TECHNICAL TRANSACTIONS 9/2018
CIVIL ENGINEERING

DOI: 10.4467/2353737XCT.18.134.8973
SUBMISSION OF THE FINAL VERSION: 9/8/2018

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The reliability of masonry structures – evaluation methods for historical buildings

Niezawodność konstrukcji murowych – metody oceny w budynkach historycznych

Abstract

In this paper, selected issues regarding the assessment of the reliability of masonry structures in existing buildings, including historical buildings, are presented. The specifics of diagnostics and computational analysis of these types of objects are highlighted. Methods of determining the masonry compressive strength in existing structures while taking into account the reliability theory are given. A combination of non-destructive and destructive tests is proposed as the basis for determining the masonry strength parameters. Practical solutions for determination of design masonry strength were given which can be used in assessing the safety of massive brick walls and pillars constituting important structural elements of existing masonry buildings.

Keywords: masonry structures, historical buildings, reliability theory, masonry compressive strength

Streszczenie

W artykule przedstawiono wybrane zagadnienia w zakresie oceny niezawodności konstrukcji murowych w obiektach istniejących w tym obiektach o charakterze historycznym. Wskazano na specyfikę diagnostyki i analiz obliczeniowych tego typu konstrukcji. Podane zostały metody określenia wytrzymałości na ściskanie murów w budynkach istniejących z uwzględnieniem teorii niezawodności konstrukcji. Kompilacja badań nieniszczących i niszczących została zalecona w artykule jako podstawowa metoda w procesie oceny parametrów wytrzymałościowych murów. W artykule podane zostały również praktyczne rozwiązania służące określeniu obliczeniowej wytrzymałości na ściskanie murów, które mogą być wykorzystane przy ocenie bezpieczeństwa masywnych ścian i filarów będących podstawowymi elementami istniejących konstrukcji murowych.

Słowa kluczowe: konstrukcje murowe, budynki historyczne, niezawodność konstrukcji, wytrzymałość muru na ściskanie

1. Introduction

Over the centuries, masonry buildings have been the basic type of buildings erected in cities. Various materials and technologies have been used for their construction. Masonry structures have been erected according to traditional rules passed down and developed by successive generations of builders. As a result of their considerable durability and resistance to fire, many masonry buildings have survived to the present day. Some of these facilities are in good technical condition and continue to successfully perform their functions. If the technical conditions of these buildings are adequate and do not raise any objections, detailed analyses of their structures and the examination of their reliability conditions are not usually necessary. A different situation exists for buildings planned for renovation and reconstruction. In such cases, it is necessary to carry out tests and calculations showing that after the completion of construction, the existing masonry structure can be safely used in its new state.

The masonry walls and pillars subjected to compression are the most important elements of the historical masonry buildings. It is necessary to take into consideration reliability methods to obtain correct compressive strength materials which the structures were erected. Due to the historical character of many masonry facilities, the possibilities for removing an appropriate number of samples are limited. For this reason, the ability to assess masonry strength distribution is also limited. In contrast to concrete structures, there are currently no codes which give procedures that allow identifying the masonry strength of a given structure. This makes the analysis of existing masonry structures very difficult. There are also only a few research works on the determination of the masonry strength of existing structures which take reliability theory into account [1–3].

The considerations presented in this article relate to homogeneous brick masonry in which brick layers are placed in a regular pattern (masonry bond).

2. Basic theory of reliability assessment

The safety and reliability of the structure depends on many factors, firstly, on the type and size of loads and the load-bearing capacity of structural elements. Whether the state of safety is sufficient depends on adherence to the relevant building codes. In codes the requirements in terms of load capacity of cross sections, deformations or displacements of the structure or its fire resistance were given. These requirements take into account the consequences of a failure, for example, the loss of human life and social, economic and environmental consequences.

An acceptable probability level of failure is most often measured in terms of the probability of its occurrence. One of the basic axioms of the reliability theory is the inability to design a structure with a failure probability equal to zero. This involves the adoption of an acceptable probability of failure. The probability of failure (P_f) is determined by the formula:

$$P_f = P\left[g(\mathbf{X}) \leq 0 \right] = \int_{g(\mathbf{X}) \leq 0} f_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x} \tag{1}$$

where:

$g(\mathbf{X})$ – performance function,

$\mathbf{x}$ – vector of basic random variables,

$f_{\mathbf{X}}(\cdot)$ – multidimensional function of variables $\mathbf{x}$.

The acceptable failure probability levels proposed in [4] depending on the anticipated failure consequences are presented in Table 1.

Table 1. The failure probability levels recommended in JCSS [4]

Costs	Failure consequences		
	Minor	Average	Major
Large	$P_f \approx 10^{-3}$	$P_f \approx 10^{-4}$	$P_f \approx 10^{-4}$
Medium	$P_f \approx 10^{-4}$	$P_f \approx 10^{-5}$	$P_f \approx 10^{-5}$
Small	$P_f \approx 10^{-5}$	$P_f \approx 10^{-5}$	$P_f \approx 10^{-6}$

The dependence of the acceptable probability of failure for historical buildings (P_f) on their values to society in [1] was proposed. The values of social criterion factors S_c are shown in Table 2.

