

Barbara Larwa (bl@chemia.pk.edu.pl)

Chair of Chemical and Process Engineering, Faculty of Chemical Engineering  
and Technology, Cracow University of Technology

## INVESTIGATION OF TEMPERATURE DISTRIBUTION IN THE GROUND INDUCED BY HEAT SOURCE AND UNDER NATURAL CONDITIONS

### BADANIA ROZKŁADU TEMPERATURY W GRUNCIE WYWOŁANEGO ŹRÓDŁEM CIEPŁA ORAZ W WARUNKACH NATURALNYCH

#### Abstract

This paper deals with theoretical and experimental analysis of heat transfer in the ground. Calculation results of heat transfer under natural conditions were presented. Temperature profiles in the ground were determined for cyclic steady state. Results of experimental studies conducted in laboratory setup were shown. Experimental studies were related to determination of temperature of the heated granular bed. The calculation and measurement results presented were used to determine a mathematical model of the ground heat exchanger.

**Keywords:** process modelling, transient heat conduction, renewable energy sources

#### Streszczenie

Artykuł dotyczy analizy teoretyczno-doświadczalnej przenoszenia ciepła w gruncie. Przedstawiono wyniki obliczeń przenoszenia ciepła w warunkach naturalnych. Wyznaczono profile temperatur w gruncie w zależności od czasu dla cyklicznego stanu ustalonego. W części eksperymentalnej przedstawiono wyniki badań prowadzonych w instalacji laboratoryjnej. Badania dotyczyły wyznaczenia temperatur w ogrzewanym złożu ziarnistym. Przedstawione wyniki badań i obliczeń zostały wykorzystane do opracowania modelu matematycznego gruntowego wymiennika ciepła.

**Słowa kluczowe:** modelowanie procesów, nieustalone przewodzenie ciepła, odnawialne źródła energii

## Nomenclature

|            |  |
|------------|--|
| $a$        | – thermal diffusivity of the ground, $\text{m}^2/\text{s}$                                   |
| $B$        | – oscillation amplitude around the temperature $T_v$ , K                                     |
| $Bi$       | – Biot number  |
| $C_1, C_2$ | – constants dependent on the Biot number   |
| $c$        | – heat capacity of the ground, $\text{J}/(\text{kgK})$                                       |
| $h_0$      | – heat transfer coefficient, $\text{W}/(\text{m}^2\text{K})$                                 |
| $k$        | – thermal conductivity of the ground, $\text{W}/(\text{mK})$                                 |
| $L$        | – damping depth, m   |
| $q_v$      | – rate of heat generation per unit of volume, $\text{W}/\text{m}^3$                          |
| $t$        | – time, days   |
| $t_c$      | – cycle time, days   |
| $t_{\max}$ | – time from the beginning of the year until the maximum ambient temperature is reached, days |
| $T$        | – temperature, $^{\circ}\text{C}$  |
| $T_a$      | – ambient air temperature, $^{\circ}\text{C}$  |
| $T_b$      | – temperature of the ground at great depth, $^{\circ}\text{C}$                               |
| $T_0$      | – temperature of the ground surface, $^{\circ}\text{C}$                                      |
| $x$        | – position coordinate, m   |
| $\rho$     | – ground density, $\text{kg}/\text{m}^3$   |
| $\omega$   | – frequency, $1/\text{s}$ .  |

## 1. Introduction

The ground is an advantageous heat source for heat pumps. In comparison with the air it has a much more stable temperature and a high specific heat.

Ground heat exchangers are essential parts of ground-source heat pumps. The ground heat exchanger is used for extraction or injection of heat from/into the ground by a heat transfer fluid which circulates in a closed cycle. There are two types of ground heat exchangers, namely horizontal and vertical systems can be used. Horizontal ground heat exchangers have a significant advantage over vertical ones. In the former, shallow layers of the ground are being cooled as a result of extraction of heat from the ground by the exchanger; these layers are in a direct contact with the environment. This results in the fact that the cold ground takes more heat from the environment during the warm period of the year and loses less heat during the cold period. Vertical exchangers do not have this beneficial feature as the cold ground at a great depth is not able to compensate the heat loss by receiving heat from the environment [1].

