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Paweł Szeptyński (pszeptynski@pk.edu.pl)
Institute of Structural Mechanics, Faculty of Civil Engineering, Cracow University of Technology

Influence of non-uniformity of cracking on calculation of deflection of reinforced-concrete elements, according to Eurocode 2

Wpływ nierównomierności zarysowania na obliczanie ugięć elementów żelbetowych według Eurokodu 2

Abstract

Formulae for the deflection of two examples of isostatic systems (simply supported beam, cantilever) were derived, accounting for influence of distribution of bending moments on cracking and beam stiffness distribution, according to EC2. Numerical analysis of the problem for the same two examples as well as for two further hyperstatic systems was performed using an iterative FEM algorithm. The influence of non-uniformity of cracking on deflection and distribution of bending moments was shown to be negligible in typical practical design problems, therefore also an estimation of deflection on the basis of the distribution of bending moments and the value of the factor α_k obtained from linear solution (before redistribution) is shown to be justified.

Keywords: reinforced-concrete, cracking, deflection, Eurocode

Streszczenie

Wyprowadzono zamknięte wzory na ugięcia dla dwóch przykładów układów statycznie wyznaczalnych (belka swobodnie podparta, wspornik), uwzględniając wpływ rozkładu momentów zginających na zarysowanie i rozkład sztywności belki, zgodnie z EC2. Wykorzystując iteracyjny algorytm MES, przeprowadzono numeryczną analizę zagadnienia dla tych samych schematów statycznych oraz dwóch przypadków układów statycznie niewyznaczalnych. Wykazano minimalny wpływ nierównomierności zarysowania na wielkość ugięcia i rozkład momentów zginających w typowych praktycznych zagadnieniach projektowych, a tym samym stwierdzono zasadność szacowania ugięć na podstawie rozkładu momentów i wartości współczynnika α_k dla rozwiązania liniowego (sprzed redystrybucji).

Słowa kluczowe: żelbet, zarysowanie, ugięcie, Eurokod

1. Motivation

Design practice indicates that in the case of large-span reinforced-concrete (RC) buildings, e.g. large commercial or shopping centres, it is the Serviceability Limit State (SLS) not the Ultimate Limit State (ULS) that determines the final amount of reinforcement and thus the total cost of construction. As it becomes more common to limit floor deflection up to 1/500 of span length in order to prevent possible cracking of in-fill masonry walls due to forced deformation, the problem of precise determination of deflection becomes even more important because of economic reasons both for designers and investors. This problem is influenced by many factors, among which cracking, creep, settlement and global deformation of the structure should be mentioned. It is cracking which will be discussed in this paper.k

The algorithm for the calculation of deflections which is presented in the EC2 [1] standard is based on the model by Rostasy, Koch and Leonhardt [5]. The code states that having determined the curvature of a beam in every section in view of cracking total deflection should be obtained via numerical integration of obtained curvatures. The problem is non-linear as any change in the structure'srigidity stiffness due to cracking results in redistribution of the bending moments in hyperstatic systems. Such problems are usually solved in an iterative manner.

Before EC2 came into force in some countries, a different approach was a common design practice, namely: calculation of deflections with the use of a single value for the reducedrigidity stiffness of the cracked beam for all of its sections. The decrease in beamrigidity stiffness due to cracking for the whole beam was calculated for a single design value of the bending moment – the same one then used to calculate deflection according to formula [3]:

$$u = \alpha_k \frac{5}{48} \frac{M_{Sd} L_{eff}^2}{B} \tag{1.1}$$

or any equivalent, in which α_k is determined using the classical methods of structural mechanics. The clue aspect of this formula, which is discussed here, is whether M_{sd} and α_k are determined before or after the redistribution of moments. The commentary given in [2] is useless as it refers to an isostatic system in which such a redistribution does not occur. However, both old [3] and modern [4] handbooks on the design of RC structures suggest using α_k resulting from the linear solution even for hyperstatic systems. The origin of M_{sd} is discussed in both standards in a general way, stating that it should be determined with the use of analysis methods proper for the case under consideration: a linear–elastic analysis accounting for cracking or a non-linear analysis.

A simplified approach, widespread among designers, was to use the design values of M_{sd} obtained from the linear solution (before redistribution). It was due to considerable difficulties in determining the actual distribution of bending moments when the FEM software was not available in the past, or when it did not enable non-linear analysis. It must be admitted, however, that even nowadays performing non-linear analysis is expensive (this concerns the cost of software licenses as some commonly used commercial FEM software used in design offices still lacks the option for such calculations) and time-consuming.

