TECHNICAL TRANSACTIONS | CZASOPISMO TECHNICZNE
MECHANICS | MECHANIKA
2-M/2016

EUGENE ALEXEEV, BORIS GOLOVUSHKIN,
ALEXANDR LABUTIN, ELENA EROFEEVA*

STUDY OF SYSTEM-WIDE AND STRUCTURAL PROPERTIES AND OPTIMAL CONTROL OF THE PRE-POLYAMIDATION TANK

ANALIZA SYSTEMOWA I STRUKTURALNA WŁAŚCIWOŚCI ORAZ OPTYMALNE STEROWANIE ZBIORNIKIEM DO PRE-POLIAMIDACJI

A b s t r a c t

The study on system-wide and structural properties of a prepolyamidation tank were performed by simulation. Control channels were selected. An algorithmical synthesis of a tank optimal control system was performed. A designed control system was simulated.

Key words: chemical tank, controllability, optimal control, control system

S t r e s z c z e n i e

Analizę systemową i strukturalną właściwości zbiornika do pre-poliamidacji przeprowadzono metoda symulacji. Wyznaczono kanały kontrolne, dokonano algorytmicznej syntezy systemu sterowania oraz zamodelowano projektowany system.

Słowa kluczowe: zbiornik, sterowalność, optymalne sterowanie, system sterowania

DOI:

* MSc. Eng. Eugene Alexeev; MSc. Eng. Boris Golovushkin, Prof. PhD. DSc. Eng. Alexandr Labutin, PhD.DSc. Eng. Elena Erofeeva, Department of Technical Cybernetics and Automation, Chemical Engineering and Cybernetics Faculty, Ivanovo State University of Chemistry and Technology.

1. Introduction

At present, the synthetic polymer polycaproamide attracts the attention of many researchers. Thanks to its properties, the polymer is widely used in some industries. Thread for industrial use, composite materials with special properties for medicine and food industry, polymer colour concentrates and thermal stabilisers can be made based on this polymer.

The main industrial method for producing polycaproamide is hydrolytic polymerisation of caprolactam in melt. With an approach to saving energy and resource at our university promising technology, for producing this polymer were developed. Extraction and energetically unfavourable stage regeneration lactam waters were changed by combined drying and removal of the residual monomer in an inert gas process.

In this work, the object is the pre-polyamidation tank in which stage of pre-polyamidation of polyamide-6 in solid phase is existing. Earlier mathematical model of this stage was derived, simulation was performed and the adequacy of the derived model was confirmed.

The aim of this work is to study system-wide and structural properties (connectivity, controllability and observability) of tank and to design an optimal controller by using mathematic and simulation methods.

2. Analysis of system-wide and structural properties

At the present time, researchers widely use simulation methods of technological processes of synthesising synthetic polymers for solving different optimisation and control tasks. It’s necessary to create an effective control system for performing the process under the economic or technical point of view’s optimal behaviours. The processes are studied as control object for solving this problem.

Analysis of connectivity

Dimensionless transfer coefficients were determined for studying the connectivity degree. Input and output variables were selected after analysing the technological process. Input variables are consumptions of pellets of polymer, nitrogen and heat transfer agent. Output variables are concentrations of caprolactam and water in pellets of polymer and the tank temperature.

Matrix of dimensionless coefficients K has been obtained as follows:

$$K = \begin{pmatrix} 0.388 & 0 & -0.0975 \\ 0.2325 & 0.001875 & 0.041 \\ -0.0855 & 0 & 0.0855 \end{pmatrix} \tag{1}$$

The method using the Bristol matrix which characterises connectivity degree in static, was used for evaluating the degree of system connectivity. Each element of the matrix is a result of the division of two derivatives: the first one is a derivative of steady state open loop system output under control, the second one is a derivative of steady state closed loop system output under control. The Bristol is as follows:

$$\lambda = K \cdot (K^T)^{-1} \tag{2}$$

where K is a transfer coefficients matrix.

In this work, we have obtained matrix λ as:

$$\lambda = \begin{pmatrix} 0.388 & 0 & -0.0975 \\ 0.2325 & 0.001875 & 0.041 \\ -0.0855 & 0 & 0.0855 \end{pmatrix} \cdot \left(\begin{pmatrix} 0.388 & 0 & -0.0975 \\ 0.2325 & 0.001875 & 0.041 \\ -0.0855 & 0 & 0.0855 \end{pmatrix}^T \right)^{-1}$$

$$\lambda = \begin{pmatrix} 0.953 & -114.06 & -0.187 \\ 0.961 & -149.706 & 1.441 \\ 0.041 & -27.892 & 1.041 \end{pmatrix} \tag{3}$$

After analysing λ, we established that our object is connected. Hence, it's necessary to compensate cross links for controlling the tank.