Horizontal ground heat exchangers have been widely used in many countries as a heat source for ground-source heat pump systems. Therefore, ground heat exchangers are the subject of many studies that are both experimental and numerical. For example Wu et. al. [2, 3] investigated the thermal performance of slinky heat exchangers for ground source

heat pump systems for the UK climate. The authors presented the results of experimental measurements as well as of numerical simulation.

In order to estimate the power gain while using a heat pump coupled with the ground heat exchanger one should analyse the temperature distribution in the ground under natural conditions.

The aim of this work is to provide important data for a ground heat exchanger design, which is temperature distribution in the ground. The temperature field was determined numerically and experimentally. In this research paper the elements of modelling of heat transfer in the ground as well as measurement results of heating process of the granular bed in laboratory setup were presented. Both aspects play an essential role in developing the mathematical model of a ground heat exchanger coupled with a heat pump.

## 2. Analysis of temperature distribution in the ground under natural conditions

It was assumed that conduction is the only mechanism of heat transfer in the ground, coefficient of thermal diffusivity is invariable over time and space as well as heat transfer taking place only in the direction of the  $x$ -axis (vertical). The equation of heat conduction has the form:

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2} + \frac{q_v}{c\rho} \quad (1)$$

Under natural conditions due to no heat exchanger installed the rate of heat generation  $q_v = 0$ . The boundary condition for the surface of the ground is as follows:

$$x=0 \quad -k \frac{\partial T}{\partial x} = h_0 (T_0 - T_a) \quad (2)$$

In the above relationship the quantity  $h_0$  is the apparent coefficient of heat transfer between the ground surface and the environment. This coefficient determines heat transfer by both: convection and radiation. Due to changes in thermal conditions on the surface of the ground, the quantity  $h_0$  is averaged over time.

The ambient temperature  $T_a$  changes periodically according to the relationship:

$$T_a = T_b + B \cdot \cos \left[ \omega (t - t_{\max}) \right] \quad (3)$$

where the cycle time  $t_c \approx 365$  days, hence the frequency  $\omega = 2\pi/t_c = 0.0172 \text{ day}^{-1}$ . On the basis on the data for Kraków, shown in [4], constants in relationship (3) were determined. These constants are as follows:  $T_b = 8.5^\circ\text{C}$ ,  $B = 10.4 \text{ K}$ ,  $t_{\max} = 198$  days.

The ground should be considered as a semi-infinite body. Hence, the second boundary condition related to a constant temperature of the ground at great depth has the following form:

$$x \rightarrow \infty \quad T = T_b \quad (4)$$

The solution of equation (1) with boundary conditions (2–3) and (4) leads to the relationship for the cyclic steady state [5]:

$$T = T_b + BC_1 \cdot \exp\left(-\frac{x}{L}\right) \cdot \cos\left[\omega(t - t_{\max}) - \frac{x}{L} - C_2\right] \quad (5)$$

The constants  $C_1$  and  $C_2$  are dependent on the Biot number:

$$C_1 = \frac{Bi}{\sqrt{(Bi+1)^2 + 1}} \quad (6)$$

$$C_2 = \tan^{-1}\left(\frac{1}{Bi+1}\right) \quad (7)$$

where  $Bi$  is the Biot number determining the ratio between the internal and external thermal resistance to heat transfer:

$$Bi = \frac{h_0 L}{k} \quad (8)$$

The internal resistance to heat transfer is located in the ground, while the external one is related to heat transfer between the ground surface and the atmospheric air.

If the external thermal resistance is slight in comparison with the internal one,  $Bi \rightarrow \infty$  and then  $C_1 = 1$ ,  $C_2 = 0$ . In this case relationship (5) simplifies to the form [6]:

$$T = T_b + B \cdot \exp\left(-\frac{x}{L}\right) \cdot \cos\left[\omega(t - t_{\max}) - \frac{x}{L}\right] \quad (9)$$