It is obvious that such a simplified approach as the one described above cannot be strictly valid at least for two reasons:

- Integration of the curvatures determined for non-uniform stiffness distribution must result in a different total displacement than in the case of uniform rigidity stiffness decrease;
- In the case of hyperstatic systems, the distribution of moments depends on the relative distribution of rigidity stiffness – if, in turn, distribution ofrigidity stiffness depends on the current distribution of moments, then the whole problem becomes non-linearly coupled.

It must also be stated clearly that the guidelines in the EC2 standard do not allow such an approach to be used. Despite this, it is still in use sometimes due to the simplicity of implementation in the calculation algorithms and due to its efficiency. The question that arises in this situation is whether the error occurring when using simplified approach mentioned above is acceptably small or not. It is often assumed that such an approach provides a "safe" solution, overestimating true deflection – as the problem is non-linear, such an assumption may at best be a plausible conjecture rather than a definite statement. However, as mentioned before, such an overestimation may emerge to be needless, unnecessarily expensive and – in some cases – even unacceptably dangerous: too large an amount of reinforcement may cause a situation in which the reinforcement steel is strained below yield strain and the concrete is compressed with a limit value. Such a situation is not allowed for bent elements as it may result in brittle destruction and sudden collapse. This might happen only in the case when the true load exceeds the design value, which should not take place at all; nevertheless, any over-reinforcement is improper at least for economic reasons.

2. Theoretical analysis

2.1. Assumptions

General relations governing the problem of determination of deflection of a bent beam (or slab in a one-direction bending state) are:

Equilibrium relation:

$$\frac{d^2}{dx^2}M(x)=-q(x) \tag{2.1}$$

Constitutive relation:

$$\frac{d^2}{dx^2}w(x)=-\frac{M(x)}{B(x)} \tag{2.2}$$

where:

q – external load,
M – bending moment,
w – deflection,
B – bending stiffness.

We assume also that the bending stiffness is reduced due to cracking according to the EC2 formula, approved by the European Committee for Concrete (CEB) and based on the propositions by Rostasy, Koch and Leonhardt [5].

$$B(x)=\begin{cases} E_c J_{\mathrm{I}} & \Leftrightarrow M<M_{cr} \\ \dfrac{E_c J_{\mathrm{II}}}{1-\beta_1\beta_2\left(\dfrac{M_{cr}}{M(x)}\right)^2\left(1-\dfrac{J_{\mathrm{II}}}{J_{\mathrm{I}}}\right)} & \Leftrightarrow M\geq M_{cr} \end{cases} \tag{2.3}$$

where:

E_c – the Young modulus of the concrete (mean or effective value, depending on the specific case),

M_{cr} – cracking moment,

$J_{\mathrm{I}}, J_{\mathrm{II}}$ – cross-section's second moment of area in uncracked and cracked state respectively,

β_1, β_2 – are factors accounting for rebar contact stress and influence of long-term loading respectively.

This provides a non-linear system of ordinary differential equations with discontinuous coefficients. Attempts to find a general solution to such a problem should be doomed to fail in advance. The problem simplifies greatly if one assumes a quadratic moment distribution – an assumption, which is usually fulfilled in most typical design problems. This account for uniform load $q(x) = q =$ const. and any boundary nodal displacements and point loads. Integration of equilibrium equations yields:

$$M(x)=-\frac{q}{2}x^2+C_1x+C_2 \tag{2.4}$$

where:

C_1, C_2 – are constants.

The whole problem reduces now only to double direct integration of (2.2) which may be carried out analytically. If cracking occurs, three cases must be considered:

- **CASE 1:** $2C_2q+C_1^2<0$

$$\begin{aligned} w(x)= & \frac{\beta}{q}\ln\left|-\frac{q}{2}x^2+C_1x+C_2\right|-\alpha\left(-\frac{qx^4}{24}+\frac{C_1x^3}{6}+\frac{C_2x^2}{2}\right) \\ & -2\beta\left[\frac{C_1-qx}{q\sqrt{-\left(2C_2q+C_1^2\right)}}\operatorname{atan}\left(\frac{C_1-qx}{\sqrt{-\left(2C_2q+C_1^2\right)}}\right)\right]+C_3x+C_4 \end{aligned} \tag{2.5}$$

- **CASE 2**: $2C_2 q + C_1^2 = 0$

$$w(x) = \frac{2\beta}{q}\ln\left|C_1 - qx\right| - \alpha\left(-\frac{qx^4}{24} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2}\right) + C_3 x + C_4 \tag{2.6}$$

