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Assessment of propagation of modelling un-certainty by the procedures for determin-ing maximum dynamic errors

Ocena propagacji niepewności modelowania przez procedury wyznaczania maksymal-nych błędów dynamicznych

Abstract

The paper discusses the method for modelling linear analogue systems of the second order in the time domain. As a result of such modelling, the parameters of the model and the associated uncertainties are obtained. Procedures for determining the absolute error and the integral-square error are presented. These procedures make it possible to determine precisely such an input sig-nal with one constraint that maximizes an error at the output of the system. Values of the parameters of an example model are determined and the propagation of the uncertainties of modelling results is assessed by the procedures for determining the maximum dynamic errors. The results of calculations presented in the paper were carried out in MathCad15.

Keywords: uncertainties of model parameters, time domain modelling, maximum dynamic error

Streszczenie

W artykule omówiono metodę modelowania w dziedzinie czasu liniowych systemów analogo-wych drugiego rzędu. Jako wynik takiego modelowania uzyskano parametry modelu oraz związane z nimi niepewności. Przedstawiono procedury wyznaczania maksymalnych błędów dynamicznych dla przypadku kryteriów błędu: bezwzględnego i całkowokwadratowego. Procedury te pozwalają w sposób precyzyjny określić taki sygnał wejściowego z jednym ograniczeniem, który maksymalizuje błąd na wyjściu systemu. Wyznaczono wartości parametrów przykładowego modelu oraz oceniono propagację niepewności wyników modelowania przez procedury wyznaczania maksymalnych błędów dynamicznych. Wyniki obliczeń przedstawionych w artykule przeprowadzono w programie MathCad15.

Słowa kluczowe: niepewności parametrów modelu, modelowanie w dziedzinie czasu, maksymalny błąd dyna-miczny

1. Introduction

An analysis of errors generated by analogue systems intended for processing undetermined dynamic signals can be realized based on a previously determined mathematical model of such a system. This model can be presented by means of a transfer function, a complex frequency response and an impulse or step function. There are mutual mathematical relations between these types of models, which allow them to transform easily from one form into another.

In the measurement technique, in automation and signal processing theory, modelling in the frequency domain is most often carried out based on measurement of the amplitude and phase characteristics [1–6]. The method for determining both the model parameters and associated uncertainties is determined by the relevant standards, choice of which depends on the type and application of the system modelled [7]. However, there are practical applications, for example in biomedical measurements, for which it is impossible to apply the abovementioned modelling. In such cases, modelling in the time domain is applied on the basis of measurement of the response to step stimulus signal [8, 9]. For the order of model dynamics which is assumed according to the type of modelled system, the characteristic parameters of the step response are determined on the basis of its time recording. In order to minimize the error of modelling method, it is desirable to perform a series of readings of the response to successive switches of step stimulus signal. In this way, an accurate determination of the model parameters and associated uncertainties is possible. In this paper, modelling was performed on an example of a serial RLC circuit with the output signal from a capacitor. This system has the properties of a second-order model. The choice of such a system results from the ability to easily check the effectiveness of the applied modelling.

The mathematical model reflecting the dynamics of the system constitutes the basis for a series of theoretical studies that include, among other things, procedures for determining maximum dynamic errors. These errors are the response to an input signals with the constraints imposed on them, that is, the amplitude or both the amplitude and the rate of change [10, 11]. In [11, 12], it has been shown that any other input signal generates an error less than or at most equal to the maximum value. Depending on purpose of the dynamic system, one uses different error criteria. The most popular are the absolute error and the integral-square error.

In this paper, above criteria of the error for the case of input signals constrained only in amplitude, which have the 'bang-bang' shape are considered. For this type of input signal, it is possible to determine precisely the switching times for both error criteria. In the case of two simultaneous constraints, it is only possible for the absolute error [12, 13]. For both criteria, the values of maximum error are determined and the propagation of uncertainties associated with the parameters of the model is assessed using the procedure for determination of the errors. This was realized by checking all possible cases of increases or decreases of the model parameters by the values of associated uncertainties. Such research has been carried out so far on the basis of modelling in the frequency domain and only for the absolute error [14].

