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An analysis of electromagnetic processes in a turbogenerator with a real rotor and a stator tooth structure in the no-load mode

Analiza procesów electromagnetychnych w turbogeneratorze ze structurami realnymi rotora i statora na biegu jałowym

Abstract

The results of the research presented in this article are 2-D mathematical field models of a turbogenerator in the no-load mode with real tooth structures on the rotor and stator in the rotor coordinate system and in the physical systems of reference. The results of the computer simulation of the electromagnetic field in the cross-sectional zones of the device in the no-load mode transition process are pre¬sented.

Keywords: electromagnetic field, vector potential, turbogenerator, tooth zones, movable and immovable me¬dia, systems of coordinates

Streszczenie

W artykule przedstawiono model 2-D polowy matematyczny turbogeneratora na biegu ja-łowym ze strukturami realnymi zębów rotora i statora w systemie koordynat rotora i fizycznych systemach współrzędnych rotora i statora jednocześnie. Przedstawiono rezultaty symulacji komputerowej pola elektromagnetycznego w plaszczyznie przekróju urządzenia dla procesu przejściowego na biegu jałowym.

Słowa kluczowe: kpole elektromagnetyczne, potencjał wektorowy, turbogenerator, strefy zębów, srodowiska poruszające się i stacjonarny, układy współrzędnych

1. Introduction

The practicality of the possibilities to perfect properties, improve characteristics, and optimise parameters of electrical engineering equipment directly depends on the depth of a multi-aspect, versatile analysis of the peculiarities of electromagnetic phenomena in the elements of the given device. Thus, in the world scientific and technical literature, much attention has been paid to the development of theoretical and methodological approaches to solving the problems arising when developing mathematical models of electric objects. In these approaches, efforts are made to take the construction of objects into account to as great an extent as possible and apply real physical laws of electromagnetic phenomena occurring in the device media [1–12]. One of the more advanced strategies for successfully tackling this problem is to develop mathematical models of electric objects exclusively based on the theory of electromagnetic fields. This article summarizes the results obtained while conducting research into the development of the mathematical field model of turbogenerators in no-load modes with different levels of detail of the tooth structures of the rotors and stators both in the rotor's own coordinate system, and in the physical reference systems of the rotor and stator simultaneously [13].

The aim of the proposed publication is the search for theoretically reasoned, practical and effective ways of solving problems that may arise while developing the mathematical field models of electrodynamic objects. The following problems are some of the most important to be solved:

- achievement of correctness of mathematical and physical justification of the methods chosen for calculating an electromagnetic field in moving and fixed media of electric devices in different systems of reference;
- the need for improved strategies for identifying boundary and initial conditions for the field equations on the basis of matching the first and higher spatial derivatives of a vector potential function with certain magnetic properties of the media on both sides of the separation line;
- the need to identify optimal ways of taking into account the non-linear characteristics of the magnetic materials with deep saturation. These provide both necessary numeric stability of the models and practical implementation of the effective algorithm for the calculation of electromagnetic processes in the complex construction of electrodynamics devices based on the models developed.

Despite certain limitations of the proposed models [13], the main limitation being the performance of field calculation in 2-D space and the necessity to use some abstract coefficients applied in the theory of circuits. The values of the coefficients may be in rather a wide range of different modes, the approaches to the design of the 2-D models that are the focus of this research have opened up the possibility to develop effective 3-D models of this class of physical objects by using the methods described.

As a result of the computer simulation of electromagnetic processes in the moving and fixed media of the turbogenerator, it can be confirmed that it is impossible to develop a single 2-D mathematical field model of any electric device that will calculate various modes of its operation. The simulation of each mode requires the development of a separate model, each of which taking into consideration the construction of the device and describing the real physical characteristics and parameters of the object media to a degree, which is sufficient for reproducing a particular given mode under simulation.

Currently, one of the significant additional disadvantages of the mathematical field models of electric devices is the time-consuming computer simulation of the transition processes. This is often used as the strongest argument against using mathematical field models without emphasizing the fact that the particularly mathematical field models are a reliable source of true, comprehensive, and detailed information on the electromagnetic phenomena in the objects. The rapid development of modern computational engineering and software in the near future will eliminate the urgency for a solution to the indicated problem; furthermore, theoretical and algorithmic issues concerning the development of the 3-D models will become much more pressing.

2. Statement of the problem

The proposed mathematical field models of the turbogenerator with real rotor and stator tooth structures both in a single moving coordinate system of the rotor, and in the physical systems of reference are based on the equations of electromagnetic field with respect to potentials in quasi-stationary approximation. They imply a direct time integration of the formed system of equations.

In the developed models of the turbogenerator, there is no summand $\mathbf{v} \times \mathbf{B}$ in the equations of electromagnetic field for moving media. In work [13], it is theoretically substantiated that the given summand can only be used to transfer determined values of the field vectors from one inertial system to another – it cannot help in calculating unknown electromagnetic quantities in moving media of electrodynamic devices.

The calculation of the electromagnetic field in the models is conducted for a no-load mode of the turbogenerator at the given voltage of the rotor power supply winding.

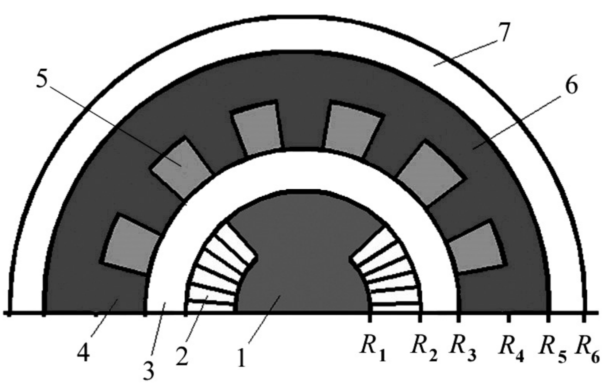

Fig. 1. Calculated zones of the turbogenerator cross section

On the pole division of the device, in the models, we reproduce twenty-four stator slots and teeth and eight slots with an excitation winding in the left and right parts of the rotor – as specified by the technical parameters of the object. The angular segment in which all rotor slots are located in each part of the winding ('2' in Fig. 1) on the pole division of the turbogenerator is approximately 60°. Practically in the same segment, there are 8 stator slots. That is why to simplify the models, the number of rotor slots at both winding sides on the stator pole division is also chosen to be equal to 8.

With regard to paying consideration to achieving accurate reconstruction of the tooth zones, it is necessary to choose the minimum possible discretisation grid size – this enables a more detailed description of the real structure of the device. Another possible way to achieve accurate reconstruction is through the application of separate rotor and stator grids and performing the recalculation of unknown electromagnetic quantities at their boundaries.

Fig. 1 shows a virtual image of the turbogenerator cross section, where 1 is the massive rotor body; 2 denotes the rotor tooth zone; 3 is the air gap between the rotor and stator; 4 is the stator tooth; 5 is slot with the stator winding; 6 is the laminated stator body; 7 is the air zone outside the turbogenerator.

The geometric dimensions of the zones formed in the models (see Fig. 1) correspond to the geometric dimensions of the construction of the real TGV-500 turbogenerator that are presented in Fig. 2.

