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QUANTITATIVE ESTIMATION OF THE IMPACT OF RANDOM  
CONFOUNDING FACTORS ON THE DURATION AND COST OF  
CONSTRUCTION WORK

KWANTYTATYWNE SZACOWANIE WPŁYWU LOSOWYCH  
CZYNNIKÓW ZAKŁÓCAJĄCYCH NA CZAS TRWANIA  
I KOSZTY ROBÓT BUDOWLANYCH

**Abstract**

Random events may interfere with the execution process and results of the work. Then, the duration and cost of work are random variables. The random characteristics of these variables are described by using the  $\beta$ -PERT probability distribution and the simplified formulas. To determine the probable changes of the duration and cost of work, depending on the random factors, the coefficients of optimism and pessimism have been used. Additionally, the coefficients of improvement and deterioration of conditions of work implementation have been introduced. The values of these coefficients are calculated depending on the type, the probability of occurrence, the intensity of random perturbations and depending on the probable changes of the work duration as well as the probability and cost consequences of the price increase. Extreme random improvement and deterioration of the work execution conditions are analysed.

**Keywords:** construction, risk, coefficients of optimism and pessimism

**Streszczenie**

Zdarzenia losowe mogą zakłócać proces realizacji i wyniki robót, wówczas czas trwania i koszty robót są zmiennymi losowymi. Losowe charakterystyki tych zmiennych są opisywane za pomocą rozkładu prawdopodobieństwa  $\beta$ -PERT i uproszczonych formuł. Aby określić prawdopodobne zmiany czasu trwania i kosztów robót, w zależności od czynników losowych, zostały użyte współczynniki optymizmu i pesymizmu. Ponadto, wprowadzono współczynniki poprawy i pogorszenia warunków wykonywania robót. Wartości tych współczynników są obliczane w zależności od typu, prawdopodobieństwa wystąpienia i intensywności losowych perturbacji, prawdopodobnych zmian czasu trwania robót oraz prawdopodobieństwa i konsekwencji kosztowych wzrostu cen. Ekstremalna poprawa i pogorszenie warunków wykonywania robót są analizowane.

**Słowa kluczowe:** budowa, ryzyko, współczynniki optymizmu i pesymizmu

## 1. Introduction

Constructions work is carried out on building sites in actual surroundings and the natural environment. The technological and organizational conditions that characterize the requirements of the object structure execution can be described by using the design characteristics model of structure  $\mathbf{S}$  as follows [1, 2]:

$$\mathbf{S} = \langle \mathbf{G}, \mathbf{L} \rangle \quad (1)$$

$\mathbf{G} = \langle \mathbf{Y}, \mathbf{U}, \mathbf{P} \rangle$  – a coherent and a-cyclic unigraph with a single initial node and a single final node that describes the interdependence and permissible sequences of works,

$\mathbf{Y} = \{y_1, \dots, y_p, \dots, y_k, \dots, y_m\}$  – set of the nodes of the graph representing:  $y_i$  initial event and  $y_i$  final event for each specific piece of work  $u_j \in \mathbf{U}$ ;

$\mathbf{U} = \{u_1, \dots, u_p, \dots, u_1, \dots, u_n\}$  – set of the arcs (arrows) of the graph representing relatively independent pieces of work constrained by initial  $y_i \in \mathbf{Y}$  and final  $y_k \in \mathbf{Y}$  nodes;

$\mathbf{P} \subset \mathbf{Y} \times \mathbf{U} \times \mathbf{Y}$ ,  $\langle y_i, u_j, y_k \rangle \in \mathbf{P}$  – a three-term relation that assigns to each arc  $u_j \in \mathbf{U}$  the initial node  $y_i \in \mathbf{Y}$  and final node  $y_k \in \mathbf{Y}$ ;

$\mathbf{L}: \mathbf{U} \rightarrow \mathbf{R}^+$  – a function defined on the set  $\mathbf{U}$  of arcs of the graph  $\mathbf{G}$  determines the expected values  $E[L_j]$  of the variables  $L_j$ , and describes the size of works  $u_j \in \mathbf{U}$ .