2. Modelling of second-order systems in the time domain

For the systems described by the following transfer function

$$K(s)=\frac{a\omega_0^2}{s^2+2\beta\omega_0 s+\omega_0^2}, \tag{1}$$

where a is the amplification coefficient, ω_0 is the non-damped natural frequency and   is the damping factor, the step response is presented by

$$h(t)=\mathcal{L}^{-1}\left(\frac{K(s)}{s}\right)=a\left\{1-\exp(-\beta\omega_0 t)\left[\cos(\omega_t t)+\frac{\omega_0\beta}{\omega_t}\sin(\omega_t t)\right]\right\}, \tag{2}$$

in which

$$\omega_d=\omega_0\sqrt{1-\beta^2} \tag{3}$$

is the damped natural frequency.

The step response becomes extreme at times when its derivative vanishes, that is, when the impulse response

$$k(t)=\frac{d}{dt}h(t)=\frac{a\omega_0^2}{\omega_d}\exp(-\beta\omega_0 t)\sin(\omega_d t) \tag{4}$$

is equal to zero.

The response $k(t)$ reaches zero if

$$\sin(\omega_d t_n)=0 \tag{5}$$

that is, for the following case

$$\omega_d t_n=n\pi,\quad n=0,1,2,\ldots. \tag{6}$$

Based on (3) and (6), we have

$$t_n=\frac{n\pi}{\omega_0\sqrt{1-\beta^2}},\quad n=0,1,2,\ldots. \tag{7}$$

Substituting t_1 into

$$\tilde{h}(t)=h(t)-a \tag{8}$$

one obtains the ratio of the maximum value of the step response to the steady state value which is called the overshoot and is calculated by

$$\Delta y=a\cdot\exp\left(\frac{-\beta\pi}{\sqrt{1-\beta^2}}\right). \tag{9}$$

Transformation of (9) gives

$$\beta = \frac{\ln\left(\frac{\Delta y}{a}\right)}{\sqrt{\ln^2\left(\frac{\Delta y}{a}\right) + \pi^2}}. \tag{10}$$

Let us present the damped natural frequency in the form

$$\omega_d = \frac{2\pi}{T_d}, \tag{11}$$

where T_d is the period of damped oscillations.

Comparing the right-hand sides of (3) and (11), we finally have

$$\omega_0 = \frac{2\pi}{T_d\sqrt{1-\beta^2}} = \frac{2\pi}{T_d\sqrt{1 - \frac{\ln^2\left(\frac{\Delta y}{a}\right)}{\ln^2\left(\frac{\Delta y}{a}\right)+\pi^2}}}. \tag{12}$$

Figure 1 shows the step response and the principle of determination of the parameters a, Δy and T_d which are the basis for determination of parameters of the model (1).

The parameters a, Δy and T_d are calculated as arithmetic means from a series of measurements obtained for the positive step responses. Fig. 2 shows the *a* voltage step stimulus signal and the positive and negative responses.

