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### CORRELATION BETWEEN CRYSTAL AND MAGNETIC STRUCTURE OF THE POLYCRYSTALLINE AND NANOPARTICLE TBMNO<sub>3</sub> MANGANITE

## ZWIĄZEK MIĘDZY STRUKTURĄ KRYSTALICZNĄ I MAGNETYCZNĄ POLIKRYSTALICZNEJ I NANOROZMIAROWYCH PRÓBEK MANGANITU TBMNO<sub>3</sub>

#### Abstract

On the basis of neutron diffraction data the Mn–O bond lengths and Mn–O–Mn bond angles for the poly- and nanocrystalline TbMnO<sub>3</sub> samples are determined. All the samples crystallize in the orthorhombically distorted perovskite structure (space group *Pnma*) and exhibit antiferromagnetic ordering below 41 K. The Tb atoms and O<sub>1</sub> atoms are in (4)c site, Mn atoms – in 4(b) site and O<sub>2</sub> atoms – in 8(d) site. The Mn–O<sub>2</sub>–Mn bond angles for the polycrystalline and nanosize samples are similar, whereas the Mn–O<sub>1</sub>–Mn bond angles for the nanoparticle samples are larger. The temperature dependencies of the Mn–O bond lengths and the Mn–O–Mn bond angles, the Jahn-Teller distortion parameter (JT) and MnO<sub>6</sub> – octahedron distortion parameter (delta) for polycrystalline sample exhibit anomalies at  $T_y$  temperature for Mn sublattice.

Keywords: crystal structure, exchange interactions, nanoparticle, grain size, Mn-O bond lengths, Mn-O-Mn bond angles, the Jahn-Teller distortion parameter

#### Streszczenie

Na podstawie wyników neutronowej dyfrakcji wyznaczono długości wiązań Mn–O oraz kąty wiązania Mn–O–Mn dla polikrystalicznej oraz nanorozmiarowych próbek manganitu TbMnO<sub>2</sub>, Wszystkie próbki krystalizują w rombowo zdystorsowanej strukturze perowskitu (grupa przestrzenna *Pnma*) i wykazują antyferromagnetyczne uporządkowanie ponizej 41 K. Atomy Tb i tlenu O<sub>1</sub> zajmują pozycję 4(c), atomy Mn po-zycję 4(b), a atomy tlenu O<sub>2</sub> pozycję 4(d). Wartości kątów wiązania Mn–O<sub>2</sub>–Mn są zbliżone dla polikrystalicznej i nanorozmiarowych próbek związku TbMnO<sub>3</sub>, podczas gdy wartości kątów wiązania Mn–O<sub>2</sub>–Mn są wyższe dla próbek nanorozmiarowych. Temperaturowe zależności: długości wiązań Mn–O, kątów wiązania Mn–O–Mn, parametru dystorsji Jahna-Tellera (JT) oraz parametru dystorsji oktaedru MnO<sub>6</sub> (delta) wykazują dla próbki polikrystalicznej anomalie w temperaturze Néela dla podsieci Mn.

Słowa kluczowe: struktura krystaliczna, oddziaływania wymiany, nanocząstki, rozmiar ziarna, długości wiązań Mn–O, kąty wiązania Mn– O–Mn, parametr dystorsji Jahna-Tellera

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#### 1. Introduction

The explanation of the complex magnetic interactions and correlation of the magnetic, structural and electron properties of the  $\text{REMnO}_3$  (RE are the rare – earth ions) manganites is of fundamental interest [1].

 $TbMnO_3$  has been attracting a lot of attention in recent years because of its strong coupling between ferroelectricity and magnetism [2].

The main motivation for performed studies was to obtain the data concerning the crystal structure and magnetic properties of the TbMnO<sub>3</sub> manganite as a function of the grain size. The model for interpretation of magnetic properties of the nanoparticle compounds is based on the ratio of ideal inner core and nonmagnetic surface, i.e., on the surface/volume ratio [3].

In this paper, we have discussed the influence of the internal structural parameters (Mn–O bond lengths and Mn–O–Mn bond angles) on the magnetic behaviour of the polycrystalline and two nanopowder TbMnO<sub>3</sub> samples. The structural distortion parameters, i.e. Jahn-Teller distortion (JT) and MnO<sub>6</sub> – octahedron distortion (delta) were found for all the samples.

