

Bożena Hoła

Mariusz Szóstak (mariusz.szostak@pwr.edu.pl)

Department of Construction Methods and Managements, Faculty of Civil Engineering,  
Wrocław University of Science and Technology

A MATHEMATICAL MODEL OF ACCIDENT EVENT DEVELOPMENT  
IN THE CONSTRUCTION INDUSTRY

---

MATEMATYCZNY MODEL ROZWOJU SYTUACJI WYPADKOWEJ  
W BUDOWNICTWIE

**Abstract**

Accidents at work constitute a heavy psychological and economic burden for companies and societies. The vast majority of cases can be prevented by appropriate and effective preventive measures. Knowledge about the course of events, their causes and consequences is required in order to carry out such actions. The knowledge gained can provide a basis for drawing conclusions of a preventive nature. The article presents a mathematical model, which was used in the study of the phenomenon of accidents in the construction industry, and provides the test results that were obtained using 350 sample cases.

**Keywords:** construction industry, accident at work, cause-and-effect model

**Streszczenie**

Wypadki przy pracy stanowią bardzo duże obciążenie psychiczne i ekonomiczne dla przedsiębiorstw i społeczeństw. Zdecydowanej większości wypadków można zapobiec poprzez odpowiednie i skuteczne działania prewencyjne. Aby takie działania prowadzić, należy mieć wiedzę o przebiegu wypadków, ich przyczynach i skutkach. Uzyskana wiedza może stanowić podstawę do formułowania wniosków o charakterze prewencyjnym. W artykule przedstawiono: matematyczny model zastosowany w badaniu zjawiska wypadkowości w budownictwie oraz zamieszczono wyniki badań uzyskane na próbie 350 wypadków.

**Słowa kluczowe:** budownictwo, wypadek przy pracy, model przyczynowo-skutkowy

## 1. Introduction

The construction sector is one of the most dangerous industries in the world. This is evidenced by the reports of organizations and agencies that deal with issues of safety, including, among others: the National Safety Council – NSC, the Bureau of Labour Statistics – BLS, the National Institute of Occupational Safety and Health – OSHA, the Commission of European Communities – CEC, the Health and Safety Executive – HSE and the Polish Central Statistical Office – GUS.

During the recent 20<sup>th</sup> World Congress on Health and Safety at Work, which was held in 2014 in Frankfurt, the International Labour Organization (ILO) stated that about 350,000 workers die each year as a result of accidents at work. At least 60.000 fatal accidents at work occur every year on construction sites all over the world. This means that on average, one employee dies every 10 minutes when at work [1]. In 2015 there were 5.776 accidents at work on Polish construction sites, of which 69 resulted in the death of an employee [2].

Accidents at work are the subject of studies and analysis by many researchers [3–6]. The aim of this study is to detect specific regularities that characterize the phenomenon of accidents, which may indicate the directions of preventive measures.

The aim of the research conducted by Drozd [3] was to show the importance of the characteristics of a construction site and also the behaviour of employees when defining the circumstances of an accident at work. Study [4] presents the results of research on the phenomenon of accidents that occurred in 2014 in 28 European Union countries. The results obtained indicate that the construction industry, among all sections of the national economy, is in third place in terms of the total number of accidents at work. Spain is the European country with the highest accident rate [7].

Up to 34.6% of the total number of accidents at work in the construction industry in Spain are serious accidents and 33.9% are fatal accidents [8, 9]. In turn, fatal accidents are most often caused by falling from a height (33.8%) and being hit by a moving vehicle (15.9%). Studies on the identification of events, which are inconsistent with the appropriate conduct of the work process and usually cause a fatal accident, indicated that the most common event that leads to an accident is a fall from a height. 30% of people were injured as a result of falling from scaffolding and 12.1% as a result of falling from a roof structure during roofing work [10].

The results obtained by the authors of study [11] confirm that accidents occur most frequently as a result of a fall from a height or a fall at the same level (43.9%). The most common injuries are fractures of limbs (35.3%) and the most vulnerable part of the body that gets injured is the head (16.3%). According to research [12], a fall most often occurs during the installation of steel structures (33.22%), the concreting of ceilings (6.67%) and the plastering or painting of exterior walls (10.00%). Understanding the mechanisms of the occurrence of accidents at work is the first step in the process of preventing accidents and improving safety in the workplace.