Table 2. Social criterion factors S_c [1]

Description of the structure	S_c
Historical structures of great importance (e.g. listed by UNESCO)	0.005
Historical structures listed as nationally important	0.05
Historical structures listed as regionally important	0.5
Non-listed historical structures	5

The acceptable probability of failure (P_f) is directly proportional to the factor S_c. Taking into account the recommendations given in Table 2, this means that values of P_f, depending on the historical value of the buildings, may differ by up to a thousandth-fold. The decisions of conservation services and conservation organisations that determine the historical value of the buildings are crucially important in establishing the acceptable probability of failure. It should be noted that the historical value of an object may depend not only on the material value of the engineering work itself, but also on the historical events related to this object that are important to society. For buildings subject to conservation protection, an acceptable probability of failure is proposed at a level of approximately 10^{-6}. This does not apply to historical structures of great importance (e.g. listed by UNESCO), for which P_f values should be determined individually. A significant proportion of masonry buildings are not subject to conservation protection. For these types of structures, it is appropriate to attribute an acceptable level of probability of failure in the range of 10^{-5}–10^{-4}.

The code EN-1990 [5] specifies the acceptable probability of failure depending on the consequences classes of failure (CC3, CC2, CC1) and the reliability classes (RC3, RC2, RC1). The recommendations of the code [5] are presented in Tables 3 and 4. An important safety parameter is the reliability index (β) which is related to the probability of failure by:

$$P_f = \Phi(-\beta) \tag{2}$$

where:

Φ – Laplace function.

The reliability assessment method associated with the reliability index (β) is often called the probabilistic method of Level II.

Table 3. Minimum values for reliability index (ultimate limit states) and maximum values of failure probability according to EN 1990 [5]

Reliability Class	**Minimum values for β_{reg}, maximum values of P_f**	
	1 year reference period	**50 year reference period**
RC3	$\beta = 5.2$; $P_f \cong 9.9 \cdot 10^{-8}$	$\beta = 4.3$; $P_f \cong 8.5 \cdot 10^{-6}$
RC2	$\beta = 4.7$; $P_f \cong 1.3 \cdot 10^{-6}$	$\beta = 3.8$; $P_f \cong 7.1 \cdot 10^{-5}$
RC1	$\beta = 4.2$; $P_f \cong 1.2 \cdot 10^{-5}$	$\beta = 3.3$; $P_f \cong 4.8 \cdot 10^{-4}$

Table 4. Definition of consequences classes according to EN 1990 [5]

Consequences class	**Description**	**Examples of buildings and civil engineering works**
CC3	**High** consequences for loss of human life, or economic, social or environmental consequences **very great**	Grandstands, public buildings where consequences of failure are high
CC2	**Medium** consequences for loss of human life, economic, social or environmental consequence **considerable**	Residential and office buildings, public buildings where consequences of failure are medium
CC1	**Low** consequences for loss of human life, and economic, social or environmental consequences **small or negligible**	Agricultural buildings where people do not enter (e.g. storage buildings), greenhouses

The recommendations given in Tables 2 and 3 indicate that most existing masonry buildings can be classified to consequences class of failure CC3 or CC2. The consequences of failure for historical buildings should be considered, not only for material reasons or for loss of human life (as given in Table 4), but also for the consequences related to the loss of the historical value of the object itself.

The vector **F** of random structure failure events is defined in the form of a function:

$$\mathbf{F} = \{g(\mathbf{x}) \leq 0\} \tag{3}$$

where:

$g(\mathbf{x})$ – a performance function:

$$g(\mathbf{x}) = R(\mathbf{x}) - E(\mathbf{x}) \tag{4}$$

where:

$R(\mathbf{x})$ – the resistance,
$E(\mathbf{x})$ – the effect of actions,
$R(\mathbf{x}), E(\mathbf{x})$ – random variables.

The structure is considered to survive if:

$$g = R - E \geq 0 \tag{5}$$

The criterion of reliability of the structure in the probabilistic method of Level II can be written by comparing the design resistance (R_d) and the design value of action effects (E_d):

$$R_d \geq E_d \tag{6}$$

Taking into account the reliability index (β), formula (6) can be transformed into the following form:

$$R_d = \mu_R - \beta_R \sigma_R \geq \mu_E + \beta_E \sigma_E \tag{7}$$

where:

$\beta_R = \beta|\alpha_R|, \ \beta_E = \beta|\alpha_E|$

α_R, α_E – the values of sensitivity factors for resistance and action effects, respectively.

Formula (7) is represented in graph form in Fig. 1.