The resource requirements and conditions for executing the work are described by using the construction technology model  $\mathcal{L}$  as follows [1, 2]:

$$\mathcal{L} = \{ \langle \mathbf{H}, \mathbf{K}, \mathbf{T} \rangle, \mathbf{S} \} \quad (2)$$

$\mathbf{H} = \{ \mathbf{H}^1, \dots, \mathbf{H}^r, \dots, \mathbf{H}^s \}$  – the set of teams  $\mathbf{H}^r$  for work  $u_j \in \mathbf{U}$  execution,

$\mathbf{T}: (\mathbf{H} \times \mathbf{U}) \rightarrow \mathbf{R}^+$  – a function defined on the set  $\mathbf{H}$  of teams  $\mathbf{H}^r$  determines the expected values  $E[T_{j,r}]$  of random variables  $T_{j,r}$ , and describes the duration of work  $u_j \in \mathbf{U}$ ,

$\mathbf{K}: (\mathbf{H} \times \mathbf{U}) \rightarrow \mathbf{R}^+$  – a function defined on the set  $\mathbf{H}$  of teams  $\mathbf{H}^r$  determines the expected values  $E[K_{j,r}]$  of random variables  $K_{j,r}$ , and describes the cost of work  $u_j \in \mathbf{U}$ .

In the proposed approach, the expected values  $E[T_{j,r}]$  and  $E[K_{j,r}]$  of random variables  $T_{j,r}$  and  $K_{j,r}$  are calculated in accordance with the simplified formulas of the PERT method:

$$E[T_{j,r}] = \frac{\underline{T} + 4\hat{T} + \bar{T}}{6} \quad \text{and} \quad E[K_{j,r}] = \frac{\underline{K} + 4\hat{K} + \bar{T}}{6} \quad [1, 2].$$

The realistic or the most probable values

$\hat{T}_{j,r}$  and  $\hat{K}_{j,r}$  for these formulas are estimated directly on the basis of the individual analysis of the technology, resources, and conditions of work execution, or determined on the basis of commonly used catalogues of standards of resource consumption and commonly used price lists. The optimistic values  $\underline{T}_{j,r}$  and  $\underline{K}_{j,r}$ , as well as the pessimistic values  $\bar{T}_{j,r}$  and  $\bar{K}_{j,r}$ , are calculated by using coefficients of optimism  $\underline{p}_{j,r}^t$  and  $\underline{p}_{j,r}^k$ , and coefficients of pessimism  $\bar{p}_{j,r}^t$  and  $\bar{p}_{j,r}^k$  according to the

$$\text{formulas: } \underline{T}_{j,r} = \hat{T}_{j,r} - \underline{p}_{j,r}^t \hat{T}_{j,r}, \quad \underline{K}_{j,r} = \hat{K}_{j,r} - \underline{p}_{j,r}^k \hat{K}_{j,r}, \quad \bar{T}_{j,r} = \hat{T}_{j,r} + \bar{p}_{j,r}^t \hat{T}_{j,r}, \quad \bar{K}_{j,r} = \hat{K}_{j,r} + \bar{p}_{j,r}^k \hat{K}_{j,r}.$$

Other approaches to the risk assessment, partly comparable to those described here, are presented in papers [3–5].

The above mentioned approach corresponds to standard or moderate conditions for the execution of the work. In practice, the conditions for the implementation of the work may be highly changeable. In terms of such conditions, the favourable and difficult conditions of the execution should be considered. Then, additionally it is proposed to apply the coefficient of conditions improvement and the coefficient of conditions deterioration of work execution, respectively.

## 2. The rules for determining the coefficients of optimism and pessimism

Coefficients of optimism  $\underline{p}_{j,r}^t$  and pessimism  $\bar{p}_{j,r}^t$  enable us to take into account the impact of random events on the duration of work  $u_j \in \mathbf{U}$  carried out by the working team  $\mathbf{H}^r \in \mathbf{H}$ . They are estimated for each part of the project for particular perturbations in a similar way. In specific situations, the aggregate coefficients of optimism and pessimism should be estimated, which take into account the total influence of all random disturbances. In such cases coefficients  $\underline{p}_{j,r}^t$  and  $\bar{p}_{j,r}^t$  are estimated depending on the probability of disturbances  $q_{j,r}$  and the time consequences of the disturbances  $t_{j,r}$ . The values  $q_{j,r}$  and  $t_{j,r}$  can be determined using the description of the situation listed in the Tables 1 and 2.