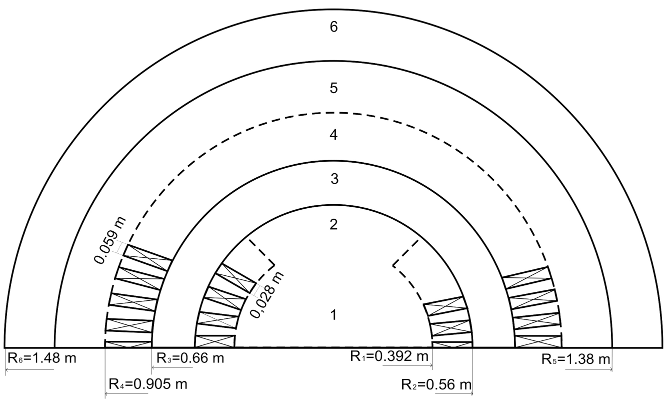

Fig. 2. Main geometric zone dimensions of the turbogenerator cross section

While developing the mathematical field models of the turbogenerator with real tooth rotor and stator structures, as with descriptions of previous works, the following main assumptions were made:

- emission currents in the device media are disregarded;
- the electromagnetic field in the turbogenerator can be considered to be plane-parallel;
- the electromagnetic phenomena on the butt are not taken into consideration;
- the hysteresis characteristics of the magnetic material of the rotor and stator are not taken into consideration.

3. Mathematical model of a turbogenerator with real rotor and stator tooth zones in the coordinate system of the moving rotor

The given model implies that the calculation of the electromagnetic processes in the turbogenerator in a no-load mode is achieved using a single moving coordinate system of the rotor. The electromagnetic phenomena in the model are considered in terms of quasi-stationary approximation in the case of a plane-parallel field. In order to obtain the minimum number of calculated equations and to simplify the task of determining boundary and initial conditions, the mathematical field model is designed on the basis of electromagnetic field potentials using the Weil's gauge $\nabla\varphi = 0$.

Work [13] shows that the calculation of electromagnetic processes in moving and fixed media can be performed both in the own coordinate systems of moving and fixed bodies, and in the transformed systems of reference. The only condition is that all the variables present in the equations must belong to the same system of coordinates.

In the suggested model, the analysis of the electromagnetic phenomena in the body, teeth and slots of the rotor – as well as in the air gap between the rotor and stator – is performed using the coordinate system of the moving rotor. The calculation of the electromagnetic field in the body and teeth of the rotor is performed using the following equation:

$$\frac{\partial \mathbf{A}}{\partial t} = -\gamma^{-1}\nabla\times(\nu\nabla\times\mathbf{A}) \quad \mathbf{A} = A_z\,\mathbf{z}_0 \tag{1}$$

where **A** is the vector potential of the electromagnetic field; γ is the static matrix of electric conductions; ν denotes the matrix of the static inverse magnetic penetrability of the medium; ∇ is the Hamiltonian operator.

In this equation, the component of the vector potential of the electromagnetic field A_z belongs to the rotor coordinate system.

To describe the electromagnetic phenomena in the slots with the excitation winding connected to an external voltage source, we use the following equation:

$$\frac{\partial \mathbf{A}}{\partial t} = -\gamma^{-1}\left(\nu_0\nabla\times\nabla\times\mathbf{A}\pm\boldsymbol{\delta}\right) \quad \mathbf{A} = A_z\,\mathbf{z}_0 \quad \boldsymbol{\delta} = \delta_z\,\mathbf{z}_0 \tag{2}$$

where ν_0 is the inverse magnetic copper penetrability; δ_z is the axis component of the extraneous current density vector.

The value of the current density in the rotor slots can be found through the relationship:

$$\delta_z = \frac{w_f i_f}{S} \tag{3}$$

where w_f, i_f are the number of windings and current of the excitation winding; S is the integral area of the rotor winding slots.

In the air gaps between the rotor and stator, the electromagnetic field can be calculated by using the relationship:

$$0=\nu_0\nabla\times\nabla\times\mathbf{A};\quad \mathbf{A}=A_z\,\mathbf{z}_0 \tag{4}$$

where ν_0 is the inverse magnetic air penetrability.

The electromagnetic processes in the body, slots and teeth of the stator, as well as outside of the turbogenerator, are considered in the reference system of the stator fixed to that of the rotor.

As conduction currents are absent in the stator slots with a winding enclosed when the turbogenerator operates in a no-load mode, the electromagnetic field in the zone is determined by the following relationship:

$$0=\nu_0\nabla\times\nabla\times\mathbf{A}';\quad \mathbf{A}'=A_z'\,\mathbf{z}_0 \tag{5}$$

where $\mathbf{A}'$ is the vector potential of the electromagnetic field in the coordinate system connected to the rotor.

In the laminated body and teeth of the stator, the electromagnetic phenomena are described by the following relationship:

$$0=\nabla\times\nu\nabla\times\mathbf{A}';\quad \mathbf{A}'=A_z'\,\mathbf{z}_0 \tag{6}$$

The electromagnetic field outside the turbogenerator is determined by dependence (5).

In the model, a fuller reproduction of the laws of electromagnetic phenomena on the external boundaries of the pole division of the turbogenerator cross section – and consequently, the improvement of numeric stability of the formed system of equations – were provided by applying an advanced approach to the formation of boundary conditions. This implies matching the first and second spatial derivatives of the function of vector potential on the external separation lines of the calculated zones and their periodicity along the angular coordinate [13]. To find the boundary conditions of the corresponding internal boundaries of the turbogenerator cross-sectional zones, we use the following known dependencies [13]:

$$H_r^-=H_r^+;\quad H_\alpha^-=H_\alpha^+ \tag{7}$$

where H_r, H_α are the radial and tangential components of the vector of magnetic force on both sides of the medium separation line, respectively.

The relationships

$$A_{z,(k)}=\frac{\nu_{r,(k-1)}\Delta\alpha_2 A_{z,(k-1)}+\nu_{r,(k+1)}\Delta\alpha_1 A_{z,(k+1)}}{\nu_{r,(k-1)}\Delta\alpha_2+\nu_{r,(k+1)}\Delta\alpha_1} \tag{8}$$

$$A_{z,(k)}'=\frac{\nu_{r,(k-1)}\Delta\alpha_2 A_{z,(k-1)}'+\nu_{r,(k+1)}\Delta\alpha_1 A_{z,(k+1)}'}{\nu_{r,(k-1)}\Delta\alpha_2+\nu_{r,(k+1)}\Delta\alpha_1}$$

where ν_r, ν_α are the levels of static inverse magnetic penetrability of the medium in the radial and tangential directions respectively; $\Delta r_1, \Delta r_2, \Delta\alpha_1, \Delta\alpha_2$ are the steps of the discretisation grids in the respective coordinates, are used to find the value of the vector potential of the electromagnetic field on the internal boundaries of the device along the radial coordinate.

