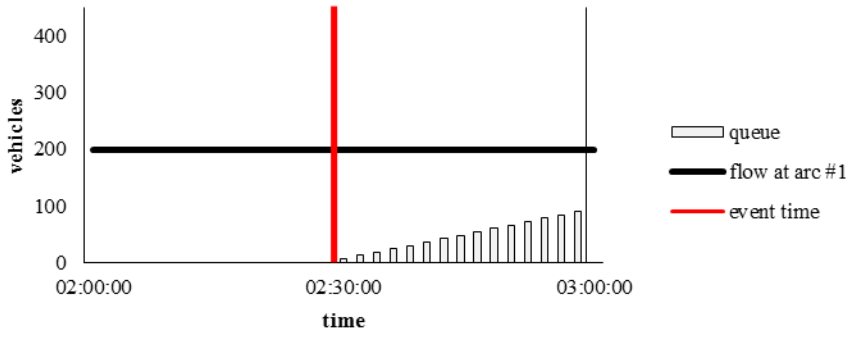

Figure 5.14 Temporal profiles of flow and queue at the toy network – case of unknown event.

If the event is known before, i.e. it is planned and communicated well in advance so that $i_\varepsilon < t_\varepsilon$, the single DTA (Schema 2) can be applied for the new perspective of costs including the known event (5.32). The lower route remains efficient up to the time when the event becomes active. The result is the rerouting flows, but only at origin and switching to the alternative route to avoid the predicted consequences of the simulated event before it took place. The queue do not form at all, as drivers had enough time to avoid the event (Figure 5.16) thanks to communicating event (blue line) early enough to change route before departing from the origin. The two above cases show the solutions that are available in state-of-the-practice DTA solutions, both of them providing unrealistic results that either overestimate, or neglect the effect of the event.

Figure 5.15 Flows (thickness of the bars) on the toy-network at $02^{\underline{35}}$. Case of fully known event.

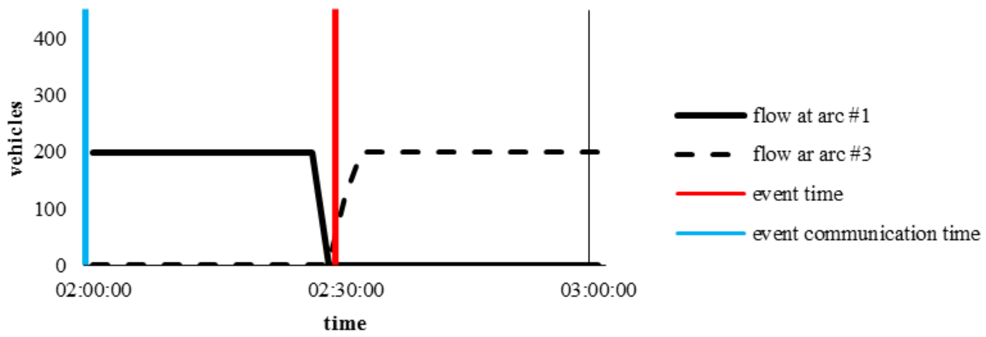

Figure 5.16 Temporal profiles of flow and queue at the toy network – case of fully known event.

5.2.3.2. *Single event with an immediate information available for everyone*

The simplest use-case of RH-DTA is a single event taking place in the network at time t_ε and every driver in the network gets instantly informed about it at time $i_\varepsilon = t_\varepsilon$. The event changes the network characteristics from $\boldsymbol{\delta}$ to $\boldsymbol{\delta}^\varepsilon$ from time instant i_ε on, by reducing the capacity at the event arc. The event is unexpected, so is the change in the network characteristics and corresponding costs. Consequently, the new demand pattern $\mathbf{p}^\varepsilon$ is unavailable for drivers before the event is known. But drivers immediately know the new costs $\mathbf{c}^{\varepsilon}$ resulting from the event. The horizon of RH-DTA can be rolled at $i_{\varepsilon}=t_{\varepsilon}$ and RH-DTA defined with (5.36) can be applied directly.

The sequence (Table 5.7) shows the consecutive steps of computations that need to be executed to obtain the final results. It starts with a DTA computation at which drivers are unaware of incoming atypical situation, resulting in the typical state-of-the-network calculated before the event. The input for the first DTA is the typical network and the typical demand pattern. Since it is the first stage, no vehicles **Q** were saved yet. Drivers change perspective of costs used for routing when they start being aware about the event i_{ε}. According to the hybrid demand pattern (4.22), they will be propagated with new demand pattern $\mathbf{p}^{\varepsilon}$. To this end, a new DTA problem is solved with the updated network $\boldsymbol{\delta}^{\varepsilon}$ and additional input **Q** pre-calculated in the first DTA**.** Such sequence results in the flows propagated with the typical demand pattern up to the time when event becomes known and then propagated with an updated (actual) demand pattern taking into account the event.

Table 5.7 RH-DTA sequence for a single event communicated immediately with input, output and results validity.

			validity of the results	
			time	t_{ε}
procedures sequence	input	output	before the event	after the event
1. typical DTA	**δ, d**	**Q**	**valid**	overwritten
2. DTA for perspective ε	**Q, δ**$^{\varepsilon}$	-	not calculated	**valid**

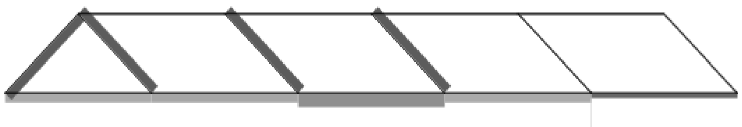

Figure 5.17 Flows (thickness of the bars) on the toy-network at 02[35]. Case of single event with immediate information available for everyone.

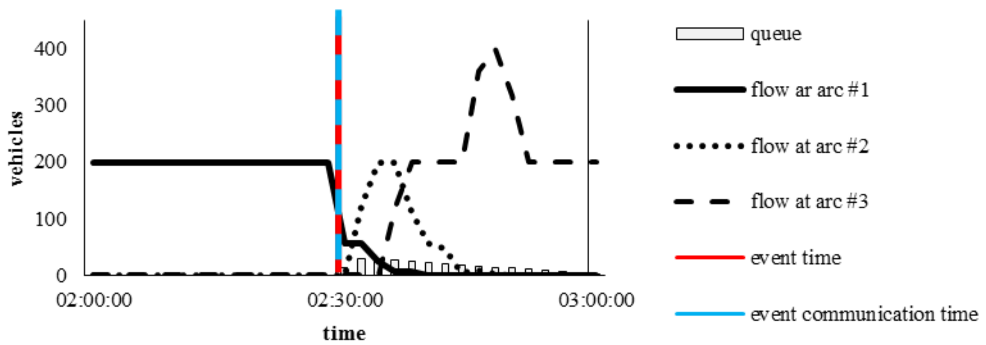

Figure 5.18 Temporal profiles of flow and queue at the toy network – case of single event with immediate information available for everyone.

The above sequence executed on the toy network resulted in the traffic flows depicted at Figure 5.18 where the drivers follow a typical demand pattern throughout the validity of the first horizon ($\tau<i_{\varepsilon}$) and are free to reroute afterwards. They escape the lower-route through an escape arc *(#2*) which has a positive flow only for a short period as the upper-route can be chosen from the origin for the flows departing at $\tau>i_{\varepsilon}$. Interestingly, for the short period the

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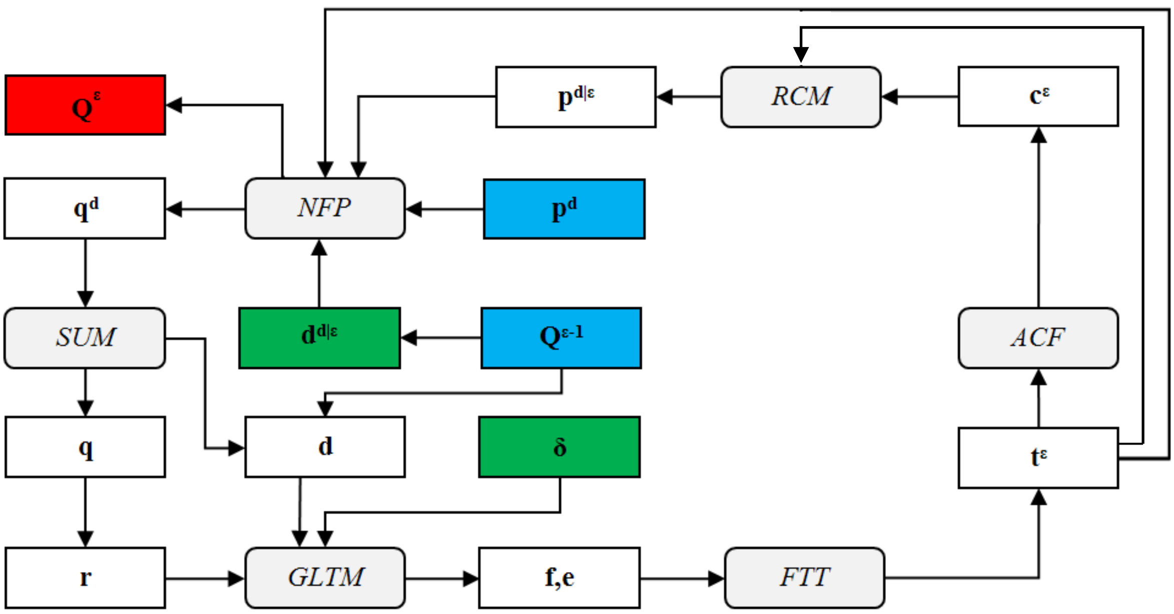
flow at the alternative is 400 veh/h although the total demand flow is only 200 veh/h, this is due to travel time variation and different timing at which drivers arrive at alternative route from escape-arcs, leading to temporal cumulation of flows. The small queue builds up even though all drivers have received immediate information, this is because some of them were already at the affected arc and has lost the opportunity to reroute.

5.2.3.3. *Single event with an immediate information available only for some drivers.*

The above use-case becomes more realistic by assuming that not everyone has access to information and only given share of drivers s_ε will know about the event ε (such heterogeneity was proposed, among others, by Watling and Hazelton, 2003). If so, only a part of flow will be able to switch to a new perspective after the event. Consequently, the flows will propagate according to the typical demand pattern up to time t_ε and then only a given share s_ε of the flow will propagate according to a new perspective $\mathbf{p}^\varepsilon$, while the remaining part $(1-s_\varepsilon)$ will continue propagation with the typical demand pattern **p.** Such modification yields two separate NFP procedures. When propagation starts again at time i_ε with saved flows **Q** the initial demand needs to be split into two classes with a given share s_ε. Each of the classes is handled independently according to the respective demand pattern. The unaware part of flow is propagated with the typical demand pattern $p_a^d(\theta)$ throughout the whole simulation period, while the aware part of flow is propagated with a hybrid route-choice model that rolls to the new perspective at time i_ε. To cover this case the sequence from Table 5.7 is extended. The demand pattern **p** obtained in first DTA problem is memorized and then used in second stage as shown in Table 5.8. The second stage of the rolling horizon is depicted on Schema 9, which extends Schema 8 with the additional input of NFP so that it can be executed for the two user-classes simultaneously.



Schema 9 DTA for rolling horizon with two user classes: informed and uninformed.

Table 5.8 Multiclass RH-DTA sequence for a single event communicated immediately with input, output and validity for the results.

Procedure	input	output	validity of the results time before the event	t_ε after the event
1. typical DTA	**δ, d**	**Q, p**	**valid**	overwritten

2. DTA for perspective ε	**p, Q, δ^ε^**	-	not calculated	**valid**

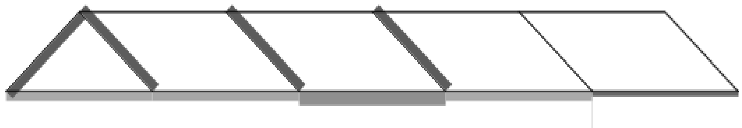


Figure 5.19 Flows (thickness of the bars) on the toy-network at 02[35]. Case of single event with immediate information available for some drivers.

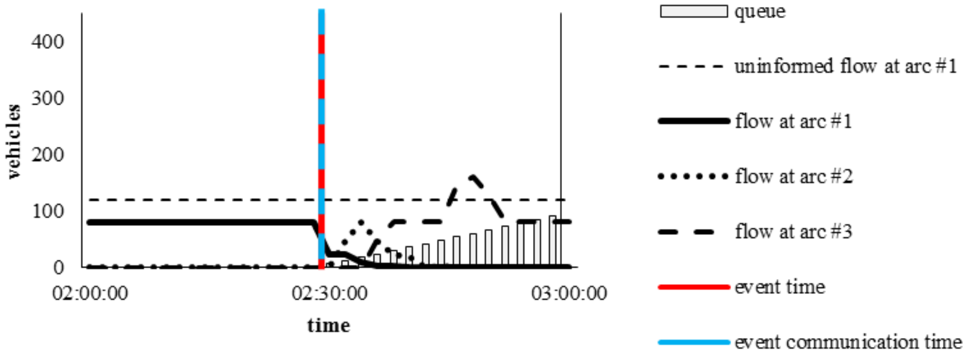


Figure 5.20 Temporal profiles of flow and queue at the toy network – case of single event with immediate information available for some drivers.

In the above case the total demand of 250 vehicles is propagated and 100 of them have access to the information. The uninformed drivers (grey dashed profile) act as if the situation was typical and do not reroute, they propagate straight towards the event and form the queue. The informed share of drivers act like in the previous example: reroute whenever they receive the information (Figure 5.19).

5.2.3.4. *Single event with multiple user classes*

The above schema of the two user-classes can be generalized for any number of user-classes. Let us define set of user-classes **U** through shares s_u such that $\sum_{u \in \mathbf{U}} s_u = 1$. Each class associated with different perspective of costs $\mathbf{c^u}$. The NFP can be further extended so that each class rolls to the new horizon at the given time i_u. At this time a given share of the flow switches to the updated demand pattern $p_a^{d|u}(\theta)$ (5.38). This way multiple classes can be handled simultaneously, i.e. classes can have various time at which they receive the information. Such sequence can model situation when number of multiple information sources are available and each of them has different information time about the event. Unfortunately, computation time of the demand model is linear with respect to the number of classes, so that introducing multiple demand classes increases computation time and can make the whole solution unsuitable for the real-time applications.

$$q_a^{d|u}(\theta) = s_u \left(d_{a^-}^d(\theta) + q_{a^-}^{d|u}(\theta) \right) \cdot \begin{cases} p_a^d(\theta) & \text{if } h_{a^-}^d(\theta) < i_u \\ p_a^{d|u}(\theta) & \text{if } h_{a^-}^d(\theta) \geq i_u \end{cases} \tag{5.38}$$

Apart from informed and uninformed users, it can be beneficial to include pre-informed users, which would allow handling the events that can be known in advance (i.e. planned road works, demonstrations, road-closures, etc.). Users of this class are free to reroute from the origin. Interestingly, they will make their route choices subject to not only to the event but also to expected rerouting of other users. Modelling the three user-classes simultaneously can be done by introducing the single event at time t_ε but two different communication times i_ε, yielding two horizons: earlier for the pre-informed users and later for others.

5.2.3.5. *Single event with a delayed information available for everyone*

In practice, the above use-cases shall be further extended as the information usually comes with some delay, so that $i_\varepsilon \geq t_\varepsilon$. Over the period $\tau \in (t_\varepsilon; i_\varepsilon)$ the event ε is active but still unknown. It impacts the supply side by changing the network from $\boldsymbol{\delta}$ to $\boldsymbol{\delta}^\varepsilon$, yet the new perspective $\mathbf{p}^\varepsilon$ is still unavailable, as the information did not reach the drivers yet. To model such case the general RH-DTA sequence has to be extended with an additional stage. In the first stage, drivers propagate according to the typical demand pattern, in the last stage they propagate according to a new perspective. The challenging time period when event is active but unknown, $\tau \in (t_\varepsilon; i_\varepsilon)$, is modelled by solving the DNL problem (Schema 3).

The DNL is used in RH-DTA to propagate the pre-calculated demand pattern on the network that has changed to $\boldsymbol{\delta}^\varepsilon$. The fixed-point iterations of DNL are needed to gain consistency between expected and actual costs. The general RH-DTA sequence from Table 5.7 is extended with the $DNL(\mathbf{p}, \mathbf{d}, \boldsymbol{\delta}^\varepsilon)$ (4.20) problem solved between the two consecutive DTA problems. At this stage, flows are propagated according to the typical demand pattern, yet the supply side calculates the travel times over the actual network, with the active event $\boldsymbol{\delta}^\varepsilon$.

Table 5.9 RH-DTA sequence for single event communicated with delay.

Procedure	input	output	results time before the event	 t_ε unknown event	 i_ε known event
1. typical DTA	**δ, d**	**p,Q**	**valid**	overwritten	overwritten
2. DNL	**δ$^\varepsilon$, d, p, Q**	**Q**	not calculated	**valid**	overwritten
3. DTA for perspective ε	**δ$^\varepsilon$, Q**	-	not calculated	not calculated	**valid**

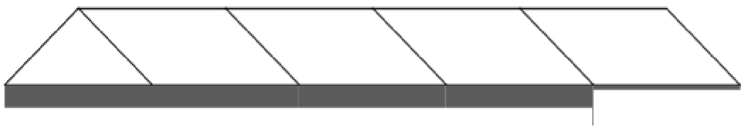


Figure 5.21 Flows (thickness of the bars) on the toy-network at 02[35]. Case of single event with delayed information available for everyone.

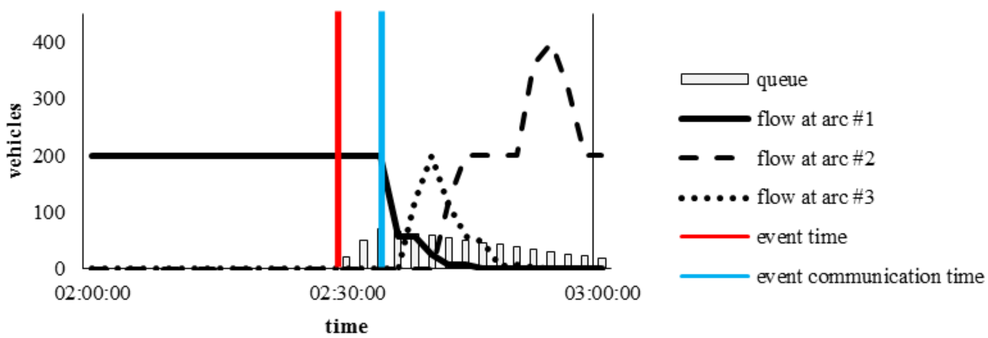


Figure 5.22 Temporal profiles of flow and queue at the toy network – case of single event with delayed information available for everyone.

At 02^{35} (Figure 5.21) the rerouting still cannot be observed as the new horizon comes with a 10-minute delay and a new demand pattern will not be available up to this time. During this time the queue builds up rapidly as the event is not known yet. The rerouting phenomenon starts at the communication time and is similar to the behavior observed in the case of immediate information (Figure 5.18). The queue grows longer, as for 10 minutes the drivers propagated towards the event, yet it slowly dissipates as no one is entering it anymore and the outflow from the impacted arc is small (50 veh/h).

5.2.3.6. *Multiple events*

Any of the above can be applied recursively for more than one event. In this case, the last stage of the framework becomes the first stage of a new sequence calculated for incoming perspective. In the case of two consecutive events with immediate information available for everyone, the sequence from the Table 5.7 extends to the sequence from Table 5.10. The horizon rolls twice, first for the $t_{\varepsilon 1}$ and then at $t_{\varepsilon 2}$.

Table 5.10 RH-DTA sequence for two events communicated immediately.

Procedure	input	output	results time before the event	$t_{\varepsilon 1}$ unknown event	$t_{\varepsilon 2}$ known event
1. typical DTA	**δ, d**	**Q1**	**valid**	overwritten	overwritten
2. DTA for perspective ε1	**$\delta^{\varepsilon 1}$, d, Q1**	**Q2**	not calculated	**valid**	overwritten
3. DTA for perspective ε2	**$\delta^{\varepsilon 2}$, d, Q2**	-	not calculated	not calculated	**valid**

The real case for the urban setting would be when there are multiple events, the information comes with some delay and is known only to some percentage of drivers. The three general cases from the above are then joined and form the following schema of Table 5.11 (for clarity just input and output is indicated). In general, for *n* events communicated with delay there are *2n* horizons, for each event the DNL is run at the time of the event and new DTA is calculated when the event is known.

Table 5.11 RH-DTA sequence for multiple events communicated with a delay.