Each accident at work occurs according to a specific scenario. The aim of the research presented in this article is to define possible accident scenarios in the construction industry on the basis of analysis of an appropriately numerous set of accidents. Moreover, the authors also aim to develop a model that illustrates the phenomenon of accidents in the construction industry.

## 2. Description of the problem

Accidents at work occur in different places and at different time periods. These events, organized in accordance with the passage of time  $t_z$ , create an infinite sequence of accidents, which can be analysed as a discrete accident process:

$$W = \{w_z; z = 1, \dots, n_w\} \quad (1)$$

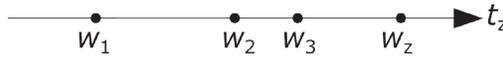


Fig. 1. Discrete accident process (own work)

Every accident event  $\{w_z; z = 1, \dots, n_w\}$  in the  $W$  process runs according to a specific scenario. An accident process model, which was proposed by the European Statistical Office (EUROSTAT), was modified for the purpose of this research and used to describe the scenario of a single accident at work. The authors' own model of an accident, in the form of a cause-and-effect chain that consists of 11 events (nodes) connected with relations, was developed using definitions, concepts and codes from the EUROSTAT model [13]. The previous event in this sequence is the cause of the following event, while the following event is a result of the previous event. The general model of a single accident is shown in Figure 2.

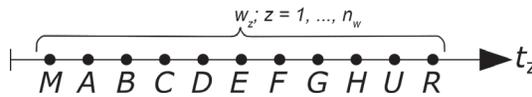


Fig. 2. The cause-and-effect chain of the course of a single accident (own work)

The following events (nodes) are separated in the model:

- $M$  – the event that initiates a single accident process,
- $A$  – the working environment,
- $B$  – the working process,
- $C$  – the specific physical activity performed by the persons injured at the time of the accident
- $D$  – the material agent of the specific physical activity performed by the persons injured at the time of the accident,
- $E$  – the event that is a deviation from the normal state,
- $F$  – the material agent of the event that is a deviation from the normal state,
- $G$  – the event that causes an injury,
- $H$  – the material agent that is the source of an injury associated with the event that causes the injury,
- $U$  – the type of injury,
- $R$  – the type of accident.

In order to consider all the circumstances of the course and consequences of an accident, two types of events were proposed in the model:

- ▶ a real event, which according to the theory of systems causes a change in the state of a system – a construction site ( $E \text{ i } G$ ),
- ▶ an apparent event, which describes the circumstances in which an accident occurred and also its consequences ( $A, B, C, D, F, H, U, R$ ).

Each individual accident is initiated by an apparent event ( $M$ ).

### 3. Description of events in the model

Each node in the model that is shown in Figure 2 can have many different meanings, depending on the type of construction site, location, type of work and equipment used. The individual specific cases are identified by the index included in the letter designation of the node. This index is a numerical code taken from the methodology of the accident course analysis proposed by EUROSTAT. The following nodes indicate:

- ▶  $A = \{a_i; i = 0.21, 0.22, \dots, 0.26, 0.29\}$  – that the place of an accident at work is a construction site and can be, among others: a building being constructed –  $a_{021}$ ; a building being demolished, repaired or maintained –  $a_{022}$  or an opencast quarry, opencast mine, excavation or trench –  $a_{023}$ . Moreover, the place of the construction can be located: underground –  $a_{024}$ ; on/over water –  $a_{025}$  or other types of working environments not listed in this group –  $a_{029}$ .
- ▶  $B = \{b_j; j = 21, 22, \dots, 25, 29\}$  – the process of work that is carried out by the injured person at the time of accident initiation. The following processes, which are characteristic for the construction industry, were distinguished: excavation –  $b_{21}$ ; a new construction – building –  $b_{22}$ ; a new construction – civil engineering, infrastructures, roads, bridges, dams, ports –  $b_{23}$ ; remodelling, repairs, extensions, building maintenance – all types of constructions –  $b_{24}$  or demolition – all types of constructions; other types of working processes not listed in this group –  $b_{29}$ .
- ▶  $C = \{c_k; k = 1, 2, \dots, 9\}$  – the specific physical activity performed by the persons injured at the time of the accident, namely: operating a machine –  $c_1$ ; working with hand-held tools –  $c_2$ ; driving/being on board a means of transport or handling equipment –  $c_3$ ; handling of objects –  $c_4$ ; carrying things by hand –  $c_5$ ; movement –  $c_6$ ; presence –  $c_7$  or other specific physical activities not listed in this group –  $c_9$ .
- ▶  $D = \{d_l; l = 00, 01, \dots, 20, 99\}$  – the material agent of the specific physical activity performed by the persons injured at the time of the accident. The following material factors can be distinguished for the construction industry: buildings, structures, surfaces – at ground level –  $d_{01}$ ; buildings, structures, surfaces – above ground level –  $d_{02}$ ; buildings, structures, surfaces – below ground level –  $d_{03}$ ; hand tools, not powered –  $d_{06}$ ; hand-held or hand-guided tools, mechanical –  $d_{07}$ ; land vehicles –  $d_{12}$ ; materials, objects, products, machine or vehicle components, debris, dust –  $d_{14}$ ; bulk waste –  $d_{19}$ ; physical phenomena and natural elements –  $d_{20}$ ; other material agents not listed in this group –  $d_{99}$  or no material agent –  $d_{00}$ .