Table 1. Probability of disturbances  $q_{j,r}$

Level	Description	$q_{j,r}$
Level 1	The probability of disturbances is extremely small	0.00–0.20
Level 2	The probability of disturbances is little	0.20–0.40
Level 3	The probability of disturbances is medium	0.40–0.60
Level 4	The probability of disturbances is high	0.60–0.80
Level 5	The probability of disturbances is extremely high	0.80–1.00

Table 2. Time consequences of disturbances  $t_{j,r}$

Level	Description	$t_{j,r}$
Level 1	Disturbances may have extremely small influence on continuity and the quality of works	0.00–0.20
Level 2	Disturbances may have small influence on continuity and slightly reduce pace but without affecting the quality of works	0.20–0.40
Level 3	Disturbances may slightly disrupt continuity and reduce pace, and cause small difficulties to ensure the quality of works	0.40–0.60
Level 4	Disturbances may highly disrupt continuity and reduce pace, and cause important difficulties to ensure the quality of works	0.60–0.80
Level 5	Disturbances may enforce periodic suspension and cause extremely high difficulties to maintain the quality of works	0.80–1.00

According to the probability of disturbances  $q_{j,r}$ , the probability of absence of disturbances  $r_{j,r} = 1 - q_{j,r}$  can be estimated. If the scope and impact of disturbances may be highly variable, it is proposed to additionally use the coefficient of variation of disturbances  $v$ . Then  $r_{j,r} = (1 - q_{j,r})v$  and  $v \in (0, 1]$ . Finally, the values of coefficients of optimism  $\underline{p}_{j,r}^t$  and pessimism  $\bar{p}_{j,r}^t$  for the random duration of the work can be calculated as follows:

$$\underline{p}_{j,r}^t = 1 - (1 - r_{j,r} t_{j,r}^t) \text{ and } \bar{p}_{j,r}^t = 1 - (1 - q_{j,r} t_{j,r}^t) \quad (3)$$

If there are random disturbances  $f = 1, \dots, g$  and some or all of them can strongly disrupt the run of work and such impact can be reliably estimated, their influence can be compiled separately. This means the factors  $q_{j,r}^f$ ,  $r_{j,r}^f$  and  $t_{j,r}^f$  should be determined for each random factor also using the descriptions in tables (1) and (2). Then, the time coefficients of optimism  $\underline{p}_{j,r}^t$  and pessimism  $\bar{p}_{j,r}^t$  should be calculated according to the formulas:

$$\underline{p}_{j,r}^t = 1 - \prod_{f=1}^g (1 - r_{j,r}^f t_{j,r}^f) \quad \text{and} \quad \bar{p}_{j,r}^t = 1 - \prod_{f=1}^g (1 - q_{j,r}^f t_{j,r}^f) \quad (5)$$

Changes in duration of work also affect the costs of the work. This impact can be estimated using time coefficients of optimism  $\underline{p}_{j,r}^t$  and pessimism  $\bar{p}_{j,r}^t$ . However, the costs of the work also depend on changes in the prices of labour, prices of machinery, and prices of products and building materials. From this perspective, it is necessary to estimate the probability of prices changes  $\delta_{j,r}$  and  $\epsilon_{j,r}$  or  $\epsilon_{j,r} = (1 - \delta_{j,r})v$ , and cost consequences of the price changes  $\kappa_{j,r}$ . Factors  $\delta_{j,r}$  and  $\kappa_{j,r}$  are independent. Their values can be estimated using descriptions in Tables 3 and 4 respectively.

Table 3. Probability of price changes  $\delta_{j,r}$

Level	Description	$\delta_{j,r}$
Level 1	The probability of price changes is extremely small	0.00–0.20
Level 2	The probability of price changes is little	0.20–0.40
Level 3	The probability of price changes is medium	0.40–0.60
Level 4	The probability of price changes is high	0.60–0.80
Level 5	The probability of price changes is extremely high	0.80–1.00

Table 4. Cost consequences of price changes  $\kappa_{j,r}$

Level	Description	$\kappa_{j,r}$
Level 1	An increase of cost is extremely small	0.00–0.20
Level 2	An increase of cost is little	0.20–0.40
Level 3	An increase of cost is medium	0.40–0.60
Level 4	An increase of cost is high	0.60–0.80
Level 5	An increase of cost is extremely high	0.80–1.00

Based on the aforementioned factors, the direct influence of price changes on the cost of work can be calculated as follows:

$$\underline{k}_{j,r} = 1 - (1 - \varepsilon_{j,r} \kappa_{j,r}) \quad \text{and} \quad \bar{k}_{j,r} = 1 - (1 - \delta_{j,r} \kappa_{j,r}) \quad (6)$$

Then, the total impact of the disruptions and price changes on the cost of the work can be estimated using the coefficients of optimism  $\underline{p}_{j,r}^k$  and pessimism  $\bar{p}_{j,r}^k$ . The coefficients can be calculated as follows:

$$\underline{p}_{j,r}^k = 1 - (1 - \underline{p}_{j,r}^t (1 - \underline{k}_{j,r})) \quad \text{and} \quad \bar{p}_{j,r}^k = 1 - (1 - \bar{p}_{j,r}^t (1 - \bar{k}_{j,r})) \quad (7)$$

The assumptions presented in the Tables 1–4, and the formulas (1)–(7) enable the random characteristics of the work to be estimated and, on this basis, to reliably analyse and assess the risk of time and risk of cost of construction project. All coefficients can be easily calculated using any spreadsheet.