Procedure	input	output
1. DTA for perspective ε1	**$\delta^{\varepsilon 1}$, d**	**$p^{\varepsilon 1}$**
1a. DNL for perspective ε1	**$\delta^{\varepsilon 2}$, d, $p^{\varepsilon 1}$**	**$Q^{\varepsilon 1}$**
2. DTA for perspective ε2	**$\delta^{\varepsilon 2}$, d, $p^{\varepsilon 1}$, $Q^{\varepsilon 1}$**	**$p^{\varepsilon 2}$**
2a. DNL for perspective ε2	**$\delta^{\varepsilon 3}$, d, $p^{\varepsilon 2}$**	**$Q^{\varepsilon 2}$**
…	**…**	**…**
n. DTA for perspective εn	**$\delta^{\varepsilon n}$, d, $p^{\varepsilon 1}$,$Q^{\varepsilon n-1}$**	**$p^{\varepsilon n}$**
na. DNL for perspective εn	**$\delta^{\varepsilon n+1}$, d, $p^{\varepsilon n}$**	**$Q^{\varepsilon n}$**

5.2.3.7. *Real-time rolling horizon*

Yet in real-time setting, the use-case can be even more complicated, i.e. number of events are happening in the real-time and the time to act is limited, therefore the following *real-time rolling horizon* is proposed. RH applied in real-time sequentially updates the perspective of the network, i.e. every 15 minutes. The new perspective updates the network with all the events that became active since the last update. For every new horizon the demand pattern is calculated with a DTA and then the DNL starting from the time of new horizon is executed on the real-network, where new events can occur dynamically impeding the traffic flow in GLTM. When horizon rolls again the number of vehicles **Q** from the DNL is used as a starting point for the new horizon at which the perspective is updated and results in the new

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demand pattern computed with a DTA and further propagated with DNL. Mind that for each new horizon the perspective includes only those events that are known, while at the DNL level all the events present in the network are included in $\boldsymbol{\delta}^{\varepsilon}$. It can happen that some events are communicated almost immediately, while others remain unknown for the users. Such implementation of RH-DTA is a particularly useful in real-time applications as it enables to automatically keep the model up-to-date and retain realistic representation of the actual state-of-the-network.

5.2.3.8. *Instantaneous travel time*

Final case, handles the specific class of users and is motivated with the parctial significance of this class. In practice a significant share of equipped drivers rely on instantaneous travel time information, i.e. current state-of-the-network provided online (i.e. Bing maps, Google maps, tom-tom, etc.). It is shown that such information can lead to disequilibrium and is not a good way of informing drivers (Bifulco et al., 2009), (Bifulco et al., 2013). Yet the instantaneous travel times are a common source of information used widely. Therefore, users making routing decisions based on instantaneous travel times (so called *Boston User Equilibrium*) should be handled within RH. They use travel costs evaluated at the time of receiving information $\mathbf{c}(i_{\varepsilon})$ instead of temporal profile of the expected future evolution of the cost. Such scalar value of costs can be inserted into multiclass assignment of Schema 9 instead of the temporal profile of costs.

5.2.4. *Summary of the rolling horizon model*

Proposed Rolling Horizon models can handle multiple scenarios probable in real traffic networks. Starting from the trivial ones up to the Real-time Rolling Horizon, where the basic RH-DTA algorithm becomes a fully-operative engine of the TMC providing valuable estimates of state-of-the-network in real-time based on the actual traffic situation. This is the main justification for the proposed solution, which may seem oversimplified at first. The Rolling Horizon concept, originating in the production planning, becomes a valuable starting point for the rerouting DTA model. RH framework coupled with the hybrid route-choice model allowed to extend a classic DTA into the RH-DTA algorithm. RH-DTA overcomes the fundamental limitations of the equilibrium algorithms. It can be applied sequentially in the rolling horizon to address number of possible scenarios as shown in the examples. The lightweight RH-DTA algorithm guarantees short computation times and low memory consumption. Consistency between consecutive horizons is guaranteed through saving the snapshot of the state-of-the-network – number of vehicles with their destinations. Which, at the same time, is a reasonable warm start for the real-time calculations based on the actual state of the network.

5.3. *Summary of the rerouting models*

In general, both solutions presented in the thesis are capable to predict when, where and how a certain flow travelling toward a destination stops routing according to the typical demand pattern and reroutes by utilizing information about a given traffic event. In the RH-DTA, the rerouting time is fixed and coincides with the communication instant of the event, while the decision involves only a fixed predefined share of informed users. On the contrary, the ICM represents the rerouting share as an multivariate function of the time elapsed from event communication and of its impact on the route towards the destination. Although ICM is better focused on the rerouting phenomenon than RH DTA, so far it is available only for a single traffic event. While the RH-DTA is designed to work in real-time environment.

6. Direct and indirect observation of rerouting phenomena in traffic networks

6.1. Introduction

This chapter supplements the proposed rerouting models by showing how they can be justified with the traffic observations. The methods to estimate the rerouting phenomena in traffic networks are proposed. Two datasets valuable for the estimation are identified: *od* paths observed during the unexpected event; and traffic flows of the network cut-set observed over the long period including atypical situation. Formal analysis of the above datasets is proposed to describe rerouting phenomena. For paths the formulas for theoretical (modelled) and observed rerouting probabilities are defined. For observed traffic flows the rerouting part of the flow with its temporal profile is defined. Both of them can form an input for the maximum likelihood estimation problem where either the theoretical rerouting probabilities, or modelled traffic flows are matched with the observed ones. Estimation problem provides parameterization of the ICM model in which rerouting is modelled through calculating probability of rerouting for a given place and time in the network subject to current situation and information (elaborated in 5.1.2.3). Detailed representation of phenomena with ICM model yields realistic results as it covers cognitive process of rerouting, unfortunately it is hard to estimate and validate its parameterization. In particular rerouting in ICM is parameterized with the: spreading profiles for various information sources (see section 5.1.2.1), utility in the compliance model (see section 5.1.2.2) and, implicitly, travel costs used to make rerouting decisions (see section 5.1.4). The estimation methods are proposed for the ICM model, which can be estimated directly as it models the actual rerouting behavior. For the RH-DTA model, which simplifies the phenomena, estimation cannot not be applied directly.

Direct estimation problem is illustrated with the synthetic data (prepared by author just to illustrate the proposed procedure) showing how a single observed path can be processes and what information it provides. Whereas indirect estimation is demonstrated on the field-data from Warsaw bridges observed over several consecutive days including day of the event. Central findings are: a) about 20% of affected traffic flow reroutes, b) rerouting flows are increasing in time, c) drivers show strategic capabilities, d) and maximize their utility while rerouting. All of those behavioral features of rerouting behavior were assumed in the thesis and are now supported with the field observations.

The chapter is organized as follows. Primarily direct estimation problem is elaborated, starting with path dataset definition and method to analyze the path set. Than estimation method is proposed followed by illustrative example. Secondly indirect estimation with traffic flows is shown with dataset of a cut-set traffic flows definition and method to analyze observed traffic flows. Direct estimation problem is then redefined to handle indirect observations. Finally the field-data of Warsaw bridges is analyzed to reveal and estimate rerouting phenomena. Chapter is concluded with remarks and future directions.

6.2. Direct observation

Rerouting phenomena can be observed directly by looking at the *od* paths, as will be shown in this section. The input for the direct observation is a set of the paths $\boldsymbol{K}=\{k\}$ observed during the unexpected event. A generic path k is defined in the network G as an ordered set of arcs $a \in A$ and nodes $i \in A$ reached at time t_i^k. Path k connects origin node $o \in N$ with the destination node $d \in N$. Such set of the paths can be collected i.e. during a long-term study of

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sample individuals recurrently travelling through the traffic network. If during this long-term study it is possible to identify an atypical day resulting from some unexpected event, then the input is satisfactory for the proposed estimation procedure. For further analysis, only a subset of paths that could possibly be affected by an event is needed.

6.2.1. Observing paths

Using the results of a DTA problem (Schema 2), for each arc *a* of path *k* the probability of choosing it while being at its tail node a^- at time *τ* subject to travelling destination *d* can be defined. Further called arc conditional probability, elaborated in section 4.1.3 and denoted as $p_a^d(\tau)$. $p_a^d(\tau)$ is a result of the sequential route choice model (RCM) with implicit path enumeration calculated for a given set of travel costs $\mathbf{c}$ and times **t** (Gentile et al., 2006). To reproduce the ICM idea, let's introduce two RCMs; one calculated with the typical travel costs and times: $\hat{p}_a^d(\tau) = RCM(\hat{c}, \hat{t})$ and one calculated using the actual costs and times: [illegible]. Typical costs (denoted with superscript ^) are the conditions observed during a typical day (when no unexpected event is present), which coincide with costs and times of dynamic user equilibrium and are, at the same time, the conditions expected by individuals to occur when making route choices. Actual conditions (denoted with superscript ~) in turn are those observed as a consequence of an unexpected event. Actual travel times and costs will be used by individual to choose a new path to avoid consequences of unexpected event (see section 5.1.4 for details on cost pattern used for rerouting).

For explicit path *k* we can define its probability with the implicit RCM as the arc conditional probabilities product along the path (Bellei et. al., 2005), formally expressed with (6.1). Let's further define for each path the rerouting point *r* as the point in time and space where an individual reroutes, i.e. where he/she makes a decision to change path to the destination to avoid consequences of the event. Technically we define *r* as a point at which individual starts using actual travel times and costs for routing, i.e. instead of using $\hat{p}_a^d(\tau)$ he switches to [illegible] to get to destination from *r*.

Location of *r* is not seen directly when looking at path *k,* yet we propose following two schemas to determine it: one more general and another one utilizing the typical realization $\hat{k}$ of rerouting path *k.* In general case, using *r* as rerouting point, we can redefine (6.1) for the case of rerouting with (6.2), where *a<r* denotes a subpath between origin *o* and rerouting point *r* and *a≥r* denotes a subpath between rerouting point *r* and destination *d*. Let's define *r* with (6.3) as the point for which p_k^r (6.2) is maximized. From (6.3) *r* can be understood as the point at which path *k* yields the highest probability computed with (6.2), in other words *r* is chosen so that path *k* is consistent with $\hat{p}_a^d(\tau)$ prior the rerouting and with [illegible] after the rerouting. For boundary cases, if *r=d* the rerouting was not observed and whole path is more consistent with $\hat{p}_a^d(\tau)$ than with any combination through p_k^r. Respectively if *r=o* the actual costs were already considered at the departure, i.e. p_k^r is maximized for [illegible]. To improve the identification of the rerouting point *r* the two criteria are added: (6.4) and (6.5). Condition (6.4) guarantees that at the rerouting point the actual arc conditional probability differs from the typical one. It prevents from placing the rerouting points at the corridor upstream of the actual rerouting point, where typical and actual probabilities can be equal for number of nodes. Condition (6.5) guarantees that the ICM model evaluated at *r* yields positive rerouting probability, this way the general consistency between observed point *r* and results of the ICM model is guaranteed. Alternatively, for the paths for which the typical (recurrently observed) realization $\hat{k}$ is known we can directly obtain *r* as the last node of overlapping part of *k* and $\hat{k}$.

In this case r can be seen as the point at which individual starts acting atypically (i.e. diverged from his typical path $\hat{k}$).

$$p_k = \prod_{a \in k} p_a^d(t_a) \tag{6.1}$$

$$p_k^r = \prod_{a \in k} \begin{cases} \hat{p}_a^d(t_a) & \text{if } a < r \\ \tilde{1} & a \geq r \end{cases} \tag{6.2}$$

$$r = \arg\max_{r \in k} \{p_k^r\} \tag{6.3}$$

$$\hat{p}_r^d(t_r^k) \neq \tilde{1} \tag{6.4}$$

$$\alpha_r^d(t_r^k) > 0 \tag{6.5}$$

Using (6.3) we extend data from ***K*** by adding the rerouting point r and the rerouting time t_r^k for each of the observed paths k. This way each realized path k is decomposed into two parts: prior and after the rerouting decision (if such decision was made), denoted respectively $i<r$, and $i>r$ with the rerouting point r in the middle. Let's define observed rerouting probability $\bar{\alpha}_r^d(t_i^k)$ for each node of the path i with formula (6.6) being zero for all the path nodes except the rerouting point r. Mind that $\bar{\alpha}$ in (6.6) is not defined for subpath $i>r$, after the rerouting point, this is because the rerouting decision process is already over at r and α has no particular meaning for $i>r$. We further define the probability of realization for path k with rerouting path $\bar{\alpha}_k$ using (6.6) in a joint probability that rerouting decision was not taken before rerouting point r and it was taken at the rerouting point r. Obviously, for each observed path k probability of observing this path equals one and is computed as in (6.7).

$$\bar{\alpha}_i^d(t_i^k) = \begin{cases} 0 & \text{if } i < r \\ 1 & \text{if } i = r \end{cases} \tag{6.6}$$

$$\bar{\alpha}_k = \prod_{i<r} \left(1 - \bar{\alpha}_i^d(t_i^k)\right) \cdot \bar{\alpha}_r^d(t_r^k) \tag{6.7}$$

6.2.2. Calibrating ICM model using observed paths

To estimate the ICM model from the observations of K paths the following method is proposed. The estimation process is defined through the theoretical and observed values. Theoretical being probabilities of rerouting for each element given by ICM model $\alpha_i^d(\tau)$ (calculated with ICM formulas defined in chapter 5.1, namely (5.27)). While the observed values are the observed rerouting probability $\bar{\alpha}_i^d(t_i^k)$ calculated with (6.6) obtained from the observed paths and their rerouting points r.

Following optimization program is proposed, with the main objective to model the rerouting probabilities $\alpha_i^d(\tau)$ matching the observed ones $\bar{\alpha}_i^d(t_a^k)$. In general, optimization program can be defined as in (6.8) where $\|x\|$ is any distance measure. Since the rerouting probability is, in fact, a function of the ICM parameters, the problem can be defined as a parameterization problem of the ICM model. Thus, the solution of the problem is searched in the space of acceptable values of ICM parameters: $\mathbf{a}, \beta_{\Delta p_{id}}, \beta_{\Delta w_{id}}, \eta$, as defined in section 5.1.2.3. Following most of the researchers (6.8) is redefined with the maximum log-likelihood

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formula using log of likelihood as shown in (6.9). In general, such estimation is advised for models with categorical outcomes (in this case a dichotomous yes/no model).

$$\{\mathbf{a}, \beta_{\Delta p_{id}}, \beta_{\Delta w_{id}}, \eta\} = \underset{\mathbf{a}, \beta_{\Delta p_{id}}, \beta_{\Delta w_{id}}, \eta}{\arg\min} \left\{ \sum_{k \in K} \sum_{a \leq r} \left\| \bar{\alpha}_i^d(t_i^k) - \alpha_i^d(t_i^k) \right\| \right\} \tag{6.8}$$

$$L(\mathbf{a}, \beta_{\Delta p_{id}}, \beta_{\Delta w_{id}}, \eta) = \sum_{k \in K} \sum_{a \leq r} \left(\bar{\alpha}_i^d(t_i^k) \cdot \ln\left(\alpha_a^d(t_i)\right) + (1 - \bar{\alpha}_i^d(t_i^k)) \cdot \ln\left(1 - \alpha_a^d(t_i)\right) \right) \tag{6.9}$$

The log-likelihood L of (6.9) can be used as an objective function in the optimization problem. The parameters $\mathbf{a}, \beta_{\Delta p_{id}}, \beta_{\Delta w_{id}}, \eta$ can be then estimated by finding the maximum of L, using numerical methods or the software package, i.e. BIOGEME (Bierlaire, 2003).

Mind that the formulation of likelihood (6.9) will sum over all elements (nodes) of paths $k \in K$ prior its rerouting points. And most of observations will be for not rerouting (i.e. having zero value of $\bar{\alpha}_i^d(t_i^k)$*),* as there are just $|k|$ rerouting points r and much more elements of the paths. This will result in optimization driven by the second part of the likelihood summand – for which the $\bar{\alpha}_i^d(t_i^k) = 0$, while the actual phenomena is observed at the rerouting points, where $\bar{\alpha}_r^d(t_r^k) = 1$. Therefore reformulation of (6.9) is advocated. Also because (6.9) uses each element of subpath $a \leq r$ as an independent observation, while it is more adequate to treat each path k as the independent observation.

Therefore, an alternative formulation of L (6.9) using $\bar{\alpha}_k$ as defined in (6.7) is proposed. To this end, an equivalent of $\bar{\alpha}_k$ from ICM model is needed, which can be defined with following:

$$\alpha_k = \prod_{i<r} \left(1 - \alpha_i^d(t_i^k)\right) \cdot \alpha_r^d(t_r^k) \tag{6.10}$$

Based on the above L can be redefined to sum only paths, not path elements, so that:

$$L(\mathbf{a}, \beta_{\Delta p_{id}}, \beta_{\Delta w_{id}}, \eta) = \sum_{k \in K} \left(\bar{\alpha}_k \cdot \ln\left(\alpha_k\right) + (1 - \bar{\alpha}_k) \cdot \ln\left(1 - \alpha_k\right) \right) \tag{6.11}$$

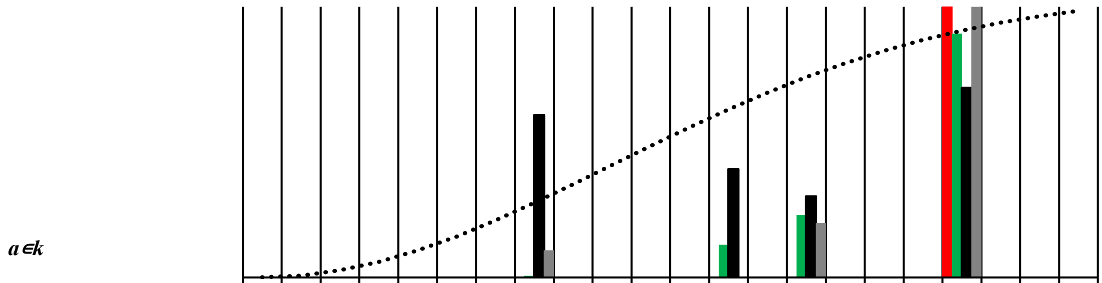
Which, as only observed paths with $\bar{\alpha}_k = 1$ are used, simplifies to:

$$L(\mathbf{a}, \beta_{\Delta p_{id}}, \beta_{\Delta w_{id}}, \eta) = \sum_{k \in K} \bar{\alpha}_k \cdot \ln\left(\alpha_k\right) \tag{6.12}$$

Above formulation of log-likelihood function reduces number of observations for calibration compared to (6.9), yet it is more consistent with the actual correlations between the observations, which are not captured in (6.9). For a single observed path k the L computed with both formulations will be the same, however formulation of (6.12) will yield a different structure of optimization program, supposedly more consistent with structure of the problem. The redefined log-likelihood maximization can be seen as maximizing the probability of realization of the observed paths produced by ICM through (6.12). Sample can be extended to cover also paths for which rerouting was not observed (for which r computed with (6.3) is placed at destination) by assuming in ICM model $\alpha_d^d(\tau) \equiv 1$.

6.2.3. Illustrative example

This section shows a synthetic example for the above method with a single path k observed during an unexpected event. The path k consists of 22 decision points $i \in N$ reached at the respective times t_i^k. Through DTA the arc conditional probabilities can be obtained, both typical $\hat{p}_a^d(t_i^k)$ and actual $\tilde{}$ of each arc a of the path. They can be then applied through (6.3) to determine the rerouting point *r*. In presented case the observed rerouting point according to (6.3) was identified at node 19th.



$a \in k$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
observed probability	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
ICM probability	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.005	0.0	0.0	0.0	0.0	0.12	0.0	0.23	0.0	0.0	0.0	0.9	0.0	0.0	0.0
Δp	0	0	0	0	0	0	0	0.6	0	0	0	0	0.4	0	0.3	0	0	0	0.7	0	0	0
Δw	0	0	0	0	0	0	0	0.1	0	0	0	0	0	0	0.2	0	0	0	1	0	0	0
share of ICM aware flow	0%	1%	3%	6%	10%	15%	21%	28%	34%	41%	49%	55%	62%	68%	74%	79%	84%	88%	91%	94%	96%	98%

Figure 6.1 Synthetic example of a single observed path observation with theoretical and observed rerouting probabilities and explanatory variables of ICM.

