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EVALUATION OF THE PRACTICABILITY OF THE DECOMPOSITION  
AND EQUIVALENT SUBSTITUTION METHOD IN THE ANALYSIS  
OF RELIABILITY OF THE SEWAGE SYSTEM

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OCENA PRAKTYCZNEGO ZASTOSOWANIA METODY DEKOMPOZYCJI  
I EKWIWALENTNEJ ZAMIANY W ANALIZIE NIEZAWODNOŚCI SYSTEMU  
KANALIZACYJNEGO

**Abstract**

The work compares the accuracy of calculations of the reliability parameters of the sewerage network using the Decomposition and Equivalent Replacement (MDE) method, proposed by Yu. A. Yermolin and M.I. Alekseev [3], definitely simpler and less onerous in relation to the graph method. Comparing the results of calculations with both methods, applied to a simple network, one can come to the conclusion that the MDE calculations in simple cases give satisfactory accuracy. However, it would be necessary to check whether, as the complexity increases, this accuracy is still satisfactory.

**Keywords:** reliability, sewage network, graph, decomposition, equivalent substitution, failure

**Streszczenie**

W porównano dokładność obliczeń parametrów niezawodnościowych sieci kanalizacyjnej metodą dekompozycji i ekwiwalentnej zamiany (MDE), zaproponowaną przez Ju. A. Jermolina i M.I. Alieksiejewa [3], zdecydowanie prostszą i mniej uciążliwą w stosunku do metody grafów. Porównując wyniki obliczeń obiema metodami, zastosowane do prostej sieci, można dojść do wniosku, że obliczenia metodą MDE w prostych przypadkach dają zadowalającą dokładność. Należałoby jednak sprawdzić, czy w miarę wzrostu złożoności, dokładność ta jest nadal zadowalająca.

**Słowa kluczowe:** niezawodność, sieć kanalizacyjna, graf, dekompozycja, ekwiwalentna zamiana, awaryjność

## 1. Introduction

In addition to the largest and most common issues relating to the everyday usage of the gravitational sewage system – such as clogging of the sewer or its siltation and sedimentation – leakages are also a very significant problem. These may occur as a result of breakages, the overgrowth of roots, the displacement of pipes at the joints, or corrosion.

Whilst the effects of failures may be visible on the surface of the area where they occurred, e.g. collapses of the roadway or flooding of properties, they can also occur without any trace on the surface. This second type of failure can lead to technical disasters of the sewage system (especially gravitational), which are characterised by an impossibility of distinguish the idle time state. Moreover, the state of waiting for repair is usually discrete due to the fact that inspections of the system are held with a certain frequency.

One of the important parameters for assessing the reliability of a sewage system is the amount of sewage that is not disposed due to a failure. This value can be estimated by using the graph method [1, 2, 6, 8] which is basing on Markov's theory of processes. However, Yu. A. Yermolin and M.I. Alekseev [3] have proposed an alternative method of calculating the sewage system parameters which has been called the decomposition and equivalent substitution method (MDE).

The aim of the present research is to compare the two above-mentioned methods by calculating the amount of sewage that is not disposed due to a failure of sewage system with a simple spatial structure.

## 2. Graph method

The graph method is based on calculating the probabilities  $P_{S_i}$  of all  $S_i$  states of a given network (or a branch of the network) and then multiplying them by the sum of effluences  $Q_{S_i}$  of those segments from which in a given state the sewage is not disposed. The sum of these products gives the desired expected value of unreleased wastewater [1].

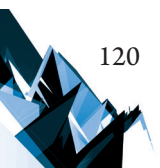
In order to calculate the probabilities of particular states, a matrix of the intensity of transitions  $\Lambda$  is constructed in which diagonal elements  $\Lambda_{ij}$  ( $i = j$ ) represent the probability of leaving this state (1 minus the probability of staying in a given state), elements above the diagonal  $\Lambda_{ij}$  ( $i < j$ ) represent the probability of entering the state  $S_i$  as a result of repairing the state  $S_j$ , elements below the diagonal  $\Lambda_{ij}$  ( $i > j$ ) represent the probability of entering the state  $S_i$  due to failure of the state  $S_j$ .