In definitions of  $Bi$  the quantity  $L$  is a damping depth defined as follows:

$$L = \sqrt{\frac{2a}{\omega}} \quad (10)$$

In Figs. 1a and b the temperature profiles in the ground, determined according to relationships (5) and (9) are presented. The ground temperature profiles are shown for 1.5-month intervals. The calculations were performed for  $Bi = 2$  and  $Bi \rightarrow \infty$ . As one can see from Figs. 1a and b, for  $x/L > 5$  the temperatures change slightly. In addition, for smaller values of  $Bi$  the temporal variations of temperatures in the sub-surface layers of the ground are insignificant. For example for  $a = 0.5 \cdot 10^{-6} \text{ m}^2/\text{s}$  the damping depth  $L = 2.24 \text{ m}$  was calculated. For this value stabilization of the ground temperature takes place at a depth below around  $5 \cdot 2.24 \approx 11 \text{ m}$ .

In Table 1 the calculation results of temperature of the ground for different dates for finite ( $Bi = 2$ ) and infinite values of the Biot number are presented. In this second case because of the lack of the external thermal resistance to heat transfer, temperature of the ground is identical with ambient air temperature. The numbers in the 4<sup>th</sup> and 5<sup>th</sup> columns correspond to values read from Fig. 1b for  $x/L = 0$ . In the last two columns the values of the ground surface temperature corresponding to  $Bi = 2$  and  $Bi \rightarrow \infty$ , respectively, are shown. The values

of the temperature for  $Bi = 2$  should be treated as more realistic in comparison with the values calculated for  $Bi \rightarrow \infty$ . Differences in the temperatures from the last two columns show that the conditions on the ground surface substantially affect its temperature and consequently the amount of heat which the ground can receive from (or transfer to) the air.

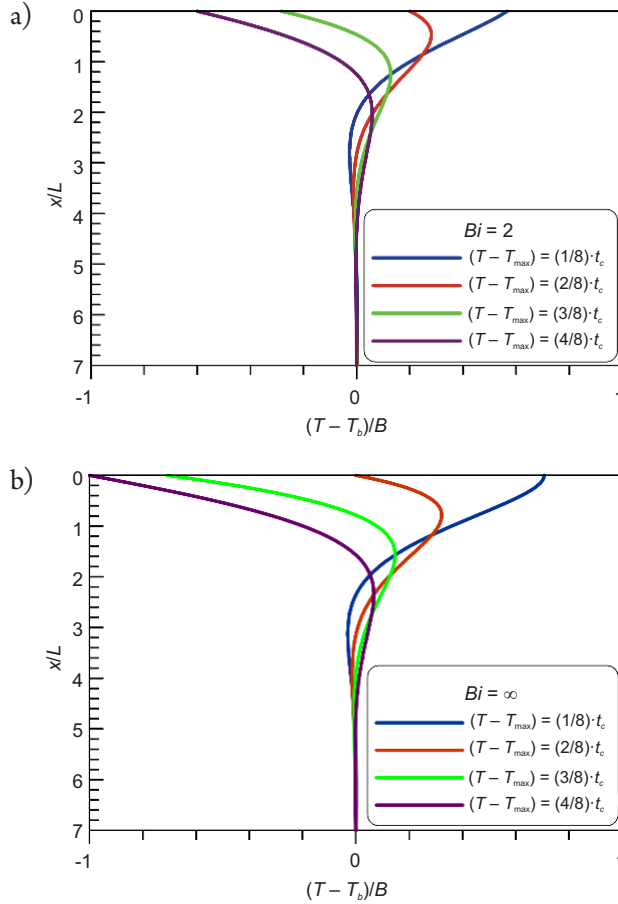


Fig. 1. Ground temperature profiles for a)  $Bi = 2$ , b)  $Bi \rightarrow \infty$

Table 1. Temperatures of the ground surface

| $(t - t_{\max})/t_c$ | $t$ [days] | date   | $(T - T_b)/B$ |                         | $T$ [°C] |                         |
|----------------------|------------|--------|---------------|-------------------------|----------|-------------------------|
|                      |            |        | $Bi = 2$      | $Bi \rightarrow \infty$ | $Bi = 2$ | $Bi \rightarrow \infty$ |
| 1/8                  | 244        | Sep 1  | 0.565         | 0.707                   | 14.4     | 15.8                    |
| 2/8                  | 289        | Oct 15 | 0.200         | 0.000                   | 10.6     | 8.5                     |
| 3/8                  | 335        | Dec 1  | -0.282        | -0.707                  | 5.6      | 1.2                     |
| 4/8                  | 380        | Jan 15 | -0.597        | -1.000                  | 2.3      | -1.9                    |