The example is depicted at Figure 6.1 which shows the consecutive decision points and the input variable s for the estimation problem. The observed *r* and theoretical (ICM) probability of rerouting as well as the ICM internal results: share of the aware flow and *Δp, Δw* used in computing the utility of rerouting. The share of aware flow modelled with ICM increases along CDF of Rayleigh distribution according to (5.2) and the respective spreading profiles of the information sources. Theoretical rerouting probability of ICM is positive only if there is utility of rerouting, resulting from positive values of *Δp,* or *Δw.* The theoretical rerouting probability is positive only for 8th, 13th, 15th and 19th decision points with various combinations of *Δp* and *Δw*. The path elements 20th to 22th (subpath after the rerouting decision was made) are not used in the estimation of (6.12), as they do not bring additional information. The theoretical probability of realizing the analyzed path is computed with (6.10) is *0.606.* Which is the probability of not rerouting prior 19th decision point and rerouting at 19th decision point. The log-likelihood of the proposed model calculated with both with (6.9) and (6.12) coincides and equals *-0.4995*. Model estimation for this single path do not make sense as this single path cannot make a sample. Nevertheless, the applied procedure resulted in $L \approx 0$ by modifying spreading profiles 5.1.2.1 of the awareness model ι so that it is zero prior t^k_{19} and quickly grows almost to 1 for t^k_{19}. This shows on one hand that the model can be estimated with the proposed method, but on the other hand, that much bigger sample is needed. In addition, it shows that additional boundary conditions for parameters should be well thought to fit the reality and preserve the actual cognitive process captured by ICM (as discussed in section 5.1.2.3).

6.3. *Indirect observation*

This section redefines the above estimation procedure with a different dataset. Now the traffic flows are observed, with emphasis on usefulness of observing an entire cut-set of the transportation network. The modified version of the estimation problem (6.12) is proposed.

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Processing framework is applied on field-data from Warsaw bridges to observe the rerouting phenomena. Framework proposed above, in which ICM model was estimated with direct observations of paths, is here redefined to work with indirect observations of traffic flows. First part of the section provides the method to analyze the traffic flows in case of the events to obtain variables meaningful for the rerouting phenomena. Well-founded framework for the proposed estimation is applied on the field-data. The traffic flows crossing Wisła River in Warsaw, Poland, are observed to see how it changes in case when one of the bridges is blocked. Though the additional input needed for estimation (real-time dynamic traffic assignment model as defined in section 3.3) was not available, the rough estimates of the most important characteristics of the rerouting phenomena were obtained, namely: a) total volume of rerouting vehicles, b) time of rerouting, c) new route choice pattern. Which is a reasonable starting point for further estimation of the proposed rerouting models.

6.3.1. Observing flows

This section defines the input of the procedure and derives desired characteristics. Traffic network G is observed through measuring the traffic flows $q_a(\tau)$ at some observed arcs $A_{obs} \subseteq A$. In practice, continuous observation of traffic flow is discretized and aggregated to a given time discretization as in (6.13). In the field-data analyzed below the traffic flows were aggregated to the hourly values.

$$q_a(H) = \int_{H-1}^{H} q_a(\tau) d\tau \tag{6.13}$$

Network is observed over several consecutive days and $\mathbf{q}_a^D = \{q_a(\tau) : \tau \in (1,24)\}$ denotes the daily traffic flow profile observed during day D, being the vector of hourly observations $q_a(\tau)$ for each hour τ. The long-term observation over set of observed days $\boldsymbol{D_{obs}}$ is represented through a set of daily profiles $\mathbf{q}_a = \{\mathbf{q}_a^D : D \in \mathbf{D_{obs}}\}$. Within observed days $\mathbf{D_{obs}}$ let's specify the subset of typical days $\mathbf{D_{typical}}$, that is the days for which nothing atypical (events, excessive demand, severe weather, road closures etc.) took place. From observations of the typical days $\mathbf{q}_a^{typical} = \{\mathbf{q}_a^D : D \in \mathbf{D_{typical}}\}$ it can be seen if a day-to-day fluctuations are significant. Although there is number of procedures in literature to obtain the mean, expected, reference, traffic flow and its variance (see i.e. Hranac et al., 2012), here the basic Student's t-statistics and the confidence intervals are used (6.14), where $\sigma\left(q_a^{typical}(\tau)\right)$ is the observed standard deviation of flows at arc a at time τ on a set of the typical days. t statistics are taken for $|D_{typical}|-1$ degrees of freedom so that the typical traffic flow can be defined with (6.14).

$$\varepsilon_a^{\max}(\tau) = t_{1-\alpha/2,|D_{typical}|-1} \frac{\sigma\left(q_a^{typical}(\tau)\right)}{\sqrt{|D_{typical}|-1}} \tag{6.14}$$

If observed values for the typical days are within the confidence interval $E(q_a^{typical}(\tau)) \pm \varepsilon_a^{\max}(\tau)$ it can be assumed that there is a typical traffic flow profile $\hat{q}_a(\tau) = E(q_a^{typical}(\tau))$ at arc a during a typical day.

Let's further decompose an observed flow $q_a(\tau)$ as a mixture of the two parts: typical $\hat{q}_a(\tau)$ and extra of flow $\varepsilon_a(\tau)$, as expressed with in (6.15). By definition the observation of arc a is typical as long as $|\varepsilon_a(\tau)| \leq \varepsilon_a^{\max}(\tau)$ and it becomes atypical otherwise. We further denote atypical flow $\tilde{}$ and measure it with an extra flow $\varepsilon_a(\tau)$. Atypical flow is the flow measured on the atypical day i.e. during the event. For atypical observation let's define the time instant $\tau_a^{e^-}$ at which the flows starts falling out of confidence interval and $\tau_a^{e^+}$ at which it becomes typical again. The impact period τ_a^e can be then defined with (6.16) as the time period during which the measured flows are atypical. To measure the atypical flow let's cumulate the event impact through (6.17) and define the total impact with a total extra flow $E_a(\tau_a^{e^+})$. In the remainder of the analysis it is assumed that traffic flow profile outside of impact period τ_a^e is typical $\hat{q}_a(\tau)$. Thanks to this analysis can be restricted to τ_a^e by using $\varepsilon_a(\tau) = 0 : \tau \notin \tau_a^e$ to isolate impact from random traffic fluctuations not related to the event.

$$\varepsilon_a(\tau) = \tilde{\ } \text{[illegible]} \tau) \tag{6.15}$$

$$\tau_a^e = \left\langle \tau_a^{e^-}, \tau_a^{e^+} \right\rangle : \tau \in \tau_a^e \Rightarrow \left| \varepsilon_a(\tau) \right| > \varepsilon_a^{\max}(\tau) \tag{6.16}$$

$$E_a(\tau) = \int_{\tau_a^{e^-}}^{\tau} \varepsilon_a(\theta) d\theta \tag{6.17}$$

In the above framework, an arc impacted by event is defined as an arc for which flows are atypical for some impacted period τ_a^e. Which can be further extended by looking at the temporal profile of $E_a(\tau)$ and distinguishing several cases for impacted arcs:

- negatively impacted arc, for which $E_a(\tau_a^{e^+}) < 0$;
- positively impacted arc, for which $E_a(\tau_a^{e^+}) > 0$;
- neutrally impacted arc, for which $E_a(\tau_a^{e^+}) = 0$;

By definition, it can be said that for the impacted arcs there is a strict relation between *E* and *Q*, so that the total traffic flow of impacted arc over the impacted period is:

- $\tilde{\ }$ [illegible] $\hat{Q}_a(\tau_a^{e^+})$ if $E_a(\tau_a^{e^+}) < 0$;
- $\tilde{\ }$ [illegible] $\hat{Q}_a(\tau_a^{e^+})$ if $E_a(\tau_a^{e^+}) > 0$;
- $\tilde{\ }$ [illegible] $\hat{Q}_a(\tau_a^{e^+})$ if $E_a(\tau_a^{e^+}) = 0$;

The above cases can be linked to the traffic situations as follows. For the negatively impacted either the capacity restriction did not allow all the typical flow to propagate through this arc and/or the traffic flows shifted (rerouted) to the positively impacted arcs for which the cumulated flow was greater than typically. The special case arises for impacted arcs for which there is an impact, but the cumulated flow remains typical. In this case only the profile of flow *$q_a(\tau)$* has changed, resulting in negative $\varepsilon_a(\tau)$ in the first, building, phase and positive in the second, unloading, phase. If the E of the building-phase equals to E of unloading phase it means that arc is neutrally impacted, and rerouting phenomena is not observed. Empirically it would correspond to the case when event causes some impact (i.e. slows down the traffic, or reduces capacity), but it doesn't alter the route-choice model, no-one waiting in queue decides to reroute. Surprisingly the unloading phase at the impacted arc was not observed in the field data.

Mind that τ_a^e is defined separately for each arc *a* and is not unique for the whole network, thanks to this the temporal dimension of the rerouting phenomena can be observed. For the negatively impacted arcs $\tau_a^{e^-}$ is the time instant at which the backward wave (Lighthill and Whitham, 1955) propagated from the place of event *e* reaches arc *a*, i.e. a moment at

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which the queue caused by event *e* reaches arc *a*. So that $\tau_a^{e^-} = \tau^e + \Delta w(e,a)$ where τ^e is the time of the event and $\Delta w(e,a)$ is the wave propagation time from arc of the event *e* to arc *a*. $\Delta w(e,a)$ can be straightforwardly derived from the LWR traffic flow model, i.e. (Gentile, 2010). The beginning of the impact time for negatively impacted arcs results from the phenomena taking place at the supply side, mainly queue formation. On contrary, the impact period at positively impacted arcs arises from rerouting phenomena, i.e. when the drivers who decide to change their routes from the negatively impacted arrive at the positively impacted arcs.

6.3.2. Cut-set observation

Based on the above general considerations of traffic flows observed in atypical days, let's move considerations to the specific case of a screenline observation which, as will be demonstrated, is valuable to estimate the rerouting phenomena. A specific subset $A_{obs} \subseteq A$ of arcs is observed, being cut-set of the graph dividing the network into two subgraphs, such that for each *od* pair with origin laying in one subgraph and destination in another there is no path connecting *o* with *d* which does not contain at least one arc of the cut-set. Practically such cut-set is obtained by looking at a screenline of some linear barrier, i.e. railway line, motorway, or the river. In dataset of this thesis all the bridges crossing the river were observed, cutting the network into left- and east-bank subgraphs.

While observing the entire cut-set it can be assured that the total traffic crossing it both on the typical and on the atypical day is observed. Moreover if the event took place at one of the cut-set arcs, the event arc *e* is directly observed. Thanks to this, the time of the event can be directly observed and, as $\Delta w(e,a) = 0$, it coincides with beginning of impact period at event arc *e* (which is also the only negatively impacted observed arc). Let's then introduce the total rerouting flow *R* due to the event. The quantity of rerouting flow defined through *R* results from sum of all rerouting decisions made by drivers throughout the space (network arcs $a \in A$) and time (τ) represented with $r_a(\tau)$ – which is the central variable of estimation model that will be proposed. Unfortunately, this is not observed directly from the flows and can only be estimated.

While observing the cut-set and the event took place at the cut-set the total rerouting flow *R* coincides with the total extra flow of event arc, $\left|E_e(\tau_a^{e^+})\right|$, as in (6.18). However, the flow conservation rule (6.19) shall be verified. If (6.19) holds true, it means that total flow crossing the cut-set is conserved, i.e. total extra flow on all positively impacted observed arcs is equal to the extra flow on negatively impacted observed arcs. Which means that the whole impacted flow has shifted to other observed arcs, so that *R* can be appropriately calculated with (6.18). Otherwise it means that some part of the impacted flow has resigned from the trip and did not cross the cut-set at all. In such case it is better to define the total rerouting *R* through (6.20) which cumulates ε only for the positively impacted arcs.

$$R \equiv \sum_{a \in A} \int_{\tau_a^{e-}}^{\tau_a^{e^+}} r_a(\tau) d\tau = -E_e(\tau_a^{e^+}) \tag{6.18}$$

$$\sum_{o \in A_{obs} \setminus \{e\}} E_o(\tau_o^{e^+}) - E_e(\tau_e^{e^+}) = 0 \tag{6.19}$$

$$R \equiv \sum_{a \in A} \int_{\tau_a^{e-}}^{\tau_a^{e^+}} r_a(\tau)\, d\tau = \sum_{o \in A_{obs} \setminus \{e\}} E_o(\tau_o^{e^+}) \tag{6.20}$$

$$r_a(\tau) = \sum_{o \in A_{obs}} r_a^o(\tau) \tag{6.21}$$

$$\varepsilon_o(\tau) = \sum_{a \in A} r_a^o\left(\tau - \Delta t(a,o)\right) \tag{6.22}$$

Unobserved rerouting flows $r_a(\tau)$ can be indirectly observed at $o \in A_{obs}$ through $\varepsilon_o(\tau)$ as in equation (6.22). This formula sums the decisions made by individuals through $r_a(\tau)$ back in time when they were made, $\Delta t(a,o)$ before observing the flow at observed arc. For this end departure time from arc a to arrive at observed arc o at time τ is used $\tau - \Delta t(a,o)$, which is a direct result of the traffic flow model. Another introduced variable is $r_a^o(\tau)$ being the decomposition of rerouting flow $r_a(\tau)$ per observed arcs $o \in A_{obs}$, as in (6.21). The rerouting flow observed at arc a will choose a new path, to get to the destination. While observing the entire cut-set it can be assured that the new paths will cross the cut-set and, in turn, will be observed through the extra flow at one of the the observed arcs of the cut-set $\varepsilon_o(\tau)$. So that the rerouting flow can be decomposed per the observed arc $r_a^o(\tau)$.

With the above framework, the estimation problem of rerouting phenomena can be defined. The proposed problem is solved through searching the rerouting flows $r_a(\tau)$ in space a and time τ with (6.22). This problem can be paraphrased as a problem of finding unknown rerouting flows $r_a(\tau)$ that will sum up to the observed extra flows at the observed cut-set. Or, more technically, to find $r_a^o(\tau)$ so that for each observed arc $\varepsilon_o(\tau) = \sum_{a \in A} r_a^o\left(\tau - \Delta t(a,o)\right)$. Which is a strongly underdetermined problem and should be further constrained. The constrains should include: a) at the supply side, the traffic flow theory to determine travel times Δt; b) characteristics of rerouting phenomena (reaction of drivers to unexpected events). Therefore in the following section $r_a(\tau)$ is defined through the ICM model.

6.3.3. Estimating ICM model

The rerouting problem proposed above through the equation (6.22) was underdetermined, though it can be now redefined using ICM model through (5.27) as shown in (6.23). Let's substitute $r_a^o(\tau)$ of (6.22) with formula (6.23) where the traffic flows at arcs q_i^d (decomposed per destination) are multiplied with rerouting probability α_i^d and with $p_{a \to o}^d(\tau)$. $p_{a \to o}^d(\tau)$ tells from the DTA route choice model the share of flow q_i^d at arc a that reaches observed arc o at time τ (it can be seen as equivalent of $r_a^o(\tau)$ resulting from the RCM model of DTA).

$$\overline{r}_a^o(\tau) = \sum_{d \in D} q_a^d(\tau) \cdot \alpha_a^d(\tau) \cdot p_{a \to o}^d(\tau) \tag{6.23}$$

$$\overline{\varepsilon}_o(\tau) = \sum_{a \in A} \overline{r}_a^o\left(\tau - \Delta t(a,o)\right) \tag{6.24}$$

Thanks to this elaboration (6.22) is no longer underdetermined as the theoretical $\overline{r}_a^o(\tau)$ of ICM model (6.23) can be used. In the theoretically modelled $\overline{r}_a^o(\tau)$ the flows q_a^d and share $p_{a \to o}^d(\tau)$ result from DTA model, while α is modelled with ICM. This way degrees of freedom in the estimation problem are limited to the set of ICM parameters $\{\mathbf{a}, \beta_{\Delta p_{id}}, \beta_{\Delta w_{id}}, \eta\}$.

With this extension (6.22) is no longer a simple linear equation, but becomes an estimation problem where objective is to parameterize ICM so that it will produce the observed extra flows ε through (6.24). First, let's distinguish the theoretical extra flow $\overline{\varepsilon}_o(\tau)$ at

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arc o defined through (6.25) and define the objective function through some distance measure $\left\|\varepsilon_o(\tau)-\bar{\varepsilon}_o(\tau)\right\|$. This way the problem (6.26) to estimate ICM model parameters can be formulated.

$$\bar{\varepsilon}_o(\tau)=\sum_{a\in A}\sum_{d\in D} q_a^d(\tau-\Delta t(a,o))\cdot\alpha_a^d\left(\tau-\Delta t(a,o)\right)\cdot p_{a\to o}^d\left(\tau\right) \tag{6.25}$$

$$(\mathbf{a},\beta_{\Delta p_{id}},\beta_{\Delta w_{id}},\eta)=\underset{\mathbf{a},\beta_{\Delta p_{id}},\beta_{\Delta w_{id}},\eta}{\arg\min}\left\{\sum_{o\in A_{obs}}\sum_{\tau\in\tau_a^e}\left\|\varepsilon_o(\tau)-\bar{\varepsilon}_o(\tau)\right\|\right\} \tag{6.26}$$

6.4. Case study of Warsaw bridges

The direct procedure introduced in chapter 6.3 is run with the real field observations. The temporal profiles of flows $q_a(\tau)$ for each bridge crossing Wisła River in Warsaw (Figure 6.2) were collected over 11 consecutive days including the day of the event. The 6 days were identified as typical, the weekend and Friday were excluded from the analysis due to atypical profile of flows.

On the last observed day there was a severe event impacting the flows in the city. The event took place at the Siekierkowski Bridge (further denoted as the event arc e) on 9th of April 2014, at about $9^{\underline{00}}$ in eastbound direction. As a result two out of three lanes were blocked until $11^{\underline{30}}$ when the bridge was cleared. The capacity during the event was reduced to one-third (ca. 1700 vehicles per hour) causing severe delays and queues. During the day of the event drivers had access to number of information sources. Majority of the drivers were equipped with smartphones and some part of them had access to the traffic information services. Several companies provided traffic forecasts based on historical data coupled with actual state (mainly from FCD data), radio broadcasted information about the event with approximately 20-minute delay. Grota-Roweckiego bridge was closed at the time of the event (due to road works), and data collected at Gdański bridge was erroneous, so they are excluded from further analysis.

The event arc e is the southernmost bridge in Warsaw and the expressway leading to it has a limited number of junctions. The topology of the network points the only one logical ‘escape-junction’ at which drivers could reroute to avoid impact of the event (marked with a cross at Figure 6.2). Such arbitral topological assumption reduces the search space of $r_a(\tau)$ to a single junction. Rerouting drivers at the ‘escape-junction’ switch to Wisłostrada – arterial parallel to the river leading to all the alternative bridges.

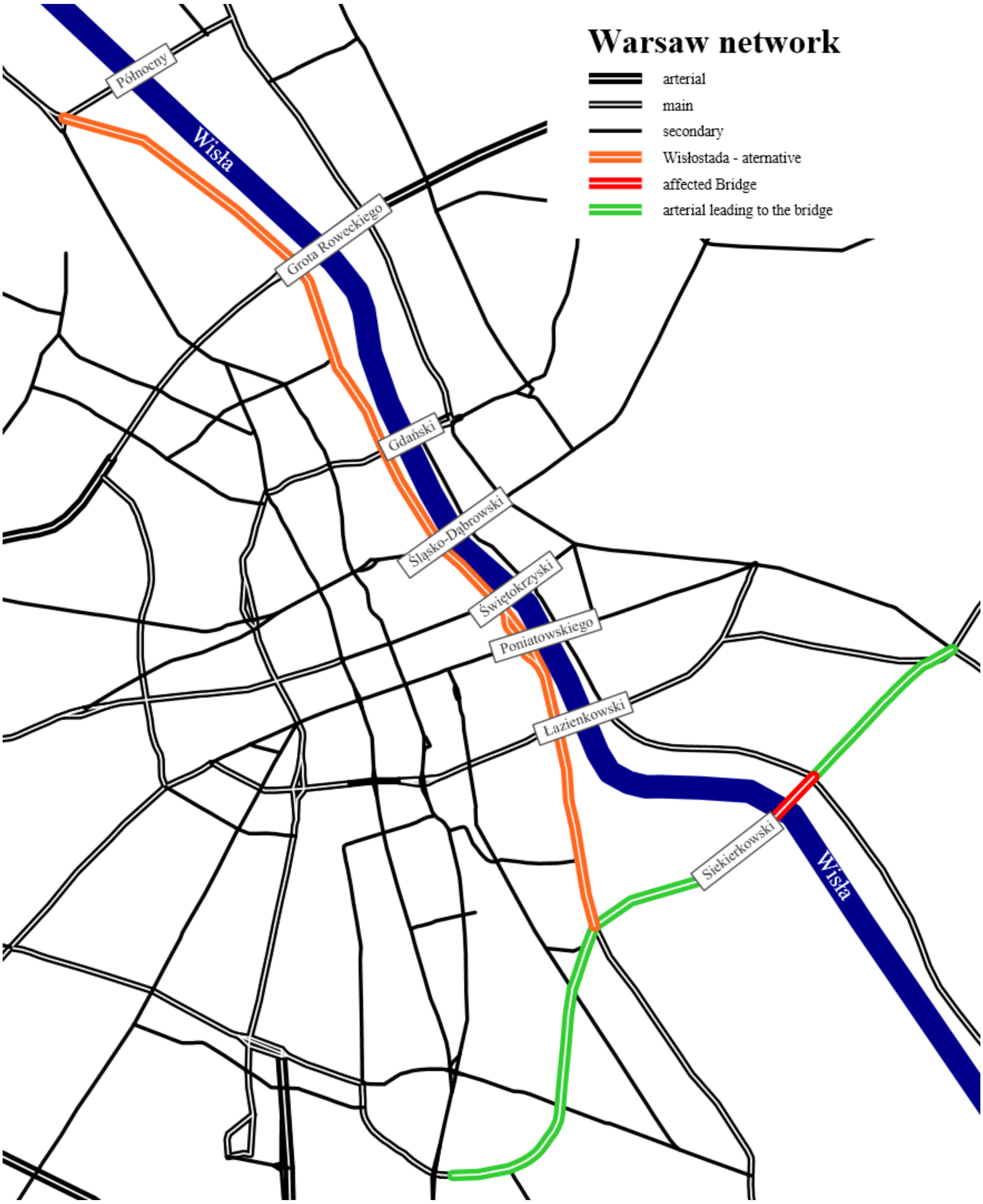

Figure 6.2 Warsaw bridges, southernmost (red) is affected. The arterial leading to the event bridge is marked green, the alternative is marked orange.