- ▶  $E = \{e_o; o = 0, 1, \dots, 9\}$  – the event that is a deviation from the normal state, therefore, an event incompatible with the appropriate conduct of the work process that initiates the occurrence of a dangerous accident. Such an event can be, among others: deviation due to electrical problems, explosions, fire –  $e_1$ ; breakage, bursting, splitting, slipping, falling, collapsing of a material agent –  $e_3$ ; loss of control of a machine, means of transport or handling equipment, hand-held tools or objects –  $e_4$ ; slipping, stumbling and falling, the fall of a person –  $e_5$ ; other deviations not listed in this group –  $e_9$ , or no information –  $e_0$ .
- ▶  $F = \{f_p; p = 00, 01, \dots, 20, 99\}$  – the material agent of the event that is a deviation from the normal state. The classification of material factors overlaps with the list of the factors distinguished in node D.
- ▶  $G = \{g_q; q = 0, 1, \dots, 9\}$  – the dangerous event that is a consequence of a deviation and causes an injury. The following events were qualified as hazardous: contact with electrical voltage, extreme temperature, hazardous substances –  $g_1$ ; being drowned, buried or enveloped –  $g_2$ ; horizontal or vertical impact with or against stationary objects –  $g_3$ ; being struck by an object in motion, collision –  $g_4$ ; contact with a sharp, pointed, rough or coarse material agent –  $g_5$ ; being trapped, crushed –  $g_6$ ; other event causing an injury not listed in this group –  $g_9$ , or no information –  $g_0$ .
- ▶  $H = \{h_s; s = 00, 01, \dots, 20, 99\}$  – the material agent that is the source of an injury associated with the event that causes the injury. The classification of material factors overlaps with the list of the factors distinguished in node D.
- ▶  $U = \{u_v; v = 000, 010, \dots, 140, 999\}$  – the type of injury caused by contact with a hazardous material agent. The following injuries were distinguished, among others: bone fractures –  $u_{020}$ ; dislocations, sprains and strains –  $u_{030}$ ; traumatic amputations –  $u_{040}$ ; concussion and internal injuries –  $u_{050}$ ; drowning and asphyxiation –  $u_{080}$ ; multiple injuries –  $u_{120}$ ; injury after falling from a height –  $u_{130}$ ; injury due to backfilling with soil –  $u_{140}$  or death of a victim –  $u_{150}$ .
- ▶  $R = \{r_x; x = 1, 2, 3\}$  – the type of accident. A result of an accident at work can lead to the death of the victim (fatal accident) –  $r_1$ ; severe body injuries (severe accident) –  $r_2$  or small body injuries (light accident) –  $r_3$ .

#### 4. Mathematical description of the model

Every accident begins with an initiating event and runs through specific indirect events that lead to the occurrence of the final event i.e. the type of accident. Based on the analysis of post-accident protocols, it should be noted that every accident follows a separate accident scenario [14]. Graphical illustration of a large number of different scenarios, in the form of a directed graph, is shown in Figure 3.

The directed graph ( $Y$ ) is defined as an ordered pair of sets  $N$  and  $T$ , as follows:

$$Y = \langle N, T \rangle, \quad (2)$$

where:

- $N$  – is a random non-empty set of vertices and nodes,
- $T$  – is a set of possible ordered pairs of adjacent nodes  $N$ , which are called directed edges or arcs.