The difference between the ground temperature  $T$  at specified depth and temperature of the ground surface  $T_0 = (T)_{x=0}$  was also analysed. Dependence between the difference  $T - T_0$  with the position for individual, considered days in a year was presented in Fig. 2. The greatest value of differences, above 10 K, falls into periods of the lowest and the highest ambient temperature. These maximum differences are reached at a fairly large depth (approx. 5 m). During transitional periods (Apr 15, Oct 15) temperature differences between the interior of the ground and its surface are slight. Maximum values, about 3 K, are reached at depth approx. 2 m.

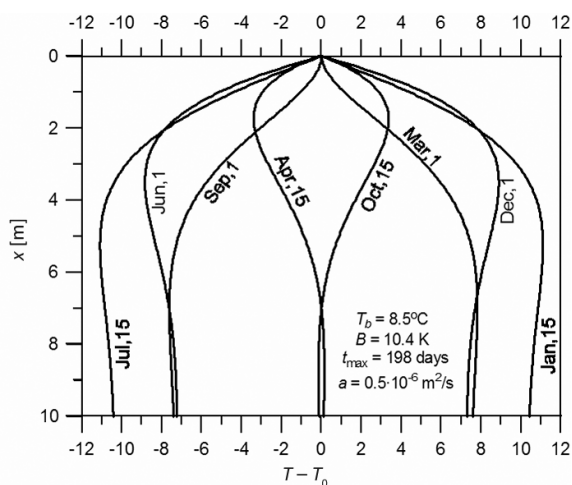


Fig. 2. Dependence of the difference of temperatures  $T - T_0$  on position and time

### 3. Experimental investigations of heat conduction in a granular bed

When a heat exchanger is installed in the ground the rate of heat generation per unit volume  $q_v$  does not equal zero, which results in changes in temperature distribution in the ground in comparison with natural conditions. In this case the heat source constitutes heat exchanger tubes located horizontally at a specified depth.

In papers [7, 8] numerical simulations of heat conduction in the ground with an arrangement of horizontal and parallel heat exchanger pipes in it were carried out using the ANSYS application. The experimental investigations described below were carried out in order to verify obtained numerical results and to examine the temperature profiles in the ground.

Experimental studies concerning the temperature distribution in granular bed were performed in a laboratory setup, which was composed of two horizontal copper pipes 10 mm in diameter placed in a grain material – dry quartz sand [7]. The distance between the pipes was equal to 300 mm. The heat transfer took place in a 650×450×150 mm organic glass cuboidal vessel. Copper pipes for almost the entire length were lagged. There was no insulation only at sections located horizontally in vessel (with 150 mm length). The scheme of this experimental setup is shown in Fig. 3.

Water at a temperature of 50°C was flowing through the pipes. Appropriately a high flow rate of water provided a practically temperature invariability between the inlet and the outlet of the vessel. The initial sand temperature was equal to 21°C. The temperature outside of the vessel with the granular bed was also 21°C.

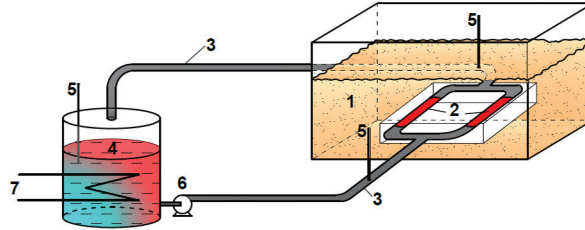


Fig. 3. Scheme of experimental setup (1 – vessel with granular bed, 2 – pipes without isolation, 3 – lagged pipes, 4 – thermostatic tank, 5 – thermometers, 6 – pump, 7 – heating element)

24 temperature sensors DS18B20 ( $\pm 0,5^\circ\text{C}$ ) were placed round the pipes according to the scheme in Fig. 4. The sensors were connected with a data processing system (Lämpömittari). The duration of measurement was equal to 10 hrs.

