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Procedia Engineering 208 (2017) 129–135

Procedia Engineering

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2nd International Joint Conference on Innovative Solutions in Construction Engineering and Management: 16th Lithuanian-German-Polish colloquium and 6th meeting of EURO working group Operational Research in Sustainable Development and Civil Engineering 24 May- 2nd International Workshop on flexibility in sustainable construction, ORSDCE 2017, 24-26 April 2017, Poznan-Puszczykowo, Poland

Linearised CPM-COST model in the planning of construction projects

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Abstract

Due to the specific nature of construction projects, non-linear and discrete dependencies are seen as appropriate for the modelling of issues of time and cost. Approximation algorithms are usually used in order to precisely solve the problem of cost minimisation. In order to find a precise solution for non-linear dependencies, the authors propose their linearisation and the use of linear programming methods in order to determine the minimum cost of a project. The proposed linearised CPM-COST model has been formally written in the form of a linear programming problem. The model is helpful in determining the duration and deadlines of the performance of tasks under the conditions of a set directive deadline, with the objective of minimising total costs. Using a computer program written in the Python language, the authors have presented the model using a calculation example. The authors are currently working on improving the linearised model for non-linear time-cost dependencies and the development of a linearised model for discrete functions.

Peer-review under responsibility of the scientific committee of ORSDCE 2017.

Keywords: CPM-COST; Time-Cost Optimisation; linearization; linear programming

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1877-7058
Peer-review under responsibility of the scientific committee of ORSDCE 2017.
10.1016/j.proeng.2017.11.030

1. Introduction

Different approaches are used in the process of planning construction projects due to the character of the data that is being encountered during it: including a deterministic, probabilistic [1] and fuzzy one [2]. Based on the manner of their optimisation, they are divided into single [3] and multi-criteria approaches [4]. During the planning and carrying out of a construction project, engineers encounter various problems, e.g. in the selection of subcontractors [5] in the case of public projects, as well as problems associated with managing a project and a construction company [6]. The problems of the time and cost of an entire project are very important.

There are many methods of time-cost optimisation (TCO). In most cases, a non-linear [7] or discrete [8,9] dependency of cost to time for each task is assumed. This is justified due to the specifics of construction projects. Metaheuristic [10,11], mixed-integer non-linear programming (MINLP)[12] or mixed-integer linear programming (MILP)[13] methods are usually used in these types of problems.

Linear dependency, which is used in the classic CPM-COST method [14,15] is idealised, which is why a linearisation of non-linear time-cost dependency has been proposed. Linearisation should be understood as an approximation of the non-linear function of cost do time, a first-degree spline linear function. Linearisation is used in order to simplify the model and the involved calculations. Linearised models are often used in the construction industry. [16] features a proposition of a linearised model in order to analyse the characteristics of noise and vibrations in the railway industry. The paper [17] features the use of linearisation to perform seismic analysis during the construction of tall telecommunications masts. [18] features the linearised mode-shape (LMS) method being used to analyse the influence of wind on tall buildings. [19] uses a linearised approach to prognosticate thermal stresses within an asphalt surface layer. The authors have not encountered the use of linearisation in the performance of time-cost analyses of construction projects using the classic CPM-COST method.

The goal of the article is the development of a linearised CPM-COST model in the planning of construction projects. The method omits the methodology of the process of the linearisation of a non-linear dependency itself, assuming that a linearised function for each task is known. Linearisation can be performed, for instance, using the least squares method.

The proposed model has been implemented in the Python programming language. The generally accessible PyMathProg environment[20], which is used to model, solve, analyse and modify linear programming tasks, has also been used. The source code of the program can be made available by the authors.

Nomenclature

r	amount of tasks within the network
t_k	task k duration
Tf_k	task k completion deadline
Tf	project completion deadline
$d_{k,i}$	time-cost dependency linearising variables for task k
tgr_k	crash time for task k
kgr_k	crash costs for task k
$t_{k,i}$	linearity segment length for task k
$a_{k,i}$	slope value for the binding line for segment $t_{k,i}$
p_k	number of linearity segments for task k
POP_k	precursor index set for task k
B	network starting task index set
L	network ending task index set

2. The linearised CPM-COST model

2.1. Model A - the minimisation of direct costs under the conditions of a determined directive deadline

The model that is presented in the article will be called model A. A CPM network is analysed in a single-point (node) version, composed of r tasks. POP_k will be a set of the indices of precursors for task k, B is going to be a set of indices of the starting tasks within the network (the tasks that do not have precursors), L - will be a set of indices of the final tasks within the network (tasks that do not have successors). The tasks are connected by a Finish-Start relation (FS).