- **CASE 3**: $2C_2 q + C_1^2 > 0$

$$\begin{aligned} w(x) &= \frac{\beta}{q}\ln\left|\left(C_1 - qx\right)^2 - \left(2C_2 q + C_1^2\right)\right| - \alpha\left(-\frac{qx^4}{24} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2}\right) \\ &- \frac{\beta}{\sqrt{2C_2 q + C_1^2}}\left(x - \frac{C_1}{q}\right)\ln\left|\frac{qx - C_1 - \sqrt{2C_2 q + C_1^2}}{qx - C_1 + \sqrt{2C_2 q + C_1^2}}\right| + C_3 x + C_4 \end{aligned} \tag{2.7}$$

where:

$$\alpha = \frac{1}{E_c J_{\mathrm{I}}}\frac{J_{\mathrm{I}}}{J_{\mathrm{II}}} > 0 \tag{2.8}$$

$$\beta = \frac{\beta_1 \beta_2 M_{cr}^2}{E_c J_{\mathrm{I}}}\left(1 - \frac{J_{\mathrm{II}}}{J_{\mathrm{I}}}\right)\frac{J_{\mathrm{I}}}{J_{\mathrm{II}}} > 0 \tag{2.9}$$

The sign of $\Delta = 2C_2 q + C_1^2$ depends on the number of distinct real roots of (2.4). In the first case ($\Delta < 0$), there are no real roots – when the load is applied downwards (gravitational load) such a situation is not likely to occur and for simple beams (single-span, without overhangs) without nodal displacements it is impossible. The second case ($\Delta = 0$) occurs when the extremum of moment distribution is at the same time the root of (2.4) – this happens, for example, at the end of a cantilever. The third case ($\Delta > 0$) is the one which describes most typical situations.

Equations (2.5)–(2.7) describe the deformation of the beam axis in the cracked area. In the uncracked version, the deflection is described with a fourth order polynomial function. The range of the cracked zone is not known in advance. In the case of isostatic systems it may be easily determined as the interval in which $\left\{x : \left|M(x)\right| > M_{cr}\right\}$, since the distribution of moments is known. Solving the problem reduces now to just finding the integration constants as a solution of the linear algebraic system given by the boundary conditions and compatibility conditions for deflection and angle of rotation at the boundaries of cracked zone. In the case of hyperstatic systems, both range of cracked zone and reaction forces (i.e. constant C_2) are unknown – finding them requires a solution to a complex non-linear algebraic system. For this reason, only two cases of isostatic systems are considered below.

2.2. Simply supported beam of length *L* under uniform load *q*

In a coordinate system with its origin in the middle of the beam span, the distribution of bending moments is given by the constants $C_1 = 0, C_2 = \frac{qL^2}{8}$. The derivation of the solution is rather schematic, yet lengthy – let us present only the results. Let us denote $EJ = E_c J_1$. Maximum deflection is as follows:

$$
\begin{aligned}
w_{\max} &= \frac{5}{384}\frac{qL^4}{EJ} - \frac{2\beta}{q}\ln\left|1+\frac{L_{cr}}{L}\right| \\
&+ \frac{5qL^4(\alpha EJ - 1)}{384EJ}\left[\frac{3}{5}\left(\frac{L_{cr}}{L}\right)^4 - \frac{4}{5}\left(\frac{L_{cr}}{L}\right)^3 - \frac{6}{5}\left(\frac{L_{cr}}{L}\right)^2 + \frac{12}{5}\frac{L_{cr}}{L}\right]
\end{aligned}
\tag{2.10}
$$

where the range of the cracked zone (located symmetrically in the middle of span) equals:

$$
L_{cr} = L\sqrt{1 - \frac{8M_{cr}}{qL^2}} \tag{2.11}
$$

2.3. Cantilever of length *L* under uniform load *q*

In a coordinate system with its origin at the distribution of bending moments is given by constants $C_1 = \frac{qL}{2}, C_2 = -\frac{qL^2}{2}$. Maximum deflection is equal to:

$$
w_{\max} = \frac{1}{8}\frac{qL^4}{EJ} + \frac{2\beta}{q}\ln\left|1-\frac{L_{cr}}{L}\right| + \frac{qL^4(1-\alpha EJ)}{8EJ}\left[\left(\frac{L_{cr}}{L}\right)^4 - 4\left(\frac{L_{cr}}{L}\right)^3 + 6\left(\frac{L_{cr}}{L}\right)^2 - 4\frac{L_{cr}}{L}\right] \tag{2.12}
$$