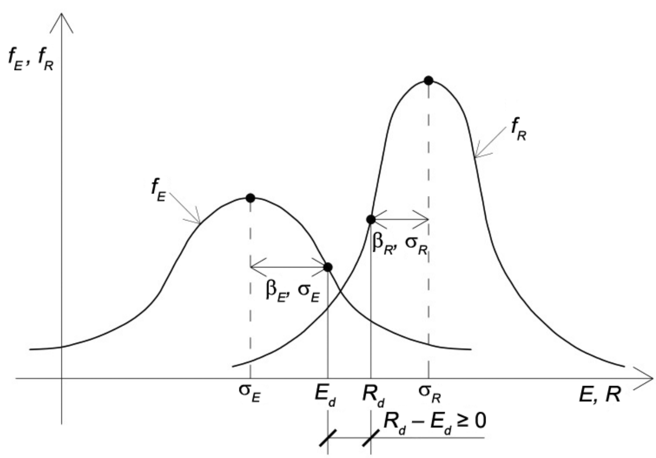

Fig. 1. Load effect (f_E) and resistance probability (f_R) density functions

The design resistance R_d can be expressed in the following form:

$$R_d = \frac{1}{\gamma_{Rd}} R(X_d; a_d) = \frac{1}{\gamma_{Rd}} R\left(\eta \frac{X_k}{\gamma_m}; a_d\right) \tag{8}$$

where:

X_d – the design value of material property,

X_k – the characteristic value of material property,

γ_{Rd} – the partial factor covering uncertainty in the resistance model,

η – the conversion factor,

γ_m – the partial factor for the material property,

a_d – the design value of geometrical data.

Assuming that:

$$\gamma_{Rd}\gamma_m = \gamma_M \tag{9}$$

the design resistance R_d can be obtained as follows:

$$R_d = R\left(\eta\frac{X_k}{\gamma_M}; a_d\right) \tag{10}$$

The basic task in the analysis of masonry structures is to determine the load-bearing capacity of walls and pillars subjected to compressive loads. In equation (10), the parameter of the material determining the resistance of the wall or pillar is therefore the masonry compressive strength ($X_k = f_k$; $X_d = f_d$). The design value of the compressive strength of masonry (f_d) is:

$$f_d = \eta\frac{f_k}{\gamma_M} \tag{11}$$

Methods of determining the characteristic compressive strength of the masonry (f_k) and the value of the partial safety factor (γ_M) are provided in code EN 1996-1-1 [6]. However, this code applies only to masonry structures designed and erected today. Currently, there are no codes on how to determine the masonry strength of existing facilities, which makes analysis of these types of structures very difficult.

In code EN 1996-1 [6] it is recommended that the values of the partial safety factor (γ_M) should be determined on the basis of classes related to execution control, the type of mortar assumed in the project and the category of bricks planned to be used in the masonry. This factor, therefore, first of all captures the difference between the strengths of the masonry selected in the design and the masonry that can be found in the structure. For the analysis of existing structures, the situation is completely different. The safety factor should be determined on the basis of the test results and should factor in the uncertainty resulting from the limitations of the applied research methods.

3. Determination of the masonry compressive strength based on laboratory and in-situ tests

The most reliable method to determine the masonry strength in an existing building is testing masonry samples cut from the structure. However, it is rarely possible to take masonry samples of sufficiently large dimensions (Fig. 2a) which are the most representative from massive brick walls or pillars. The collection of such samples causes significant damage to the masonry structure and decreases its bearing capacity. Such large instances of damage to the structures are also unacceptable for conservation reasons. Therefore, it is best to try to determine the masonry strength on samples made in the laboratory using materials with characteristics similar to those from which the structures were constructed. Research of these samples, however, may give false results, because it is extremely difficult to apply modern technologies to recreate original brick masonry erected several dozen or several hundred years ago. Historical bricks formed and fired in a specific way are definitely different from bricks produced today, even if they have similar compressive strengths. Similarly, historical mortars were made from binders and additives which are currently difficult to precisely define and precisely reproduce.

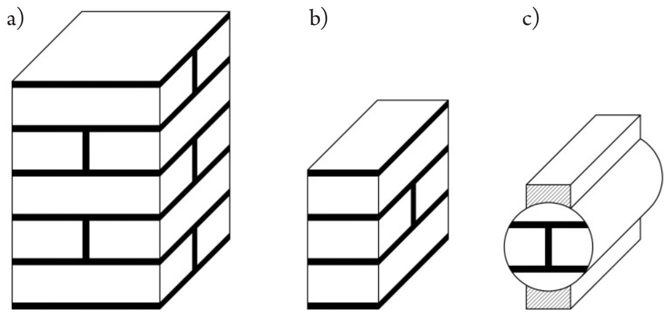

Fig. 2. Samples cut from the structure to determine masonry compressive strength: a) masonry pillars; b) masonry prisms; c) cylindrical samples (150 mm diameter)

For this reason, the basic method to identify the masonry strength is testing performed on original material samples taken from the structure. Most often, smaller samples (prismatic or cylindrical) are cut from the masonry walls (Fig. 2b, c). Due to the dimensions of small samples and the method of applying load to them, the test results should be recalculated using appropriate correction factors. For example, for core samples with a diameter of 150 mm tested under compression perpendicular to the bed joints (Fig. 2c), the correction factor is 1.8 [7, 8]. The error in assessing the masonry strength depends on the dimensions of the samples and the test method. The processes of cutting out, transporting and preparing masonry samples for testing also have an impact on the test results. The samples cut from masonry made on weak mortars are very sensitive and susceptible to damage during their transport to the laboratory; thus, it is necessary to provide them with adequate protection. Samples that are damaged during transport are not suitable for strength tests as their cohesion has been compromised.