6.4.1. Analysis

In this and following sections, field data is analyzed to obtain the characteristics of the rerouting phenomena that can be derived from the observations. In the first step, representative subset of typical days $\mathbf{D}_{\mathbf{typical}}$ is selected by excluding atypical observations (i.e. excessive demand on Friday). This way typical flow $\hat{q}_a(\tau)$ can be determined and observed extra flow $\varepsilon_a(\tau)$ in most cases falls within the confidence interval computed with (6.14). As depicted on Figure. 6.3 , day-to-day traffic flow fluctuations of the event arc e are insignificant and typical flow well represents the expected (mean) flow profile. The traffic flows observed in the typical days were within the confidence intervals computed with (6.14) for α=10%. The same holds true for most of the bridges, typical traffic flow profiles $\hat{q}_a(\tau)$ and their standard deviations over all bridges of a cut-set are shown in Table 6.1 and at Figure. 6.3

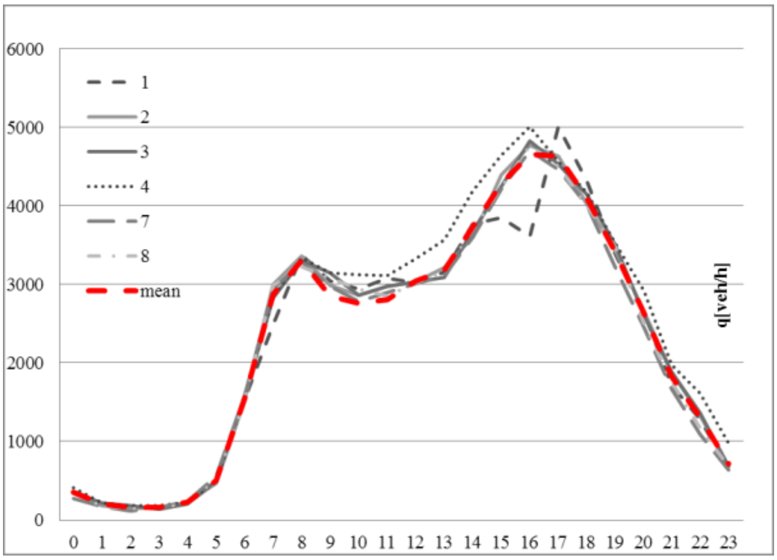

Figure. 6.3 Temporal profile of flow throughout the day observed on the typical days at the affected bridge *e* .

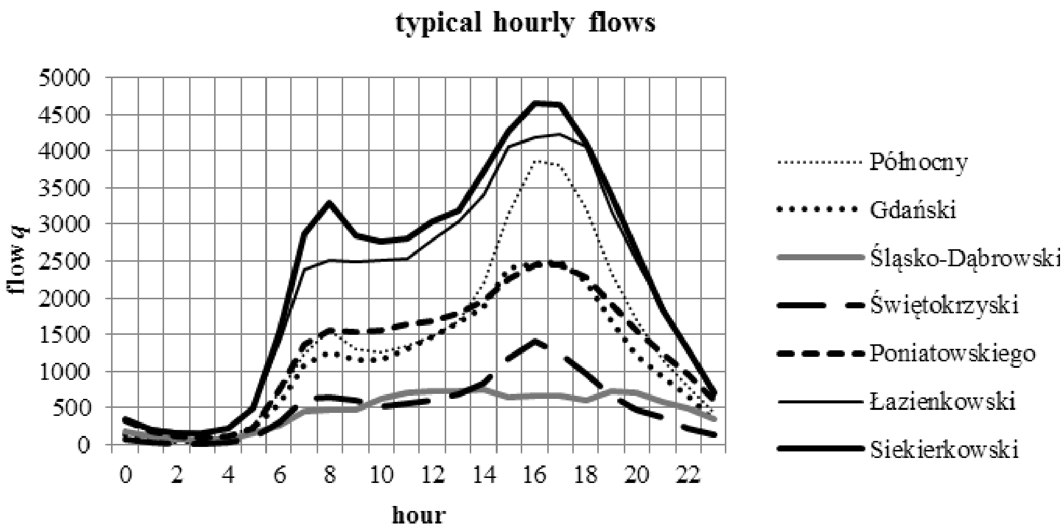

Figure. 6.4 Typical observed temporal profile of traffic flow $\hat{q}_a(\tau)$ throughout the day at the cut-set.

Table 6.1 Typical hourly traffic flows and their standard deviations at observed bridges.

bridge	Północny		Śląsko-Dąbrowski		Świętokrzyski		Poniatowskiego		Łazienkowski		Siekierkowski	
hour	**mean**	**std**	**mean**	**std**	**mean**	**std**	**mean**	**std**	**mean**	**std**	**mean**	**std**
0	**200**	*24*	**185**	*16*	**68**	*5*	**335**	*51*	**325**	*47*	**353**	*44*
1	**105**	*11*	**110**	*22*	**36**	*7*	**179**	*32*	**228**	*48*	**200**	*29*
2	**75**	*10*	**81**	*27*	**20**	*8*	**119**	*33*	**188**	*56*	**156**	*23*
3	**69**	*9*	**69**	*12*	**20**	*5*	**106**	*17*	**149**	*30*	**160**	*14*
4	**103**	*9*	**77**	*10*	**26**	*5*	**107**	*10*	**224**	*57*	**223**	*15*
5	**261**	*10*	**156**	*14*	**87**	*3*	**224**	*14*	**503**	*87*	**504**	*33*
6	**714**	*20*	**266**	*8*	**315**	*21*	**751**	*25*	**1408**	*95*	**1542**	*29*
7	**1240**	*47*	**463**	*13*	**618**	*23*	**1375**	*18*	**2388**	*108*	**2871**	*184*
8	**1550**	*26*	**471**	*21*	**640**	*10*	**1553**	*36*	**2517**	*86*	**3304**	*58*
9	**1295**	*49*	**475**	*29*	**600**	*22*	**1536**	*42*	**2500**	*111*	**2852**	*564*
10	**1267**	*66*	**630**	*27*	**523**	*32*	**1551**	*64*	**2504**	*134*	**2758**	*442*
11	**1349**	*37*	**700**	*15*	**556**	*20*	**1643**	*67*	**2527**	*122*	**2804**	*485*

12	**1478**	*71*	**726**	*30*	**610**	*50*	**1694**	*70*	**2790**	*175*	**3042**	*140*
13	**1684**	*196*	**724**	*39*	**685**	*49*	**1797**	*65*	**3040**	*218*	**3184**	*182*
14	**2188**	*272*	**745**	*52*	**846**	*39*	**1971**	*34*	**3402**	*205*	**3731**	*216*
15	**3143**	*200*	**647**	*16*	**1172**	*71*	**2261**	*29*	**4050**	*183*	**4277**	*240*
16	**3873**	*84*	**669**	*30*	**1417**	*34*	**2453**	*92*	**4184**	*235*	**4656**	*473*
17	**3806**	*101*	**666**	*16*	**1250**	*97*	**2439**	*59*	**4236**	*293*	**4629**	*182*
18	**3206**	*108*	**602**	*28*	**957**	*10*	**2276**	*90*	**4058**	*225*	**4097**	*116*
19	**2338**	*277*	**739**	*31*	**665**	*26*	**1909**	*108*	**3182**	*234*	**3409**	*104*
20	**1704**	*220*	**703**	*16*	**467**	*24*	**1560**	*91*	**2507**	*276*	**2630**	*150*
21	**1165**	*96*	**579**	*68*	**369**	*27*	**1230**	*54*	**1876**	*208*	**1831**	*114*
22	**803**	*113*	**504**	*57*	**214**	*24*	**962**	*90*	**1278**	*118*	**1307**	*165*
23	**426**	*103*	**348**	*81*	**134**	*34*	**594**	*62*	**673**	*152*	**717**	*119*

The event arc *e* has the highest total cumulative flow *Q*. Bridges in eastbound direction are more congested in the afternoon peak than in the morning. At the time of the event at the alternative bridges there is still some capacity in eastbound direction available i.e. for rerouting vehicles.

Typical flows $\hat{q}_a(\tau)$ can be compared with atypical flows observed during the day of the event $\tilde{\tau}$ to derive extra part $\varepsilon_a(\tau)$. At the event arc *e* the negative extra flow $\varepsilon_e(\tau)$ is evident (see Figure 6.5, and Table 6.2), both at the level of flows and their cumulatives. In total during the day 3705 vehicles less has passed the bridge than typically. $E_e(\tau) = -3705$ which stems for about 6% of the daily flow and about 25% of flow during impact period. Impacted period has a strict beginning at $09^{\underline{00}}$ with clearly identifiable loading-phase from $09^{\underline{00}}$ to $13^{\underline{00}}$. Interestingly the queue dissipation after the road is cleared is not observed. What was expected is that the queue which formed upstream of the event will dissipate at the level of capacity until it reaches back the typical values when queue diminishes. While what was observed is the flow recovering to typical levels ($13^{\underline{00}}$), reaching capacity much later, during afternoon peak ($16^{\underline{00}}$). Traffic flows are slightly above the typical values only for a short period from $13^{\underline{00}}$ to $16^{\underline{00}}$. Therefore, the limitation of impacted period τ_e^e is not obvious. Here it was assumed that impact period coincides with the capacity reduction time (loading-phase) and the flows above typical in the unloading phase are neglected. Nevertheless, the observation at event arc *e* fits into definition of negatively impacted arc proposed in former section. For the remainder of calculations τ_e^e from $09^{\underline{00}}$ to $13^{\underline{00}}$ is used.

Having estimated the impact period τ_e^e , *R* according to (6.18) can be estimated. In total during the impact period *R= –3 705* vehicles less has travelled through the bridge. What shall, however, be tested is the conservation rule (6.19), for this end typical and atypical flows at the remainder of cut-set are compared and $E_e(\tau)$ for each bridge is obtained (depicted at Figure 6.12). What is interesting to see is the flow profile crossing the whole cut-set during the impacted time (typical and atypical) at Figure 6.6. Traffic crossing the river drops significantly at time of the event ($09^{\underline{00}}$) which coincides with the drop at the event arc *e* at $09^{\underline{00}}$, yet at $10^{\underline{00}}$it is recovers approaching to typical and at $11^{\underline{00}}$ it goes above, going back to typical values at $13^{\underline{00}}$. Based on the above the total impact time of the whole cut-set is identified as $09^{\underline{00}}$ –$13^{\underline{00}}$.

Having a rough overview of the entire cut-set the situation at single alternative bridges can be looked at. From the charts showing typical and atypical flows, extra flows ε of each bridge can be identified. At the neighboring bridge (Łazienkowski, Figure 6.11) positive ε appears at $10^{\underline{00}}$ and reaches back the typical level at $13^{\underline{00}}$, analogous behavior can be seen at the three consecutive bridges (Figures 6.8 – 6.10). While on the northernmost bridge: Most Północny (Figure 6.7) the observed flows are typical throughout the whole day–they are not impacted by the event.

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Table 6.2. flow q and cumulative flow Q of affected bridge, typical and atypical, impact period in bold.

	Traffic flows q		Cumulatives Q	
hour	typical	atypical	typical	atypical
0	353	391	353	391
1	200	245	553	636
2	156	167	708	803
3	160	174	868	977
4	223	240	1091	1217
5	504	543	1595	1760
6	1542	1505	3137	3265
7	2871	3017	6009	6282
8	3304	3238	9312	9520
9	2852	**1583**	12164	11103
10	2758	**1784**	14922	12887
11	2804	**1724**	17725	14611
12	3042	**2850**	20768	17461
13	3184	**2994**	23952	20455
14	3731	3566	27683	24021
15	4277	4348	31960	28369
16	4656	4911	36615	33280
17	4629	4693	41245	37973
18	4097	4034	45342	42007
19	3409	3354	48751	45361
20	2630	2632	51382	47993
21	1831	1919	53213	49912
22	1307	1294	54520	51206
23	717	729	55237	51935

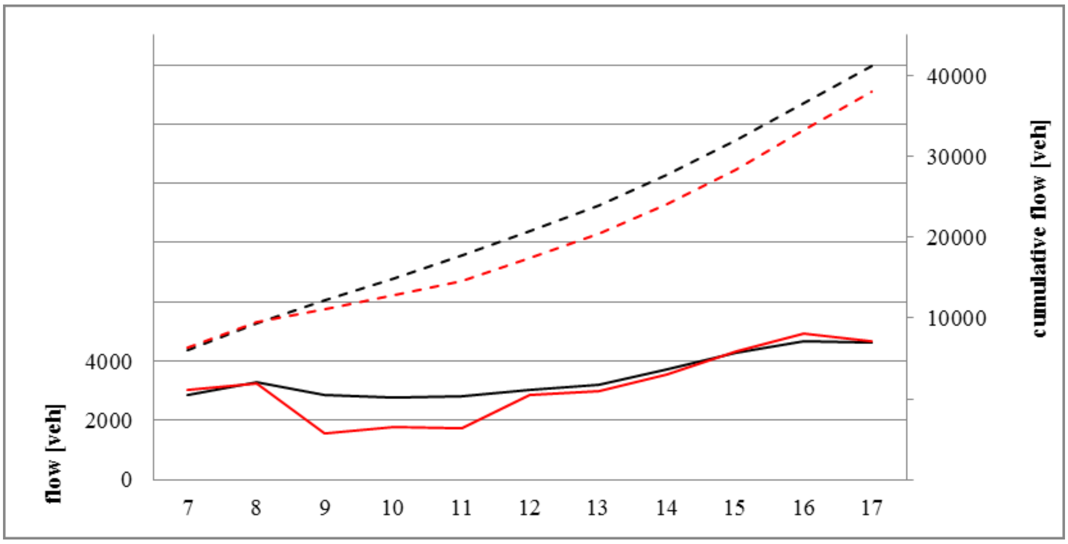
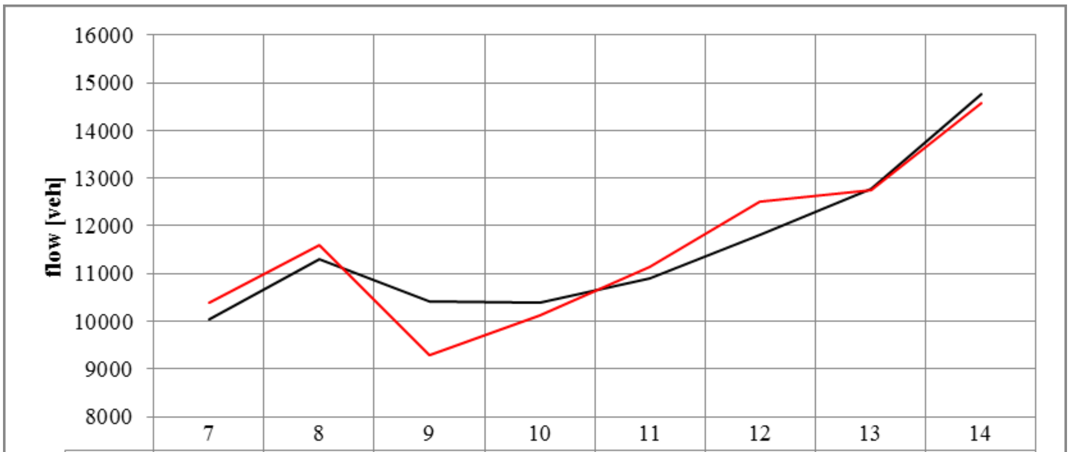
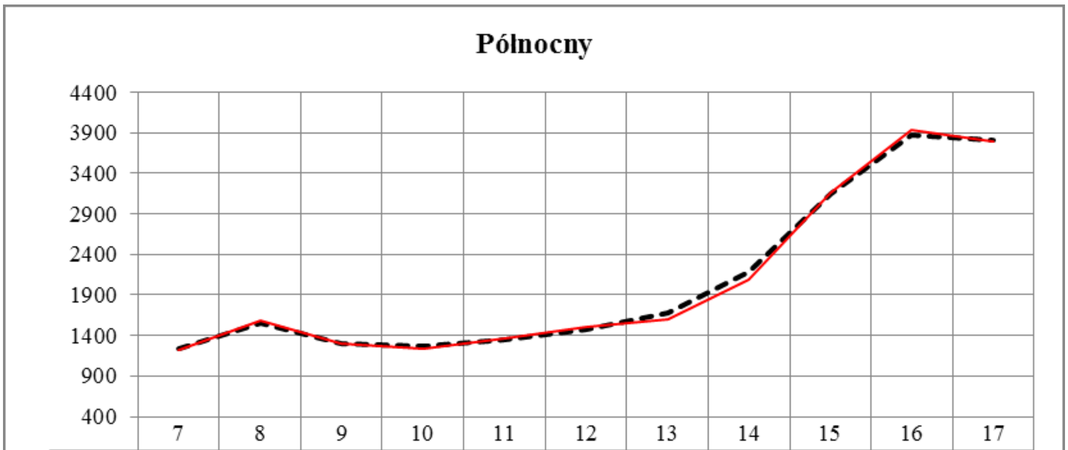


Figure 6.5. Temporal profile of the flow and the cumulative flow (dashed) of affected bridge, typical and atypical (red).



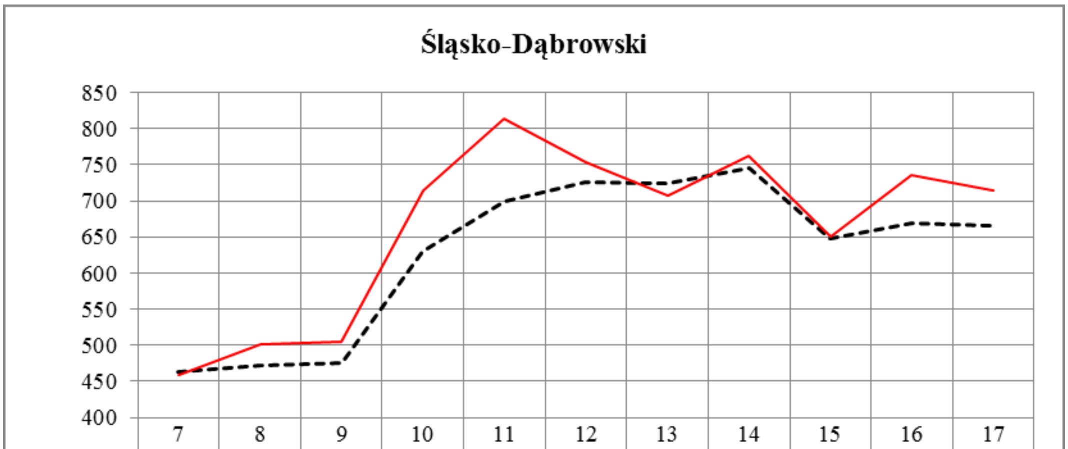
	7	8	9	10	11	12	13	14
typical	10050	11293	10406	10397	10887	11807	12780	14768
atypical	10399	11585	9288	10130	11136	12498	12745	14581

Figure 6.6 Temporal profile of the flow of the whole screenline typical and atypical (red).