Each activity within the network is described by *(tgr,kgr,TA)* parameters. The parameter *tgr* defines the crash (minimal) task completion time, while parameter *kgr* defines the crash (maximum) costs that need to be paid in order to perform the task in crash time. The *TA* parameter is going to be defined in the following manner: $TA=(t_1,a_1,t_2,a_2...t_p,a_p.)$, where the t_i coefficient determines the length of the segment inn which a negative linear function is dominant and whose slope is equal to a_i. The a_i value needs to take on negative values and in the case when p>1 then a_i coefficients for the successive segments need to meet the condition shown in formula (1).

$$a_{i+1} > a_i, \forall i \in \{2...p\}. \tag{1}$$

Figure 1 shows a sample time-cost dependency for every task within the network and a graphical interpretation for parameters t_i and a_i.

tgr	3
kgr	1200

k	t	a
1	3	-200
2	1	-100
3	4	-25

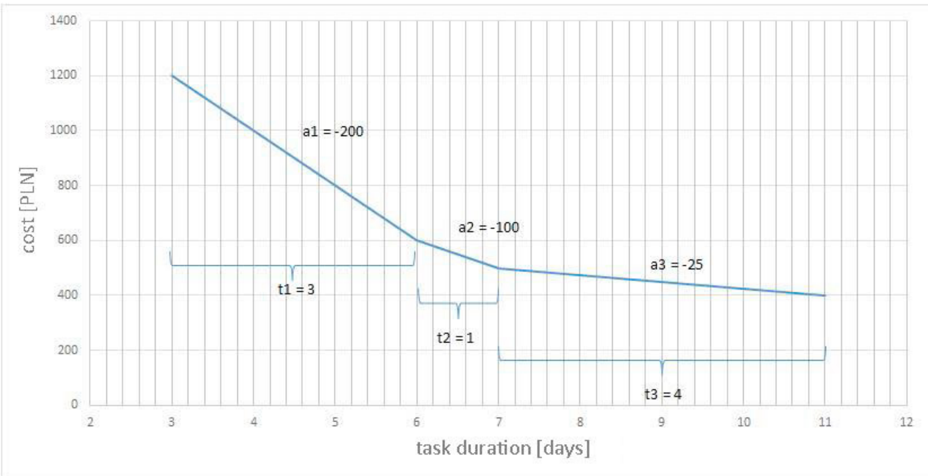

Fig. 1. Time-cost dependency for tasks along with an interpretation of parameters

A task can be completed during a t duration, that meets the condition outlined in formula (2).

$$t \in \left\langle tgr; tgr + \sum_{i=1}^{p} t_i \right\rangle. \tag{2}$$

For these conditions, the problem can be described in the form of a linear programming model in the following manner:

Decision-making variables: $t_k, Tf_k, Tf, d_{k,i}$, where $i \in \{1,2....p_k\}, k \in \{1,2...r\}$

Goal function: $K \to \min : K = \sum_{k=1}^{r} \sum_{i=i}^{p_k} d_{k,i} a_{k,i} + \sum_{k=1}^{r} kgr_k$ (3)

Limitations:

$$t_k \geq tgr_k, \forall k \in \{1,2...r\} \tag{4}$$

$$t_k \leq tgr_k + \sum_{i=1}^{p_k} t_{k,i}, \forall k \in \{1,2...r\} \tag{5}$$

$$d_{k,i} \leq t_{k,i}, \forall i \in \{1,2....p_k\}, \forall k \in \{1,2...r\}, \tag{6}$$

$$tgr_k + \sum_{i=1}^{s} d_{k,i} \leq t_k, \forall s \in \{1,2...p_k\}, \forall k \in \{1,2...r\} \tag{7}$$

$$Tf_b \geq t_b, \forall b \in B \tag{8}$$

$$Tf_k - Tf_{pop} \geq t_k, \forall k \in \{1,2...r\}, \forall pop \in POP_k \tag{9}$$

$$Tf \geq T_l, \forall l \in L \tag{10}$$

$$Tf = td \tag{11}$$

$$d_{k,i} \geq 0 \tag{12}$$

The decision variables of the model are: t_k - which is the duration of task k, Tf_k - which is the time of the completion (deadline) of the entire project, $d_{k,i}$ - are linearising variables for the time-cost dependency for task k.