The expressions

$$A_{z,(i)} = \frac{\nu_{\alpha,(i-1)}\Delta r_2 A_{z,(i-1)} + \nu_{\alpha,(i+1)}\Delta r_1 A_{z,(i+1)}}{\nu_{\alpha,(i-1)}\Delta r_2 + \nu_{\alpha,(i+1)}\Delta r_1} \qquad (9)$$

$$A'_{z,(i)} = \frac{\nu_{\alpha,(i-1)}\Delta r_2 A'_{z,(i-1)} + \nu_{\alpha,(i+1)}\Delta r_1 A'_{z,(i+1)}}{\nu_{\alpha,(i-1)}\Delta r_2 + \nu_{\alpha,(i+1)}\Delta r_1}$$

are used to calculate the value of A_z on the internal boundaries along the angle α, with the index i corresponding to the nodes of the grids along the radii, and k corresponding to the nodes of the grids along the angular coordinate.

In terms of the other objective physical condition of the spatial periodicity of the electromagnetic field on the pole division of the turbogenerator [13]

$$\left.\frac{\partial H_r}{\partial \alpha}\right|_{\alpha=0} = -\left.\frac{\partial H_r}{\partial \alpha}\right|_{\alpha=180} \qquad (10)$$

we obtain the following calculated dependencies for determining boundary conditions along the radii:

$$A_{z,(k=1)} = 2A_{z,(k=2)} + 2A_{z,(k=n-1)} - A_{z,(k=3)} - A_{z,(k=n-2)} - A_{z,(k=n)} \qquad (11)$$

$$A_{z,(k=n+1)} = A_{z,(k=2)} + A_{z,(k=4)} + A_{z,(k=n-1)} - 2A_{z,(k=3)} - 2A_{z,(k=n)}$$

$$A'_{z,(k=1)} = 2A'_{z,(k=2)} + 2A'_{z,(k=n-1)} - A'_{z,(k=3)} - A'_{z,(k=n-2)} - A'_{z,(k=n)}$$

$$A'_{z,(k=n+1)} = A'_{z,(k=2)} + A'_{z,(k=4)} + A'_{z,(k=n-1)} - 2A'_{z,(k=3)} - 2A'_{z,(k=n)}$$

where k is the index corresponding to the nodes of the discretisation grid along the angular coordinate.

By the equation:

$$A'_{z,(i=m+1)} = 2A'_{z,(i=m)} - A'_{z,(i=m-1)} \qquad (12)$$

we determine the boundary conditions for the equations of the field on the external boundary of the calculated air zone, with i being the index corresponding to the spatial grid nodes in the cylindrical system of coordinates along the radius.

As the results of the computer calculation of the no-load mode transition processes based on the models developed in [13] showed, numerical stability of the solutions also depends on the way in which the non-linear characteristics of magnetic material are presented in the models. For this, using the following cubic splines proved to be the most effective:

$$\nu(B) = \sum_{m=0}^{3} a_m^{(k)} \left(B - B_k\right)^m, \quad k = 1, 2, \ldots, n \tag{13}$$

where n is the number of segmentations along the axis B. The module value of the magnetic induction vector in the nodes of the discretisation grid of the cylindrical system of coordinates in the corresponding systems of reference is found from the given relationships:

$$B_r = \frac{1}{r}\frac{\partial A}{\partial \alpha}; \quad B_\alpha = -\frac{\partial A}{\partial r}; \quad B = \sqrt{B_r^2 + B_\alpha^2} \tag{14}$$

where B_r, B_α, B are the radial and tangential components and the module of a magnetic induction vector in the grid nodes of both the physical reference system of the rotor, and the transformed coordinate system of the stator, respectively.

The non-linear electromagnetic characteristics of the rotor and stator materials in the model are considered by means of expression (13) with respective coefficients for the specified media.

The recalculation of the magnetic characteristics of the equivalent medium of the laminated stator body was performed on the basis of the following relationship:

$$\nu_\alpha = \nu_r = \frac{d_f + d_0}{d_f + \nu_f \cdot d_0 / \nu_0} \cdot \nu_f \tag{15}$$

where ν_f, ν_0 are the inverse magnetic penetrability of the ferromagnetic sheet and isolation of the laminated stator; d_f, d_0 are the width of the sheet and isolation, respectively.

To remove ambiguity in the balance of electrical and magnetic quantities in the case of non-linear variants of the models, an equation for the excitation winding is formed in the form:

$$\frac{di_f}{dt} = \left(u_f - r_f i_f - \frac{d\psi_f}{dt}\right) \Big/ L_f \tag{16}$$

where

$$\frac{d\psi_f}{dt} = w_f k_f l_r \sum_{i=1}^{n} \frac{\partial A_{zRi}}{\partial t} \tag{17}$$

with w_f being the number of the rotor winding turns; l_r is the axis length of the winding; A_{zRi} is the value of the vector potential function in the rotor coordinate system in the grid nodes being located in the winding zone; and k_f being the coefficient involving the number of nodes along the angle α which are found in the rotor winding zone.

The equations for determining the stator winding voltages have the following form:

$$u_i = \frac{d\Psi_i}{dt} = w_i k_i l \sum_{m=1}^{n} \frac{\partial A'_{zSi}}{\partial t}, \quad i = A, B, C. \tag{18}$$

where w_i is the number of the stator windings in each phase; l denotes the axis winding length; k_i being the coefficient involving the number of nodes along the angle α, which is found in the stator winding zone; A'_{Si} is the value of the vector potential function in the nodes of the spatial discretization grid connected with the coordinate system of the moving rotor within the stator windings.

Equations (1)–(18) make up the basis of a mathematical field model designed for calculating the no-load mode of the turbogenerator with the real tooth zones of the rotor and stator at the specified voltage of the excitation winding in the reference system of the moving rotor.

4. Mathematical model of a turbogenerator with real tooth zones of the rotor and stator in the rotor and stator systems of coordinates simultaneously

One of the most important directions of research into the peculiarities of electromagnetic phenomena is creating mathematical field models of electrodynamic devices in physical systems of coordinates of moving and stationary bodies.

This section deals with the development of a mathematical field model for calculating the no-load mode of the turbogenerator at the pre-set voltage of the excitation winding with the real tooth zones of the rotor and stator in the physical systems of reference.

The mathematical modelling of the electromagnetic field in the massive body of the rotor, teeth and slots with the rotor winding, and in the air gap between the stator and rotor is performed in the system of coordinates connected to the moving rotor. At the same time, in the slots with the stator windings, teeth and stator body, and in the air gap outside the turbogenerator, this is performed in the reference system of the static stator. The given model provides a positional relationship of the rotor construction elements relative to the media of the stator along the angular coordinate.

The analysis of electromagnetic phenomena in the cross section of the body and teeth of the rotor is performed using equation (1); analysis of phenomena in the slots with electrically conductive rotor winding connected to the external source of voltage by using expression (2); analysis in the air gap between the rotor and stator by means of ratio (4). In all the zones mentioned, an axis component of the function of vector potential of the electromagnetic field A_z belongs to the physical coordinate system of the moving rotor.

As the device is in a no-load mode, there are no conduction currents in the stator winding, the calculation of the electromagnetic field in the zone is performed by using the following equation:

$$0 = \nu_0 \nabla \times \nabla \times \mathbf{A}; \quad \mathbf{A} = A_z \mathbf{z}_0 \tag{19}$$

To describe the electromagnetic phenomena in the teeth and body of the stator, we use the equation below:

$$0 = \nabla \times \nu \nabla \times \mathbf{A}; \quad \mathbf{A} = A_z \mathbf{z}_0 \tag{20}$$

We also find the value of the function of the vector potential **A** outside the turbogenerator by means of expression (20).