	7	8	9	10	11	12	13	14	15	16	17
typical	1240	1550	1295	1267	1349	1478	1684	2188	3143	3873	3806
atypical	1218	1580	1307	1239	1360	1506	1601	2087	3161	3940	3789

Figure 6.7 Temporal profile of the typical and atypical (red) flow at Północny Bridge.



	7	8	9	10	11	12	13	14	15	16	17
typical	463	471	475	630	700	726	724	745	647	669	666
atypical	458	501	505	715	813	753	708	762	651	735	714

Figure 6.8 Temporal profile of the typical and atypical (red) flow at Śląsko-Dąbrowski Bridge.

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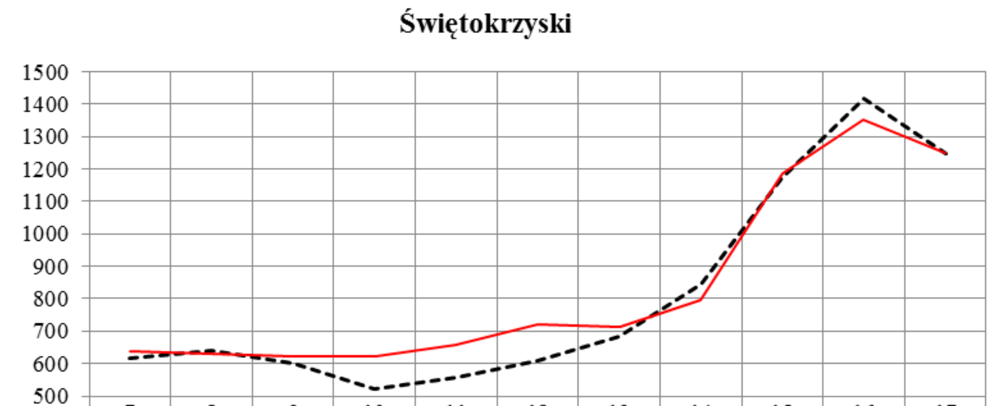
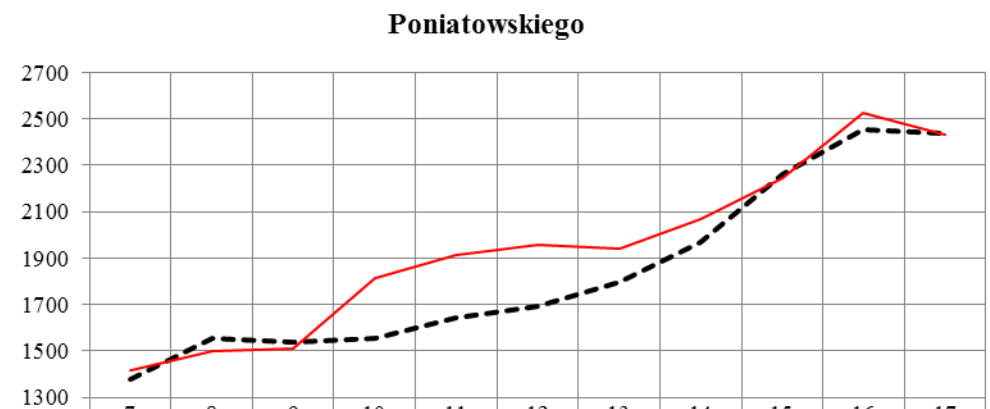


	7	8	9	10	11	12	13	14	15	16	17
typical	618	640	600	523	556	610	685	846	1172	1417	1250
atypical	638	631	623	621	657	721	715	794	1188	1353	1248

Figure 6.9 Temporal profile of the typical and atypical (red) flow at Świętokrzyski Bridge.



	7	8	9	10	11	12	13	14	15	16	17
typical	1375	1553	1536	1551	1643	1694	1797	1971	2261	2453	2439
atypical	1415	1501	1511	1815	1911	1957	1940	2070	2242	2525	2430

Figure 6.10 Temporal profile of the typical and atypical (red) flow at Poniatowskiego Bridge.

	7	8	9	10	11	12	13	14	15	16	17
typical	2388	2517	2500	2504	2527	2790	3040	3402	4050	4184	4236
atypical	2411	2692	2512	2691	3194	3180	3078	3290	3729	4218	4567

Figure 6.11 Temporal profile of the typical and atypical (red) flow at Łazienkowski Bridge.

Table 6.3 extra flow ε at impacted bridges.

hour	Północny	Śląsko-Dąbrowski	Świętokrzyski	Poniatowskiego	Łazienkowski	Siekierkowski
9	12	30	24	-25	12	-1269
10	28	85	**98**	**264**	**187**	-974
11	-11	**113**	**101**	**268**	**667**	-1080
12	-29	27	**112**	**263**	**390**	-192
13	83	-16	30	**143**	38	-190
total	**83**	**239**	**364**	**912**	**1294**	**-3705**

Table 6.3 shows extra flows at impacted bridges, based on this the total rerouting flow R computed with (6.20) equals 2809, which confronted with -3705 computed with (6.18) shows that flow conservation did not hold true and ca. 24% of flow did not cross the river at all during the impact period. According to consideration from chapter 6.3.2 the total rerouting flow computed with (6.20) is used as more appropriate, which yields lower total rerouting share of $19,2\%$.

The field data discretized every hour did not allow to precisely estimate the time dimension of rerouting flows $r_a(\tau)$. However, a delay in reaction can be seen between the event and time when rerouting flows are observed at the impacted bridges. This delay is visible at Figure 6.12 when ε is negative for event arc at $09^{\underline{00}}$ and becomes positive for alternatives at $10^{\underline{00}}$. Which, taking into account the approximated travel time from 'escape-junction' to closest alternative (Most Łazienkowski) of 13 minutes, leads to the conclusion that rerouting starts less than hour after the event but not immediately. The above is all that can be stated this data discretization about timing of rerouting.

The extra flows $\varepsilon_a(\tau)$ are increasing in time reaching the top-level ca. 2 hours past the event. Which is coherent with the ICM assumption that the rerouting flows increase in time due to increasing awareness (reflected in information spread model of ICM (5.2)) and increasing delay the event causes (reflected in the ICM model – through $M, \Delta t, \Delta w, \Delta p$).

Lastly, the new route choice pattern being impact of the event can be approximated as shown in Table 6.4. The closest alternative takes the biggest share of rerouting flows, and the share decreases as the distance (cost to reach the alternative) increases. This fits the assumption that users make rational choices while making rerouting decisions (they minimize the travel costs), as well as the assumption that users take into account the choices made by others (not only single optimal alternative is used), see section 5.1.4. Those observations support the ICM assumptions that a) rerouting is driven by cost minimizing formula, i.e. drivers reroute according to $\tilde{I}$ b) more than one iteration of DTA is needed to reproduce rerouting route-choice pattern, with $\tilde{I}$ being updated every iteration (see section 4.4.5).

Table 6.4 route-choice between bridges of a cut-set.

share	Północny	Śląsko-Dąbrowski	Świętokrzyski	Poniatowskiego	Łazienkowski	Siekierkowski
typical	12.6%	5.8%	5.3%	14.6%	23.7%	26.0%
atypical	12.7%	6.3%	6.0%	16.5%	26.5%	19.8%
extra	2.9%	8.3%	12.6%	31.5%	44.7%	-

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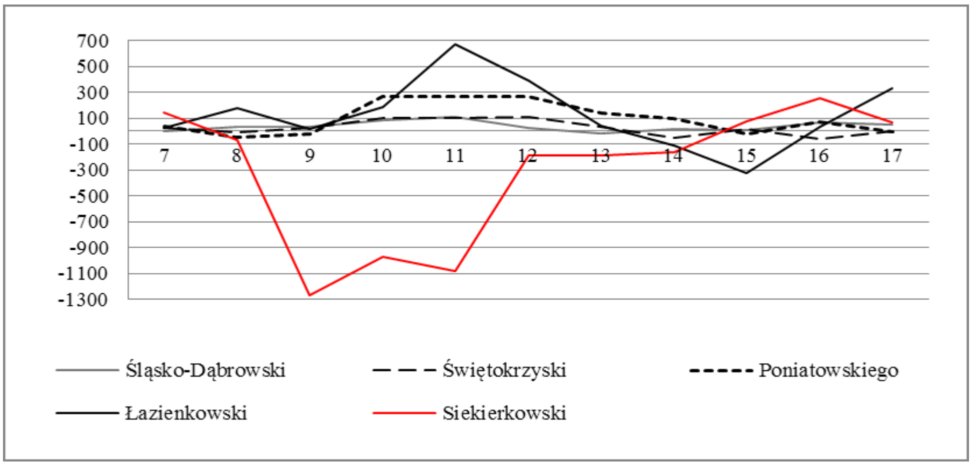

Figure 6.12 extra flow ε at impacted bridges.

6.5. Summary of the rerouting observations

In this chapter evidence of rerouting phenomena was found, which was observed at the field-data. An extra flow at alternative bridges was identified when major bridge was blocked. Most important characteristics of rerouting were approximated: around 20% of impacted flow reroutes, rerouting decisions starts to be made around one hour after the event. The emerging route-choice pattern for rerouting flows shows that not only alternative with the lowest cost is chosen when making rerouting decisions, but also other (further) alternatives. This supports the hypothesis on the strategic capabilities of (at least Polish) drivers. What is more, the relation between additional travel cost of rerouting through alternative and its share among rerouting flows was observed. Share of alternative decreases while the cost to reach it increases.

Furthermore, a theoretical framework to estimate rerouting model both using observation of traffic flows crossing the cut-set of the network (in this case the river) as well as the path observations was proposed. Full estimation requires both the densely time-discretized observation data and established, real-time dynamic traffic assignment model that is yet not available. Thanks to the method proposed here it is possible to understand ICM parameters and, in turn, the rerouting phenomena itself. Namely: significance of information and observation, shape of information spread function, sensitivity to experienced atypical delay, utility formula for rerouting and form of a discrete choice model. The method can be used to identify the points at which rerouting takes place and to approximate the magnitude of the phenomena. The case study shows that unexpected events significantly impact the state-of-the-network and rerouting phenomena should definitely be included in real-time DTA models. This part of thesis should be further extended with the real dataset of observed paths, or a detailed traffic flows data discretized every 5 minutes in the city with established real-time DTA model.

7. Conclusions

In the thesis the vital issue of how the unexpected events impact the traffic networks is addressed. The study of traffic flows crossing the Wisła river in Warsaw revealed the scale of this impact: almost 3 000 vehicles decided to reroute – change the route when one of the bridges was blocked. Those vehicles (20% of the affected flow) significantly impact the expected state of the traffic network, as they use different routes and generate different congestion pattern than typically. To estimate the congestion pattern arising in such cases the traffic managers need to understand and model the rerouting phenomena. Yet handling rerouting phenomena requires thinking out-of-the box and understanding the limitations of the common traffic assignment algorithms. Most importantly 'Wardropian' equilibrium algorithms, which resemble the learning process in the recurrent situations. Rerouting requires different machinery, first of all the phenomena itself is dynamic, so it should be handled in the dynamic environment, and secondly it has to model the actual drivers' behavior in cases of unexpected events.

The main contributions of the thesis are the two rerouting, macroscopic DTA models: Information Comply Model that exposes the phenomena and handles it in details, and the Rolling-Horizon that simplifies it so that it can be applied in real-time environment. The two models are illustrated with the examples showing their capabilities. ICM examples on the toy-network show various cases of informing the drivers: radio, VMS at two different locations, no information only observation, on-line sources becoming viral, etc. RH-DTA examples include single and multiple events, immediate and delayed information and multiple user-classes.

The rerouting behavior is identified in the Information Comply Model, which can be seen as the main contribution of the thesis. ICM handles the phenomena with two phases: awareness and compliance. Drivers need to become aware of the event before they decide to reroute (comply with the information). In the nowadays traffic networks the awareness can come from the multiple sources of information, i.e. available online, broadcasted on the radio, transmitted through the VMS, etc. It creates a complex information context of where and how the drivers are notified about the event. Proposed awareness model approximates the probability of becoming aware subject to the spreading profiles of social media, broadcasted information, VMSes passed by, experienced delay etc. This allows a fair approximation of who and where becomes aware of the event and who can (potentially) decide to reroute. This leads to a final stage of rerouting process when drivers decide to reroute based on the utility they see in rerouting, quantified with the possible gains and possible losses of the rerouting decision. Such utility is then used in the binomial yes/no discrete choice model yielding the rerouting probability for a given space, time and utility. This way the rerouting phenomenon is defined behaviorally and the general structure of the rerouting models is defined.

Parallel to the above behavioral description of the phenomena, the algorithmic structure for the rerouting model needs to be proposed. The effort was made to propose models that can be easily integrated with the macroscopic DTA algorithms, will use the same concept, data structure, algorithms, and modify the acknowledged solutions only when it is necessary. The thorough examination of the DTA algorithm revealed the limitations in respect to the rerouting phenomena: "*Limit of pre-trip decision based on expected future costs*", and "*Limit of pre-trip routing based on costs from previous iterations*". The hybrid route-choice model (Pel et al., 2009) was redefined and in the thesis to the macroscopic DTA and proved to conserve the demand. It happened to be an efficient way to overcome the identified DTA limitations. It founded the formal representation of the rerouting phenomena inside the DTA algorithm. With a hybrid model, the propagation in a single DTA problem can be simultaneously modelled with the two sets of demand patterns: typical (for those who do not reroute) and actual (for those who reroute). This way drivers can switch form the typical to the actual while travelling, which is the technical way to represent the rerouting phenomena in DTA. To complete the solution three states of flow are identified: unaware, aware and rerouted. The transitions between them are handled with the Markov chain to gain spatiotemporal consistency. Thanks to this an ICM model is proposed as a fully operative extension of the DTA algorithm, which realistically represent the rerouting phenomena and can be easily parameterized to fit the actual observations. The fundamental constrain of the ICM model is that it addresses only one event at the time. The utility formulas of the compliance model are computed strictly subject to the event and the delay that it causes. Including more than one event is impossible and would result in wrong computation of the utility. Moreover, in ICM the event need to be known in advance at the beginning of computation, it cannot be added in real-time.

The above shortcomings of the ICM model justified the need for the Rolling Horizon model, which is the second contribution of the thesis. RH-DTA utilizes the idea of the hybrid model inside a DTA through saving the *snapshot* of the directed vehicle flows in the network. This way a simulation can be saved and restarted consistently, moreover a restart can use a new, updated demand pattern. RH-DTA is a flexible algorithm that can handle number of real-time scenarios. The most valuable RH implementation is the real-time rolling horizon that can handle any possible traffic situation: multiple events taking place in real-time known only to some drivers. Such functionality is satisfactory for the traffic management applications providing a fair input to forecast traffic pattern in real-time. Yet it is at the cost of losing most of the ICM details in Rolling Horizon: the awareness is simplified to a single time instant and rerouting decision is not driven by the utility of rerouting.

To make the models practically applicable they shall be further estimated and verified. Thesis proposes the estimation method for the ICM models from the two datasets: direct (observing paths) and indirect (observing flows). For both datasets the formal analysis of the information they provide allows to formulate the max log-likelihood estimation problem for the ICM model. The indirect observation method is illustrated with a case study where rerouting phenomena is revealed from the flows crossing Wisła river in Warsaw.

7.1. Further research directions

This thesis contributes to the ongoing development of the real-time DTA, where number of research groups try to make the established DTA solutions applicable in the real-time traffic environment. What is addressed here is one part of this broad problem: modelling the atypical demand pattern arising from unexpected events. Presumably, as the new DTA advancements are developed, the contributions of the thesis will be gradually adjusted and modified to be efficient within the developing DTA environment. This points several DTA streams of the research that can be integrated with the work of the thesis, namely:

- The theory behind a recent day-to-day traffic evolution models provides a detailed structure of drivers behavior and adaptation, which is closely related to the rerouting problem (where en-route route-choices are adapted), so that the two problems can be successively integrated (Ahmed, et al.,2014)(Iryo and Ishihara, 2014)
- The stream of research addressing the travel time variability and uncertainty in route-choices provides the framework where drivers identify the vulnerable parts of the network and avoid them in route-choices. This can provide a nice extension of the rerouting models, where the actually experienced delays will build up a drivers network knowledge (Gao et al., 2008), (Gao et al., 2010)

- The reaction of drivers in the thesis is limited to the route choice, while the full spectrum identified by (Dobler, 2013) would include also resigning from the trip, changing destination, changing departure time, etc. This way the rerouting model would become part of the fully elastic demand model addressing one aspect of elasticity.

Apart from the theoretical advancements possible for the proposed models, they show a big practical potential. They fill be fully revealed only when applied in the real-time ITS systems with DTA model, which at this moment are not common. The full functionality of the rerouting models will be gained at the established dynamic macroscopic model working in real-time inside the ITS system, which represents the reality. Such systems are gradually applied throughout the world, yet are still at the early stage of the development and their full functionality will be obtained in the near future. Applying in real-time systems will allow to further integrating them with the real-time traffic estimation, so that the actual state-of-the-network can come from the actual estimate coming from the real-time data (automated traffic counts, floating car data, etc.). This way both the ICM and RH models can use the accurate estimate of state-of-the-network as an input. To make it operative, the more accurate methods estimating the state-of-the-network, especially travel times, congestion and queue length, i.e. from work of (Meschini and Gentile, 2010), shall be used.

In the applied ITS systems the rerouting models could be properly calibrated and estimated from the observations as defined in Chapter 6, especially the dataset of the observed paths. Even small sample of paths, coupled with the real-time DTA data would be a valuable input for the estimation. Such process would require observing full paths, i.e. through a GPS, or a cellular data. Such researches are already conducted; see (Rasmussen and Nielsen, 2013). The ICM model is built with the assumptions that shall be further verified. One of them is the costs that are used by the rerouting drivers – the way they perceive and are aware of the actual travel times. Namely, what should be verified is the: formula of cost perceived by the rerouting drivers to make a rerouting decision. Maybe users perceive the cost differently and act according to the instantaneous, maximal, or exaggerated travel costs. This way a rerouting would be made not based on actual costs, but the updated expectation of costs. Such perception can be either optimistic or pessimistic leading to over- or under-estimation of the delay caused by event. Such heterogeneity can be handled with further researches.

Another aspect is the forecasting capabilities of drivers, which is somehow cumbersome to verify, as it is implicit within the iterative DTA algorithm without a closed form. The hypothesis that the iterations resemble forecasting capabilities can be incorrect, it can happen that the convergence of the relative gap is faster, has different shape than the actual strategic capabilities of the drivers. Number of iterations of the DTA algorithm and contraction formula (MSA, or some more complex) used in the ICM model is unsupported with observations. In addition, the cost updating formula (handled internally in MSA, or explicitly by some cost-updating rule) can be fitted to the actual behavior to represent the cost used for the strategic rerouting decisions. Likewise, a way of handling the variability among choices in the probabilistic model shall be better looked at. Presumably, a variance η for the logit rerouting model will be higher than for the recurrent, typical routing decision basing on experience. The rerouting decisions are uncommon and the drivers will probably act more randomly, leading to higher values of η, or even a different kind of route-choice model. The assumptions on the information spreading processes are easier to define and verify directly. Number of social media researches is being conducted, also in transport related topics, so that the full ATIS analyses can be included to support the awareness model of ICM. This links i.e. to work of (Bifulco et al., 2014) on ATIS impact on state-of-the-network and information spread research by (Procter et al., 2013). Another interesting research direction is to integrate the model with social-media real time system monitoring virality of information. Such systems are able to detect the trending news spreading through the social media, this way the

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traffic manager would be informed about which traffic-related events are communicated virally. So that magnifying the Rayleigh spread profile with the total delay in (5.10) would not be needed anymore as virality will be known directly: some events are known by everyone, some are neglected.

Furthermore, the machinery to model the rerouting decision in ICM (compliance model) can be further developed. First of all the rerouting utility quantified with the possible gains and possible losses can be unjustified, maybe users will perceive their rerouting utility differently this can be revealed in the estimation process based on revealed preferences. Similarly handling the decision process with the binomial logit model, can be inappropriate. Maybe a more complex discrete-choice model would be better suited to what is revealed in the stated and revealed preference surveys. Specifically promising would be to utilize the losses/gains representation of the utility with the prospect theory approach, where the users react asymmetrically to losses and gains. This links to work of (Kahneman and Tversky, 1979) (Ben-Elia and Shiftan, 2010) (Avineri, 2014).