The error (Δ_a) of assessing the average strength of the masonry (f_{mean}) depending on the research methods is estimated to be from 10 to 20% [7, 8]:

$$\Delta_a = \pm\,(0.1 - 0.2) \tag{12}$$

This error also includes differences resulting from the dimensions of the tested masonry samples in relation to the dimensions of the masonry wall or the pillar. As the research shows, the masonry strength determined on whole walls is less than the strength estimated on the basis of tests of cylindrical samples or small masonry prisms [1, 8]. A minimum of 3 samples cut from masonry should be tested. With a larger sample size ($n > 3$) it is possible to more precisely determine the mean compressive strength of the masonry (f_{mean}) and the coefficient of variation of masonry strength (V_f). If the number of samples taken from the structure is small, complementary non-destructive tests (NDT) are very important. Firstly, NDT can provide an impression of the homogeneity of the wall. These tests do not cause damage to the masonry structure. They can therefore be conducted in many places. The size of tests is only limited in such cases by the availability of the structure and the costs of the research. The ultrasonic pulse method (UPM), the infrared thermography method (IRT) and tests using geo-radar (GPR) provide the best results in determining the homogeneity of masonry. The Schmidt hammer can also be used for the preliminary assessment of wall homogeneity in external layers, whereas the impact-echo method (IEM) can be used to search for discontinuity of the masonry structure (e.g. cracks and scratches). The choice of research methods depends on many factors and should be made by the expert, in relation to the specific objectives, after a preliminary visual assessment of the structure.

If it is not possible to cut out masonry samples from the existing structure, masonry strength can be estimated using the flat-jack method [9, 10, 11]. In the first step, the mortar from two bed joints is removed; flat jacks are then placed in these joints. The increase of pressure in the flat jacks causes loading of the masonry between them. The disadvantage of this method is local damage to the tested wall. For this reason, the flat-jack method is rarely used to assess masonry strength. However, this method is often used to determine the masonry modulus of elasticity and the level of compressive stress in walls and pillars. The error in assessment of the masonry strength using the flat-jack method is estimated to be around 20%.

The effect of the laboratory and in-situ tests is the identification of the distribution of the masonry strength. The characteristic value of masonry compression strength, understood as 5% quantile, is determined in the case of normal distribution from formula:

$$f_k = f_{mean}\left(1 - k_n V_f\right) \tag{13}$$

If the distribution of the masonry compressive strength is log-normal, the characteristic value of the masonry compressive strength is:

$$f_k = \frac{f_{mean}}{\sqrt{1+V_f^2}} \exp[-k_n V_f] \tag{14}$$

where:

k_n – a factor dependent upon the sample size,

f_{mean} – mean masonry strength:

$$f_{mean} = \frac{1}{n}\sum_{i=1}^{n} f_i \tag{15}$$

Values of k_n according to code EN 1990 [5] are presented in Table 4.

Table 5. Values of k_n for the 5% characteristic value

N	3	4	5	6	10	20	∞
k_n	1.89 (3.37)	1.83 (2.63)	1.80 (2.33)	1.77 (2.18)	1.72 (1.92)	1.68 (1.76)	1.64 (1.64)
() values of k_n if V_x unknown							

The reliability index (β) introduced in the probabilistic method of Level II can be used to calibrate partial factors. Assuming as given in [5] the sensitivity factor $\alpha_R = 0.8$ for $n \to \infty$ partial factor γ_M can be expressed as follows:

for normal distribution $$\gamma_M = \gamma_{Rd}\gamma_m = \gamma_{Rd}\eta_d \frac{1-1.64V_f}{1-0.8\beta V_f} \tag{16}$$

for lognormal distribution $$\gamma_M = \gamma_{Rd}\eta_d \exp[V_f(0.8\beta - 1.64)] \tag{17}$$

where:

η_d – conversion factor strongly dependent upon the type of testing method.

In the current state of testing masonry structures, determining the value of the factor γ_{Rd} which expresses the model error is very difficult. The code [5] recommends for designed structures $\gamma_{Rd} = 1.1$. For existing masonry structures, whose detailed identification is limited, it is proposed to adopt slightly larger values, for example, $\gamma_{Rd} = 1.15$.

It is also difficult to precisely determine the error resulting from tests of the specified type of masonry samples (see Fig. 2). Due to the size of the sample and the way it is loaded, the largest error is made for the samples shown in Fig. 2c. The value of this error is estimated at around 20% ($\Delta_a = 0.2$) and therefore $\eta_d = 1.0 + \Delta_a = 1.2$ can be accepted. Results closest to the real values of the masonry compressive strength are obtained by testing masonry pillars (Fig. 2a, $\eta_d = 1.1$).

The values of the partial factor γ_M for masonry structures in historical buildings classified to reliability classes RC2 (β = 3.8) and RC3 (β = 4.3) are presented in Fig. 3.