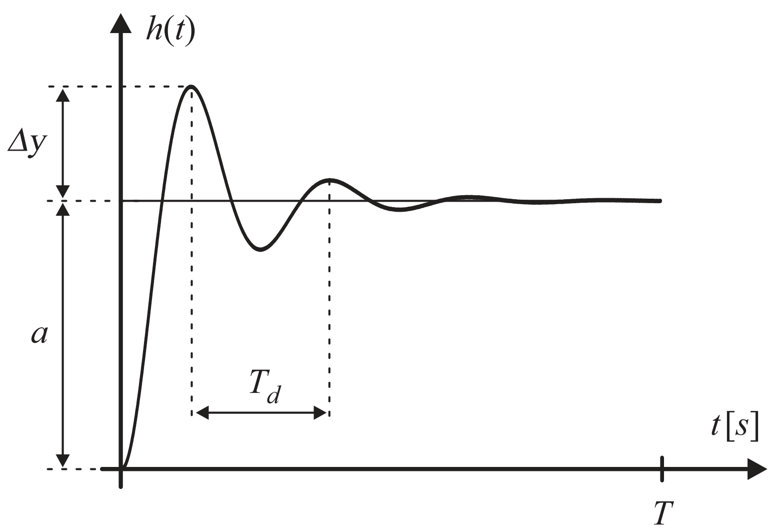

Fig. 1. Step response and its associated parameters

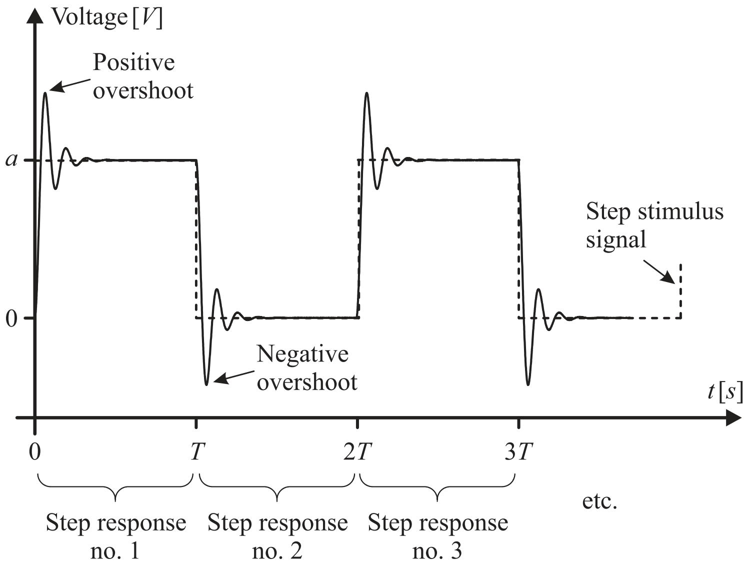

Fig. 2. Successive step responses

3. Determination of uncertainties associated with model parameters

The standard uncertainties $u(a)$, $u(\Delta y)$ and $u(T_d)$ associated with the parameters a, Δy and T_d as well as the expanded uncertainties related to the parameters of model (1) are calculated based on methods A and B [15, 16].

Utilizing method A, we have

$$u_A(x)=\sqrt{\frac{1}{M(M-1)}\sum_{m=1}^{M}(x_m-\bar{x})^2}, \tag{13}$$

where M is the number of measurement data, and

$$\bar{x}=\frac{1}{M}\sum_{m=1}^{M}x_m. \tag{14}$$

Value of M corresponds to the number of positive step responses.

Uncertainty of B type is calculated based on

$$u_B(x)=\sqrt{B_1^2+B_2^2}, \tag{15}$$

where

$$B_1=c_f\cdot\sigma_x/\sqrt{M} \tag{16}$$

and c_f and σ_x are the coverage factor and the standard deviation calculated for measurement data, respectively.

The variable B_2 calculated for a and Δy is

$$B_2 = \sqrt{u_a^2 + u_g^2 + u_o^2}\,, \tag{17}$$

where: u_a, u_g and u_o are the absolute, gain and offset uncertainties, respectively. The uncertainties under the root in (17) are calculated by

$$u_a = R_V \cdot u_g + R_V \cdot u_o + u_n \tag{18}$$

$$u_g = u_{rg} + g_t \cdot t_{chlic} + r_t \cdot t_{chlec}\,, \tag{19}$$

$$u_o = u_{ro} + o_t \cdot t_{chlic} + u_{INL}\,, \tag{20}$$

where: R_V, u_n, u_{rg}, g_t, t_{chlic}, r_t, t_{chlec}, u_{ro}, o_t and u_{INL} are the reading voltage range $[V]$ the noise uncertainty $[\mu V]$ the residual gain uncertainty [ppm of range] the gain tempco [ppm/°C] the temperature change from last internal calibration [°C], the reference tempco [°C], the temperature change from last external calibration [°C], the residual offset uncertainty [ppm of range] the offset tempco [ppm of range/°C] and the integral nonlinearity uncertainty[ppm of range], respectively.

The noise uncertainty is calculated, as follows

$$u_n = r_n\,, \tag{21}$$

where r_n is a standard deviation of random noise $[\mu V]$

In the case of parameter T_d the variable B_2 is

$$B_2 = 1/s_r\,. \tag{22}$$

where s_r is the sampling rate.

The standard uncertainties $u(a)$, $u(\Delta y)$ and $u(T_d)$ are calculated based on (13) and (15), utilizing the following formula

$$u(x) = \sqrt{u_A(x)^2 + u_B(x)^2}\,. \tag{23}$$

The combined uncertainties associated with β and ω_0 are calculated utilizing the total derivative, as follows

$$u_c(\beta) = \sqrt{\left[\frac{\partial\beta}{\partial a}u(a)\right]^2 + \left[\frac{\partial\beta}{\partial\Delta y}u(\Delta y)\right]^2} \tag{24}$$

and

$$u_c(\omega_0) = \sqrt{\left[\frac{\partial\omega_0}{\partial a}u(a)\right]^2 + \left[\frac{\partial\omega_0}{\partial\Delta y}u(\Delta y)\right]^2 + \left[\frac{\partial\omega_0}{\partial T_d}u(T_d)\right]^2}\,. \tag{25}$$