The matrix is quasi stochastic and its columns satisfy the property:

$$\sum_i \Lambda_{ij} = 0 \quad (1)$$

The direct result of this property is the fact that diagonal elements satisfy the dependence:

$$\Lambda_{ii} = -\sum_{i \neq j} \Lambda_{ji} \quad (2)$$



In practical usage, one assumes asymptotic (stationary) values of probabilities  $P_{Si}$ , assuming that the process is ergodic. In this case, they can be calculated as a solution of set of equations which can be written as a matrix equation:

$$\mathbf{\Pi}\mathbf{\Lambda} = \mathbf{0} \quad (3)$$

where  $\mathbf{\Pi}$  represents the vector of limit probabilities.

Such a set is of course indeterminate; therefore, to eliminate uncertainty, one of the equations is replaced with the following condition:

$$\sum_i P_{Si} = 1 \quad (4)$$

which leads to substitution of the appropriate row of matrix  $\mathbf{\Lambda}$  by a vector of ones and substitution of the appropriate position in vector  $\mathbf{0}$  by the value 1.

Further proceedings are already a simple issue of the matrix account.

### 3. The decomposition and equivalent substitution method

The decomposition and equivalent substitution method (MDE) [3, 8], which treats the sewerage network as a tree-like graph [5], is based on subsequent replacement of Y-shaped structures which are composed of two external edges (leaves of the graph) connected in the parent node and an edge having end at this node by an equivalent edge. Examples of such structures are shown in Fig. 1.

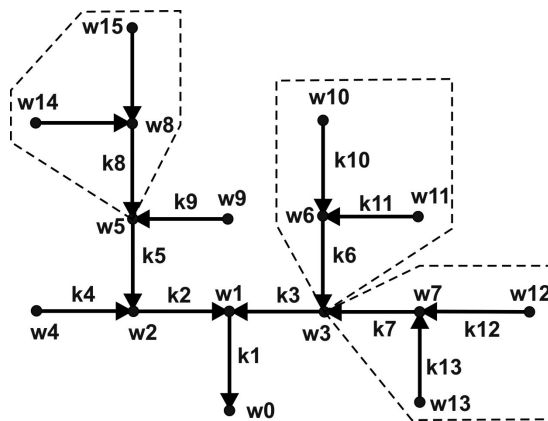


Fig. 1. Sewerage network graph with outlined Y-shaped structures that are to be substituted by equivalent edges

In the next step, after receiving the new network structure, it is necessary to repeat the procedure described above until the whole network (or the branch of interest) is replaced with one equivalent channel.

At the substitution stage, the equivalent effluence  $q_e$  of the Y-shaped structure is calculated being a simple sum of effluences  $q_i$  of all sewers constructing the substituted structure. The expected value  $Q_e$  of unreleased wastewater is also calculated. As an auxiliary value, a dimensionless  $\gamma_e$  coefficient is also calculated, which for a single sewer is understood as quotient of failure rate  $\lambda$  and renewal  $\mu$  intensities [8]:

$$\gamma_i = \frac{\lambda_i}{\mu_i} \quad (5)$$

While calculating the parameters of the equivalent sewer, it is assumed that the probability of simultaneous failure of two or more sewers is significantly smaller than the probability of the failure of a single sewer; thus, in the graph structure of the states of the Y-shaped structure, such states are not considered (Fig. 2).

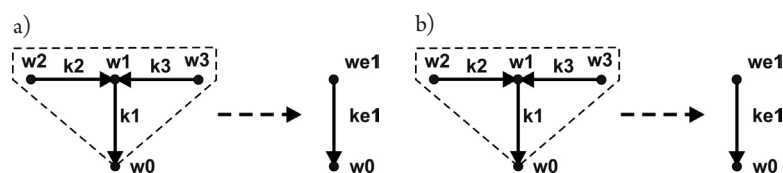


Fig. 2. Scheme of substituting the Y-shaped structure with one equivalent sewer (a) and the corresponding graphs of states of these structures (b)