#### 2. Experiment and results

The polycrystalline TbMnO<sub>3</sub> manganite was prepared by the solid-state reaction. The final sintering treatment was performed at 1150°C for 15 h. For preparation of the nanosize TbMnO<sub>3</sub> manganite the sol-gel method has been used. The two samples of the nanopowders were obtained after annealing at 800 and 850°C [4]. The crystal structure of the samples was obtained by X-ray powder diffraction at room temperature using the Philips PW-3710 X'PERT diffractometer with CuK<sub> $\alpha$ </sub> radiation. The obtained data were analysed with the Rietveld-type refinement soft-ware Fullprof program [5].

The X-ray diffraction data indicate that all the samples studied have orthorhombic crystal structure (space group *Pnma*). In this structure the Tb and O<sub>1</sub> atoms occupy 4(c) site: (x, y, 1/4), O<sub>2</sub> atoms are in 8(d) site: (x, y, z) and Mn atoms are in 4(b) site: (1/2, 0, 0) (Fig.1).

The obtained data indicate that the lattice constants and positional parameters  $x_i$ ,  $y_i$ ,  $z_i$  slightly change with changing grain size [6]. The data for the nano-samples indicate that the *a*-constant is smaller and the *b* and *c* are larger than ones for the polycrystalline sample. All parameters have minimum at T = 30 K and quickly increase with increasing temperature.



Fig. 1. The orthorhombic crystal unit cell

The grain sizes of nano-samples (800 and 850°C) were determined using the Scherrer relation  $d = \lambda/B\cos\theta_{B}$ , where d is the grain size,  $\lambda$  is the X-rays wavelength,  $\theta_{B}$  is the corresponding angle of the Bragg diffraction and B is the difference between half-widths of the Bragg reflex of the nanopowder and the standard sample [7]. The grains sizes were calculated using the experimental X-ray data and the following relation:  $B = \beta - \beta_{0}$ , where  $\beta$  is the half-widths of the Bragg reflex of the investigated sample and  $\beta_{0}$  the similar value for the standard sample of Si powder with the grain size of 10 µm. The exact method of determination of grain size is described in [8]. The average grain size values determined there are: 60 nm and 45 nm for 850-nano and 800-nano samples, respectively.

In the next step, the grain sizes and strain effects were determined based on the Williamson-Hall method [9]. In this method, the broadening of Bragg peak is a sum of grain size broadening  $\beta_d = K\lambda/d\cos\Theta$  and strain broadening  $\beta_s = \epsilon tg\Theta$ , where the shape factor K is close to 1, *d* is a value of grain size and  $\epsilon$  is a strain constant.

Thus, the resulting total broadening:  $\beta_{\text{total}} = \beta_s + \beta_d = \epsilon \operatorname{tg}\Theta + K\lambda/d\cos\Theta$ . Multiplication of the above equation by  $\cos\Theta$  leads to

$$\beta_{\text{total}}\cos\Theta = \epsilon \sin\Theta + K\Theta/d.$$

Therefore, the grain size d can be determined from the intercept of line fitted with linear regression as applied to the  $\beta_{total} \cos \Theta$  versus  $\sin \Theta$  data.

The experimental  $\beta_{total}$  values have been determined from the relation:

$$\beta_{\text{total}} = [(\beta_{\Theta})^2_{\text{sample}} - (\beta_{\Theta})^2_{\text{Si}}]^{1/2}$$

where  $(\beta_{\Theta})_{\text{sample}}$  is a half – width of selected Bragg reflection of the investigated sample, while  $(\beta_{\Theta})_{\text{Si}}$  is a similar value found for the standard sample of Si powder.

The values of the grain size d are equal to 57 nm and 51 nm for 850-nano and 800--nano samples, respectively. Presented data indicate that the value of grain size increases with increasing annealing temperature.

The analysis presented in this paper based on the neutron diffraction powder data collected using the E2 and E6 diffractometers installed at the BERII reactor (Helmholtz--Zentrum Berlin) within the temperature range from 1.6 to 260 K. The data were processed using the program FullProf.

Neutron diffraction data [10] indicate that all the samples have orthorhombic crystal structure. Determined values of the lattice constants and atomic positions parameters are presented in Table I in [10]. Low temperature data indicate that the magnetic ordering of Mn and Tb sublattice for polycrystalline TbMnO<sub>3</sub> is sinusoidal modulated described by the propagation vector  $\mathbf{k} = (k_x, 0, 0)$ . The magnetic moments in Mn sublattice order below 41 K, while in Tb one they order below 9 K.