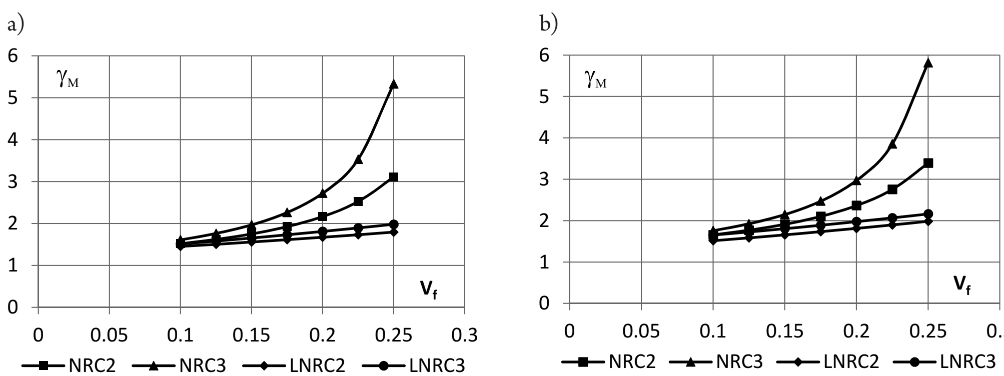

Fig. 3. The partial factor γ_M for historical masonry structures ($n \to \infty$); $\eta_d = 1.1$; b) $\eta_d = 1.2$
NRC2, NRC3 – normal distribution, reliability class RC2 and RC3, respectively;
LNRC2, LNRC3 – lognormal distribution, reliability class RC2 and RC3, respectively

The comparison presented in Fig. 3 shows that the differences between values of γ_M for the RC3 and RC2 reliability classes, especially for masonry characterised by a high coefficient of variation of strength, can be very significant. The proper assessment of an existing facility and the establishment of an appropriate reliability class of the building is therefore a fundamental issue in programming the scope of tests and calculations of the structure. It is also important to assess the type of probability distribution of the masonry compressive strength. In the testing of a historical masonry structure, only a small set of samples can be taken. For this reason, the determination of factor γ_M for a relatively small sample size is of practical importance. In this case, formulas (16) and (17) are changed to the following forms:

$$\gamma_M = \gamma_{Rd}\eta_d \frac{1-k_n V_f}{1-k_{d,n} V_f} \tag{18}$$

$$\gamma_M = \gamma_{Rd}\eta_d \exp[V_f (k_{d,n} - k_n)] \tag{19}$$

The values of $k_{d,n}$ determined for $\beta = 3.8$ and $\beta = 4.3$ (for 95% confidence level according to SKJ [12]) are shown in Tables 5a and 5b. The values for $\beta = 3.8$ are the same as those recommended in the code [5].

Table 5a. Values of $k_{d,n}$ for the ULS design value ($\beta = 3.8$)

n	3	4	5	6	10	20	∞
$k_{d,n}$	3.56 (–)	3.44 (11.4)	3.37 (7.85)	3.33 (6.36)	3.23 (4.51)	3.16 (3.64)	3.04 (3.04)
(…) values of $k_{d,n}$ if V_x unknown							

Table 5b. Values of $k_{d,n}$ for the ULS design value ($\beta = 4.3$)

n	3	4	5	6	10	20	∞
$k_{d,n}$	4.02 (–)	3.88 (16.5)	3.80 (10.21)	3.76 (7.91)	3.65 (5.41)	3.52 (4.19)	3.44 (3.44)
(…) values of $k_{d,n}$ if V_x unknown							

In laboratory reports, the values of k_n i $k_{d,n}$ for 'V_f known' are generally accepted and could be similarly performed in the case of diagnostic tests. However, the opposite views are also expressed in discussions. Previous studies, the results of which are presented in [1, 2, 13], indicate that the distributions of strength for masonry as well as for bricks and mortars are in the form of log-normal distributions. A summary of the calculation results based on formula (19) are shown in Tables 6a, 6b, 6c and 6d.

Table 6a. Values of partial factor γ_M ($\beta = 3.8$; V_f known)

V_f	*n*						
	3	**4**	**5**	**6**	**10**	**20**	**∞**
0.100	1.50 (1.63)	1.49 (1.62)	1.48 (1.61)	1.48 (1.61)	1.47 (1.61)	1.47 (1.60)	1.46 (1.59)
0.125	1.56 (1.70)	1.55 (1.69)	1.54 (1.68)	1.54 (1.68)	1.53 (1.67)	1.52 (1.66)	1.51 (1.64)
0.150	1.63 (1.77)	1.61 (1.76)	1.60 (1.75)	1.60 (1.74)	1.59 (1.73)	1.58 (1.72)	1.56 (1.70)
0.175	1.69 (1.85)	1.68 (1.83)	1.67 (1.82)	1.66 (1.81)	1.65 (1.80)	1.64 (1.79)	1.62 (1.76)
0.200	1.77 (1.93)	1.75 (1.90)	1.73 (1.89)	1.73 (1.89)	1.71 (1.87)	1.70 (1.86)	1.67 (1.83)
0.225	1.84 (2.01)	1.82 (1.98)	1.80 (1.96)	1.80 (1.96)	1.78 (1.94)	1.77 (1.93)	1.73 (1.89)
0.250	1.92 (2.10)	1.89 (2.06)	1.87 (2.04)	1.87 (2.04)	1.85 (2.01)	1.83 (2.00)	1.80 (1.96)
(…) values of γ_M for $\eta_d = 1.2$							