Under these assumptions, the Y-shaped structure can have 4 states: 0 – all three sewers are operational; 1 – sewer  $k1$  is inoperable; 2 – sewer  $k2$  is inoperable; 3 – sewer  $k3$  is inoperable. Transitions between these states occur with the appropriate failure ( $\lambda_1, \lambda_2, \lambda_3$ ) or renewal ( $\mu_1, \mu_2, \mu_3$ ) intensities. For such a structure of the graph of states, the probabilities of these states are expressed by the formulas [3]:

$$\begin{aligned} P_{S0} &= \frac{1}{1 + \gamma_1 + \gamma_2 + \gamma_3} \\ P_{S1} &= \frac{\gamma_1}{1 + \gamma_1 + \gamma_2 + \gamma_3} \\ P_{S2} &= \frac{\gamma_2}{1 + \gamma_1 + \gamma_2 + \gamma_3} \\ P_{S3} &= \frac{\gamma_3}{1 + \gamma_1 + \gamma_2 + \gamma_3} \end{aligned} \quad (6)$$

By knowing the probabilities of states and effluences of individual sewers, one is able to calculate the expected value of unreleased wastewater  $Q_e$ . The same can be done for the equivalent structure which has only two states: 0 – sewer is operational; e1 – sewer is inoperable. By determining the streams of failure and renewal intensities for the equivalent sewer as  $\lambda_{e1}$  and  $\mu_{e1}$ , respectively, and the corresponding dimensionless parameter as  $\gamma_{e1}$ , the following probabilities of states are obtained:

$$P_{S0} = \frac{1}{1 + \gamma_{e1}} \quad (7)$$

$$P_{S_{e1}} = \frac{\gamma_{e1}}{1 + \gamma_{e1}}$$

The expected value of unreleased wastewater  $Q_e$  is calculated in the same way as before. The adoption of the key assumption:

$$Q = Q_e \quad (8)$$

and the comparison of the relevant analytical formulas which facilitate the calculation of these quantities as functions  $\gamma_i$  and  $q_i$ , enables calculating both  $\gamma_{e1}$ , as well as  $Q_e$  and  $P_{S_{e1}}$ . The quantity of  $P_{S_{e1}}$  should be understood as the probability of failure of the entire substituted structure.

At this point, analytical formulas obtained in this way will be deliberately withheld because the authors of the method have applied unnecessarily strong assumptions that reduce the usage of the method. These overly strong assumptions are [3]: 1) only those sewers which are the leaves in the graph structure of the network have effluence  $q_i$  greater than 0; 2) the network is a binary tree, i.e., apart from the nodes which are the ends of the leaves, each node joins only two inflow sewers and one outflow sewer.

As has been demonstrated [5], assumption 1) is not mandatory and disregarding it only slightly complicates the formulas used to calculate the values  $\gamma_{e1}$ , as well as  $Q_e$ . One can also disregard assumption 2) by introducing dummy complementary sewers to the graph structure with effluence and parameter  $\gamma$  with a value of '0'. This also enables to analysis of those nodes that have more than two inflow sewers as well as those nodes with only one inflow sewer.

For such modified assumptions, appropriate analytical formulas to calculate the values  $\gamma_{e1}$ , and  $Q_e$  are as follows [5]:

$$Q_e = \frac{(\gamma_2 + \gamma_1)q_2 + (\gamma_3 + \gamma_1)q_3 + \gamma_1q_1}{1 + \gamma_1 + \gamma_2 + \gamma_3} T \quad (9)$$

and

$$\gamma_{e1} = \frac{(\gamma_2 + \gamma_1)q_2 + (\gamma_3 + \gamma_1)q_3 + \gamma_1q_1}{(1 + \gamma_3)q_2 + (1 + \gamma_2)q_3 + (1 + \gamma_2 + \gamma_3)q_1} \quad (10)$$

where  $T$  is time, for which the expected value of unreleased wastewater is calculated.

Considering that:

$$Q_e = P_{S_{e1}}(q_1 + q_2 + q_3)T \quad (11)$$

it is also easy to calculate the value of  $P_{S_{e1}}$ .



For the purpose of further analysis, solutions should be introduced for a structure where there is only one outflow sewer; therefore, sewer  $k_3$  in Fig. 2a and state 3 in Fig. 2b are not present. For such a system, value  $\gamma_{e1}$  is a simple consequence of equation (10):

$$\gamma_{e1} = \frac{(\gamma_2 + \gamma_1)q_2 + \gamma_1 q_1}{q_2 + (1 + \gamma_2)q_1} \quad (12)$$

whereas  $Q_e$  results directly from equation (9):

$$Q_e = \frac{(\gamma_2 + \gamma_1)q_2 + \gamma_1 q_1}{1 + \gamma_1 + \gamma_2} T \quad (13)$$

by a simple substitution of  $q_3$  and  $\gamma_3$  with '0'.