The boundary conditions for the main system of equations on the external perimeter of the calculated zones in the physical coordinate systems along the radii are used in the form of equation (11).

On all the formed internal boundaries of the cross sectional zones of the turbogenerator between the media with different electromagnetic characteristics in the physical systems of reference, the boundary conditions are determined on the basis of relations (8), (9).

The non-linear characteristics of the magnetic materials of the rotor and stator in the model are considered by using dependence (13). At the same time, the values of the module and the components of the magnetic induction vector in the nodes of the rotor and stator grids are determined using (14). The magnetic characteristics of the equivalent anisotropic medium of the stator body are calculated using expression (15).

Equation (16) is used to determine the values of the current in the excitation winding of the turbogenerator; the values of the stator phase voltages are obtained by means of relationship (18).

The mathematical field model of the turbogenerator designed for calculating the no-load mode at the pre-set voltage of the excitation winding with the real rotor and stator tooth zones implies a reciprocal displacement of the grid nodes along with the corresponding physical media of the moving and stationary system of coordinates if the following condition is satisfied:

$$\Delta\alpha = \omega \Delta t \tag{21}$$

where $\Delta\alpha$ is the step of the discretisation grids along the angular coordinate; Δt represents the step of the time integration of the system of differential equations; ω denotes the angular speed of the rotor rotation.

5. Results of computer simulations

Computer simulations of the transition process in no-load mode were conducted at a level of at u_f = 141 V (u_f denotes the voltage of the excitation winding) using the mathematical field model of the turbogenerator with the real tooth zones of the rotor and stator. The model was developed in the reference system connected to the moving rotor, taking the parameters of the real TGV-500 turbogenerator in to consideration. To perform a more comprehensive analysis of the electromagnetic processes occurring in the no-load mode of turbogenerator at the chosen step of the discretisation grid along the angular coordinate, it is sufficient to calculate the field for eight possible cases of the reciprocal location of the rotor and stator, since the given number corresponds to the number of grid discretization individually for a slot and a tooth of the stator in the tangential direction.

As a result of the computer calculations of the transition process of the no-load mode, using the model described above we obtained spatial-time values of the **E**, **H**, **B**, **D**, and so comprehensive information on the character of the electromagnetic processes in the zones of the device cross section.

The spatial distributions of electromagnetic quantities shown in Fig. 4–12 correspond to the following nodes of the coordinate grid of the turbogenerator cross–sectional zones: 0–54 (along the angular coordinate of the ferromagnetic rotor body zone); 54–75 (the rotor tooth zone); 75–87 (the air gap zone between the rotor and stator); 87–117 (the stator tooth zone); 117–174 (the stator body zone); 174–186 (the air gap outside the turbogenerator); 4–8 (along the angular coordinate of the rotor tooth structure); 12–16; 20–24; 28–32; 36–40; 44–48; 52–56; 60–64 (slots on the right side of the winding); 132–136; 140–144; 148–152; 156–160; 164–168; 172–176; 180–184; 188–192 (slots on the left side of the winding); 2–4; 8–12; 16–20; 24–28; 32–36; 40–44; 48–52; 56–60; 136–140; 144–148; 152–156; 160–164; 168–172; 176–180; 184–188; 192–194 (rotor teeth); 64–132 (large rotor tooth); 2–4 (along the angular coordinate of the stator tooth structure); 8–12; 16–20; 24–28; 32–36; 40–44; 48–52; 56–60; 64–68; 72–76; 80–84; 88–92; 96–100; 104–108; 112–116; 120–124; 128–132; 136–140; 144–148; 152–156; 160–164; 168–172; 176–180; 184–188; 192–194 (laminated teeth of the stator body); 4–8; 12–16; 20–24; 28–32; 36–40; 44–48; 52–56; 60–64; 68–72; 76–80; 84–88; 92–96; 100–104; 108–112; 116–120; 124–128; 132–136; 140–144; 148–152; 156–160; 164–168; 172–176; 180–184; 188–192 (stator slots).

Figure 3 depicts a time dependence of the excitation winding current of the turbogenerator rotor in the transition process in a no-load mode.

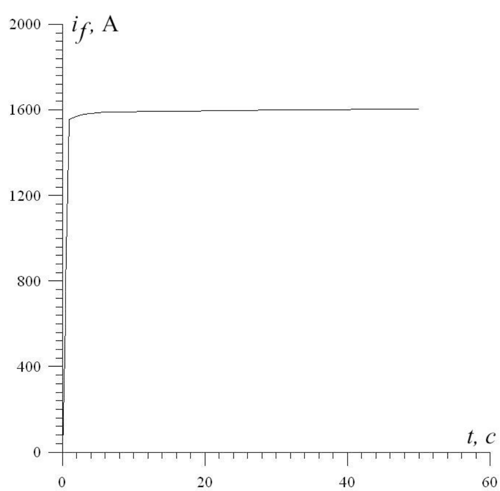

Fig. 3. Calculated values of the current in the turbogenerator excitation winding

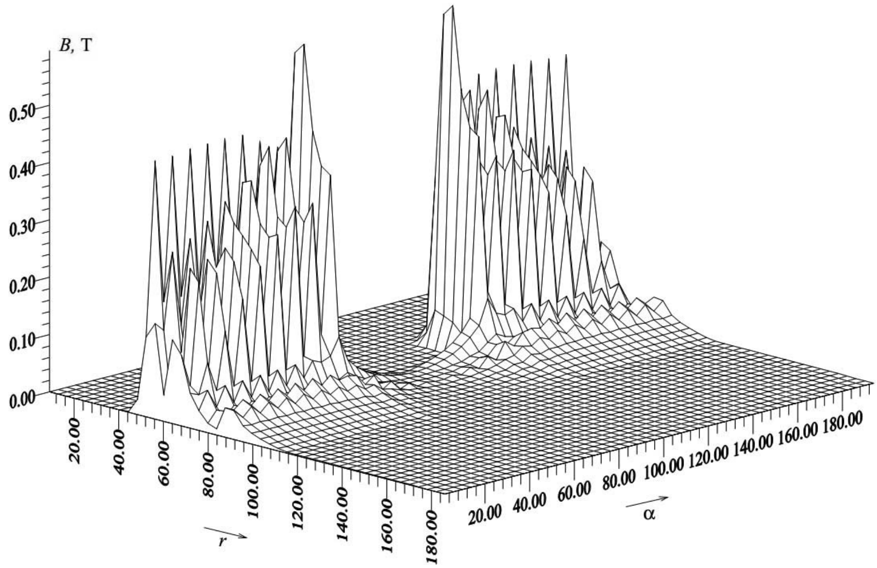

Fig. 4. Spatial distribution of the magnetic induction vector module in the rotor coordinate system on the turbogenerator pole division at t = 1s of the transition process in a no-load mode