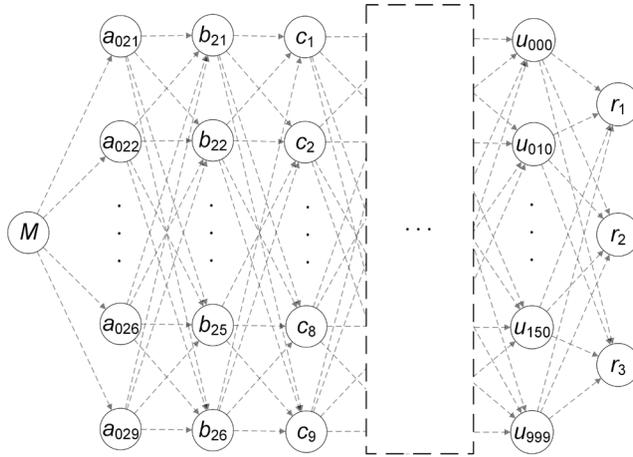


Fig. 3. Model of the development of an accident event in the construction industry (own work)

The  $N$  set consists of 11 subsets that include events classified into the specified following groups:  $M, A, B, C, D, E, F, G, H, U, R$  each of which contains selected detailed information on the course of an accident:

$$N = M \cup A \cup B \cup C \cup D \cup E \cup F \cup G \cup H \cup U \cup R \quad (3)$$

The  $T$  set of the ordered pairs of adjacent  $N$  events (nodes) can be written as follows:

$$T = \{(m, a_i), (a_i, b_j), (b_j, c_k), (c_k, d_l), (d_l, \varepsilon_o), (\varepsilon_o, f_p), (f_p, g_q), (g_q, h_s), (h_s, u_v), (u_v, r_x)\}$$

$$T = \{(M, A), (A, B), (B, C), (C, D), (D, E), (E, F), (F, G), (G, H), (H, U), (U, R)\} \quad (4)$$

For the phenomenon of accidents examined, the sought function is a function describing the course of events in a hypothetical case, for which it is possible to obtain the maximum value of the probability of a particular scenario. The sought objective function can be represented as follows:

$$P(T) = P(M \cap A \cap B \cap C \cap D \cap E \cap F \cap G \cap H \cap U \cap R) \rightarrow \max \quad (5)$$

The probability of the occurrence of scenario  $P(T)$  may be presented using the formula of the conditional probability of dependent events [15]:

$$\begin{aligned}
P(T) &= P(M \cap A \cap B \cap C \cap D \cap E \cap F \cap G \cap H \cap U \cap R) = \dots \\
&= P(R | M \cap A \cap B \cap C \cap D \cap E \cap F \cap G \cap H \cap U) \cdot P(U | M \cap A \cap B \cap C \cap D \cap E \cap F \cap G \cap H) \cdot \\
&P(H | M \cap A \cap B \cap C \cap D \cap E \cap F \cap G) \cdot P(G | M \cap A \cap B \cap C \cap D \cap E \cap F) \cdot \quad (6) \\
&P(F | M \cap A \cap B \cap C \cap D \cap E) \cdot P(E | M \cap A \cap B \cap C \cap D) \cdot \\
&P(D | M \cap A \cap B \cap C) \cdot P(C | M \cap A \cap B) \cdot P(B | M \cap A) \cdot P(A | M)
\end{aligned}$$

## 5. Test results obtained on the basis of accident analysis

An accident process that consists of 350 accidents at work, which happened in the Polish construction industry in the years 2008–2014, was simulated. The study involved accidents that happened in 4 Polish provinces: Lower Silesia, Kujawsko-Pomorskie, Lubelskie and Lubuskie. Post-accident protocols, obtained from branches of the State Labour Inspectorate, were the source of information on accidents. The investigated accidents happened during the erection of new buildings and also renovation works.

Table 1 shows part of the matrix of the relations between successive nodes *A* and *B*, and *C*, and Figure 4 presents the course of the most often occurring accident processes. Each scenario shows the specified path in the graph, which begins in node *M* and ends in node *R*. Each node is described by: sign, numerical code, number of events. The numbers included on the arches define multiplicity of the transition in the path.