In the crystal unit cell (space group *Pnma*) the Mn<sup>3+</sup> and Tb<sup>3+</sup> sublattices can be described by four modes proposed by Bertaut [11]: one ferromagnetic ordering:  $\mathbf{F} = m_1 + m_2 + m_3 + m_4$ and three antiferromagnetic arrangements:  $\mathbf{A} = m_1 - m_2 - m_3 + m_4$ ,  $\mathbf{C} = m_1 + m_2 - m_3 - m_4$ and  $\mathbf{G} = m_1 - m_2 + m_3 - m_4$ .

Below 41 K, neutron diffraction patterns for the polycrystalline sample exhibit additional magnetic peaks connected with the antiferromagnetic modulated ordering with  $k_x = 0.28$  in Mn sublattice described by  $C_x - \text{mode}$  (see Fig. 1a in [6]).

The Mn magnetic moments, parallel to the *a*-axis, form a collinear incommensurate structure of  $C_x - \text{mode}$ . At T = 16 K a noncollinear magnetic structure described by  $C_x A_z - \text{mode}$  with the Mn moment in the *a*-*c* plane was observed (see Fig. 2).

The Tb sublattice exhibits the antiferromagnetic incommensurate ordering of the  $\mathbf{F}_{x}\mathbf{A}_{z}$  – type at T = 5 K. The Tb magnetic structure is described by propagation vector- $\mathbf{k} = (k_{x}, 0, 0)$  where  $k_{x}$  is equal to 0.423(1) (Fig. 2). At the same temperature, the Mn moments still form the  $\mathbf{C}_{x}\mathbf{A}_{z}$  structure described by propagation vector  $\mathbf{k} = (k_{x}, 0, 0)$  where  $k_{x}$  is equal to 0.282(1).

The refinement of the magnetic peaks intensities for the nano-800 and nano-850 samples below  $T_N$  shows that the Mn moments form a collinear incommensurate magnetic structure of  $C_x$  – type described by the propagation vector  $\mathbf{k} = (k_x, 0, 0)$  (see Fig. 3). The corresponding patterns for the nano-800 and nano-850 samples are presented in Figs. 1b and 1c in [6]. At 1.6 K, the additional peaks connected to the Tb moments ordering are visible. The Tb structure can be described by the  $\mathbf{A}_z$  – mode with propagation vector  $\mathbf{k} = (k_x, 0, 0)$  (see Fig. 3), while for the polycrystalline sample the  $\mathbf{F}_x\mathbf{A}_z$  – mode was evidenced.

The Mn magnetic moments values for nano-samples (at 1.6 K  $\mu$ (Mn) = 2.94(2)  $\mu_{\rm B}$  and 3.03(4)  $\mu_{\rm B}$  for nano-800 and nano-850, respectively) are smaller than for the polycrystalline sample (at 5 K  $\mu$ (Mn) = 4.06(2)  $\mu_{\rm B}$ ), whereas for the nano-samples the  $k_x$  components equal to 0.321(2) and 0.328(2) for nano-800 and nano-850, respectively, are larger than in the polycrystalline sample (0.282(1)).

Similar conclusions concern the parameters characterizing the ordering in Tb sublattice. At 1.6 K  $\mu$ (Tb) = 3.68(11)  $\mu_{B}$  and 4.43(7)  $\mu_{B}$  for nano-800 and nano-850, respectively.

For polycrystalline sample  $\mu$ (Tb) is equal to 6.55(4)  $\mu_{\rm B}$  at 5 K. The values of  $k_x$  component for Tb sublattice are larger for nano-samples (0.443(5) and 0.451(3) for nano-800 and nano--850, respectively.

The  $T_N$  Néel temperature connected with the Tb sublattice is lower for nano-samples (6.7 K) in comparison to polycrystalline sample (9 K).