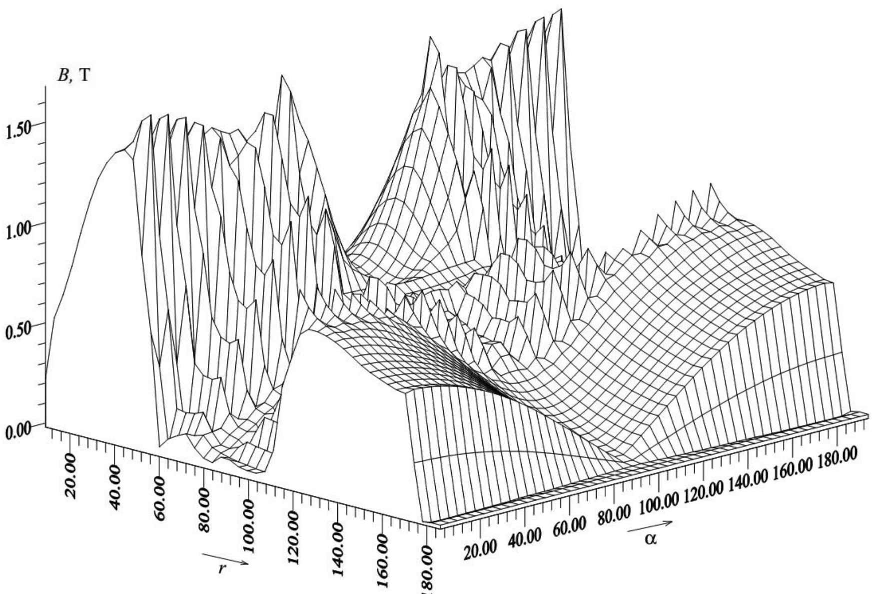

Fig. 5. Spatial distribution of the magnetic induction vector module in the rotor coordinate system on the turbogenerator pole division at t = 100s of the transition process in a no-load mode

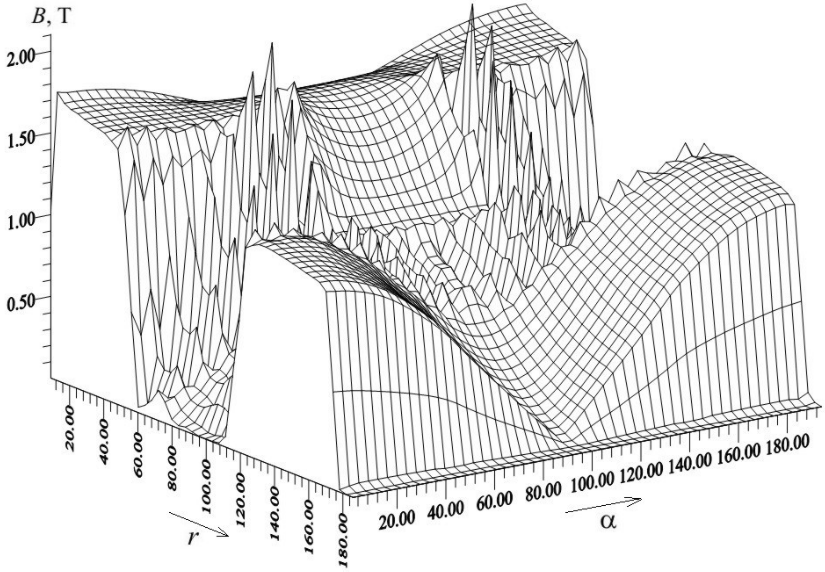

Fig. 6. Spatial distribution of the magnetic induction vector module in the rotor coordinate system on the turbogenerator pole division at t = 650s of the transition process in a no-load mode

Figure 4 shows spatial distributions of the magnetic induction vector module on the turbogenerator pole division at the point of time t = 1s of the no-load mode transition process. This was obtained on the basis of the mathematical field model developed in the coordinate system of the moving rotor.

Figure 5 demonstrates a spatial distribution of the magnetic induction vector module on the turbogenerator pole division at the point of time t = 100s of the no-load mode transition process. This was obtained on the basis of the mathematical field model developed in the moving rotor coordinate system.

Figure 6 illustrates a spatial distribution of the module of a magnetic induction vector on the turbogenerator pole division at the point of time t = 650s of the no-load transition process. This was obtained on the basis of the mathematical field model developed in the moving rotor coordinate system.

Figure 7 represents a spatial distribution of the vector potential of electromagnetic field in the rotor system of coordinates on the turbogenerator pole division at the point of time t = 650s of the no-load mode transition process. This was obtained on the basis of the mathematical field model developed in the moving rotor coordinate system.

Figure 8 depicts a spatial distribution of the radial component of the vector of electromagnetic field induction in the rotor system of coordinates on the turbogenerator pole division at the point of time t = 650s of the no-load mode transition process. This was obtained on the basis of the mathematical field model in the moving rotor coordinate system.

Figure 9 shows a spatial distribution of the tangential component of the vector of electromagnetic field induction in the rotor system of coordinates on the turbogenerator pole division at the point of time t = 650s of the no-load mode transition process. This was obtained on the basis of the mathematical field model in the moving rotor coordinate system.

The character of time variation of the excitation current in the rotor winding in the no-load mode transition process (see Fig. 3), and the spatial-time distributions of the module of a magnetic induction vector (see Figs. 4-6) demonstrate different periods of electromagnetic phenomena duration both in the electric winding of the rotor and in the magnetic system of the turbogenerator.

Having analysed the results of the of electromagnetic quantities distribution in real tooth zones of the rotor and stator calculated using the proposed model and compared them with those obtained employing the mathematical field models of the turbogenerator suggested in previous articles [13] where the tooth structures were substituted for the equivalent media, one may notice a discrepancy in the physical processes in the equivalent zones and the electromagnetic phenomena in the real tooth structures. This again confirms the conclusion that there is the necessity for full consideration of the existing construction of the devices while developing their mathematical field models.

The model developed in the reference systems of the rotor and stator, and the values of the electromagnetic field vector potential obtained on the basis of the model in the rotor coordinate system contributed to the calculation of the no-load mode transition process of the turbogenerator in the physical reference systems. Having the spatial-time values of the electromagnetic field vector

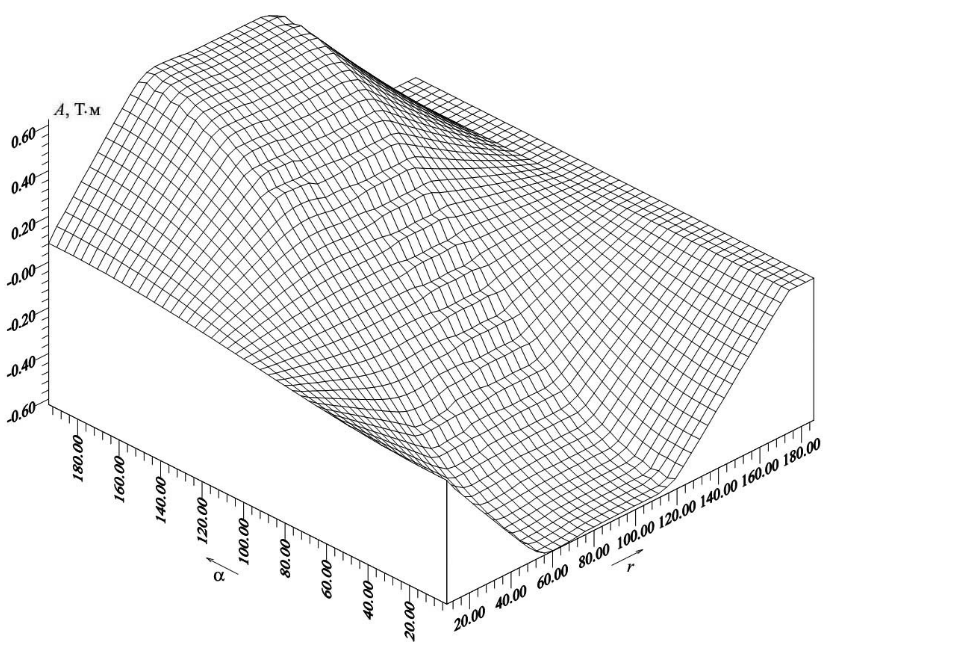

Fig. 7. Spatial distribution of the vector potential of electromagnetic field in the rotor system of coordinates on the turbogenerator pole division at the point of time t = 650s of the no-load mode transition process