Finally, the most meaningful further research direction would be to overcome the limitations of the ICM model. Namely to make it operative in real-time environment when more than one event is present. Further research will exploit possibility of integrating the RH framework with the ICM model with the proposed solution. To fill the gap between ICM and RH let's propose a new DTA model where the ICM rerouting algorithm can be used for multiple events that are communicated in rolling horizon. It is specially designed for the cases where 1) users do not know about all events and 2) some events have significantly greater impact than others do on the route choice toward a destination at single decision points. For simplification, it is assumed that events are acknowledged in the same sequence by all users. At each decision point, the user can choose among different demand patterns computed for the respective perspectives (acknowledging respective events). They can be obtained with a classic route choice model using the travel times expected by users who know the first n events. Expected travel times need to be calculated through a heuristic based on users' expectations subject to the information they have. Finally, to aggregate the arc conditional probabilities of the different events let's determine at the decision point the utility of rerouting due to each event. For this purpose a binomial logit model of ICM becomes a multinomial logit where the ICM rerouting utility for each event is calculated. The computational issue arises while computing utility as it is still unclear how to estimate the expected delay due to each event without having several DTA computations and explicit supply models in GLTM. If this issue is solved the events could be added in the rolling horizon and simulations can be restarted with destination flows as in RHM.

8. Literature

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9. Streszczenie w języku polskim

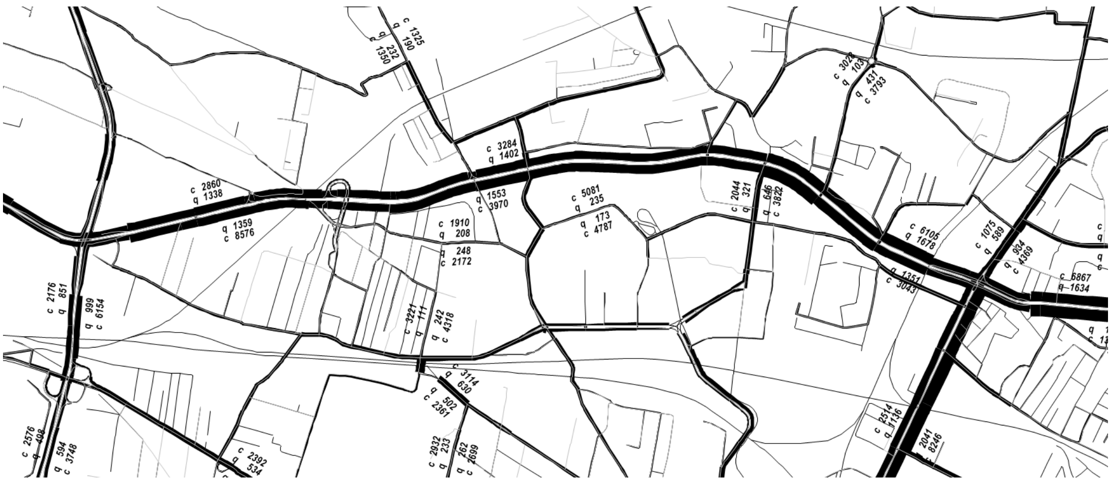
9.1. Abstrakt

Praca zawiera autorskie metody określania stanu sieci drogowej w sytuacjach nietypowych. Zaproponowano tu dwie modyfikacje dostępnych metod określania stanu sieci tak, by mogły odwzorowywać również sytuacje nietypowe. W szczególności, by możliwe było określenie stanu sieci w następstwie zdarzeń nieoczekiwanych: wypadków, zamknięć, demonstracji, awarii, itp. Cel ten osiągnięto dzięki uwzględnieniu zjawiska zmiany trasy przejazdu przez użytkownika w modelu rozkładu ruchu na sieć.

W pracy przedstawiono dwie nowe metody makroskopowego dynamicznego rozkładu ruchu na sieć (DTA). W przeciwieństwie do istniejących metod, zaproponowane metody pozwalają określać stan sieci transportowej w następstwie zdarzeń nieoczekiwanych. Przeprowadzona analiza algorytmu dynamicznego rozkładu ruchu na sieć pokazała możliwe sposoby uwzględnienia sytuacji nietypowych i ich wpływu. Umożliwiło to sformułowanie dwóch metod, które określają stan sieci po zdarzeniu nieoczekiwanym. Model przyswajania informacji reprezentuje faktyczne zachowanie użytkownika przy zmianie trasy w reakcji na zdarzenie nieoczekiwane i w realistyczny sposób określa stan sieci. Model przesuwającego się horyzontu upraszcza zjawisko, ale dzięki swojej strukturze może być używany w systemach czasu rzeczywistego. Obydwie metody wdrożono w środowisku dynamicznego modelowania ruchu (DTA). Dodatkowo, w pracy pokazano sposoby obserwacji zjawiska zmiany trasy: bezpośrednie (ścieżki) i pośrednie (potoki), oraz procedurę analizy danych pozwalającą sformułować problem estymacji dla zaproponowanych metod.

9.2. Problem

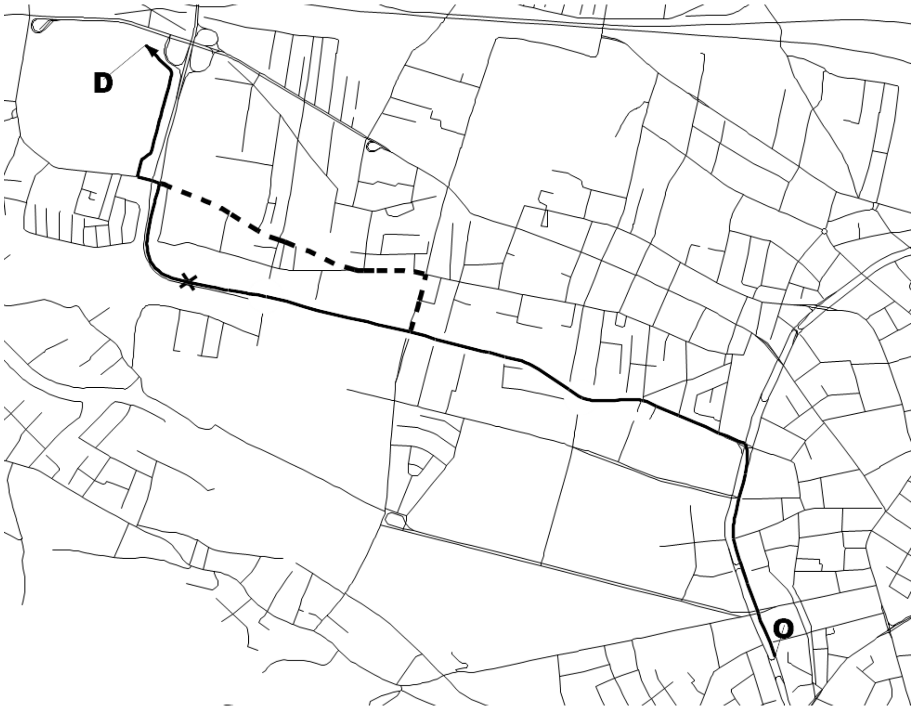
Miejska sieć transportowa ma swój ustalony stan. Jej użytkownicy realizują swoje podróże w określony sposób; dobierając cele podróży, środki transportu i trasy przejazdu. Codziennie planują łańcuch podróży tak, by podróżowanie wiązało się z jak najmniejszym kosztem: by było jak najkrótsze i jak najmniej uciążliwe. Z czasem użytkownicy nabierają doświadczenia i wiedzą, jakie decyzje podjąć by uniknąć opóźnień i strat czasu. Sieć osiąga wówczas równowagę (equilibrium), którą Wardrop (Wardrop, 1952) wyraził jako stan w którym żaden z użytkowników nie może podjąć decyzji, która wiązałaby się dla niego z mniejszym kosztem niż ta którą właśnie podjął (w szczególności decyzji co do trasy przejazdu).



Rysunek 1 Przykładowy stan sieci opisany poprzez: przepływ pojazdów *q* i warunki tego przepływu *c*

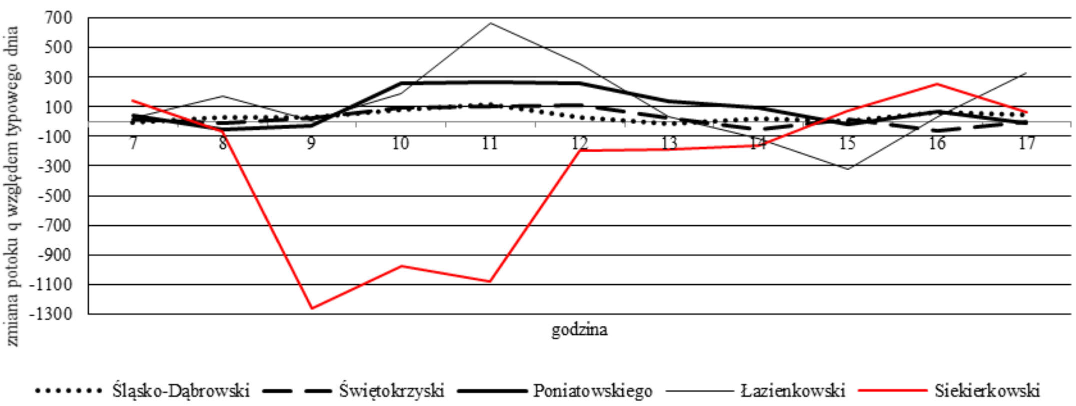
Jednak w zatłoczonych miejskich sieciach drogowych stan równowagi często zostaje zaburzony. Codziennością są zdarzenia drogowe przez które podróże nie mogą być realizowane tak, jak były zaplanowane. Jeśli na planowanej trasie przejazdu zdarzy się coś nieoczekiwanego, to opóźni to czas dotarcia do celu i zwiększy koszty podróży. Może to być wypadek samochodowy, roboty drogowe, nowy plan sygnalizacji, demonstracja, czy wydarzenie sportowe. Jeśli jest ono nieoczekiwane, to burzy plany podróży.

Użytkownicy w sieci transportowej zachowują się racjonalnie: minimalizują koszty i maksymalizują oczekiwaną użyteczność podróży (Bernoulli, 1783). Racjonalne decyzje są podejmowane nie tylko w momencie wyboru trasy, ale również przy decyzji o jej zmianie. Użytkownik w trakcie podróży może zrewidować swoje postanowienia i podjąć lepsze decyzje. Tak może się stać gdy otrzyma informacje o niespodziewanym zdarzeniu, które wydłuży jego podróż. W momencie otrzymania informacji ponowi on proces planowania (wyboru trasy) i, być może, zmieni trasę na taką, która pozwoli uniknąć negatywnych konsekwencji (opóźnień). Sytuacja taka przedstawiona jest na rys. 2, gdzie użytkownik zmienia pierwotnie wybraną trasę, aby uniknąć negatywnych konsekwencji zdarzenia drogowego, o którym nie wiedział w momencie rozpoczynania podróży.



Rysunek 2 Typowa ścieżka (pogrubiona) łącząca źródło *O* i cel *D* podróży, na której miało miejsce zdarzenie nieoczekiwane ×. Użytkownik po otrzymaniu informacji o zdarzeniu nieoczekiwanym zmienia trasę na inną (linia przerywana) by uniknąć dodatkowych kosztów dotarcia do celu.

Analiza danych z ekranu Wisły w Warszawie potwierdza hipotezę o tym, że użytkownicy zmieniają ścieżki by uniknąć konsekwencji zdarzeń drogowych. Gdy Most Siekierkowski (jeden z ważniejszych) został zablokowany przez wypadek samochodowy, użytkownicy zaczęli używać alternatywnych mostów. Stacje ciągłego pomiaru ruchu zarejestrowały wówczas zwiększony potok pojazdów na moście Łazienkowskim, Poniatowskiego i Świętokrzyskim. Pojawili się tam użytkownicy, którzy zazwyczaj przekraczają Wisłę mostem Siekierkowskim, dotyczyło to około 20% z nich. Wyniki tych obserwacji przedstawiono na rys. 3, gdzie pokazano wpływ zdarzenia nieoczekiwanego na zmianę typowych potoków pojazdów. Przez most Łazienkowski w dwie godziny po zdarzeniu przejechało o 700 pojazdów na godzinę więcej niż zazwyczaj. To duży wzrost, który znacząco zmienia stan sieci.



Rysunek 3 Zmiana przepływu pojazdów przez mosty w Warszawie w sytuacji, gdy Most Siekierkowski był zamknięty

Obserwacje te potwierdzają tezę, że użytkownicy reagują na zdarzenia nieoczekiwane i zmieniają trasę. W następstwie zdarzenia nieoczekiwanego można się więc spodziewać nie tylko korków bezpośrednio związanych ze zdarzeniem, ale również innego schematu przepływu pojazdów przez sieć. Gdy faktyczny stan sieci (czasy i koszty przejazdu) znacząco odbiega od typowego, użytkownik może zareagować zmieniając ścieżkę na bardziej atrakcyjną (o mniejszym koszcie dotarcia do celu). Reprezentacja tego zjawiska jest szczególnie trudna, gdy informacja dociera do użytkownika już po rozpoczęciu podróży. W języku angielskim problem nazywany jest *rerouting*, przez co rozumie się zmianę pierwotnie wybranej przez użytkownika ścieżki po otrzymaniu informacji o zdarzeniu. Zjawisko to jest kluczowe dla opisu stanu sieci w następstwie zdarzenia nieoczekiwanego.

Zarządzający systemem transportowym ma obecnie całą paletę środków z których może korzystać, by regulować przepływ pojazdów przez sieć i reagować na sytuacje nietypowe. Są to:

- informacje przekazywane do użytkowników (np. przez internet, radio);
- znaki zmiennej treści:
    - ograniczające prędkość,
    - zamykające pasy,
    - informujące o stanie sieci;
- zmiana planów sygnalizacji (optymalizująca przepływ pojazdów wzdłuż arterii) itp.

Aby skutecznie zarządzać ruchem drogowym środki te muszą być dopasowane do aktualnego stanu sieci, czyli do faktycznego przepływu pojazdów przez sieć. Wobec tego pojawia się potrzeba określenia stanu sieci nie tylko dla stanu równowagi, ale również sytuacji nietypowych i ich następstw.

Znaczenie tego problemu jest o tyle istotne, że zdarzenia nieoczekiwane występują powszechnie w miejskich sieciach drogowych i, paradoksalnie, to co uznawane jest za typowy stan miejskiej sieci transportowej, występuje niezwykle rzadko. Według danych z Systemu Ewidencji Wypadków i Kolizji w Krakowie w 2012 roku miało miejsce prawie 9000 zdarzeń, co daje średnio około 30 zdarzeń na typowy dzień roboczy. Każde z tych zdarzeń wpływa na stan sieci. W związku z tym koniecznością wydaje się możliwość opisu stanu sieci w sytuacji nietypowej.

W praktyce coraz powszechniejszy jest dynamiczny opis stanu sieci, w którym przepływ pojazdów i koszty z nim związane są wyrażone jako zmienne zależne od czasu. Zjawiska zachodzące w miejskiej sieci transportowej mają dynamiczną naturę: zarówno potoki pojazdów, jak i warunki ruchu zmieniają się w czasie (np. w ciągu doby). Właśnie dlatego metody dynamicznego rozkładu ruchu (Dynamic Traffic Assignment - DTA) zyskują popularność i coraz częściej są stosowane do modelowania przemieszczeń w miastach.

Sposobem rozwiązania problemu stawianego w pracy jest stworzenie dynamicznego modelu ruchu uwzględniającego zdarzenia nieoczekiwane. Jednak, jak pokazano w opisie aktualnego stanu wiedzy (rozdział 3.3), dostępne metody modelowania ruchu nie są w stanie uchwycić zjawiska *rerouting* i w realistyczny sposób określić stan sieci w następstwie zdarzeń nieoczekiwanych.

9.3. Cel i tezy pracy

W sytuacjach nietypowych (np. wypadków, zatorów, czy demonstracji) istotna część użytkowników sieci wybierze inną trasę niż zazwyczaj, zmieniając obciążenie sieci. Pożądane jest określenie jaki będzie stan sieci drogowej gdy część użytkowników, świadoma zdarzenia nieoczekiwanego, zmieni pierwotnie wybraną trasę.

Celem pracy jest, wobec tego, stworzenie metody pozwalającej na określenie stanu sieci (w sensie ustalenia przepływu potoków ruchu przez sieć w czasie) dla zadanego zdarzenia nieoczekiwanego, np. wypadku drogowego. Zaproponowana metoda powinna być zgodna z charakterystyką zjawiska obserwowaną w rzeczywistości, oraz powinna umożliwiać łatwą weryfikacje na podstawie dostępnych pomiarów.

W pracy przyjęte zostały następujące tezy badawcze:

1. Teza naukowa: Użytkownicy miejskiej sieci drogowej reagują na informacje o zdarzeniach wpływających na ich drogę do celu i mogą zmieniać ścieżkę w trakcie podróży. Możliwy i pożądany jest opis tego zjawiska i jego wpływu na stan sieci.
2. Teza praktyczna: Model popytu w dynamicznym modelu ruchu może być rozszerzony tak, by uwzględniał reakcję użytkowników na zdarzenia nieoczekiwane. Modyfikacja taka może zostać wprowadzona bez znacznego zwiększenia czasu obliczeń modelu.

9.4. Zakres pracy

Praca niniejsza rozpoczyna się od opisu problemu i omówienia jego znaczenia praktycznego. Po ogólnym zdefiniowaniu dynamicznych zjawisk zachodzących w sieci transportowej wprowadzone zostaje pojęcie stanu sieci wraz z omówieniem jak zdarzenie nieoczekiwane wpływa na stan sieci.

W przeglądzie literatury pokazano tło badawcze problemu. Opisano stan badań i metod dynamicznego rozkładu ruchu na sieć. W szczególności skupiono się na sposobie modelowania zachowania użytkowników w sieci transportowej, głównie wyboru ścieżki. Pokazano dostępne metody wyboru ścieżki i powszechnie przyjmowane założenia. Dokonano przeglądu dotychczasowych badań i propozycji rozwiązania problemu zmiany trasy przejazdu, co pozwoliło na identyfikacje luki badawczej: brak dynamicznego modelu rozkładu ruchu określającego stan sieci w następstwie zdarzenia nieoczekiwanego. Co można wyrazić równoważnie jako brak narzędzi opisujących stan sieci niebędącej w stanie równowagi. Na tej podstawie zdefiniowano problem w kontekście dynamicznego modelowania ruchu.

Proponowane rozwiązania rozszerzają metody dynamicznego rozkładu ruchu na sieć, co wymaga obszernego wprowadzenia teoretycznego. W rozdziale 4 przedstawiono podstawowe definicje i algorytmy, omówiono sposób rozwiązania problemu dynamicznego rozkładu ruchu na sieć. Nacisk położono na opis modelu popytu, w szczególności skupiono się na modelach wyboru trasy i dynamicznej propagacji ruchu przez sieć. Dla lepszego zrozumienia proponowanych metod wprowadzono i zdefiniowano hybrydowy model wyboru trasy, który pozwala na zmianę percepcji kosztów w trakcie podróży (np. po otrzymaniu informacji o

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zdarzeniu). Dzięki temu możliwe było formalne zdefiniowanie proponowanych rozwiązań autorskich w rozdziale 5.

W pierwszej kolejności opisano metodę przyswajania informacji (Information Comply Model - ICM), która jest bardziej intuicyjna i pozwala na uchwycenie istoty zjawiska. Po opisie założeń i przedstawieniu ogólnej idei omówiono części składowe: model uświadamiania i model reakcji. Formalnie zdefiniowano model uświadamiania (z procesami rozprzestrzeniania się informacji z różnych źródeł) oraz model podjęcia decyzji o zmianie trasy (z definicją użyteczności zmiany trasy i sformułowaniem dwumianowego modelu logitowego). Po zdefiniowaniu dwóch modeli składowych wprowadzone zostaje rozszerzenie modelu propagacji potoków modelu DTA poprzez łańcuch Markova, który modeluje procesy uświadamiania i zmiany trasy przez użytkowników. Opis metody kończy przedstawienie sposobu integracji jej z dynamicznym modelem ruchu. Działanie metody ilustrują przykłady obliczeniowe, w których pokazano szczegółowy opis działania z uwzględnieniem różnych źródeł informacji, sposobu informowania, wrażliwości, itp.