Analysis of controllability

Before we synthesise the control algorithm, it is necessary to study such properties of object as the stability of free movement, controllability and observability. An analysis of the results of these studies allows us to conclude the ability to control the object. Presentation of the dynamic object in the state-space model is used for this task.

An evaluations of the stability of free movement, controllability and observability are performed in the neighbourhood of operating point. Strong conditions of controllability and stability have been found only for some classes of nonlinear objects. Availability or non-availability of these properties can be established by using linearisation of nonlinear equations describing the object [1]. If the linearised system is controllable in the neighbor-hood of some steady state, then we may assume that the original nonlinear system is controllable. Consumptions of pellets of polymer, nitrogen and heat transfer agent were previously chosen as control actions. The control task is to fully remove the monomer and water from pellets of polymer and to support the tank temperature at the desired level by changing the control actions.

The original nonlinear model of object with distributed parameters was presented by using its discrete analogue (cell's model). Linearisation of the object in the neighbourhood of the working point was performed in Simulink app «Linear Analysis Tool», MATLAB. Matrix of state **A** (dimension 21×21), matrix of control **B** (dimension 21×3) and matrix of observe **C** (dimension 3×21) were derived.

A study on the stability of the unperturbed system in the state-space model was performed. If all real parts of all eigenvalues α_i of matrix **A** are negative, then the system is stable:

$$\det(\alpha_i \mathbf{I} - \mathbf{A}) = 0 \tag{4}$$

where **I** is identity matrix.

Results show that our object is stable in the neighbourhood of the working point. This way, it has a property of stabilizability [2].

Analyses of the controllability were performed by using the controllability matrix $\mathbf{N_c}$ [1-3]:

$$\mathbf{N_c} = \left[B : AB : A^2B : \ldots : A^{n-1}B\right] \tag{5}$$

where n is an order of system.

If rank of $\mathbf{N_c}$ is equal to n, then the linear system is fully controllable. If rank of $\mathbf{N_c}$ is smaller than n and bigger than 0, then the system is partly controllable. If rank of $\mathbf{N_c}$ is equal to 0, then the system is non-controllable. The controllability matrix and its rank were derived and calculated by using inbuilt MATLAB functions. The rank of $\mathbf{N_c}$ is 12. It is smaller than the order of the system that equals 21, hence our pre-polyamidation tank is not fully controllable. This way, there are such initial conditions in the phase state when the object cannot be transferred to the specified final condition. Also it was shown that our object is fully controllable in the space of outputs.

Analysis of observability

The observation equation is $\mathbf{y} = \mathbf{C} \cdot x$, where $\mathbf{y} = (y_1, \ldots, y_m)^T$ is a vector of observation variables, $\mathbf{x} = (x_1, \ldots, x_n)^T$ is vector of the output variables, $\mathbf{C}$ is an observation matrix (dimension $m \times n$). The observability matrix is written as follows [1-3]:

$$N_o = \left[C^T : A^T C^T : (A^T)^2 C^T : \ldots : (A^T)^{n-1} C^T\right] \tag{6}$$

where n is order of system.

The observability matrix and its rank was derived and calculated by using inbuilt MATLAB functions. The rank of N_o is 15. It shows that our object is not fully observable.

The task of controlling the object was solved in next step.

3. Designing the optimal controller

Control of tubular chemical reactors is probably the most difficult among all of reactor systems. It is caused by some technical and design factors.

Controllers using methods of optimal control theory are widely used on tubular reactors. At present, optimal control is one of the perspective direction of automatic control theory development. There are many works about the application of optimal control in some technical and technological objects.

Literature presents a wide range of criteria for the estimation of control quality [1-4]. Among them, criteria of maximum speed and minimum of control error dispersion are widely used. In this work we will use both criteria in dynamics.

Criterion of minimum error dispersion

There is a closed-loop control system (Fig. 1) where stationary centered random signal is used as disturbance. The interference signal is formed by a filter with transfer function $W_f(s)$ from white noise $\upsilon(t)$.

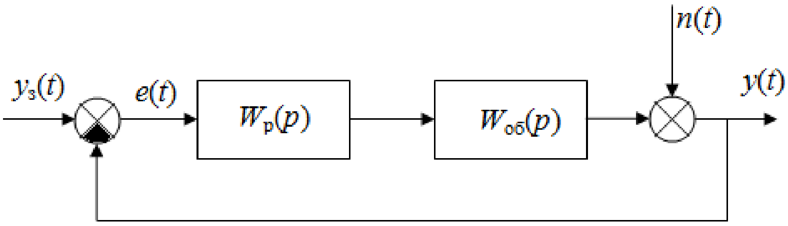

Fig. 1. Closed-loop control system

Dispersion of error can be found as:

$$De = \int_0^\infty \left[k_f(\tau) - k_1(\tau)\right]^2 d\tau \tag{7}$$

where $k_f(\tau)$ is a pulsed transition function of the filter; $k_1(\tau)$ is a pulsed transition function of the system consisting of two dynamic elements:

$$W_1(s) = W_f(s) \cdot F_y(s) \tag{8}$$

where $F_y(s)$ is a transfer function of the closed-loop system on the control channel.