Finally, the expanded uncertainties associated with parameters of the model (1) are

$$U(a) = c_f \cdot u(a), \tag{26}$$

$$U(\beta)=c_f \cdot u_c(\beta) \tag{27}$$

and

$$U(\omega_0)=c_f \cdot u_c(\omega_0). \tag{28}$$

4. Procedure for determining the absolute error

The absolute error represents the maximum possible value of the output signal $y(t)$ over the interval $[0,T]$ for the linear system described by the total impulse response

$$k(t)=k_r(t)-k_s(t), \tag{29}$$

where $k_r(t)$ and $k_s(t)$ are the impulse responses of the real system and its standard.

The standard is represented by a high-order low-pass filter with a transfer function defined by

$$K_s(s)=\frac{a}{\prod_{l=1}^{L}\left(\frac{s}{\omega_c}-e^{\frac{j(2l+L-1)\pi}{2L}}\right)}, \tag{30}$$

where L is the order of the filter.

The impulse responses $k_r(t)$ and $k_s(t)$ are obtained as the inverse Laplace transform of (1) and (30), respectively.

The maximum transient value of $y(t)$ can be achieved only for $t=T$. However, it is necessary to determine the input signal $x_0(t)$ of "bang-bang" type which generates the output $y(T)$. This output signal represents the absolute error denoted below by D.

The signal that maximizes the error D is determined based on a simple formula, as follows

$$u_0(t)=A \cdot sign\left[k(T-t)\right]. \tag{31}$$

The absolute error is calculated by means of

$$D=A\int_0^T |k(t)|dt, \tag{32}$$

where A denotes the magnitude constraint of signal $x(t)$ [12–14]. This constraint was assumed as equal to the amplification coefficient of the model (1).

5. Procedure for determining the integral-square error

Let us define by

$$\varsigma[k,x,y]=\frac{E[y]}{E[x]} \tag{33}$$

the energy transfer ratio over the interval $[0,T]$ and corresponding to the linear system described by the impulse response $k(t)$ defined by (29), input signal $x(t)$ and output $y(t)$.

The energy of the output signal $y(t)$ is represented by

$$E[y]=\int_0^T y(t)^2\,dt=\int_0^T\left[\int_0^T \mathrm{K}(t,\tau)x(\tau)\,d\tau\right]x(t)\,dt, \tag{34}$$

where

$$K(t,\tau)=\int_0^T k(t,v)k(v,\tau)\,dv \tag{35}$$

is the energy kernel which is the autocorrelation function calculated based on the impulse response $k(t)$.

It is obvious that there is such a signal $x_0(t)\in L^2$ that maximizes ς. This signal has the "bang-bang" shape and satisfies the Fredholm integral equation of the second kind, as below

$$\int_0^T K(t,\tau)x(\tau)\,d\tau=\varsigma x(t) \tag{36}$$

with the kernel defined by (35) [17].

The procedure for determining the signal $x_0(t)$ is carried out in three main steps:

1. Calculate the autocorrelation function

$$\mathrm{K}(t)=\int_0^T k(\tau)k(t+\tau)\,d\tau. \tag{37}$$

2. Determine the initial input signal

$$x^0(t)=A\cdot sign\,[\mathrm{K}(t)]. \tag{38}$$

3. Determine the signal $x_0(t)$ which has one constraint and maximizes the output energy based on the iteration algorithm, as follows

$$x^{i+1}(t)=A\cdot sign\left[\int_0^t \mathrm{K}(t-\tau)x^i(t)\,d\tau\right]\quad for\ i=0,1,2,\ldots. \tag{39}$$

This output energy corresponds to the integral-square error I_2 of a linear system [18].

The iteration algorithm is terminated when any switching times of signals $x^{i+1}(t)$ and $x^i(t)$ differ by the value of the assumed discretization step Δ. Based on the maximizing signal $x_0(t)$ the integral-square error can be calculated by

$$I_2=\int_0^T\left[\int_0^t k(t-\tau)x_0(\tau)\,d\tau\right]^2 dt. \tag{40}$$

6. Results

The parameters of the model (1) were determined based on measurement of 34 positive step responses for a equals $1[V]$. The step stimulus signal was generated by NI-6221 measuring card with a sampling rate s_r equals $100[kS/s]$. This card was also used to determine the responses shown in Fig. 2.