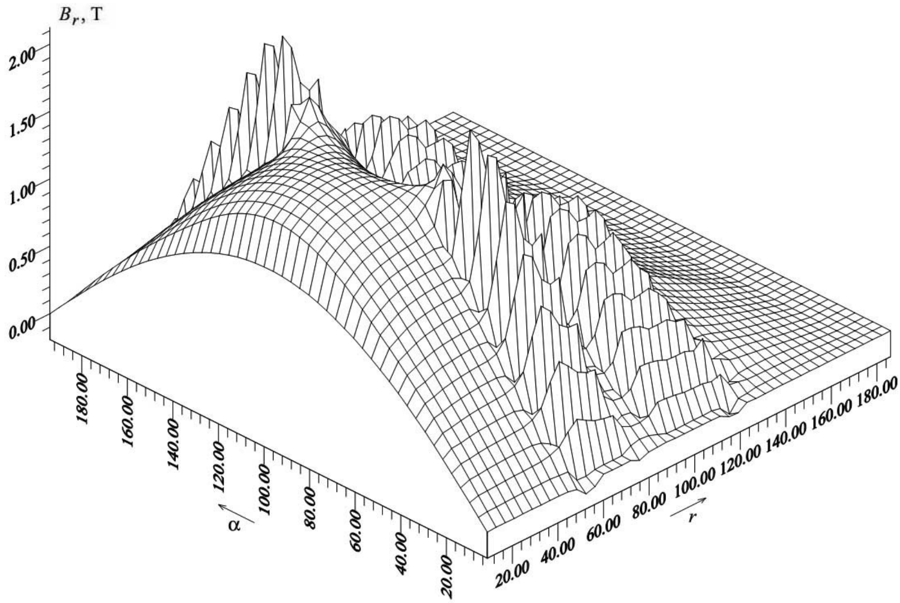

Fig. 8. Spatial distribution of the radial component of the vector of electromagnetic field induction in the rotor system of coordinates of the turbogenerator pole division at the point of time t = 650s of the no-load mode transition process

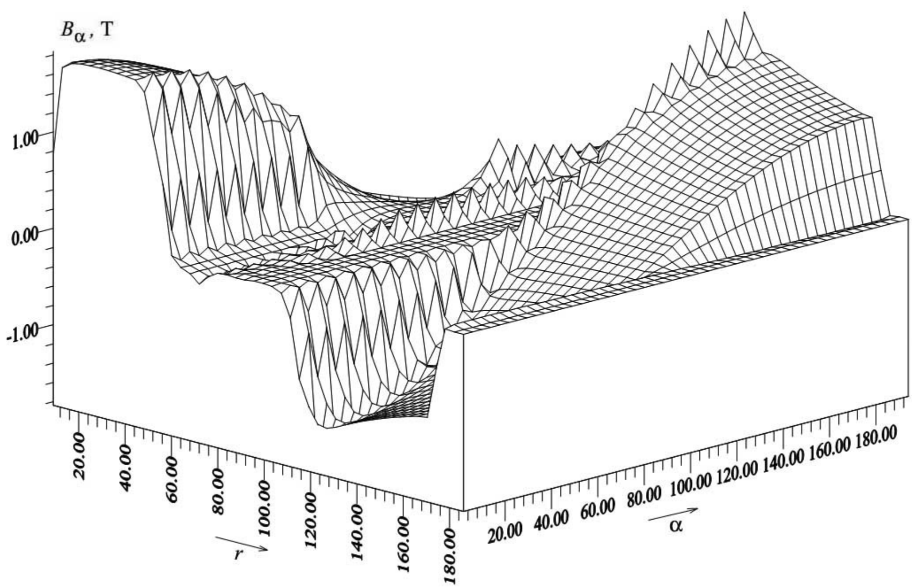

Fig. 9. Spatial distribution of the tangential component of the vector of electromagnetic field induction in the rotor system of coordinates of the turbogenerator pole division at the point of time t = 650s of the no-load mode transition process

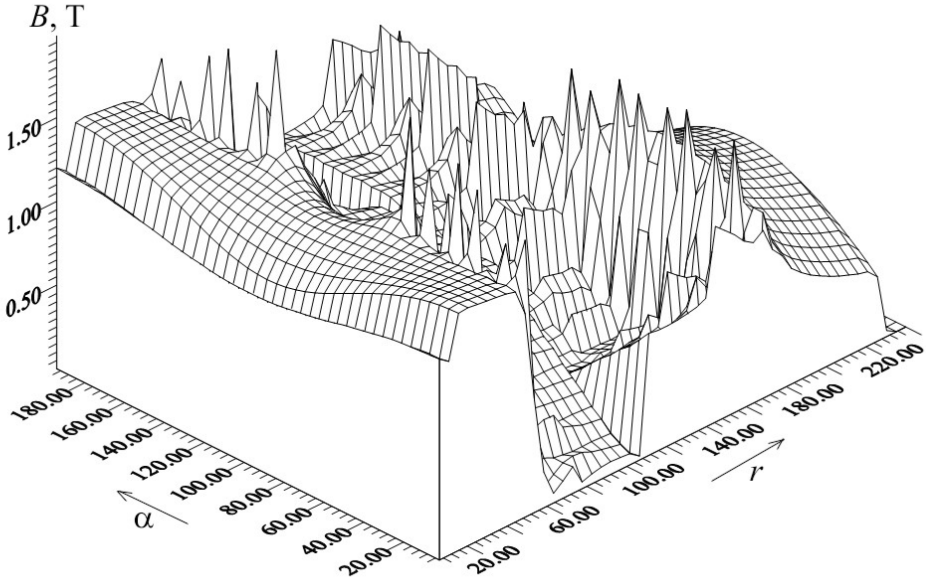

Fig. 10. Spatial distribution of the magnetic induction vector module in the phase coordinate systems of the rotor and stator simultaneously on the turbogenerator pole division at t = 645.01325s of the transition process in a no-load mode