It is necessary to select transfer functions $F_y(s)$ and $W_1(s)$ so that we allow to minimise the expression (4). It is difficult because almost always there is a transport delay in the object. In that cases, the minimum can be achieved only when $k_f(\tau) = k_1(\tau)$ and $\tau > \tau_d$.

Having found the optimal transition function $k_1^{opt}(\tau)$ and $W_1^{opt}(s)$, we can derive the optimal transfer function of the closed-loop system $F_y^{opt}(s)$ from expression (5). The transfer function of the optimal controller can be written as [4]:

$$W_p^{opt}(s) = \frac{F_y^{opt}(s)}{1 - F_y^{opt}(s)} \cdot \frac{1}{W_{o\delta}(s)} \tag{9}$$

It is known that for the system having stationary random signal as a disturbance, the transfer function of the optimal controller is:

$$W_p^{opt}(s) = \frac{e^{-\alpha\tau_d} \cdot e^{-s\tau_d}}{1 - e^{-\alpha\tau_d} \cdot e^{-s\tau_d}} \cdot \frac{1}{W_{o\delta}(s)} \tag{10}$$

where τ_d is the transport delay.

Criterion of the maximum speed

There are many ways to solve the task of seeking optimal control under the criterion of maximum speed. The Hamilton variational calculus, phase space method, Viener optimal control theory are widely used. There are some disadvantages of using these methods. One can distinguish such defects as difficulty of solving equations and their cumbersome, high probability of error, difficulty of expression for computing control actions.

The “Trajectories docking” method [5], based on the Feldbaum theorem, is the simplest method for solving the task of maximum speed. Let’s consider it on the object with the transfer function:

$$W_{ob}(s) = \frac{k}{(T_1 \cdot s + 1) \cdot (T_2 \cdot s + 1)} \tag{11}$$

The general solution of the corresponding differential equation is:

$$y(\tau) = c_0 + c_1 \cdot e^{\alpha_1 \tau} + c_2 \cdot e^{\alpha_2 \tau} \tag{12}$$

where α_1 and α_2 are roots of general solution of the homogeneous equation; c_0 is derived from seeking particular solution; c_1 and c_2 are derived from solving the Koshi task.

The entire time interval of changing the control variable and control action is divided into three sectors. The first sector characterises the beginning of the transient process, time interval $[0; \tau_r)$. The control action has a maximum value. The time interval of the second sector is $[\tau_r; \tau_f)$. This sector describes movement of the object to steady state. The control action equals 0. Since $t = \tau_f$ the object is at steady state or finish movement to steady state. The control action at time interval $[\tau_f; \infty)$ has a nominal value.

This way, the optimal control task is to find switching times τ_r and τ_f between maximal, minimal and nominal values of the control action. The system consisting of six nonlinear equations is solved for this. In general, this system may be written as:

$$\begin{cases} c_0 + c_1 \cdot e^{\alpha_1 \tau_0} + c_2 \cdot e^{\alpha_2 \tau_0} = 0, \\ \alpha_1 \cdot c_1 \cdot e^{\alpha_1 \tau_0} + \alpha_2 \cdot c_2 \cdot e^{\alpha_2 \tau_0} = 0, \\ c_{00} + c_{11} \cdot e^{\alpha_1 \tau_f} + c_{22} \cdot e^{\alpha_2 \tau_f} = y_d, \\ \alpha_1 \cdot c_{11} \cdot e^{\alpha_1 \tau_f} + \alpha_2 \cdot c_{22} \cdot e^{\alpha_2 \tau_f} = 0, \\ c_0 + c_1 \cdot e^{\alpha_1 \tau_r} + c_2 \cdot e^{\alpha_2 \tau_r} = c_{00} + c_{11} \cdot e^{\alpha_1 \tau_f} + c_{22} \cdot e^{\alpha_2 \tau_f}, \\ \alpha_1 \cdot c_1 \cdot e^{\alpha_1 \tau_r} + \alpha_2 \cdot c_2 \cdot e^{\alpha_2 \tau_r} = \alpha_1 \cdot c_{11} \cdot e^{\alpha_1 \tau_f} + \alpha_2 \cdot c_{22} \cdot e^{\alpha_2 \tau_f} \end{cases} \tag{13}$$

where y_d is the desired value of the control variable; τ_r and τ_f are switching times; τ_0 is an initial time.