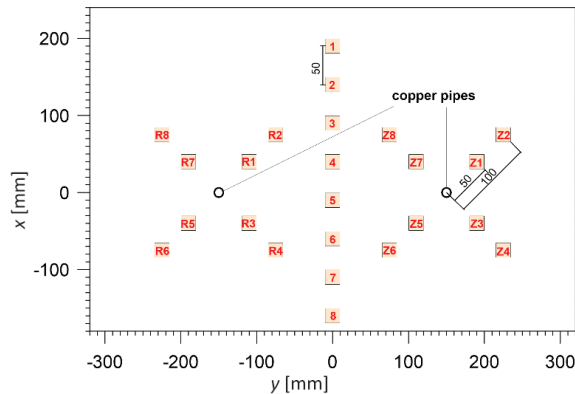


Fig. 4. Scheme of temperature sensor locations

In Fig. 5 the temperature variations of individual sensors were shown. Red indicates the courses designated by the sensors located closest the pipes (50 mm), blue – by the sensors located at a distance of 100 mm from the tube, and brown – by the sensors arranged vertically in the middle of the vessel between the tubes (as presented in Fig. 4). The highest temperature ( $29\text{--}30^\circ\text{C}$ ) were registered by sensors placed in close proximity to the pipe. Sensors located around the pipe, but at a greater distance, recorded a lower temperature – an average of about  $3^\circ\text{C}$ . The lowest temperature were recorded by sensors located between the pipes.

In Fig. 6 experimental maps of isotherms in granular bed for different heating process duration time were shown. The figures are related to the cross section of the system of pipes. The lines of constant temperature of the bed expressed as number (in  $^\circ\text{C}$ ) were presented. The axes correspond to the respective coordinate positions within the container.