The goal function (formula 3) is the sum of the direct costs of completing all tasks. The value of the goal function is minimised.

The duration of each task is longer or equal to the crash time (formula 3) and at the same time lower than or equal to maximum time (formula 5) in accordance with the dependency outlined in formula (2). The auxiliary variables $d_{k,i}$ have upper limits set through appropriate lengths of the segments of the linearised function (formula 6). The dependency described in formula (7) allows us to determine the value of the $d_{k,i}$ variables. The successive three limitations define the relationships within the CPM network. Formula (8) defines the possible completion dates for the starting tasks within the network. Formula (9) defines the dependency of the dates of the completion of tasks connected with a Finish-Start relation. Formula (10) allows us to determine the date of the completion of the entire project (marked as Tf). The date of the completion of the entire project needs to be equal to the directive deadline (formula 11). The condition of the non-negativity of variables t_k, Tf_k and Tf is executed by other conditions. Thus we should only assume that the auxiliary variable $d_{k,i}$ is either higher than or equal to zero (formula 12).

2.2. Model B - the minimisation of the direct and indirect costs of a project

The model described in this subchapter is going to be called model B.
Model A can be modified in order to be used to determine the task completion time that allows for the sum of direct and indirect costs to be minimal. The following changes need to be applied to the model:
The goal function (formula 2) will take on a form described in formula (13), while formula (11) must be removed.
.

$$K \to \min : K = \sum_{k=1}^{r} \sum_{i=i}^{p_k} d_{k,i} a_{k,i} + \sum_{k=1}^{r} kgr_k + Tf \cdot k_{poś} \tag{13}$$

The share of indirect costs has been included in the goal function. Indirect costs have been calculated as the product of parameter k_{pos}, which defines indirect unit costs, times the project duration Tf. The condition of the equality of the project duration and the directive deadline had to be removed.

3. Calculation example

3.1. Example A - the minimisation of direct costs with a set directive deadline

The proposed model A has been used to optimise the costs of a project composed of 10 tasks. The names of the tasks have been marked with the successive letters of the alphabet. The single-point network has been depicted on figure 2. All tasks are connected by a FS (Finish-Start) relation. The directive deadline was set at 35 days.

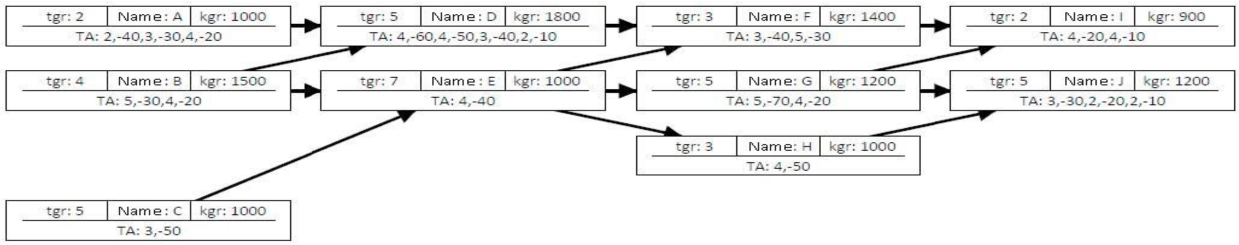

Fig. 2. Example of a network

The network used in the example yielded costs equal to 10030 PLN. The CPM network for the obtained completion times have been presented in figure 3. The critical path has been shown in red.