The first two equations of (13) show system condition at an initial time. The next two equations of (13) characterise the system at the steady state. The final equations describe the process of “trajectories docking”. So, we may write system (13) as:

$$\begin{cases} ku_m + c_1 + c_2 = 0, \\ \alpha_1 c_1 + \alpha_2 c_2 = 0, \\ c_{11} e^{\alpha_1 \tau_f} + c_{22} e^{\alpha_2 \tau_f} = y_d, \\ \alpha_1 c_{11} e^{\alpha_1 \tau_f} + \alpha_2 c_{22} e^{\alpha_2 \tau_f} = 0, \\ ku_m + c_1 e^{\alpha_1 \tau_r} + c_2 e^{\alpha_2 \tau_r} = c_{11} e^{\alpha_1 \tau_f} + c_{22} e^{\alpha_2 \tau_f}, \\ \alpha_1 c_1 e^{\alpha_1 \tau_r} + \alpha_2 c_2 e^{\alpha_2 \tau_r} = \alpha_1 c_{11} e^{\alpha_1 \tau_f} + \alpha_2 c_{22} e^{\alpha_2 \tau_f} \end{cases} \tag{14}$$

where k is a gain; u_m is a maximal value of the control action.

System (14) can be easily solved by using the numerical method. It should be noted that a description of the method can be used for designing an optimal controller providing the fastest possible shutdown of the process.

Combined system

The control system consisting of two optimal controllers for controlling the tank temperature was designed and presented in figure 2.As we can see, this system has a combined structure.

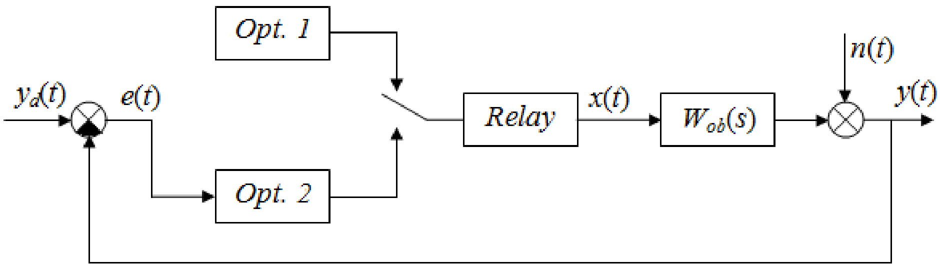

Fig. 2. Combined system for controlling temperature of pellets of polymer

Block *Relay* switches between controllers on the following condition:

$$x(\tau) = \begin{cases} Opt.1, \tau \le \tau_f, \\ Opt.2, \tau > \tau_f \end{cases}$$

For channel “consumption of heat transfer agent – temperature of pellets” transfer function without transport delay is written as follows:

$$W_{ob}(s) = \frac{47502059}{(1200 \cdot s + 1) \cdot (15 \cdot s + 1)} \tag{15}$$

Having solved system (14) with our own parameters, we have found switching times τ_r and τ_f, to equal 703 *s* and 715 *s* respectively.

The transfer function of *Opt*. 2 has the following form (16):

$$W_p^{opt}(s) = \frac{e^{-\alpha\tau_d} \cdot e^{-s\tau_d}}{1 - e^{-\alpha\tau_d} \cdot e^{-s\tau_d}} \cdot \frac{1}{k} \cdot (T_1 \cdot s + 1) \cdot (T_2 \cdot s + 1) \tag{16}$$

Having approximated the autocorrelation function of the disturbance signal formed by the filter from the white noise, we derived parameter α. The filter transfer function was derived.

Optimal controllers for channels "consumption of pellets of polymer – concentration of caprolactam in polymer" and "consumption of nitrogen – concentration of water in polymer" were derived analogically.

4. Conclusion

The carried out evaluation of system-wide and structural properties of pre-polyamidation tank allowed us to choose the control actions and measure the state variables. The problem of structural, algorithmic and parametric synthesis of the control system minimises energy consumption for performing the process synthesis of polyamide-6 was solved.

References

[1] Pupkov K. A., Egupov N. D., *Teoria optimizacii sistem abtomaticeskogo upravlenia*, Izdatelstvo MGTU im. N. E. Baumana 2004.

[2] Gubin G. K., Grebe S. F., Salgado M. E., *Proektirovanie system upravlenia*, BINOM Laboratoria znanij 2004.

[3] Rej U., *Metody upravlenia tehnologiceskimi processami*, Per s angl., Mir 1983.

[4] Besekerskij W. A., Popov E. P., *Teoria system abtomaticeskogo upravlenia*, Spd., Professia 2003.

[5] Golovushkin B., Volynskii V. U., Zaitzev V. A., *Optimal management of the process of convective drying of thin fabric materials*, Shenyang Institute of Chemistry technology, China 2004.