Cracked zone length (located by support) is equal to:

$$
L_{cr} = L\left[1 - \sqrt{\frac{2M_{cr}}{qL^2}}\right] \tag{2.13}
$$

3. Numerical analysis

3.1. Range of variation of problem parameters

Let us write explicitly the formulae for the second moment of area of the uncracked and cracked rectangular cross-section:

$$J_{\mathrm{I}} = \frac{bh^3}{12} + bh\left(x_{\mathrm{I}} - \frac{h}{2}\right)^2 + \alpha_e bd\left[\rho_1\left(x_I - d\right)^2 + \rho_2\left(x_I - a_2\right)\right] \tag{2.14}$$

$$J_{\mathrm{II}} = \frac{bx_{\mathrm{II}}^3}{3} + \alpha_e bd\left[\rho_1\left(x_{\mathrm{II}} - d\right)^2 + \rho_2\left(x_{\mathrm{II}} - a_2\right)\right] \tag{2.15}$$

$$x_{\mathrm{I}} = \frac{bh^2 + 2\alpha_e bd\left(\rho_1 d + \rho_2 a_2\right)}{2\left[bh + \alpha_e bd\left(\rho_1 + \rho_2\right)\right]} \tag{2.16}$$

$$x_{\mathrm{II}} = d\left[\sqrt{\alpha_e^2\left(\rho_1 + \rho_2\right)^2 + 2\alpha_e\left(\rho_1 + \frac{a_2}{d}\rho_2\right)} - \alpha_e\left(\rho_1 + \rho_2\right)\right] \tag{2.17}$$

where:

- h – beam height,
- b – beam width,
- d – effective height,
- a_2 – distance from compressed reinforcement centroid to the compressed edge,
- ρ_1, ρ_2 – index of stretched and compressed reinforcement respectively,
- $\alpha_e = E_s/E_c$ – ratio of Young moduli of steel and concrete.

We shall now consider what factors influence the result narrowing our considerations to single reinforced $(\rho_2 = 0)$ rectangular cross-sections. We may express effective height as $d = 0.95h$ which is not precise but a fair and useful approximation. In this way, the ratio $\frac{J_{\mathrm{II}}}{J_{\mathrm{I}}}$ becomes independent of the h/b ratio. Introducing a generalized surface load $p = q/b$ we may eventually consider our solution as dependent on four dimensionless factors:

- $\chi_1 = \frac{L}{h}$ (geometry of system),
- $\chi_2 = \frac{f_{ct}}{p}$ (strength to load ratio),
- $\chi_3 = \rho_1$ (reinforcement index),
- $\chi_4 = \alpha_e$ (steel to concrete stiffness ratio).

This is not a complete set of independent variables influencing the solution. We must also notice that α_e is itself a function of the shape of the cross-section, concrete class etc. Anyway, these four factors may be used in order to estimate the quantitative influence of non-uniformity of cracking on deflection. We shall now determine the range of variation of values of those parameters.

Considering the length-to-height ratio we may fairly assume that:

$$\chi_1 \in \langle 4; 30 \rangle$$

The lower bound is determined by the limitation that shorter beams should be considered as membrane shells. The upper bound corresponds to typical values for plates. Considering the strength-to-load ratio we may assume that typical RC structures are usually made nowadays of concrete of C20/25 up to C30/37 class. This corresponds with the scope of values of average tensile strength $f_{ct} \approx 2\text{–}3$ MPa. Surface load (characteristic value) in turn may vary from 1.5 kPa (apartments – live load only) up to ca. 15 kPa (e.g. car park or mall live load + dead load of flooring and 30 cm thick RC plate). This gives us:

$$\chi_2 \in \langle 130;2000 \rangle$$

The range of variation of the reinforcement index is limited the by the code requirements on maximal and minimal reinforcement:

$$\chi_3 \in \langle 0,0013;0,04 \rangle$$

Possible values of α_e factor range usually from ca. 7 (immediate elastic deflection) to ca. 30 (final creep deflection):

$$\chi_4 \in \langle 7;30 \rangle$$

As the SLS most often concerns the final creep deflection, we will assume long-term load ($\beta_2 = 0.5$). We will also assume ribbed reinforcement ($\beta_1 = 1.0$) as the smooth one in practically almost out of use as a longitudinal reinforcement.