### 3. Explanation examples

As an example, some varieties of work have been presented. They are part of a construction project. All estimations and calculations have been made using the formulae (1)–(7) and the assumption determined in Tables 1–4 and factor  $\nu=0,15$ .

Table 5. Land development and preconstruction works

$f$	Descriptions of disturbances	$r_{j,r}^f$	$q_{j,r}^f$	$t_{j,r}^f$		
1	Weather conditions	0.10	0.35	0.40		
	$\underline{p}_{j,r}^t$	0.039	$\bar{p}_{j,r}^t$	0.140		
	<b>Description of cost</b>	$\varepsilon_{j,r}^f$	$\delta_{j,r}^f$	$\kappa_{j,r}^f$	$\underline{k}_{j,r}^f$	$\bar{k}_{j,r}^f$
	Characteristic of price changes	0.12	0.20	0.30	0.36	0.60
	$\underline{p}_{j,r}^k$	0.038	$\bar{p}_{j,r}^k$	0.192		

Table 6. Abutment right

$f$	Descriptions of disturbances	$r_{j,r}^f$	$q_{j,r}^f$	$t_{j,r}^f$
1	Weather conditions	0.10	0.35	0.40
2	Failure of equipment	0.12	0.20	0.35
	$\underline{p}_{j,r}^t$	0,079		

$\bar{p}_{j,r}^t$	0,200				
<b>Description of cost</b>	$\varepsilon_{j,r}^f$	$\delta_{j,r}^f$	$\kappa_{j,r}^f$	$\underline{k}_{j,r}^f$	$\bar{k}_{j,r}^f$
Characteristic of price changes	0,11	0,25	0,35	0,39	0,88
$\underline{p}_{j,r}^k$	0.076				
$\bar{p}_{j,r}^k$	0.270				

Table 7. Bridge span

$f$	<b>Descriptions of disturbances</b>	$r_{j,r}^f$	$q_{j,r}^f$	$t_{j,r}^f$			
1	Precipitation	0.10	0.35	0.20			
2	Temperature	0.11	0.25	0.15			
	Wind	0.11	0.30	0.30			
	Humidity	0.12	0.20	0.20			
	Failure rate of equipment	0.14	0.10	0.40			
	Disruption on the energy supply	0.11	0.25	0.04			
	$\underline{p}_{j,r}^t$	0.141					
	$\bar{p}_{j,r}^t$	0.256					
	<b>Description of cost</b>	$\varepsilon_{j,r}^f$	$\delta_{j,r}^f$	$\kappa_{j,r}^f$	$\underline{k}_{j,r}^f$	$\bar{k}_{j,r}^f$	
	Characteristic of price changes	0.13	0.15	0.25	0.32	0.38	
	$\underline{p}_{j,r}^k$	0.137					
	$\bar{p}_{j,r}^k$	0.284					

#### 4. Final conclusions

The presented approach is simple and easy to apply. The examples given are part of a test task. Analysis of the solutions to this task has confirmed the validity of the assumptions made. The method does not require any changes in standard operating procedures of scheduling and cost estimation. Thanks to this, it can be used as part of other risk analysis methods in which the  $\beta$ -PERT probability distribution is used.

Taking into account the results obtained, we can ascertain that in many cases it is worth considering whether the work should be executed in very difficult conditions. When the probabilities and the time consequences of disturbances and the probability of price changes and cost consequences of these changes are estimated at level 3, it is necessary to prepare additional measures to ensure the execution of work in accordance with the project design and technical

specifications for the execution of work. If disturbances and their consequences or the probability of price changes and cost consequences of these changes are estimated at the level 4 or 5, it is worth considering changing the period of implementation of work or, if possible, changing the technology of work to those more suitable for executing the work in a difficult situation. Moreover, when work is executed in spite of extremely difficult conditions or very large changes in the impact of random disturbances on the course and results of the work, it is reasonable to consider favourable, moderate and difficult conditions of implementation. To estimate the impact of such disturbances on the duration and cost of work, the coefficients of improvement and deterioration of conditions of work implementation are used. Coefficients of condition improvement are used for favourable conditions for work execution. They are used to correct optimistic values for the duration and cost of work. Coefficients of condition deterioration are applied to difficult conditions for work execution. They are used to correct pessimistic values of duration and cost of work. In both cases, the values of the coefficients can be estimated directly on the basis of an analysis of the change of strength and kind of random perturbations.

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