Full procedures for obtaining analytical formulas for appropriate cases, including the case with more than two inflow sewers, have already been provided in the mentioned research [5].

#### 4. The accuracy of the decomposition and equivalent substitution method

In spite of such relaxed assumptions of the method, it contains simplifications that can potentially affect the final result of calculating the expected value of unrecovered sewage. Above all, the assumption of negligible low probability of simultaneous damage of two or more channels in the wound structure is still valid. Also the very fact of gradual reduction through the decomposition of the network structure means that in the next steps we are considering, we do not take into account the states of the remaining part.

Despite mitigation the method's assumptions, it contains simplifications that can potentially affect the final result of calculating the expected value of unreleased wastewater. Firstly, the assumption that there is a negligibly low probability of simultaneous failure of two or more sewers in the reduced structure still remains. Additionally, the fact that there is an incremental reduction through decomposition of the network's structure means that in the steps considered next, the state of the remaining parts of the aforementioned structure is not taken into account.

In order to validate the accuracy of the method, an analysis was performed of a simple sewage system structure of a small town in southern Poland [4] which is reduced to three collectors. By keeping the sewage system markings consistently as a rooted tree-like graph (as in Fig. 1), the numbering is carried out inversely to the direction of sewage runoff (Fig. 3a). Therefore, the node  $w_0$  is the ending tank of the network, sewer  $k_1$  runs from node  $w_1$  to  $w_0$ , sewer  $k_2$  runs from node  $w_2$  to  $w_1$ , and sewer  $k_3$  runs from node  $w_3$  to  $w_2$ . It can be observed that this is a case in which all structures reduced through decomposition contain only one inflow sewer – this corresponds to a situation using formulas (12) and (13).

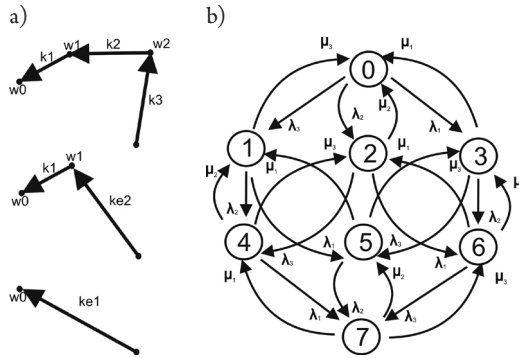


Fig. 3. Diagram of the analysed network and subsequent stages of its reduction (a) and the corresponding graph of states (b)

For such a network structure, the considered states in the graphs method are as follows:

- $S_0$  – state where all sewers are operational
- $S_1$  – state where sewer  $k_3$  I is inoperable
- $S_2$  – state where sewer  $k_2$  is inoperable
- $S_3$  – state where sewer  $k_1$  is inoperable
- $S_4$  – state where sewers  $k_3$  and  $k_2$  are inoperable
- $S_5$  – state where sewers  $k_3$  and  $k_1$  are inoperable
- $S_6$  – state where sewers  $k_2$  and  $k_1$  are inoperable
- $S_7$  – state where all sewers are inoperable

The matrix of intensity of transits for such a case is  $8 \times 8$  and appears as follows:

$$\Lambda = \begin{pmatrix} -(\lambda_3 + \lambda_2 + \lambda_1) & \mu_3 & \mu_2 & \mu_1 & \dots \\ \lambda_3 & -(\mu_3 + \lambda_2 + \lambda_1) & 0 & 0 & \\ \lambda_2 & 0 & -(\lambda_3 + \mu_2 + \lambda_1) & 0 & \\ \lambda_1 & 0 & 0 & -(\lambda_3 + \lambda_2 + \mu_1) & \\ 0 & \lambda_2 & \lambda_3 & 0 & \\ 0 & \lambda_1 & 0 & \lambda_3 & \\ 0 & 0 & \lambda_1 & \lambda_2 & \\ 0 & 0 & 0 & 0 & \\ \dots & 0 & 0 & 0 & \\ \mu_2 & \mu_1 & 0 & 0 & \\ \mu_3 & 0 & \mu_1 & 0 & \\ 0 & \mu_3 & \mu_2 & 0 & \\ -(\mu_3 + \mu_2 + \lambda_1) & 0 & 0 & \mu_1 & \\ 0 & -(\mu_3 + \lambda_2 + \mu_1) & 0 & \mu_2 & \\ 0 & 0 & -(\lambda_3 + \mu_2 + \mu_1) & \mu_3 & \\ \lambda_1 & \lambda_2 & \lambda_3 & -(\mu_3 + \mu_2 + \mu_1) & \end{pmatrix} \quad (14)$$