Magnetic structures of the polycrystalline and nanoparticle TbMnO<sub>3</sub> compounds are presented in Figs. 2 and 3, respectively. These magnetic structures are incommensurate



Fig. 2. Sinusoidal magnetic ordering in Mn sublattice – violet ( $C_xA_z$  – mode,  $k_x = 0.282(1)$ ) and in Tb sublattice – black ( $F_yA_z$  – mode,  $k_x = 0.423(1)$ ) for polycrystalline TbMnO<sub>3</sub>



Fig. 3. Sinusoidal magnetic ordering in Mn sublattice – violet ( $C_x$  – mode,  $k_x = 0.326(4)$ ) and in Tb sublattice – black ( $A_z$  – mode,  $k_x = 0.443(5)$ ) for 800-nano TbMnO<sub>3</sub>

in comparison with the crystal one. The periods of modulation of the magnetic structure are equal to 3.54a (Mn sublattice) and 2.36a (Tb sublattice) for polycrystalline sample and 3.06a (Mn) and 2.25a (Tb) for nano-samples, respectively.

In this paper we have focused on the behaviour of the internal structural parameters in the polycrystalline and two nanoparticle samples (Mn–O bond lengths and Mn–O–Mn bond angles) as a function of temperature. In the orthorhombic unit cell there are the three crystallographically independent (Mn–O<sub>1</sub>(4c) =  $r_1$ , Mn–O<sub>2</sub>(8d)<sub>1</sub> =  $r_2$ , Mn–O<sub>2</sub>(8d)<sub>2</sub> =  $r_3$ ) bond lengths and the two (Mn–O<sub>1</sub>–Mn =  $\alpha$ , Mn–O<sub>2</sub>–Mn =  $\beta$ ) bond angles (Fig. 4). The temperature dependencies of the Mn–O bond lengths and Mn–O–Mn bond angles for the polycrystalline and two nanoparticle TbMnO<sub>3</sub> samples are presented in Fig. 5.



Fig. 4. The orthorhombic crystal unit cell with the marked Mn–O bond lengths  $(r_1, r_2, r_3)$  and Mn–O–Mn bond angles  $(\alpha, \beta)$  and the exchange integrals  $J_1, J_2$ 

![](_page_5_Figure_0.jpeg)

Fig. 5. Mn–O bond lengths  $(r_1, r_2, r_3)$  as a function of temperature for polycrystalline and 800 and 850-nano-samples of TbMnO<sub>3</sub>

The temperature dependencies of the  $r_1$  and  $r_2$  bond lengths show that  $r_1$  and  $r_2$  are smaller for the nanosize samples as compared to the polycrystalline sample (see Fig. 5). This suggests that in these samples there are a greater overlap of p and d orbitals.

We have observed an increase of the  $r_1$  and  $r_2$  bond lengths for the nanosize samples with approaching to the Néel temperature. For the polycrystalline sample above T = 50 K the stabilization of all three  $r_1$ ,  $r_2$  and  $r_3$  bond lengths is visible. The dependence of  $r_3(T)$  exhibits an inverse behaviour as compared to  $r_2(T)$  (see Fig. 5).

Fig. 6 presents a gradual increase of the  $\alpha$  bond angle vs temperature for the polycrystalline sample, whereas for the nanosize samples a decrease of  $\alpha$  till to the Néel temperature and an increase beyond  $T_N$  is observed. The  $\alpha$  bond angles are larger for the nanoparticle samples as compared to  $\alpha$  for the polycrystalline sample.

![](_page_6_Figure_3.jpeg)

Fig. 6. Mn–O–Mn bond angles (α, β) as a function of temperature for the polycrystalline and 800-nano and 850-nano TbMnO<sub>3</sub> samples

This suggests an increase of superexchange interactions along the *b*-axis. Values of  $\beta$  bond angle are similar for the nano- and poly-TbMnO<sub>3</sub> samples. For both types of samples an increase of  $\beta$  is observed till the Néel temperature. Beyond this temperature the  $\beta$  bond angle value substantially drops. Using the  $r_1$ ,  $r_2$  and  $r_3$  bond lengths the Jahn-Teller parameter [12] for the polycrystalline and nanosize samples has been determined according to the formula [13]:

$$JT = \sqrt{\frac{1}{3} \sum_{i=1}^{3} [(r_i) - \langle r \rangle]^2}$$

where  $r_i$  are the experimentally determined values of (Mn–O) interatomic lengths (see Fig. 4) and < r > is the average value of these lengths.