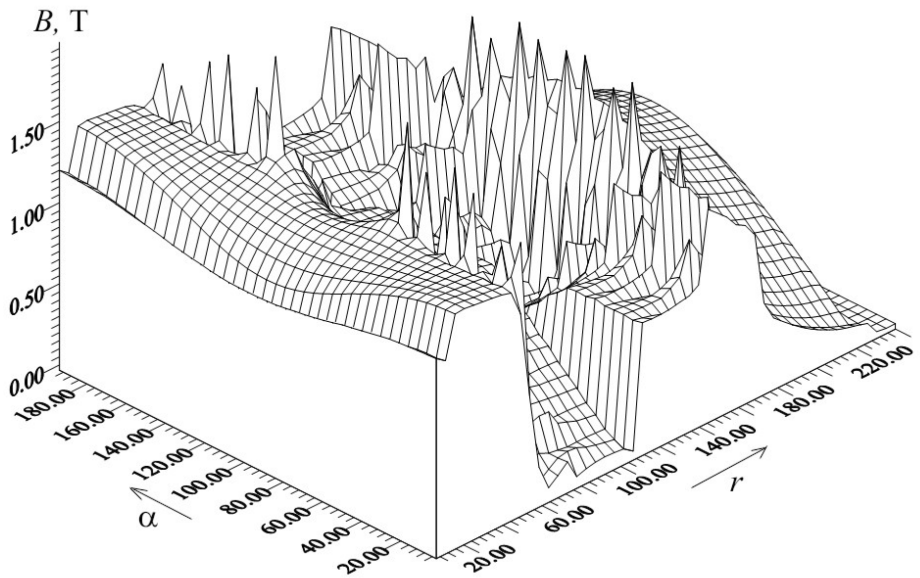

Fig. 11. Spatial distribution of the magnetic induction vector module in the phase coordinate systems of the rotor and stator simultaneously on the turbogenerator pole division at t = 645.01548s of the transition process in a no-load mode

potential in the systems of coordinates of movable and immovable media enabled calculation of the values of the current in the excitation winding and those of the phase voltages of the stator windings in the transition process. Some of the obtained results are given in Figs. 10–12.

Figure 10 presents the spatial distribution of the module of a magnetic induction vector on the pole division of the turbogenerator at the point of time $t = 645.01325$s of the no-load mode transition process. This was obtained on the basis of the mathematical field model developed in the coordinate systems of the rotor and stator simultaneously.

Figure 11 shows the spatial distribution of the module of a magnetic induction vector on the pole division of the turbogenerator at the point of time $t = 645.01548$s of the no-load mode transition process. This was obtained on the basis of the mathematical field model developed in the coordinate systems of the rotor and stator simultaneously.

Figure 12 demonstrates the spatial distribution of the module of the magnetic induction vector on the pole division of the turbogenerator with a conductive tooth zone of the stator at the point of time $t = 645.01216$s of the no-load mode transition process. This was obtained on the basis of the mathematical field model developed in the physical reference systems.

When comparing the results displayed in Figs. 6, 10 and 11, it may be observed that the spatial distribution of the module of the magnetic induction vector in the tooth zone and the rotor body (see Fig. 6) is displaced relative to the angular coordinate. The value of the displacement corresponds to the real relative positions of the rotor and stator at a prescribed fixed moment of time (see Fig. 10 & 11). In actuality, this is clear evidence of compatibility of the results obtained with the help of the models developed in different reference systems.

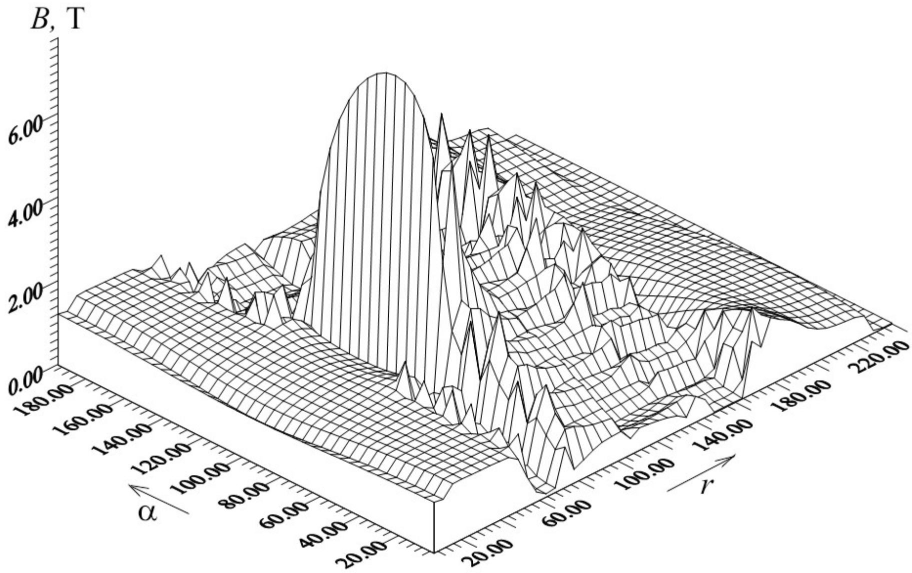

Fig. 12. Spatial distribution of the module of a magnetic induction vector on the pole division of the turbogenerator with a conductive tooth zone of the stator in phase coordinate systems at the point of time $t = 645.01216$s of the no-load mode transition process

In the considered Figures, we can see that there is a sharp change in spatial distributions in the values of the magnetic induction in the angular points of the slots with copper rotor and stator windings. Such changes are caused by an insufficient number of segments on the grid of the slot and tooth zones of the rotor and stator. This results in insufficient stability with regard to numerical solutions in the angular nodes on the boundary of nonlinear media. However, even a slight increase in the number of nodes is the cause of a significant increase in the basic system of calculation equations of the mathematical field model; furthermore, it affects the accuracy and quality of general calculation results.

Attention should especially be paid to the spatial distribution of the module of the magnetic induction vector depicted in Fig.12. The given distribution was obtained on the basis of the mathematical field model of the turbogenerator developed in phase systems of coordinates implying a reciprocal mechanical displacement of the rotor media relative to that of the stator. In this variant of the device model, we used equation (1), and not equation (4) to calculate electromagnetic processes in electrically conductive slots of the stator tooth zone. The achieved result demonstrates that in the case of calculating an electromagnetic field in the stator slots by means of expression (1), the model reproduces a short circuit mode of the stator winding rather than a no-load mode. A significant rise in the magnetic induction on the surface of the rotor, on the internal surface of the tooth zone of the stator, and in the air gap between the rotor and stator, as well as a fall of magnetic induction in the body of the stator (as the stator becomes demagnetised) can be seen in Fig. 12. These fluctuations are characteristic of the processes in the devices operating in a short-circuit mode of the stator windings and are not unusual for turbogenerators in a no-load mode. Such an effect is a direct result of the mechanical motion of the electrically conductive zone of the rotor slots relative to the magnetised stator body and does not conflict with the physical nature of electromagnetic phenomena. A detailed analysis of this discovered law is presented in work [13].

The calculations of the no-load device mode at equal values of the excitation winding voltage were carried out with the use of the developed mathematical field modes of the TGV-500 turbogenerator with different levels of tooth structure detailing and in different systems of reference. This provided all the developed models with equal by value magnetomotive forces $(w_f\, i_f)$. The results show that for all the models, the character of the change in the current value of the excitation winding in the transition process remains both quantitatively and qualitatively equal. However, there is a considerable difference in the spatial distributions of the electromagnetic field vectors in each group of the developed models.