Adoption of empirically determined the failure and renewal unit intensities [4]:

$$\lambda_0 = 0.4 \cdot 10^{-4} \text{km}^{-1} \text{h}^{-1} = 0.35 \text{km}^{-1} \text{a}^{-1}$$

$$\mu_0 = 0.1 \text{h}^{-1}$$

leads to the following sewer parameters (Table 1):

Table 1. Sewer parameters of the analysed sewage system necessary for calculations in the two presented methods [4]

Sewer	Lenght [km]	Effluence [ $Q_n$ ]	Intensity of failure [ $\text{h}^{-1}$ ]	Intensity of renewal [ $\text{h}^{-1}$ ]	Parameter $\gamma$ [-]
k1	1.0	0.16	0.000040	0.1	0.0000040
k2	0.8	0.21	0.000032	0.1	0.0000032
k3	5.5	0.63	0.000220	0.1	0.0000220

For such a set of parameters, the calculated probabilities of appropriate states as well as sum of effluences from those sewers from which, in a given state, the wastewater would not be released are as follows (Table 2):

Table 2. Values of probabilities of the state of the analysed sewage system, sum of effluences of not disposed sewage as well as products of these values

State	Probability of state $P_{si}$	Sum of effluences $Q_{si}$ [ $Q_n$ ]	$P_{si} \cdot Q_{si}$ [ $Q_n$ ]
$S_0$	0.99708679925	0.00	0
$S_1$	0.00219359096	0.63	$1.381962 \cdot 10^{-3}$
$S_2$	0.00031906778	0.84	$2.680169 \cdot 10^{-4}$
$S_3$	0.00039883472	1.00	$3.988347 \cdot 10^{-4}$
		Sum $S_0$ to $S_3$	0.001650569
$S_4$	0.00000070195	0.84	$5.896372 \cdot 10^{-7}$
$S_5$	0.00000087744	1.00	$8.774364 \cdot 10^{-7}$
$S_6$	0.00000012763	1.00	$1.276271 \cdot 10^{-7}$
$S_7$	0.00000000028	1.00	$2.807796 \cdot 10^{-10}$
		Sum $S_0$ to $S_7$	0.002050409

Finally, the value of sewage that is not disposed due to failures calculated by the graph method as a sum of values from Column 4 of Table 2 is equal to  $Q = 0.002050409 Q_n$ .

It is easy to observe that in the analysed example, MDE needs to use reduction only twice. If the parameters of the first reduction (sewers k3 and k2 to sewer ke2) are marked as  $\gamma_{e2}$ ,  $Q_{e2}$  and  $q_{e2}$  and, simultaneously, the parameters of the second reduction (sewers ke2 and k1 to sewer ke1) are marked as  $\gamma_{e1}$ ,  $Q_{e1}$  and  $q_{e1}$ , then the results of these two reductions are as follows (Table 3):



Table 3. Results of calculations of the appropriate parameters of the KDE method in the analysed sewage system

Equivalent sewer	$\gamma_{ei}$	$q_{ei} [Q_n]$	$Q_{ei} [Q_n]$
$e_2$	0.0019689171	0.84	0.001650640
$e_1$	0.0020532435	1.00	0.002049036

It should be observed that  $Q_{e1}$  corresponds to the sought expected value of unreleased wastewater and  $Q_{e2}$  corresponds to the sum of values from Column 4 of Table 2 for states 1, 2, and 3 which gives the result of 0.001650569.

It can be observed that in both cases, values  $Q_{e1}$  and  $Q_{e2}$  are almost identical to the values calculated when using graph method.

## 5. Conclusions

By comparing the results obtained by using both methods, one can conclude that MDE provides results that are sufficiently satisfactory for calculating the expected value of unreleased wastewater. It is obvious that the analysed case is very simple and one can deliberate whether possible deviations from the values obtained by more precise methods would become greater with increasing amount of elements in the network or with its structural complexity. Further analysis is therefore needed with special attention given to the influence of network complexity on the precision of calculations.

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