Table 1. Part of the matrix of the relations between nodes

						$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$	$c_7$	
	$b_{21}$	$b_{22}$	$b_{23}$	$b_{24}$	$b_{25}$	$b_{21}$	–	6	1	1	3	6	1
$a_{021}$	17	<b>148</b>	37	–	–	$b_{22}$	3	21	1	43	24	<b>55</b>	1
$a_{022}$	–	–	–	126	15	$b_{23}$	4	7	–	9	4	11	3
$a_{024}$	1	–	–	3	–	$b_{24}$	2	29	4	31	7	54	4
$a_{025}$	–	–	1	2	–	$b_{25}$	1	8	–	5	1	–	–

The following conclusions can be drawn on the basis of the results of the simulation of the process that consisted of 350 accidents at work in the construction industry:

- ▶ 202 cases of accidents at work, which were assigned to the construction sector, occurred during the construction of new buildings ( $a_{021}$ ), while 141 cases occurred during the renovation and demolition of existing building objects ( $a_{022}$ ),
- ▶ the process that causes the most accidents and is performed during the initiation of an accident is the construction of new buildings ( $b_{22}$ ); there were 148 such situations identified. 131 accidents happened during the rebuilding, repair, extension and maintenance of building objects ( $b_{24}$ ), and 38 during the construction of infrastructure facilities ( $b_{23}$ ),



- ▶ accidents occurred most commonly during the movement of an employee along a flat area, ascending or descending to another level, entering or leaving other spaces and also during jumping or moving ( $c_6$ ); there were 126 such reported cases. In 89 cases, the handling of objects ( $c_4$ ) was the activity that was performed by the victim and in 71 cases the work involved the use of hand tools ( $c_2$ ),
- ▶ the largest group of material factors related to accidents in the construction industry, which was identified in 176 cases, consisted of buildings, structures and their components above ground level ( $d_{02}$ ), including among other things: roofs of buildings, stationary or mobile scaffolding and also work platforms or ladders; another often identified material factor was heavy transport vehicles ( $d_{12}$ ), which was seen in 39 cases,
- ▶ the most common events that are incompatible with the appropriate course of the work process include slipping, stumbling or the falling of a person ( $e_3$ ), which was recorded in 149 cases, and also the damage, bursting, breaking off, slipping, falling and collapsing of a material factor ( $e_3$ ), which occurred in 118 cases,
- ▶ the falling of a victim from a height and from buildings or structures located above ground level ( $f_{02}$ ) occurred in 136 accidents,
- ▶ the most common event that caused injury was a collision with an immovable object ( $g_3$ ); such events were reported in 204 cases, out of which the hitting a horizontal surface located at ground level ( $h_{01}$ ) occurred in 173 cases,
- ▶ 141 analysed cases were followed by the death of a victim ( $r_1$ ); serious injuries happened in 201 cases ( $r_2$ ), and light injuries in 8 cases ( $r_3$ ).

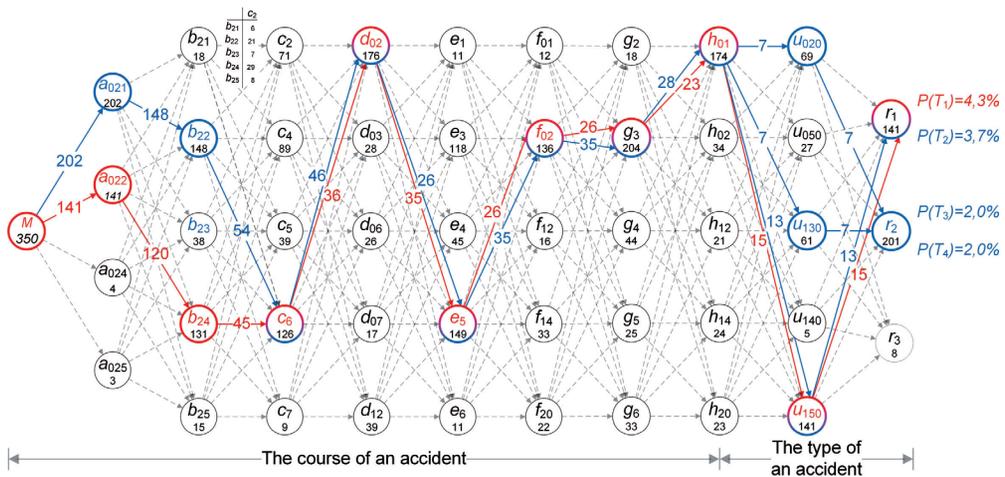


Fig. 4. The course of the most often occurring accident processes (own work)