Table 1 includes the values of the parameters a, Δy and T_d obtained in accordance with Fig. 1. The values of these parameters were tabulated with an accuracy of three significant digits M=1, 2, ..., M, where M was assumes as equal to 34. The last row in Table 1, denoted as ***Mean***, contains the mean values of all three parameters.

These values are assumed to be the estimates of parameters a, Δy and T_d and are indicated below as $\tilde{a}$, $\Delta\tilde{y}$ and $\tilde{T}_d$ The standard uncertainties associated with these parameters, determined based on (13), are as follows: $u_A(\tilde{a})=27[\mu V]$, $u_A(\Delta\tilde{y})=45[\mu V]$ and $u_A(T_d)=1{,}5[\mu s]$.

Table 1. Parameters based on successive step responses

m	a [mV]	Δy [mV]	T_d [ms]	m	a [mV]	Δy [mV]	T_d [ms]
1	994.026	440.113	1.290	18	994.288	439.689	1.290
2	994.041	439.613	1.280	19	994.302	439.513	1.280
3	994.051	440.250	1.280	20	994.311	439.666	1.280
4	994.056	439.760	1.300	21	994.337	439.478	1.290
5	994.076	440.063	1.290	22	994.349	439.628	1.280
6	994.115	439.862	1.280	23	994.362	439.453	1.270
7	994.113	440.025	1.280	24	994.387	439.267	1.290
8	994.113	439.703	1.290	25	994.397	439.257	1.300
9	994.151	440.150	1.290	26	994.406	439.410	1.270
10	994.163	439.652	1.290	27	994.428	439.710	1.290
11	994.158	439.657	1.300	28	994.430	439.548	1.280
12	994.202	439.452	1.280	29	994.465	439.512	1.290
13	994.200	439.616	1.290	30	994.488	439.489	1.270
14	994.232	439.422	1.270	31	994.493	439.484	1.280
15	994.260	439.556	1.290	32	994.506	439.148	1.280
16	994.261	439.554	1.300	33	994.533	439.283	1.300
17	994.261	439.393	1.290	34	994.559	439.418	1.280
				Mean	994.280	439.612	1.286

The coverage factor c_f was assumed as equal to 2, according to a 95% confidence level. The standard deviations calculated based on Table 1 are: σ_a=0,16 mV, $\sigma_{\Delta y}$=0,26 mV and σ_{T_d}=8,8 μs.

The following values: $t_{chili}=1[°C]$, $t_{chle}=25[°C]$, $u_{rg}=95[ppm$ of range], $g_t=25[ppm/°C]$, $r_t=5[°C]$, $u_{ro}=25[ppm$ of range], $o_t=79$ $[ppm$ of range/°C], $u_{INL}=76$ $[ppm$ of range] and $r_n=30\mu V$ are provided by the manual of measuring card [19].

For the above data and based on (15–22), the uncertainties calculated by method B are: $u_B(\tilde{a})=1.984$ [mV], $u_B(\Delta\tilde{y})=1.985$ [mV] and $u_B(T_d)=11$ [μs]. Then the standard uncertainties, calculated based on (23), are as follows: $u(\tilde{a})=1.984$ [mV], $u(\Delta\tilde{y})=1.985$ [mV] and $u(T_d)=11$ [μs]. It can be seen that these uncertainties are affected by the uncertainty determined by method B. The combined uncertainties associated with β and ω_0 are: $u_c(\beta)=0{,}0014$ [mV] and $u_c(\omega_0)=42$ [rad/s]. Finally, the expanded uncertainties associated with the model parameters are calculated. These uncertainties and the model parameters are tabulated in Table 2. The parameter $\tilde{a}$ corresponds to the mean value included in Table 1. The parameters $\tilde{\beta}$ and $\tilde{\omega}_0$ were calculated on the basis of relations (24) and (25).

Table 2. Estimates of the model parameters and associated uncertainties

Model parameters			Associated uncertainties		
$\tilde{a}$ [mV]	$\tilde{\beta}$ [–]	$\tilde{\omega}_0$ [rad/s]	$U(\tilde{a})$ [mV]	$U(\tilde{\beta})$ [–]	$U(\tilde{\omega}_0)$ [rad/s]
994.280	0.2514	5050	3.968	0.0029	83

Table 3 contains the values of errors D and I_2 determined based on (32) and the iterative procedure executed utilizing relations (37)–(40), respectively. The fifteenth-order low-pass filter with a cut-off corresponding to a 10% decrease of the amplitude-frequency characteristic relative to its value for ω=0 was adopted as a standard. This characteristic was determined based on the model (1) for the parameters contained in Table 2. The order of the model adopted constitutes the maximum value for which it is possible to carry out the calculations in the program MathCad15. The value of the amplitude constraint A was assumed to be equal to the value of estimate $\tilde{a}$.