Table 6b – Values of partial factor γ_M ($\beta = 4.3$; V_f known)

V_f	*n*						
	3	**4**	**5**	**6**	**10**	**20**	**∞**
0.100	1.57 (1.71)	1.55 (1.69)	1.55 (1.69)	1.54 (1.68)	1.53 (1.67)	1.52 (1.66)	1.52 (1.65)
0.125	1.65 (1.80)	1.64 (1.78)	1.62 (1.77)	1.62 (1.77)	1.61 (1.76)	1.59 (1.74)	1.55 (1.73)
0.150	1.74 (1.90)	1.72 (1.88)	1.71 (1.86)	1.71 (1.86)	1.69 (1.84)	1.67 (1.82)	1.66 (1.81)
0.175	1.84 (2.00)	1.81 (1.98)	1.80 (1.96)	1.79 (1.96)	1.77 (1.93)	1.75 (1.91)	1.73 (1.89)
0.200	1.94 (2.11)	1.91 (2.08)	1.89 (2.06)	1.88 (2.05)	1.86 (2.03)	1.83 (1.99)	1.81 (1.98)
0.225	2.04 (2.23)	2.01 (2.19)	1.98 (2.16)	1.98 (2.16)	1.95 (2.13)	1.91 (2.09)	1.90 (2.07)
0.250	2.16 (2.35)	2.11 (2.30)	2.09 (2.28)	2.08 (2.27)	2.05 (2.26)	2.00 (2.19)	1.94 (2.16)
(…) values of γ_M for $\eta_d = 1.2$							

Table 6c. Values of partial factor γ_M ($\beta = 3.8$; V_f unknown)

V_f	***n***						
	3	**4**	**5**	**6**	**10**	**20**	∞
0.100	–	2.40 (3.04)	2.20 (2.40)	1.92 (2.10)	1.64 (1.79)	1.53 (1.67)	1.46 (1.58)
0.125	–	2.99 (3.79)	2.52 (2.75)	2.13 (2.33)	1.75 (1.91)	1.60 (1.75)	1.51 (1.64)
0.150	–	–	2.90 (3.16)	2.37 (2.58)	1.87 (2.04)	1.68 (1.83)	1.56 (1.70)
0.175	–	–	3.32 (3.63)	2.63 (2.87)	1.99 (2.17)	1.75 (1.92)	1.62 (1.76)
0.200	–	–	–	2.92 (3.18)	2.12 (2.32)	1.83 (2.01)	1.67 (1.83)
0.225	–	–	–	3.24 (3.54)	2.27 (2.47)	1.91 (2.11)	1.73 (1.89)
0.250	–	–	–	–	2.42 (2.64)	2.00 (2.21)	1.80 (1.96)
Table 6. (...) values of γ_M for $\eta_d = 1.2$							

Table 6d – Values of partial factor γ_M ($\beta = 4.3$; V_f unknown)

V_f (cov)	***n***						
	3	**4**	**5**	**6**	**10**	**20**	∞
0.100	–	–	2.78 (3.04)	2.24 (2.45)	1.64 (1.79)	1.61 (1.76)	1.52 (1.65)
0.125	–	–	2.68 (3.39)	2.59 (2.82)	1.75 (1.91)	1.71 (1.87)	1.55 (1.73)
0.150	–	–	3.26 (4.13)	2.99 (3.26)	1.87 (2.04)	1.82 (1.99)	1.66 (1.81)
0.175	–	–	–	3.45 (3.76)	1.99 (2.17)	1.94 (2.11)	1.73 (1.89)
0.200	–	–	–	3.99 (4.34)	2.12 (2.32)	2.06 (2.24)	1.81 (1.98)
0.225	–	–	–	–	2.27 (2.47)	2.19 (2.38)	1.90 (2.07)
0.250	–	–	–	–	2.42 (2.64)	2.32 (2.53)	1.94 (2.16)
(...) values of γ_M for $\eta_d = 1.2$							

The above data shows the dependence of the value of partial factor γ_M on the sample size. This relationship is particularly clear for masonry characterised by a high variability of strength characteristics. For this type of masonry, the estimation of the partial factor γ_M should be based on the testing of a large number of samples. If it is not possible to take a sufficiently large number of samples, supplementary non-destructive tests should be provided. Non-destructive tests should also be performed in places where samples for destructive testing are to be later cut out in order to determine appropriate correlation coefficients. The combination of non-destructive and destructive tests is the basic method for determining the masonry strength and the coefficient of variation for this parameter [8, 9].

If the partial factor γ_M is determined taking into account the test results on a small sample size, the values of this factor can be significantly higher than three and therefore greater than the maximum value of γ_M given for contemporary masonry structures in code [6].

The values of factor γ_M given in Tables 6a, 6b, 6c and 6d are in the range of approx. 1.5 to approx. 4.3. It should be emphasised that these values are appropriate for brick masonry which has a COV of not more than 0.25. It should also be remembered that the masonry strength determined on the samples cut out of the structure and possibly based on supplementary non-destructive tests refer to places that constitute a small part of the masonry structure. Therefore, the final values of the partial factor γ_M should depend on the decision of the expert performing the structural analysis. The values of γ_M given in this chapter should be treated as auxiliary in such analyses.