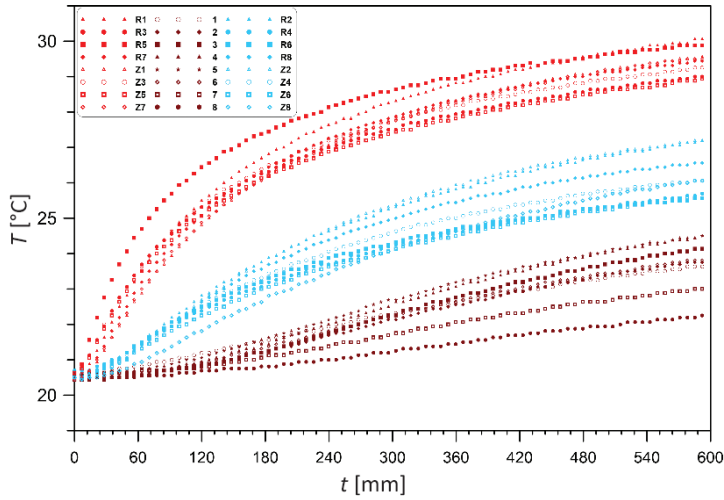


Fig. 5. Measured temporal courses of temperature of granular bed

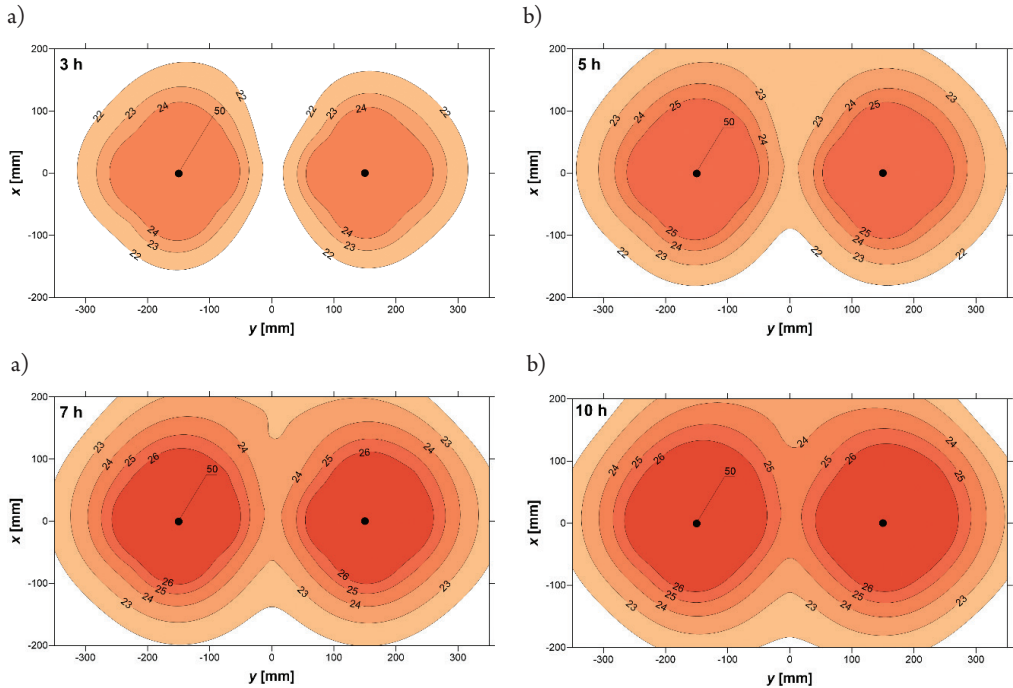


Fig. 6. Experimental maps of temperatures for a) 3 h, b) 5 h, c) 7 h, d) 10 h of heating process of granular bed

As you can see, at the beginning (3 h) each pipe heats the ground individually, independently of the other pipes. The isotherm shapes approximate a circle. However, after some time, the temperature fronts come together and their shape is flattened. After 5 hours of heating process the isotherms of 22°C are linked, isotherms of 23°C are connected after 7 hours and isotherms of 24°C – after 10 hours.



#### 4. Conclusions

The resistance to heat transfer between the surface of the ground and the environment (external thermal resistance to heat transfer) strongly affects the temperature distribution of the sub-surface layer of the ground.

Performed experimental results confirmed the numerical simulations [7, 8] – at the beginning each pipe heats the ground individually and after some time the temperature fronts come together and an increasing part of lines of constant temperature has more rectilinear and parallel to the horizontal position. Thus, the system behaves similarly to the heating the ground by infinite plate. The similarity is even larger when there are more pipes installed and the more densely they are arranged side by side. That provides the basis for utilizing the one-dimensional equations of heat conduction for modelling of horizontal ground heat exchangers.

On the base of analysis of received courses of temperature isolines in the ground a simplified mathematical model of heat transfer in a horizontal ground exchanger has been worked out and verified with experimental measurements [9].

#### References

- [1] Gan G., *Dynamic thermal modelling of horizontal ground-source heat pumps*, International Journal of Low-Carbon Technologies 8, 2013, 95–105.
- [2] Wu Y., Gan G., Gonzalez R.G., Verhoef A., Vidale P.L., *Prediction of the thermal performance of horizontal-coupled ground-source heat exchangers*, International Journal of Low-Carbon Technologies 0, 2011, 1–9.
- [3] Wu Y., Gan G., Verhoef A., Vidale P.L., Gonzalez R.G., *Experimental measurement and numerical simulation of horizontal-coupled slinky ground source heat exchangers*, Applied Thermal Engineering, 30, 2010, 2574–2583.
- [4] [www.pogodynka.pl/polska/daneklimatyczne/](http://www.pogodynka.pl/polska/daneklimatyczne/)
- [5] Carslaw H.S., Jaeger J.C., *Conduction of Heat in Solids*, second ed., Clarendon Press, Oxford 1959.
- [6] Staniszewski B., *Wymiana ciepła – podstawy teoretyczne*, WNT, Warszawa 1979.
- [7] Larwa B., *Numerical and experimental analysis of heat transfer in ground exchangers (Analiza numeryczna i eksperymentalna przenoszenia ciepła w wymiennikach gruntowych)*, PhD Thesis, Kraków 2016.
- [8] Larwa B., Teper M., Kupiec K., *Horizontal ground heat exchangers – analytical solutions and numerical simulations*, Technical Transactions, vol. 1-Ch/2016, 57–67.
- [9] Kupiec K., Larwa B., Gwadera M., *Heat transfer in horizontal ground heat exchangers*, Applied Thermal Engineering 75C, 2015, 270–276.