The first row of Table 3 contains the errors without taking into account the influence of the uncertainties, whereas other rows include the errors for any case increase or decrease in the parameters by the values of the associated uncertainties.

Table 3. Parameters of the model and the maximum values of errors D and I_2

Change of parameters			D [Vs]	I_2 [mV^2s]
$\tilde{a}$	$\tilde{\beta}$	$\tilde{\omega}_0$	4.001	24.499
$\tilde{a}-U(\tilde{a})$	$\tilde{\beta}-U(\tilde{\beta})$	$\tilde{\omega}_0-U(\tilde{\omega}_0)$	3.995	24.586
$\tilde{a}-U(\tilde{a})$	$\tilde{\beta}+U(\tilde{\beta})$	$\tilde{\omega}_0-U(\tilde{\omega}_0)$	3.944	24.015
$\tilde{a}-U(\tilde{a})$	$\tilde{\beta}-U(\tilde{\beta})$	$\tilde{\omega}_0+U(\tilde{\omega}_0)$	3.996	24.747

Change of parameters			D $[Vs]$	I_2 $[mV^2s]$
$\tilde{a}-U(\tilde{a})$	$\tilde{\beta}+U(\tilde{\beta})$	$\tilde{\omega}_0+U(\tilde{\omega}_0)$	3.944	24.065
$\tilde{a}+U(\tilde{a})$	$\tilde{\beta}-U(\tilde{\beta})$	$\tilde{\omega}_0-U(\tilde{\omega}_0)$	4.060	24.981
$\tilde{a}+U(\tilde{a})$	$\tilde{\beta}+U(\tilde{\beta})$	$\tilde{\omega}_0-U(\tilde{\omega}_0)$	4.007	24.401
$\tilde{a}+U(\tilde{a})$	$\tilde{\beta}-U(\tilde{\beta})$	$\tilde{\omega}_0+U(\tilde{\omega}_0)$	**4.061**	**25.145**
$\tilde{a}+U(\tilde{a})$	$\tilde{\beta}+U(\tilde{\beta})$	$\tilde{\omega}_0+U(\tilde{\omega}_0)$	4.007	24.452

The highest values of both errors are obtained for the case when the estimates of parameters $\tilde{a}$ and $\tilde{\omega}_0$ are increased by the values of associated uncertainties, while the estimate of parameter $\tilde{\beta}$ is decreased by the uncertainty. These values are 1.50% and 2.64% higher than the values without taking into account the uncertainties for the errors D and I_2 respectively.

7. Conclusions

The paper presents an assessment of the propagation of modelling uncertainty in the time domain by the procedure for determining both the absolute error and the integral-square error for the input signals constrained only in amplitude.

Based on 34 identification tests, the estimates of the model parameters are determined and the associated uncertainties are calculated. Then, the maximum dynamic errors were determined both for the model parameters only and for all cases of increases or decreases in the model parameters by the values of associated uncertainties. These errors were determined based on modelling of a serial RLC circuit as an example of a second-order system. As a standard that constitutes the reference for determination of the errors, the fifteenth-order low-pass filter was adopted.

The results showed that modelling uncertainty has an influence on dynamic errors, particularly for the integral square criterion. In comparison with modelling results in the frequency domain presented in [14], the time-domain modelling increases the uncertainty of parameter $\tilde{a}$ more than 16-fold, but decreases the uncertainties of the parameters $\tilde{\beta}$ and $\tilde{\omega}_0$ more than 100-fold. Thus, in the case of modelling in the time domain presented in this paper, the influence of uncertainties on the absolute error was approximately 80 times lower. This is due to the fact that the method of modelling in the time domain is more accurate but cannot always be used, in particular for examination of sensors intended for measurement of vibration. However, the most important conclusion is that for both methods of modelling, the uncertainties associated with the model parameters have an essential influence on the value of errors and this effect should be taken into account during the calibration process based on the maximum dynamic errors.

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