Druga metoda proponowana w pracy, metoda przesuwającego się horyzontu jest definiowana najpierw w zastosowaniu ogólnym (głównie w procesach planowania), a następnie przedstawiona jest propozycja zastosowania jej w modelowaniu ruchu. Pokazano jak problem DTA można rozwiązać metodą przesuwającego się horyzontu z użyciem hybrydowego modelu wyboru trasy. Na tej podstawie przedstawiono podstawowy algorytm metody RH-DTA i pokazano jak stosować go do określania stanu sieci. Kolejne przykłady pokazały możliwości zastosowania podstawowego algorytmu w coraz bardziej złożonych sytuacjach: jedno zdarzenie, spóźniona informacja o zdarzeniu, informacja dostępna tylko dla części użytkowników, różne grupy użytkowników, wiele zdarzeń, aż do pełnego algorytmu nadającego się do zastosowań w systemach czasu rzeczywistego.

W ostatniej części pracy przedstawiono metody kalibracji i weryfikacji zaproponowanych modeli. Zidentyfikowano dwa źródła obserwacji, które mogą być wykorzystane dla potrzeb kalibracji: potoki i ścieżki. Zaproponowano analizę formalną kolejno: ścieżek, a następnie potoków, która pozwala w sposób ilościowy opisać zjawisko zmiany trasy w czasie i przestrzeni. Na tej podstawie sfomułowano problem estymacji metody ICM pozwalający uzyskać zgodność wyników modelu z wynikami obserwacji. Analiza potoków ruchu mierzonych w stanie równowagi oraz w sytuacji zamknięcia jednego z mostów w Warszawie potwierdziła hipotezy o zachowaniu użytkowników założone w zaproponowanych metodach.

Pracę kończy podsumowanie, sformułowanie wniosków z przeprowadzonych analiz i propozycja dalszych kierunków badań.

9.5. Aktualny stan wiedzy i luka badawcza

Dzięki niedawnym postępom algorytmicznym dynamiczne modele ruchu (DTA) mogą być stosowane dla sieci dużych metropolii (takich jak np. Kraków). Dzięki znacznemu zmniejszeniu czasu obliczeń (Gentile, 2010) pojawiły się również nowe możliwości zastosowania dynamicznych modeli ruchu. Poza klasycznym zastosowaniem do celów analitycznych (np. symulacja i testowanie proponowanych wariantów) pojawiła się możliwość zastosowania operacyjnego (np. w centrach zarządzania ruchem działających w czasie rzeczywistym) (Meschini i Gentile, 2010). Jednak użycie dostępnych modeli dynamicznych do działania operacyjnego w czasie rzeczywistym wymaga rozwiązania szeregu problemów (związanych np. ze zbieraniem i wykorzystaniem danych rzeczywistych, ograniczonym czasem obliczeń, przekazywaniem informacji użytkownikom, uwzględnieniem zjawisk losowych, itd.). Niniejsza praca dotyczy jednego z nich: rozszerzenia modelu popytu tak, by reprezentował nie tylko typowy dzień, ale również sytuacje nietypowe, w szczególności nieoczekiwane zdarzenia i ich konsekwencje.

Nieoczkiwane zdarzenie jest tu definiowane jako niespodziewane zdarzenie wpływające na przepływ pojazdów w sieci. Może to być np. wypadek, zwężenie, zamknięcie, przebudowa, demonstracja, impreza sportowa, itd. Precyzyjną definicję zdarzeń nieoczkiwanych podaje (Dobler, 2013). Nieoczekiwane zdarzenia wpływają zarówno na podaż, jak i na popyt w sieci transportowej, sieć przestaje być w stanie równowagi. Po stronie podażowej (sieć transportowa) pojawiają się dodatkowe utrudnienia. Pojawiają się one nie tylko w miejscu nieoczekiwanego zdarzenia, ale również w pozostałych częściach sieci: przed miejscem zdarzenia (w formie kolejek rosnących w przeciwną stronę niż kierunek ruchu) oraz za nim (w formie zmniejszonych potoków pojazdów wypływających z miejsca zdarzenia). Co istotne, utrudnienia zmieniają się w czasie, kolejki pojawiają się w momencie zdarzenia i zazwyczaj rosną w czasie. Przy założeniu niezmiennego popytu zjawiska po stronie podażowej można stosunkowo łatwo określić korzystając z modeli przepływu ruchu, co pokazał (Corthout i in., 2009). Znacznie trudniej uchwycić zjawisko po stronie popytu, czyli reakcje na zdarzenia nieoczekiwane (w tym zjawisko rerouting). W literaturze zjawisko *rerouting* nazywane jest również: en-route rerouting (Snowdon i in., 2012), adaptation (Gao i in. 2010), lub hyperpath (Trozzi i in. 2013). Warto podkreślić, że praca nie dotyka bliźniaczego tzw. problemu *route-swapping* (Watling i Hazelot, 2003), czyli zmiany ścieżki przed rozpoczęciem podróży.

Dla zobrazowania problemu warto posłużyć się jedną z definicji rozkładu ruchu, w której jest on definiowany jako proces uczenia w ciągu kolejnych dni (ang. *day-to-day*, Watling i Hazelton, 2003). Taka interpretacja pozwala uchwycić istotę problemów przy modelowaniu zjawiska rerouting. W szczególności pozwala ona unaocznić dwa założenia, które wymagają weryfikacji:

1) założenie o tym, że użytkownicy korzystają z wyników poprzedniej iteracji, tzn. znają wyniki uprzednio podjętych decyzji i mogą je weryfikować. W ujęciu *day-to-day* iteracje rozumiane są jako kolejne dni, a modyfikacja jest rozumiana jako uczenie się z doświadczeń dnia poprzedniego. Równowaga, to stan w którym użytkownicy nauczyli się już zjawisk zachodzących w sieci i są usatysfakcjonowani swoimi decyzjami na tyle, że ich decyzje w kolejnym dniu nie będą się różnić od tych z dnia poprzedniego. Takie założenie nie może być przyjęte dla zdarzeń nieoczekiwanych, gdzie użytkownicy nie mają szansy na nabranie doświadczenia. W sytuacjach zdarzeń nieoczekiwanych zamiast wielu iteracji w których udoskonalane są wybory mamy jedną możliwość podjęcia decyzji przy dużej niewiadomej konsekwencji podejmowanych decyzji.

2) Założenie niezmienności danych wejściowych. W DTA proces uczenia się oparty jest o założenie, że kolejny dzień będzie identyczny jak dzisiejszy w sensie parametrów sieci i wielkości popytu. Każda kolejna iteracja problemu DTA symulowana jest dla tych samych parametrów sieci i tej samej więźby ruchu. W związku z tym DTA jest odpowiednim narzędziem do określania stanu sieci dla powtarzalnego i przewidywalnego dnia (np. typowy dzień roboczy, typowy piątek, typowy dzień świąteczny) dla którego użytkownicy mieli szansę nauczyć się swoich zachowań (dzień ten powtarza się). Ponadto nawet jeśli w danym typowym dniu rzeczywiste warunki odbiegną od typowych zachowanie użytkowników (ścieżki) nie zmieni się dopóki użytkownicy nie otrzymają informacji, że faktyczne warunki ruchu różnią się od typowych. Założenie o niezmienności danych wejściowych również nie może być przyjęte dla sytuacji nietypowych, gdzie nieoczekiwane zdarzenia z definicji wpływają na parametry sieci (np. zmniejszenie przepustowości).

Proponowane rozwiązania uwzględniają powyższe problemy i proponują alternatywne podejście zgodne z charakterystyką zjawiska *rerouting*.

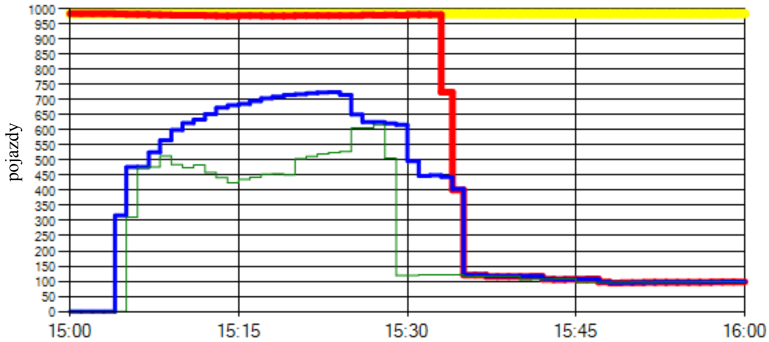
9.6. Narzędzia

Zaproponowane w pracy doktorskiej metody są rozwinięciem modelu TRE (Traffic Realtime Equilibrium), dynamicznego modelu ruchu działającemu w czasie rzeczywistym w dużych aglomeracjach miejskich. Model ten stworzony jest przez prof. Guido Gentile wraz z zespołem z uniwersytetu La Sapienza w Rzymie i rozwijany przez firmę SISTeMA, spin-off tego uniwersytetu. Rozdział 3.5 pracy przedstawia definicje i algorytmy Dynamicznego Modelu Rozkładu Ruchu (DTA), skrótowo przedstawione poniżej.

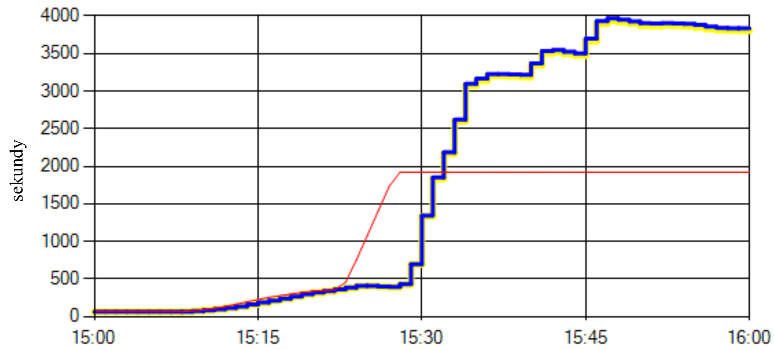
Rozwiązaniem problemu DTA jest określenie stanu sieci, a więc przepływu potoków w sieci i warunków ruchu związanych z tym przepływem. DTA rozwiązywane jest metodami rozkładu ruchu na sieć – zazwyczaj wykorzystującymi koncepcje równowagi w sieci, tzw. *equilibrium* w którym koszty przejazdu są w stanie równowagi (Wardop, 1952). W ujęciu dynamicznym *equilibrium Wardop'a* jest rozumiane jako stan w którym żaden z użytkowników nie znajduje lepszej, alternatywnej trasy, ani czasu rozpoczęcia podróży. Stan ten numerycznie osiągany jest przez rozwiązanie problemu DTA, zazwyczaj opisanego w formie problemu punktu stałego (Banach, 1922). W procesie tym wybory użytkowników (ścieżki i czas rozpoczęcia podróży) są cyklicznie modyfikowane na podstawie kosztów uzyskanych w wyniku decyzji podjętych w poprzedniej iteracji aż do stabilizacji procesu. W pracy zastosowano makroskopowe ujęcie DTA zaproponowane przez (Bellei i in., 2005). Problem DTA składa się tam z dwóch części: popyt będący funkcją podaży i podaż będącą funkcją popytu. W części popytowej określane są optymalne ścieżki przejazdu na podstawie kosztów i czasów przejazdu uzyskanych z modelu podaży. Następnie w części podażowej określane są koszty i czasy przejazdu dla potoków (zagregowanych ścieżek) uzyskanych w modelu popytu. Proces powtarzany jest cyklicznie aż do uzyskania stabilizacji.

W pracy skorzystano z aktualnej wersji algorytmu, zgodnej z definicją z (Gentile i in., 2013). Wykorzystany model popytu oparty jest o sekwencyjny model wyboru ścieżki (Gentile, 2006) będący rozwinięciem probabilistycznego algorytmu (Dial, 1971) z logitowym modelem wyboru ścieżki (Ben-Akiva i Lerman, 1985). Podaż modelowana jest uogólnionym modelem przepływu – GLTM (opisany w języku polskim w Kucharski, 2013) z pełną aktualną definicją w (Gentile, 2010). W pracy skupiono się na części popytowej DTA, w szczególności na modelu wyboru ścieżki (Route Choice Model - RCM) i modelu propagacji (Network Flow Propagation - NFP), model przepływu ruchu nie jest przedmiotem badań. Poza podstawowymi definicjami modelu DTA opisano hybrydowy model wyboru ścieżki (Mahmassani i in., 1994), (Pel, 2009), który jest punktem wyjścia do formalnego wyprowadzenia obydwu zaproponowanych metod.

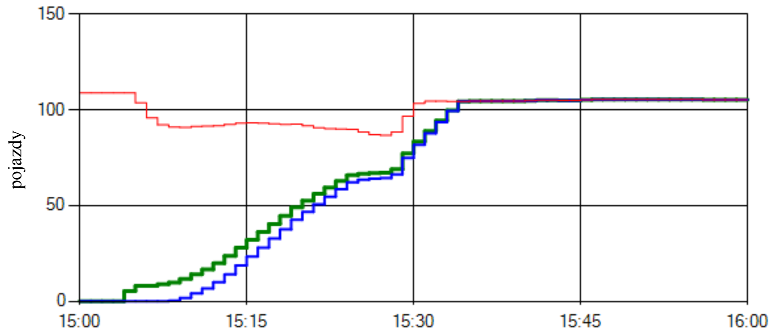
Przykładowe wyniki dynamicznego modelu ruchu dla pojedynczego odcinka przedstawiają rysunki 4 do 6, gdzie pokazano podstawowe charakterystyki odcinka (potoki, przepustowości, czasy, koszty, zapełnienie i długość kolejki) i ich zmienność w czasie.



Rysunek 4 Zmienność w czasie potoku pojazdów wjeżdżających (niebieski) i wyjeżdzających (zielony) z danego odcinka w czasie, oraz przepustowość początkowa (czerwony) i końcowa (żółty) odcinka [pojazdy].



Rysunek 5 Zmienność czasu przejazdu (niebieski) i kosztu przejazdu (czerwony) odcinka w czasie [sekundy].



Rysunek 6 Zmienność długości kolejki (niebieski), liczby pojazdów (zielony) i pojemności (czerwony) odcinka w czasie [pojazdy].

9.7. *Rozwiązania*

W pracy zaproponowano dwie nowe metody rozwiązania problemu dynamicznego rozkładu ruchu na sieć, szeroko opisane w rozdziale 0. Są to autorskie propozycje autora rozprawy, bazują one na klasycznych metodach dynamicznego rozkładu ruchu opisanych powyżej. Jednak w odróżnieniu od nich pozwalają one uwzględnić zdarzenia nieoczekiwane i modelować ich wpływ na stan sieci.

W pierwszej metodzie (Information Comply Model – model przyswajania informacji) skupiono się na procesie decyzyjnym jaki towarzyszy zmianie trasy przejazdu. W szczególności odwzorowano procesy rozprzestrzeniania informacji i podejmowania decyzji o zmianie trasy. Wyszczególniono dwie fazy: uświadamianie i reakcja. Użytkownik uświadamia sobie zdarzenie na podstawie otrzymywanych informacji, a następnie podejmuje decyzję na podstawie szacowanych zysków i strat. W procesie uświadamiania uwzględniono wiele źródeł informacji i związanych z nimi procesów rozprzestrzeniania się (internet, sieci społecznościowe, radio, znaki zmiennej treści, obserwacja). Pozwala to w realistyczny sposób odwzorować proces zmiany trasy z uwzględnieniem faktycznego procesu decyzyjnego, gdzie informacja poprzedza reakcje, a reakcja jest efektem racjonalnej decyzji mającej na celu minimalizację kosztów. Proces decyzji o zmianie trasy zamodelowano jako dwumianowy proces wyboru dyskretnego opisany modelem logitowym. Użyteczność zmiany trasy w danym miejscu i w danym czasie opisana jest jako funkcja możliwych zysków i możliwych strat. Na podstawie użyteczności określa się prawdopodobieństwo zmiany trasy w danym punkcie. Aby dwie główne składowe ICM: model uświadamiania i model reakcji mogły być zastosowane w Dynamicznym Modelu Ruchu wprowadzono trzy możliwe stany w których może znajdować się użytkownik: nieświadomy, uświadomiony i po zmianie trasy. Te trzy stany tworzą łańcuch Markova, gdzie przejścia pomiędzy stanami opisuje prawdopodobieństwo kolejno: uświadomienia i zmiany trasy. Technicznie odbywa się to poprzez zmodyfikowanie modelu propagacji potoku pojazdów w czasie tak, by propagacja pojazdów w czasie i przestrzeni uwzględniała reakcje kierowców na zdarzenia nieoczekiwane. Pozwala to określić stan sieci w następstwie zdarzeń nieoczekiwanych.

Druga metoda (Rolling-Horizon – przesuwający się horyzont) powstała z myślą o systemach czasu rzeczywistego, w szczególności centrach zarządzania ruchem w systemach ITS. W systemach tych kluczowy jest czas reakcji i możliwość stosowania w dowolnej sytuacji drogowej (np. wiele zdarzeń na raz). Wykorzystano tu metodę przesuwającego się horyzontu i uproszczono reprezentację procesu decyzyjnego. Wynik powstaje poprzez rozwiązanie sekwencji kolejnych problemów dynamicznego rozkładu ruchu na sieć, pomiędzy którymi zapamiętany zostaje chwilowy stan sieci (położenie pojazdów). Pozwala to na spójność i ciągłość wynikowego stanu sieci, pozostawiając przy tym możliwość zmiany trasy przejazdu przez kierowców w momencie otrzymania informacji. W odróżnieniu od modelu przyswajania informacji, metoda przesuwającego się horyzontu pozwala uwzględnić wiele zdarzeń nieoczekiwanych naraz. Metoda przesuwającego się horyzontu zastosowana w czasie rzeczywistym pozwala na wiarygodne prognozowanie stanu sieci dla dowolnej sytuacji w sieci (wiele zdarzeń pojawiających się w czasie rzeczywistym), co jest kluczowe dla zastosowania w systemach czasu rzeczywistego.

Obydwie metody wdrożono w środowisku dynamicznego modelowania ruchu (Dynamic Traffic Assignment – DTA) możliwym do stosowania w czasie rzeczywistym. Metody zaimplementowane są w systemie TRE (Traffic Realtime Equilibrium), będącym trzonem środowiska Optima, produkowanego przez Uniwersytet La Sapienza w Rzymie, firmę SISTeMA, oraz grupę PTV. Metoda przesuwającego się horyzontu dostępna jest zarówno w pakiecie Optima (do aplikacji w czasie rzeczywistym), jak i w środowisku PTV Visum (do aplikacji w celach strategicznych). Model przyswajania informacji jest dostępny w środowisku PTV Visum do celów analitycznych.

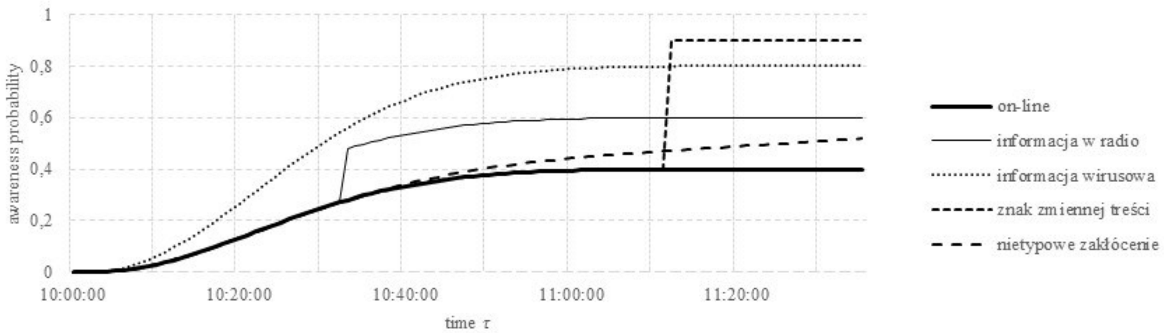
9.7.1. ICM - model przyswajania informacji

Model Przyswajania Informacji reprezentuje zachowanie użytkowników (kierowców) w sytuacjach nietypowych poprzez odwzorowanie procesu otrzymywania informacji i reakcji na nią. Użytkownicy najpierw uświadamiają sobie o zdarzeniu nieoczekiwanym a potem na nie reagują – taka jest struktura zjawiska zmiany trasy i taka też jest struktura modelu przyswajania informacji. Dzięki temu reprezentacja zjawiska realistyczna i może być łatwo interpretowana i weryfikowana. Model ma dwa kluczowe elementy: model uświadamiania o zdarzeniu i model reakcji na nie. Model uświadamiania uwzględnia wiele źródeł jednocześnie: informacje (internet, radio, znaki zmiennej treści), oraz obserwowanie nietypowego stanu sieci przez doświadczonych użytkowników. Proponowany jest tu probabilistyczny model określający oczekiwaną liczbę uświadomionych użytkowników jako funkcję czasu i położenia w sieci. Wykorzystuje on rozkłady prawdopodobieństwa otrzymania informacji z różnych źródeł jako funkcje czasu i/lub przestrzeni (zob. rysunek 7.). Tylko uświadomieni użytkownicy mogą podjąć decyzję i zareagować na zdarzenie – zmienić trasę. Liczba użytkowników zmieniających trasę jest określana zależnie od czasu i położenia w sieci na podstawie użyteczności (zob. przykład obliczeniowy na rysunku 8.). Użyteczność jest funkcją dwóch czynników: potencjalnych zysków („O ile szybciej dotrę do celu, jeśli tu i teraz zmienię trasę?") i strat („O ile dłużej zajmie mi podróż, jeśli nie zmienię tu i teraz trasy?"). Obliczona wartość użyteczności jest wykorzystana w logitowym, dwumianowym modelu wyboru dyskretnego, który określa prawdopodobieństwo zmiany trasy jako funkcję użyteczności.