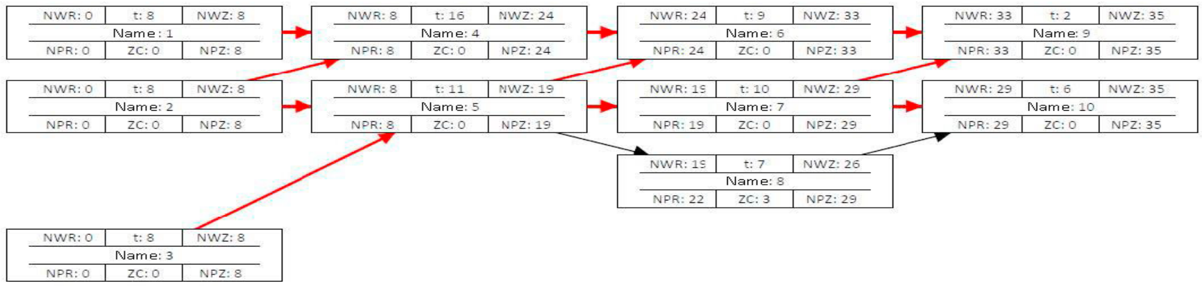

Fig. 3. An optimal solution for a directive deadline of 35

The minimum directive deadline that can be met is 22 days (assuming crash times are used), while the maximum is 52 days (assuming the maximum task durations are used). Calculations have been performed for every possible directive deadline. The graph of the costs in relation to the directive deadline has been shown on figure 4. According to expectations, it is a negative function that resembles an exponential function.

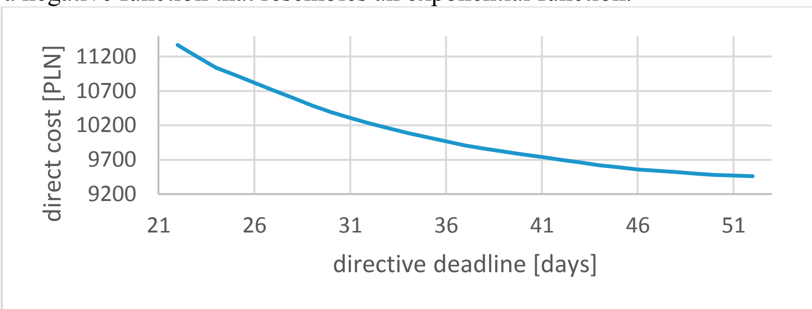

Fig. 4. The dependency between the diret cost of a task and the directive deadline

3.2. Example B - the minimisation of the direct and indirect costs of a project

The operation of model B was verified as well. Example B was identical to example A, with the exception of taking into account additional indirect unit costs k_{pos} at a level of 65 PLN/day. These parameters yielded a completion time of 34 days and the respective total cost of 12300 PLN.

In order to verify whether model B works properly, calculations using model A have also been performed.

Optimisation using model A was performed for every possible directive deadline, with indirect costs being factored in afterwards. Figure 5 shows the minimum total costs for all possible completion times (between 22 and 52 days). The minimum total cost of 12300 PLN has been attained for a completion time of 34 days (marked on the graph), which is compliant with the result obtained using model B.

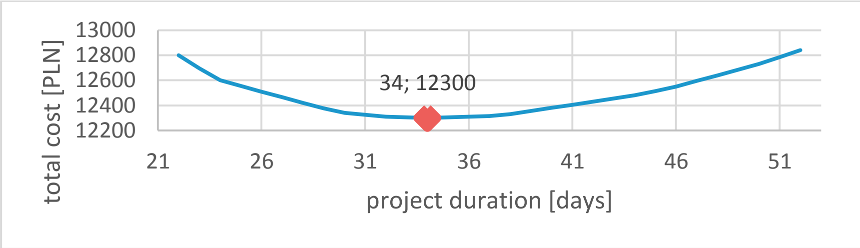

Fig. 5. The dependency of the minimum total cost of a task to its completion time.

4. Conclusion

The linearised CPM-COST model proposed in the article has been verified using calculation examples. The greatest advantage of the proposed model is its linearity, which makes calculations simpler and allows optimal solutions to be determined. A solution is considered optimal for an approximated linearised time-cost dependency, while there is no limit to increasing the density of linearity segments, which allows increasingly precise results to be obtained until the attainment of the required precision. The disadvantage of the proposed model is that it is convergent only when the derivative of the function of cost is a negative function for every task. Work is ongoing on improving the linearised model so that it can be applied to non-linear time-cost dependencies, as well as on the creation of a linearised model for discrete functions. The proposed linarised CPM-COST model is going to be applied in extending the model for use in the time-cost planning of the carrying out of multiple-structure construction projects.

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