3.2. Analysed cases

In order to verify the obtained theoretical formulae an iterative FEM script was written solving a single-span beam under uniform load. Four static schemes were considered:

- beam fixed at both ends,
- simply supported beam,
- cantilever,
- beam fixed at one end and pinned at the second one.

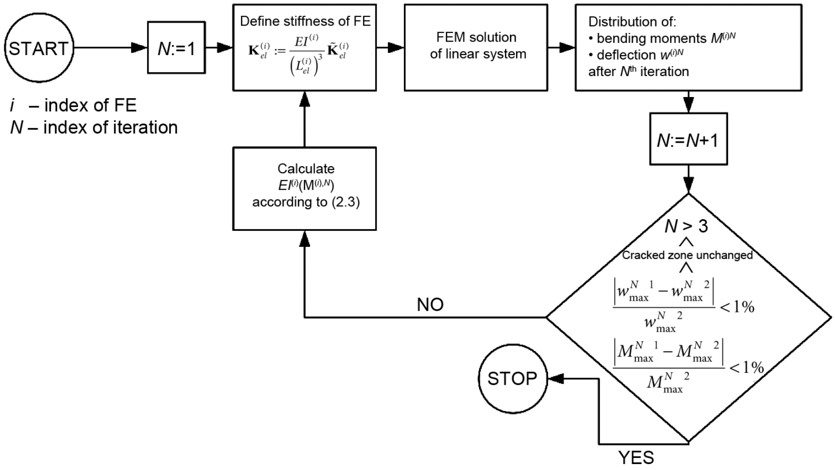

Fig. 1. Flowchart of iteration algorithm used for solution of non-linear problem

The loop end condition was a conjunction of three conditions: that last step increment in both deflection and extremal bending moment should be less than 1% of deflection or the extremal moment obtained in the previous step and that cracked zone in the last step should be the same as in the previous one. Additionally no fewer than 3 iterations must be performed. The beam was divided into 100 finite elements. After each iteration, each element's stiffness was recalculated according to formula (2.3), in which the value of bending moment for the element was taken as equal to an average value of its end nodes' bending moment values. A flowchart showing the general scheme of this algorithm is shown below.

16 basic cases were considered – each one corresponding with one combination of the extremal values of four factors introduced in previous sections. It is clear that such an "edge-value" analysis does not account for possible local extrema unless the joint influence of all of them provides a solution that depends on those factors in a monotonic way. The results, discussed below, indicate, however, that every solution (except for cantilever) corresponding with each combination of extremal values is almost identical with the simplified solution based on the moment and α_e factor determined for the uncracked system. For this reason, it may be supposed that no extremely distinct response of the system should be expected for any intermediate values. Such a supposition is of practical importance since any theoretically justified conclusions cannot be made in the case of such a complex, non-linear and self-coupled problem. These cases referred to extremal values of influence factors as follows:

Table 1. Edge-values of independent dimensionless parameters governing the problem for each analysed case

Case	χ_1	χ_2	χ_3	χ_4
1	30	2000	0.0013	30
2	30	130	0.0013	30
3	4	2000	0.0013	30
4	4	130	0.0013	30
5	30	2000	0.04	30
6	30	130	0.04	30
7	4	2000	0.04	30
8	4	130	0.04	30
9	30	2000	0.0013	7
10	30	130	0.0013	7
11	4	2000	0.0013	7
12	4	130	0.0013	7
13	30	2000	0.04	7
14	30	130	0.04	7
15	4	2000	0.04	7
16	4	130	0.04	7

Beam width – which has no major qualitative effect on the results obtained – was set at a constant numerical value $b = 1$ [m]. Each case was calculated four times for different sets of basic parameters, namely: span length (6 m or 9 m), tension strength and Young modulus of concrete (respective for C30/37 and C20/25 classes):

Table 2. Values of basic parameters governing the problem for each analysed sub-case

Case	L [m]	f_{ct} [MPa]	E_c [GPa]
A	6	2.9	32
B	6	2.2	30
C	9	2.9	32
D	9	2.2	30

A total number of 64 cases for each static scheme were analysed.

3.3. Results

The results obtained from the numerical solution were the same as those provided by the theoretical formulae. It emerged however that subcases A-D of any case 1–16 did not influence the result in any way. For most of the cases, there were only 3 iterations needed to fulfil the loop end conditions – maximum number of iterations needed in particular cases was 6. Cracking occurred only in cases 2, 6, 10 and 14 (long and low beam /thin plate/, small strength, strong load) and additionally for cantilever in cases 1, 5, 9, 13 (long and low beam / thin plate/, high strength, small load).