Aby dwa opisane wyżej modele mogły być użyte w algorytmie DTA zmodyfikowano model propagacji ruch przez sieć w DTA. Całkowity potok pojazdów podzielono na trzy stany: nieświadomy, uświadomiony i po zmianie trasy. Prawdopodobieństwa przejścia pomiędzy stanami, czyli uświadamianie i reakcja są wyrażone w formie łańcuchu Markova obliczanego na każdym węźle sieci. Modyfikacja zachowuje strukturę pierwotnego algorytmu propagacji, co umożliwia bezpośrednie użycie w algorytmie DTA. Tak zmodyfikowany model propagacji ruchu rozwiązuje problem doktoratu, czyli pozwala uwzględnić reakcje użytkowników na zdarzenie nieoczekiwane w modelowaniu stanu sieci.

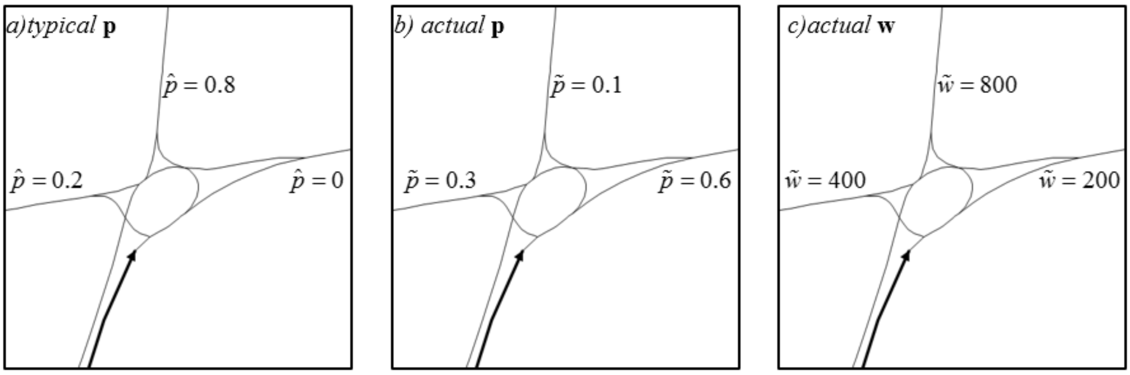
W tabeli 1 i na rysunku 9. pokazano przykład obliczeniowy modelu ICM dla przykładowego potoku 100 pojazdów przemierzającego korytarz siedmiu kolejnych punktów decyzyjnych i w czasie τ. 100 użytkowników rozpoczyna przejazd w czasie przez korytarz drogowy, w momencie rozpoczęcia podróży w źródle o są oni nieświadomi zdarzenia (oznaczeni przez $\hat{q}$). W miarę upływu czasu, w wyniku działania modelu uświadamiania, użytkownicy zaczynają być świadomi zdarzenia, zmieniają stan na świadomy (oznaczony przez a). Prawdopodobieństwo uświadomienia obliczane jest dla wielu źródeł informacji: informacja w radio (*NEWS*)*,* znak zmiennej treści (*VMS*)*,* informacja dostępna on-line (*O-L*) i obserwacja nietypowego zakłócenia (*O*). Uświadomieni użytkownicy a szacują użyteczność zmiany trasy κ na podstawie potencjalnych zysków Δp i potencjalnych strat Δw, część z nich postanawia zmienić trasę – zmieniają stan ze świadomych a na tych, którzy zmienili trasę (oznaczeni przez $\tilde{\iota}$).

Model przyswajania informacji to narzędzie do opisu zjawiska zmiany trasy i uwzględnienia go w opisie stanu sieci. Model ten znajdzie zastosowanie w procesie planowania, analiz, określania strategii informowania użytkowników, itp. W zaproponowanej tutaj formie pozwala on na uwzględnienie jedynie jednego zdarzenia, co ogranicza jego stosowanie w systemach czasu rzeczywistego. Czas obliczeń w stosunku do modelu bazowego zwiększa się nieznacznie (około 25% zwiększenia czasu obliczeń dla sieci średniej wielkości). Stąd potrzeba generalizacji modelu tak, by mógł być stosowany w czasie rzeczywistym.

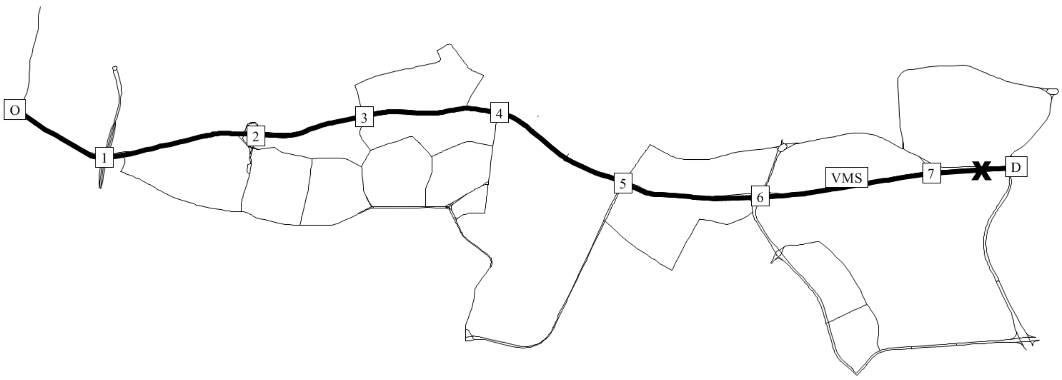


Rysunek 7 Skumulowane prawdopodobieństwo otrzymania informacji w czasie o zdarzeniu w czasie w bazowym wariancie otrzymania informacji on-line wzmacniane w kolejnych scenariuszach:

a. gdy dodatkowo podano informację w radio o 10:33,
b. gdy informacja rozprzestrzenia się ‘wirusowo’ – szybciej i z większym zasięgiem,
c. gdy na trasie przejazdu jest znak zmiennej treści informujący o zdarzeniu
d. gdy użytkownicy doświadczają konsekwencji zdarzenia (nietypowe opóźnienie).



Rysunek 8 Zmienne objaśniające potrzebne do obliczenia użyteczności zmiany trasy na przykładowym punkcie decyzyjnym - węzeł sieci do którego z dołu wjeżdża użytkownik (pogrubiona strzałka). Typowe (z lewej) i aktualne (w środku) prawdopodobieństwo wyboru ścieżki obliczone w modelu wyboru ścieżki dla sytuacji typowej (z lewej) i aktualnej (w środku), oraz aktualne koszty dotarcia do celu z danego węzła (z prawej).



A path (bold) connecting origin O with destination D, passing through the 7 descision points and one VMS sign; unexpected event marked with X.

Rysunek 9 Korytarz łączący źródło *O* i cel *D* podróży. W ostatniej części trasy ma miejsce zdarzenie nieoczekiwane.

Tabela 1. Przykład działania modelu ICM

Punkt decyzyjny		Potok			Model uświadamiania							Model decyzji		
i	τ	$\hat{q}$	a	$\tilde{\iota}$	ι	$\iota_a^{NEWS}(\tau)$	$\iota_a^{VMS}(\tau)$	$\iota_a^{O-L}(\tau)$	$M(\tau)$	$\iota_a^{O}(\tau)$	Δt_a	κ	Δp	Δw
1	7:00	100	0	**0**	**0%**	-	-	0%	1	-	-	-	0.00	1.00
2	7:10	97	3	**0**	**3%**	-	-	3%	1	-	-	**1%**	0.01	1.01
3	7:15	93	7	**0**	**3%**	-	-	3%	1	-	-	**1%**	0.01	1.00
4	7:20	62	37	**0**	**33%**	30%	-	4%	1.01	-	-	**0%**	0.00	1.00
5	7:25	60	29	**11**	**4%**	-	-	4%	1.01	-	-	**28%**	0.61	1.47
6	7:40	54	8	**27**	**10%**	-	-	10%	1.02	-	-	**78%**	0.76	2.65
7	8:20	5	1	**55**	**91%**	-	80%	6%	1.04	50%	0.5	**98%**	1.00	4.00

i - punkt decyzyjny
τ - czas w którym potok dociera do *i*-tego punktu decyzyjnego
$\hat{q}$ - potok nieświadomy
a - potok uświadomiony
$\tilde{\iota}$ - potok po zmianie trasy

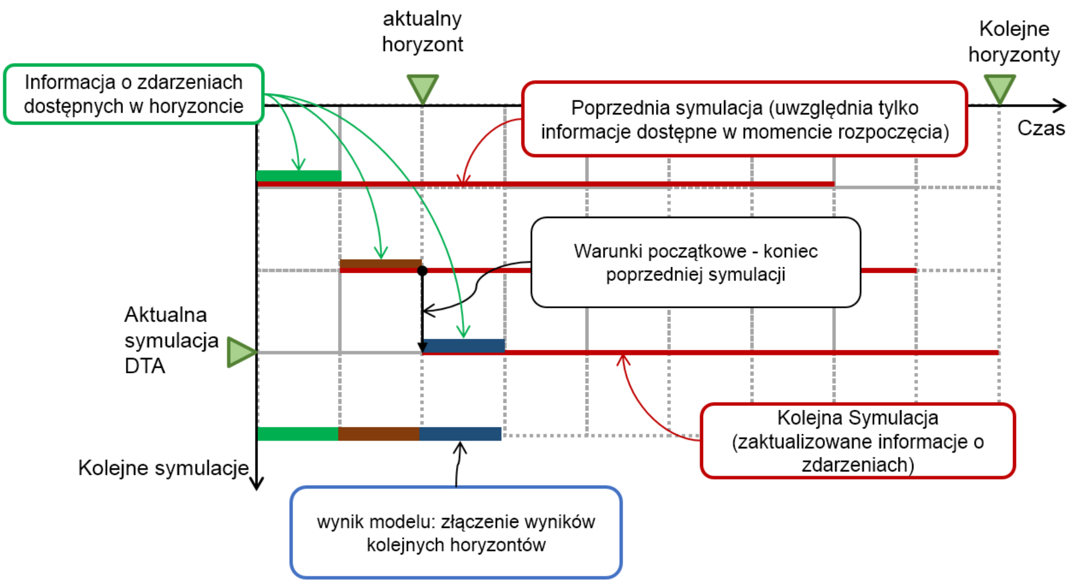
9.7.2. RH-DTA –Model przesuwającego się horyzontu

W rzeczywistych sieciach transportowych dużych aglomeracji zdarzenia są powszechne, a na stan sieci równocześnie może wpływać więcej niż jedno zdarzenie. Co więcej zdarzenia są nieoczkiwane nie tylko dla użytkowników, ale także dla zarządzającego ruchem. Stąd potrzeba zachowania reprezentacji zjawiska z modelu ICM przy poszerzeniu zakresu jego stosowania do systemów czasu rzeczywistego.

Narzędziem do osiągnięcia tego celu jest model przesuwającego się horyzontu, stosowany przy podejmowaniu decyzji w sytuacjach niepewności. W dynamicznie rozwijającym się, niepewnym otoczeniu decyzja jest weryfikowana i zmieniana za każdym razem gdy pojawia się nowa, aktualna informacja – gdy zmienia się horyzont wiadomych informacji o stanie sieci. Rozwijając koncepcję (Mahmassani i in., 1999) rozszerzono model DTA zakładając, że użytkownicy mogą zmienić swoją decyzję gdy otrzymają nową informację, inną od tej którą mieli w momencie rozpoczynania podróży. Decyzje co do wyboru trasy mogą być aktualizowane za każdym razem gdy pojawia się nowa informacja. Pojawia się wówczas nowy horyzont, który zastępuje poprzedni.

W pracy zaproponowano autorską metodę RH-DTA, rozszerzenie modelu DTA wykorzystujące koncepcję przesuwającego się horyzontu w makroskopowym, dynamicznym modelu ruchu pozwalające określić stan sieci w następstwie zdarzenia nieoczekiwanego. Stan sieci uzyskiwany jest poprzez złączenie kolejnych rozwiązań problemu DTA dla kolejnych horyzontów. Wykorzystano tutaj hybrydowy model wyboru ścieżki (opisany w rozdziale 4.3), a spójność pomiędzy kolejnymi horyzontami uzyskuje się zapamiętując liczbę i cel pojazdów w momencie zmiany horyzontu.

Podstawowy algorytm RH-DTA jest stopniowo rozszerzany w kolejnych przykładach obliczeniowych ilustrujących problem. Poczynając od najprostszej sytuacji: jednego zdarzenia o którym od razu wiedzą wszyscy użytkownicy, aż do najbardziej ogólnej sytuacji: wiele zdarzeń pojawiających się w czasie rzeczywistym o których wie tylko część użytkowników, a informacja pojawia się z opóźnieniem.



Rysunek 10 Schemat działania modelu przesuwającego się horyzontu, kolejne symulacje DTA korzystające z dostępnych w danym horyzoncie informacji i rozpoczynające się od stanu zakończenia poprzedniej symulacji. Wynikowy stan sieci powstaje poprzez złączenie wyników kolejnych symulacji.

9.8. Obserwacje zjawiska i estymacja modeli

W ostatniej części pokazano sposoby obserwacji zjawiska zmiany trasy przejazdu w sieciach transportowych. Na podstawie możliwych źródeł obserwacji opracowano metody estymacji modeli zaproponowanych w pracy. Zidentyfikowano dwa potencjalne źródła obserwacji zjawiska: bezpośrednie i pośrednie. Bezpośrednie to zapis ścieżek przejazdu dla podróży źródło-cel zebranych dla reprezentatywnej próby podróży w mieście. W próbie powinny znaleźć się trajektorie obserwowane w sytuacji nietypowej (gdy na planowej trasie przejazdu ma miejsce zdarzenie nieoczekiwane). Pośrednio zjawisko obserwować można poprzez mierzenie potoków ruchu w sytuacjach nietypowych. W szczególności dobrym poligonem badawczym jest obserwacja pełnego ekranu dzielącego sieć transportową na dwie rozłączne części (tzw. cięcie grafu), na przykład ekran rzeki w mieście. Metoda obserwacji potoków jest mniej precyzyjna niż obserwacja ścieżek, ale jest jednocześnie znacznie bardziej dostępna.

Dla obydwu metod obserwacji przedstawioną formalną analizę wyników, która pozwala opisać zjawisko zmiany trasy przejazdu. Dla obserwacji bezpośrednich określono teoretyczne (modelowane) i obserwowane prawdopodobieństwo zmiany trasy w danym miejscu w danym czasie. Dla obserwacji pośrednich zdefiniowano typową i nietypową część zmierzonego potoku. Zarówno zaobserwowane prawdopodobieństwo zmiany trasy, jak i zaobserwowane nietypowe potoki mogą służyć jako zmienna objaśniana w problemie estymacji modelu zmiany trasy przejazdu. W odniesieniu do ścieżek teoretyczne prawdopodobieństwo zmiany trasy (uzyskane w modelu ICM) dopasowywane jest do prawdopodobieństwa obserwowanego w trajektoriach. W wypadku obserwacji potoków, z kolei, zamodelowane potoki nietypowe (potoki pojazdów zmieniających trasę przejazdu) są dopasowywane do zmierzonych potoków nietypowych. Dzięki temu możliwe było sformułowanie problemu estymacji metod zaproponowanych w pracy.

W celu pokazania zjawiska rerouting na przykładzie rzeczywistym przeprowadzono analizę potoków mierzonych na ekranie Wisły w Warszawie. Analizowano potoki z kilku dni typowych, oraz z dnia w którym most Siekierkowski był zamknięty. Pozwoliło to na obserwację zjawiska zmiany trasy przejazdu. Najważniejsze wnioski z przeprowadzonej analizy to: a) około 20% potoku będącego pod wpływem zdarzenia zmienia trasę przejazdu, b) udział pojazdów zmieniających trasę rośnie w czasie, c) użytkownicy ujawniają podejście strategiczne i d) wybierają ścieżki optymalne po zmianie trasy. Obserwacje te potwierdzają założenia co do zachowania użytkowników przyjęte przy definiowaniu modeli zmiany trasy przejazdu.

9.9. Wnioski

Praca doktorska pokazuje sposób rozwiązania problemu dynamicznego modelu ruchu w sytuacjach nietypowych. Dwa zaproponowane modele dynamicznego rozkładu ruchu na sieć pozwalają opisać stan sieci w sytuacji nietypowej, co do tej pory nie było możliwe. Modele są skonstruowane tak, by można je było stosować ramach istniejących dynamicznych modeli ruchu, również tych działających w czasie rzeczywistym. Stworzone modele są pierwszymi makroskopowymi modelami rozkładu ruchu na sieć, które pokazują jak użytkownicy adaptują trasę przejazdu. Zaproponowane modele uświadamiania pozwalają opisać prawdopodobieństwo otrzymania informacji w czasie w sytuacji gdy informacje są dostępne z wielu źródeł jednocześnie, co jest nowatorskim podejściem. Zaproponowany model podejmowania decyzji i propozycja funkcji użyteczności zmiany trasy pozwala na opis zjawiska zmiany trasy w formie funkcji kosztu podróży z wykorzystaniem zmiennych dostępnych w dynamicznym modelu ruchu, co do tej pory nie było bezpośrednio możliwe.

9.10. Dalsze kierunki badań

Pokazana w pracy procedura kalibracji zaproponowanych modeli powinna być przeprowadzona na obserwacjach z rzeczywistych sieci drogowych, aby zweryfikować czy zaproponowane modele odpowiadają rzeczywistemu zachowaniu kierowców. Będzie to jednak możliwe dopiero po przeprowadzeniu obserwacji ścieżek którymi podróżują użytkownicy w sytuacji nietypowej. Obserwacje takie muszą być przeprowadzone w mieście w którym w czasie rzeczywistym działa dynamiczny model ruchu. Zaproponowany model uświadamiania jest oparty o założenia, które powinny być zweryfikowane w oparciu o dane empiryczne. W szczególności założenie o niezależności źródeł informacji, oraz o rozkładach prawdopodobieństwa dla zidentyfikowanych źródeł informacji. Przyjęte założenie co do wyboru ścieżki przez użytkowników, którzy zmieniają trasę również powinno być dalej badane. Podobnie przyjęta w pracy hipoteza, że strategiczne decyzje kierowców (uwzględniające decyzje pozostałych kierowców) mogą być modelowane poprzez rozwiązanie kolejnych iteracji problemu dynamicznego modelu ruchu powinna być potwierdzona odpowiednim badaniem (np. w badaniu zachowań kierowców). Najistotniejszym jednak kierunkiem dalszych badań powinno być połączenie dwóch zaproponowanych modeli tak, by reprezentacja zjawiska z modelu przyswajania informacji mogła być użyta w środowisku czasu rzeczywistego dla wielu zdarzeń jednocześnie

9.11. Podsumowanie

W pracy doktorskiej zdefiniowano problem na podstawie zidentyfikowanej luki badawczej: brak dynamicznego modelu rozkładu ruchu określającego stan sieci w następstwie zdarzenia nieoczekiwanego. W zebranych obserwacjach potoków pojazdów przekraczających Wisłę w Warszawie potwierdzono tezę naukową, że użytkownicy miejskiej sieci drogowej reagują na informacje o zdarzeniach wpływających na ich drogę do celu i zmieniają ścieżkę w trakcie podróży. Na podstawie analizy teoretycznej dynamicznych modeli ruchu określono sposoby rozwiązania problemu i zaproponowano dwie metody. Obydwie zaimplementowano w dynamicznym modelu ruchu, przetestowano i przedstawiono wyniki. Potwierdzono tym samym tezę praktyczną, że model popytu w dynamicznym modelu ruchu może być rozszerzony tak, by uwzględniał reakcję użytkowników na zdarzenia nieoczekiwane.

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