Figure 4 shows that the following scenario has the highest probability: an accident occurring in buildings that are being dismantled, demolished, or renovated ( $a_{022}$ ) during their rebuilding, repair, expansion or maintenance ( $b_{24}$ ) and while there is movement of an employee ( $c_6$ ) along surfaces that are at a height above ground level ( $d_2$ ). During this operation a person slips, trips or falls ( $e_3$ ) on elements of a building or structure that are located on

a lower level, but above ground level ( $f_{02}$ ). The falling is followed by a vertical collision with a stationary object or by hitting an immovable object ( $g_3$ ) that is located at ground level ( $h_{01}$ ). The most common type of injury, which results from these events, was the death of a victim ( $u_{150}$ ), i.e. a fatal accident ( $r_1$ ). For the 350 analysed accidents at work, the probability of such a scenario, which is marked in red in Figure 4, is equal to 4.3%. The course of the subsequent scenarios with the highest probability of the occurrence of scenario is marked in blue.

## 6. Summary

The paper presents a mathematical and graphical model in the form of a directed graph of the development of an accident event in the construction industry. Each path in the graph represents a possible course of a single accident. The course of a single accident consists of a sequence of apparent and real events that identify the circumstances, causes and consequences of the accident. The simulation of a process that consists of 350 accidents was carried out using the computer model constructed. The model makes use of a database containing data of accidents at work in the construction industry.

## References

- [1] International Labour Organization, *Safety and Health at Work: A Vision for Sustainable Prevention*, XX World Congress on Safety and Health at Work 2014, Global Forum for Prevention, 24–27 August, Frankfurt 2014, Germany.
- [2] Główny Urząd Statystyczny, *Wypadki przy pracy w 2015 r. Informacje i Opracowania Statystyczne*, Warszawa 2016.
- [3] Drozd W., *Charakterystyka terenu budowy w aspekcie zagrożeń bezpieczeństwa pracy*, Czasopismo Inżynierii Lądowej, „Środowiska i Architektury”, Vol. XXXIII, Issue 63 (1/1/16), 2016, 165–172.
- [4] Hoła B., Szóstak M., *Analysis of the State of the Accident Rate in the Construction Industry in European Union Countries*, Archives of Civil Engineering, Vol. 1, Issue 4, 2015, 19–34.
- [5] Khan F., Rathnayaka S., Ahmed S., *Methods and models in process safety and risk management Past, present and future*, Process Safety and Environmental Protection, Vol. 98, 2015, 116–147.
- [6] Wu W. Yang, H. Li Q, Chew D., *An integrated information management model for proactive prevention of struck-by-falling-object accidents on construction sites*, Automation in Construction, Vol. 34, 2013, 67–74.
- [7] Camino López M.A., Ritzel D.O., Fontaneda I. Gonzales, Alcantara O.J., *Construction industry accidents in Spain*, Journal of Safety Research, Vol. 39, 2008, 497–507.
- [8] Pérez-Alonso J., Callejón-Ferre A.J., Carreño-Ortega A., Sánchez-Hermosilla J., *Approach to the evaluation of the thermal work environment in the greenhouse-construction industry of SE Spain*, Building and Environment, Vol. 46, 2011, 1725–1734.

- [9] López Arquillos, Rubio Romero J.C., Gibb A., *Analysis of construction accidents in Spain, 2003–2008*, Journal of Safety Research, Vol. 43, Issue 5–6, 2012, 81–388.
- [10] Lin Y.H., Chen C.Y., Wang T.W., *Fatal occupational falls in the Taiwan construction industry*, Journal of the Chinese Institute of Industrial Engineers, Vol. 28, No. 8, 2011, 586–596.
- [11] Chi S., Han S., *Analyses of systems theory for construction accident prevention with specific reference to OSHA accident reports*, „International Journal of Project Management”, Vol. 31, Issue 7, 2013, 1027–1041.
- [12] I. M. Razwanul I., Tarek M., *Safety Practices and Causes of Fatality in Building Construction Projects: A Case Study for Bangladesh*, „Jordan Journal of Civil Engineering”, 11(2), 2017.
- [13] Eurostat European Commission, *European Statistics on Accident at Work (ESAW). Summary methodology*, Eurostat Methodologies & Working papers, 2013.
- [14] Hoła B., Szóstak M., *Analysis of the Development of Accident Situations in the Construction Industry*, Procedia Engineering, Vol. 91, 2014, 429–434.
- [15] Hebda A., *Metoda techniczno-ekonomicznej oceny składników oraz uciążliwości ryzyka wystąpienia wypadków przy pracy w kopalniach węgla kamiennego*, Uczelniane Wydawnictwa Naukowo-Dydaktyczne, Kraków 2005.