4. Determination of the masonry compressive strength based on the strength of bricks and mortar

Assessment of the masonry compressive strength based on the tests of masonry samples cut out of the structure is a labour-intensive method requiring the involvement of specialist equipment and people with extensive experience in conducting research on masonry structures. Attempts are also made to assess masonry strength based on strength tests of bricks and mortar. For example, a power relationship is used in which the compressive strength of the wall (f) is a function of the compressive strength of the bricks (f_B) and the compressive strength of the mortar (f_M):

$$f = A \cdot f_B^u \cdot f_M^w \tag{20}$$

where:

A, u, w – constants.

This form also has the formula given in code EN 1996-1-1 [6]:

$$f_k = K \cdot f_b^{0.7} \cdot f_m^{0.3} \tag{21}$$

where:

f_k – characteristic compressive strength of masonry;
f_b – normalised compressive strength of masonry units (bricks);
f_m – mortar compressive strength;
K – constant.

The characteristic strength of masonry determined from formula (21) is a function of the mean brick compressive strength and the mean mortar compressive strength determined in the tests. The test procedures are given in the relevant codes [16, 17]. Formula (21) is an empirical relation and has been determined based on extensive experimental studies. The dimension of the constant K is chosen so that the final result of the masonry strength calculations is in MPa. For brick masonry, in code EN 1996-1-1 [6] a value of K equal to 0.55 was adopted. This value applies to brick masonry without longitudinal joints. For masonry with longitudinal joints, the code EN 1996-1-1 [6] recommends a reduction of the K coefficient by 20%, hence $K = 0.46$. It should be noted that the recommendations of EN 1996-1-1 [6] apply to constructions designed from modern materials. For masonry structures erected several dozen or several hundred years ago made on weak mortars, the use of formula (16) requires additional reduction factors [8].

From formula (21), the characteristic compressive strength of masonry is obtained on the basis of the mean strength of bricks and the mean strength of mortar, i.e. deterministic parameters. This is due to the fact that until now, it has not been determined to what extent the brick strength variability and the mortar strength variability affect masonry strength. It should also be noted that in the case of existing structures, it is not possible to determine the compressive strength of the mortar in accordance with [17], because samples of dimensions $40 \times 40 \times 160$ mm cannot be cut out of masonry joints, the thickness of which is usually in the range of 10 mm to 25 mm. For this reason, appropriate correction factors should be applied.

The power formula (20) can be treated as a function determining the manner of assigning a random variable f (compressive strength of the masonry) of two random variables: f_B (compressive strength of bricks) and f_M (compressive strength of the mortar). It can be considered that the issue of determining the parameters of the probability distribution of the masonry compressive strength is a function of two random variables with known distributions. If it is assumed that f_B, f_M are independent random variables and have log-normal distributions, then the mean value of the natural logarithm $\ln(f)$ is:

$$\mu_{\ln(f)} = \mu_{\ln(A)} + u\mu_{\ln(f_B)} + w\mu_{\ln(f_M)} \tag{22}$$

while the standard deviation σ of natural logarithm $\ln(f)$ is:

$$\sigma_{\ln(f)} = \sqrt{(u\sigma_{\ln(f_B)})^2 + (w\sigma_{(f_M)})^2} \tag{23}$$

The mean value of random variable $\ln(f)$ and the standard deviation of this random variable can be directly defined based on mean values and standard deviations of the random variables $\ln(f_B)$ and $\ln(f_M)$, provided that the values of material constants A, u and w are known. In [8, 9, 14] and [15], the results of experimental tests on masonry samples cut from existing masonry are given. Based on the analysis of these results, the graph shown in Fig. 4 has been prepared.

Taking in account as recommended by EN 1996-1-1 [6], $u = 0.7$, $w = 0.3$, the value of constant A was obtained with a range of 0.26–0.65 (mean 0.43). The values of constant A were calculated by performing an inverse task, i.e. on the basis of the random variables $\ln(f)$, $\ln(f_B)$ and $\ln(f_M)$ determined in the tests, the value of constant A was calculated. From the graph in Figure 4, increases to the values of constant A can be seen with increasing masonry compressive strengths. For masonry with weak lime and lime-cement mortars, the value of A is about 0.35, while for walls with cement-lime mortars, the values of A are significantly higher. The large variability of the A-value results from, among other factors, the different workmanship of the masonry, although it is difficult to precisely determine this relationship. The detrimental effect of poor workmanship is due to the improper filling of joints. The thickness of joints also plays an important role. If the dimensions of the bricks vary, the dimensions of the mortar joints will also vary. The result is non-uniform joint thicknesses which create bending moments and stress concentrations in the brick. The decrease in the compressive masonry strength due to poor workmanship can be up to 40%.