With the level of detail of the mathematical descriptions of electromagnetic phenomena in the turbogenerator construction, there is a change in the character of reproducing the peculiarities of the electromagnetic processes occurring in separate zones of the device. This is evidence of the direct dependence of the results obtained while calculating the mathematical field models on the extent to which the turbogenerator construction details are taken into consideration in the models. It also serves as evidence of the discrepancy between the electromagnetic phenomena in the equivalent media of the models and the processes in the real tooth structures.

For simplifying the obtained results, the mathematical field models of the turbogenerator with equivalent zones of the rotor and stator are labelled M1. Models with the equivalent tooth zones of the rotor and the real tooth structure of the stator are labelled M2. Those with equivalent tooth zones of the stator and real tooth structures of the rotor are labelled M3, and the turbogenerator mathematical models that take into consideration the tooth structures of the device are labelled M4 [13].

The amplitude values of the phase voltages in the stator windings of the real TGV-500 turbogenerator in the no-load mode equal 16300 V.

The calculated values of the established amplitude phase voltages of the stator in the no-load mode determined on the basis of the models developed are presented in Table 1.

Table 1. Amplitudes of stator winding phase voltages (B)

Turbogenerator mathematical model	Linear variant of the model	Nonlinear model
M1	22000	20000
M2	21500	18250
M3	16340	16315
M4	16050	15950

Table 2 contains the relative errors of the amplitude values of the stator phase voltages, which were obtained on the basis of the developed models, with regard to the specified voltages of a real turbogenerator in the mode under investigation.

Table 2. Values of relative errors of the models

	Turbogenerator mathematical model							
	Linear variant				Nonlinear variant			
	M1	M2	M3	M4	M1	M2	M3	M4
Relative error (%)	35	31.9	0.1	1.5	23	11.9	0.01	2.1

Table 2 shows that an increase in the degree to which the real construction of the device tooth zones is taken into account (if the current flowing in an excitation winding is of the same value) causes a decrease in the value of electromagnetic quantities in the zones of the device cross section and, unsurprisingly, in the stator winding voltage, with the calculation accuracy being enhanced.

Given the data presented in Table 2, we also can observe that M4 (being the most detailed model of the turbogenerator) demonstrates a 1.5–2.1% error, and this error is greater than that of model M3. To identify the causes of such a discrepancy, it is necessary to analyse the initial parameters used in the computer simulation by involving the models developed.

Calculation results are always directly influenced by all linear and nonlinear coefficients present in the equations of mathematical models, in this particular case, this refers to the coefficients that are involved in the expressions of the main equations of the field and the auxiliary relations. Such parameters as γ, ν, ν_0 characterise the physical properties of the media, their values are precise and well-known – this is why they cannot be the cause of the obtained errors. The parameters w_f, w_i are the constants. The coefficients k_f, k_i and the values Δr_1, Δr_2, $\Delta\alpha_1$, $\Delta\alpha_2$ depend only on the number of nodes in discretisation grids. In the eight models presented, these parameters are identical. As the series of calculations showed, an increase in the number of nodes is substantial up to a certain value only, after which, a decrease in the grid step values has an insignificant influence on the results (within a few per cent).

Let us enlarge on other parameters.

The coefficient L_f in the developed models enabled considering the flows of dispersion of the front parts of the rotor winding. This parameter is often used in the mathematical description of electromagnetic processes in electric windings of devices by employing the theory of electromagnetic circuits. The range of possible values of L_f is fairly broad and depends on the mode being modelled. In the considered mode, the flows of dispersion of the excitation winding are low. Moreover, the value of L_f does not affect the established spatial distribution of the electromagnetic field in the zones of the device in no-load mode. The value of L_f was equal for all the models; consequently, this coefficient cannot cause a difference in the computer simulation results.

The assumption on the field's plane-parallelism in the device zones along its axis coordinate is a weakness of 2-D models. As far as we know, electromagnetic processes in the end parts of a turbogenerator differ from those in the cross section of the device in the middle of its length. The peculiarity mentioned can be taken into consideration only by using 3-D models.

In the real turbogenerator, the axial length of the rotor winding is l_r = 5.44m, and that of the stator is l_s = 6.2m. Since in the presented mathematical field models, the electromagnetic processes are considered the same throughout the length of the device, the values of lr and ls used for the calculations are relative approximate quantities. In all the cases of computer simulation of transition processes occurring in the turbogenerator in a no-load mode, the values of those parameters were $l_r = l_s$ = 5.44m.

If we accept that the assumed length of the rotor winding is l_r = 5.44m, the assumed value of ls is then within 5.44–6.2m. The parameter of l_s makes it possible to calibrate the developed 2-D mathematical field models of the turbogenerator in the no-load mode. If l_s = 5.56m, miscalculation of the nonlinear models will consequently be equal to M4 – 0%; M3 – (0.01+2.1)%; M2 – (11.9+2.1)%; M1 – (23+2.1)% at the same (unchanged) values of the spatial distributions of the electromagnetic field.

Certainly, for the models M1, M2, M3 it is also possible to select such nonlinear dependences of the values of γ and ν in continuous and equivalent media such that the miscalculation with respect to integral variables is equal to 0%. However, the character and the parameters of the electromagnetic phenomena reproduced by the developed models in the device zones would not correspond to the processes in the media of the real turbogenerator.

It is necessary to emphasise that in mathematical models of electric devices developed on the basis of the theory of circuits, there exists a much larger number of abstract coefficients – when manipulated, these coefficients make it possible to reach the necessary level of accuracy with the calculation. In the proposed mathematical field models of the turbogenerator and in the majority of the completed mathematical field models of other electric devices, the number of these quantities (abstract coefficients) is minimised.

6. Conclusion

The results obtained from the computer simulation of electromagnetic processes in the turbogenerator in a no-load mode based on the developed models clearly demonstrate both a theoretical and practical necessity for taking into account the systems of coordinates of moving and stationary bodies when developing mathematical field models of electrodynamics devices. It is impossible to either avoid or evade this issue in practical calculations because one of the components of the essence of an electromagnetic phenomenon is the movement of the media that are the sources of an electric or magnetic field and consequently, it is inadmissible to ignore the physical systems of reference of such media when analysing electromagnetic phenomena.

Both the developed mathematical field models of the turbogenerator with real tooth structures and the model considered in previous works [13] have proved that to analyse a no-load mode similar to the other states of electrodynamic devices in which the sources of electromagnetic fields are present in the construction elements in one reference system only, it is optimum to build the mathematical field models in this very same system of coordinates. Such an approach would considerably simplify the model because we would eliminate mechanical movement of the media, and at the same time, achieve a fairly high level of calculation accuracy.

In the case of more comprehensive research into electromagnetic phenomena in electrodynamic devices, it would be necessary to design the mathematical field models in physical systems of reference. The computer simulation of electromagnetic processes in the turbogenerator using models of this class affirms that while developing the models, it is necessary to take into consideration the coordinate systems of both moving and stationary elements of their construction and devise an appropriate way of describing characteristics and properties of the media with respect to the mode under analysis.

To overcome the present weaknesses of the proposed mathematical models of the turbogenerator, it is necessary to take the next logical step, namely to move on to the development of 3-D mathematical field models of an electrodynamic object.

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