In all the cases considered, extremal values for the bending moment after redistribution did not change more than 5.369% (beam fixed at both ends, case 10) of the respective value obtained from the linear solution – it may be stated that in typical design problems the influence of cracking on the distribution of bending moments is of minor importance.

Also the α_k factor was calculated according to the formula:

$$\alpha_k = \frac{48}{5}\frac{w_{\max} B}{M_{\max} L^2} \tag{3.1}$$

Analytical values of α_k for chosen static schemes are:

- beam fixed at both ends, $\alpha_k = 0.6$
- simply supported beam, $\alpha_k = 1.0$
- cantilever, $\alpha_k = 2.4$
- beam fixed at one end and pinned at the second one.

$$\alpha_k = \frac{55\sqrt{33}+39}{420} \approx 0.73948$$

In all cases except for cantilever, the obtained α_k values varied from those derived from the linear problem by less than 3.377% (beam fixed at both ends, case 10). For all schemes, the true maximum deflection was always smaller than the one obtained in the simplified approach.

An interesting conclusion emerges from the analysis of cases 1, 5, 9 and 13 for cantilever, which concern long and low beams (or thin plates) with high tensile strength under a small load. True maximal deflections were always smaller than those estimated with the simplified approach – in case 9, the reduction of deflection was as high as 54.174%. Such a situation occurs when the range of the cracked zone is very small, namely when the cantilever is only cracked at a short distance close to the support. The simplified approach assuming constant decrease of stiffness along whole beam must provide a considerable overestimation; however, it must be admitted that these cases correspond with very small absolute values of deflection (small load, high strength) – much smaller than the limit value.

In order to verify this result, a typical example of a balcony cantilever was analysed. We consider a 1.7 m long plate 20 cm thick, made of C20/25 class concrete. The load is 5 kPa of live load and 6.25 kPa of dead load (with flooring). Assuming reinforcement with A-IIIN class steel with $\alpha_1 = 3.5$ cm the ULS requires $\rho_1 = 0.001755$. The resultant final creep deflection ($\alpha_e = 3.5$) corresponds with ($\alpha_k = 0.9437$), which is more than 2.5 times as small as the 2.4 derived from the linear solution. In fact, the deflection obtained, equal to 2.81 mm, as well as the one estimated using the simplified approach, which equals 7.14 mm, are far smaller than the limit value $L/150 = 11.33$ mm. The cracked zone length is only 8.53 cm long, which is approximately 0.05 L.

Despite the fact that such a large overestimation concerns mostly those deflections which are very small, smaller than the limit permissible values (and thus may be fairly disregarded), it may be of interest to determine the conditions for which such overestimation occurs. This is the situation when the cracked zone length is very small. In the case of isostatic systems, the answer is obvious – it is when the bending moment exceeds the cracking value only slightly. A precise answer for hyperstatic systems is difficult to obtain due to the non-linearity of the problem. However, as the above analysis indicates, the distribution of bending moments corresponding with the linear solution is affected by cracking only to a small extent. For this reason, in the case of hyperstatic systems it may be stated that high overestimation of deflection, when using the simplified approach, occurs also when the bending moment (obtained for linear problem) exceeds the cracking value but is still very close to it. This may only be treated as a general guideline – any design process must verify the true deflection via numerical integration of the curvatures along the beam.

4. Conclusions

The analysis performed indicates that in most typical design conditions (material, load, static scheme, geometry) the influence of non-uniformity of cracking has minor influence on both the deflection and distribution of the bending moments. Differences between the solution obtained by numerical integration of curvatures in non-uniformly cracked element and the solution assuming constant stiffness decrease along the element are not significant. In such situations, it is justified to use the simplified approach approximating deflection with formula (1.1) in which both the bending moment M_{Sd} and the α_k factor are the same as in the linear solution (disregarding the influence of cracking) while only bending stiffness B accounts for cracking according to the EC2 formula (2.3).

It must be noted that the above conclusions may be valid only in the case of typical designs – single span beams of average span length which are loaded uniformly with typical live load values and made of common materials. They were derived assuming that no qualitatively or quantitatively distinct response of the system occurs for intermediate values of parameters influencing the solution. As the problem is highly non-linear, such an assumption is – strictly speaking – not justified, yet it seems to be a probable supposition as the results obtained for combinations of "edge" (extreme) values do not differ one from another in a serious way. Any generalization of those statements for multiple-span beams or for two-way reinforced slabs is an extrapolation which requires numerical verification.

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