![](_page_7_Figure_3.jpeg)

Fig. 7. Temperature dependences of the Jahn-Teller parameter (JT) and the parameter delta for the polycrystalline and nanosize samples of TbMnO<sub>3</sub>

The parameter delta, which describes the distortion of  $MnO_6$  octahedron is calculated using the formula:

delta = 
$$\frac{1}{3} \sum_{i=1}^{3} \left[ \frac{r_i - \langle r \rangle}{\langle r \rangle} \right]^2$$

Temperature dependences of the Jahn-Teller parameter (JT) and the parameter delta for the polycrystalline and nanosize samples of TbMnO<sub>3</sub> are presented in Fig. 7.

The values of both the Jahn-Teller parameter and the delta parameter indicate the MnO<sub>6</sub> octahedron distortion. Distortion is much smaller for the nanocrystalline samples than for polycrystalline one. For polycrystalline sample the Jahn-Teller parameter has anomaly at Néel temperature.

#### 3. Discussion

The data presented in this paper indicate that the magnetic properties of the nanoparticle samples strongly depend on the grain size. This manifests itself in decrease of the value of both magnetic moments in the ordered state and magnetic ordering temperature with decreasing grain size.

The TbMnO<sub>3</sub> manganite exhibits a para- antiferromagnetic phase transition at 41 K, where the Mn<sup>3+</sup> ions develop a sinusoidal incommensurate ordering propagating along the a – direction of the unit cell, described by  $C_x A_z$  – mode. Magnetic order in the Mn sublattice is collinear of  $C_x$  – type in the temperature range of 21–41 K. For the investigated nano-samples a magnetic ordering in the Mn sublattice is described by collinear  $C_x$  – mode only.

Observed antiferromagnetic order in the Mn sublattice is result of the superexchange mechanism (cation-anion-cation) which exists in manganites. The superexchange interaction depends on the Mn–O–Mn bond angles ( $\alpha$ ,  $\beta$ ) and is joined with partial overlap of the *p* (O) and *d* (Mn) orbitals. The interactions between Mn moments are based on the exchange integrals discussed by Bertaut [14].

At temperature 1.6 K, the values of  $\alpha$  and  $\beta$  bond angles are equal to 142° and 146° for the polycrystalline sample while they are equal to 145° and 145.5° for the nanoparticle samples, respectively.

The obtained values of the Mn–O–Mn bond angles ( $\alpha$ ,  $\beta$ ) are smaller than 180°. This fact indicates the moderate ferro- or antiferromagnetic interaction between magnetic moments of Mn according to the Goodenough-Kanamori rules [15, 16].

Analysis of interactions in the orthorhombic manganites with magnetic structure described by the propagation vector  $\mathbf{k} = (k_x, 0, 0)$  gives the following dependence between  $k_x$  and exchange integrals:  $\cos(\pi k_x) \approx (2J_2 - J_1)$  [17], where  $J_1$  is the exchange integral in the basal a-c plane  $[t_{2g}(Mn) - 2p_{\pi}(O) - t_{2g}(Mn)]$  and  $J_2$  is the exchange integral along the *b*-axis  $[e_{\alpha}(Mn) - 2p_{\pi}(O) - e_{\alpha}(Mn)]$ .

The inelastic neutron scattering for the bulk TbMnO<sub>3</sub> yields the positive value of  $J_1 \approx 0.15(1)$  meV and negative one of  $J_2 \approx -0.31(2)$  meV [18]. This result confirms, that for the TbMnO<sub>3</sub> manganite the superexchange interaction between Mn–O<sub>2</sub>–Mn spins in the *a*-*c* plane (J<sub>1</sub>) is ferromagnetic, while the interaction Mn–O<sub>1</sub>–Mn along the *b*-axis

is antiferromagnetic  $(J_2)$  (see Fig. 4). An increase of the  $k_x$  component observed in the nanoparticle TbMnO<sub>3</sub> sample indicates the decrease of the exchange integrals in nano-samples.

The presented results suggest that the nanoparticle size plays an important role in the formation of magnetic properties. The influence of deformation of the  $MnO_6$ -octahedron on the magnetic structure of TbMnO<sub>3</sub> manganite is observed. The values of  $Mn-O_2-Mn$  bond angles in the polycrystalline and nanosize samples are similar and the temperature dependences exhibit anomalies at  $T_N$  temperature. The values of the  $Mn-O_1-Mn$  bond angles are larger for the nanoparticle samples.

For nano-samples the Jahn-Teller distortion parameter (JT) and  $MnO_6$ -octahedron distortion parameter (delta) are lowered in comparison to the polycrystalline sample.

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