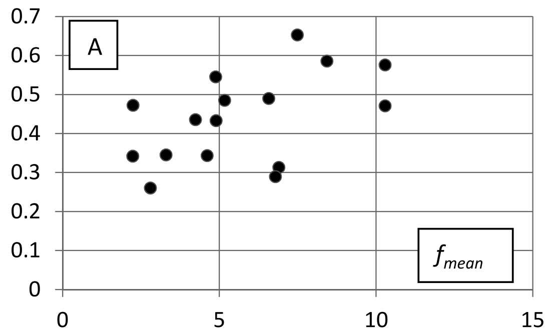

Fig. 4. Values of factor A

Taking into account relationships:

$$\sigma_{\ln(f_B)} = \sqrt{\ln(1+V_{f_B}^2)} \approx V_{f_B} \tag{24}$$

$$\sigma_{\ln(f_M)} = \sqrt{\ln(1+V_{f_M}^2)} \approx V_{f_M} \tag{25}$$

where:

$V_{fB,}$ V_{fM} – coefficients of variation of brick strength and mortar strength, respectively, a formula to determine the coefficient of variation of the masonry strength (V_f) can be obtained:

$$V_f = \sqrt{(uV_{f_B})^2 + \left(wV_{f_M}\right)^2} \tag{26}$$

A comparison of the calculations made using formula (26) with the results of experimental tests (V_f^*) are shown in Fig. 5. The values of parameters u and w were adopted in accordance with EN 1996, $u = 0.7$, $w = 0.3$.

Differences in the values of the COV calculated from formula (26) with values obtained from experimental studies can be significant (up to 60%). This is due to the fact that it is not only the coefficients of variation of bricks strength and mortar strength that have a significant impact on the coefficient of variation of masonry strength in the structure. In [18], it was stated that the workmanship of the masonry is of fundamental importance for the COV of masonry strength, while the other parameters are of secondary importance. This issue requires further research and analysis. It should also be emphasised that determining the workmanship of masonry based only on visual inspection of the structure in the external layers may lead to incorrect applications. As a rule, non-plastified brick facing layers were made more thoroughly than the rest of the masonry wall.

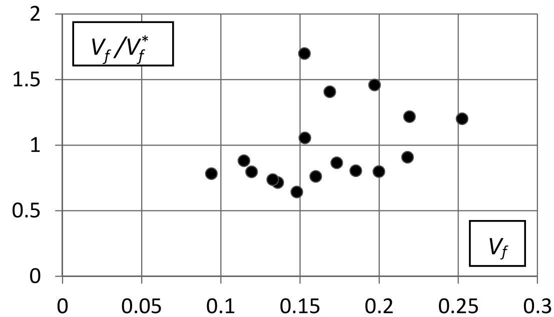

Fig. 5. Coefficient of variation (COV) of the masonry strength – comparison of test results (V_f^*) according to [8, 9, 13] and [14] and results of calculations (V_f) based on formula (26)

COV of masonry strength determined in experimental tests on samples cut from the structure were in the range of:

$$V_f^* = 0.12\text{–}0.24 \tag{27}$$

while calculated on the basis of formula (26):

$$V_f = 0.10\text{–}0.25 \tag{28}$$

The mean values of COV of masonry strength obtained from experimental investigations given in [8, 9, 14] and [15] and calculations based on formula (26) are:

$$V_{f,mean}^* = 0.178 \tag{29}$$

$$V_{f,mean} = 0.166 \tag{30}$$

The difference between $V_{f,mean}^*$ and V_{fmean} is not significant and amounts to 7%.

The function of probability distribution of the masonry compressive strength and the parameters of this function determined on the basis of formulas (24) and (25) can be used in the structural reliability analysis. In practice, the use of this method requires the testing of an appropriate number of samples of bricks and mortar, which is a prerequisite for determining the strength distributions of masonry components. The Austrian code [19] requires that this assessment should be based on brick and mortar tests in three places of the analysed section of the structure, whereby at least five samples of bricks and ten samples of mortar should be taken at each place. The samples should be representative of the entire section of the structure, so they should also be taken from internal layers of masonry walls or pillars. This is not a simple matter from a technical point of view.

For this reason, in determining the compressive masonry strength based on the strength of bricks and mortar, high partial factors should be applied ($\gamma_M \geq 2.5$).

5. Summary

In the paper, selected issues related to determining the compressive strength of masonry in existing structures, taking into account the reliability theory, have been presented. The research methods and methods for analysing their results used in determining the basic parameters of the probability distributions of masonry strength have been given. The methodology of test methods on samples cut from masonry and non-destructive testing methods have been discussed. Specifics have been highlighted with respect to performing this research on existing masonry structures, including historical structures. On the basis of the conducted analysis, a method for determining the masonry strength and the partial factor (γ_M) in existing facilities has been proposed.

The method of determining masonry compressive strength based on strength tests of bricks and mortar was also presented. The power formula was used, in which the compressive strength of masonry is a function of the compressive strength of bricks and the compressive strength of mortar.

The presented solutions can be used in practice in the analysis of walls and pillars constituting the basic structural elements of existing masonry buildings.

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If you want to quote this article, its proper bibliographic entry is as follow: Domański T., Matysek P., *The reliability of masonry structures – evaluation methods for historical buildings*, Technical Transactions, Vol. 9/2018, pp. 91–108.