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METHOD OF EMF TOTAL HARMONIC DISTORTION CALCULATION OF THE SALIENT POLE SYNCHRONOUS GENERATOR (using 2D simulation package)

METODA OBLICZENIA WSPÓŁCZYNNIKA ZAWARTOŚCI HARMONICZNYCH W SILE ELEKTROMOTORYCZNEJ DLA WYDATNOBIEGUNOWEGO GENERATORA SYNCHRONICZNEGO (z użyciem pakietu symulacyjnego 2D)

Abstract

This paper presents the method of total harmonic distortion factor calculation, which could be applied for the salient pole synchronous generator with electromagnetic or permanent magnet excitation. The method takes into account the saturation of the magnetic circuit and actual winding scheme (including winding with fractional value of slots per pole and phase Q). The calculations are based on 2D FEM simulation that produces the radial (normal to stator surface) component of the magnetic flux density in the air gap. Total harmonic distortion factor is determined as a function of the shape of the pole shoe. The analysis is performed at a constant value of equivalent air gap. The task is to find the shape of the pole shoe that guarantees the minimal value of the distortion factor K_{DIST}.

Keywords: voltage harmonics, salient pole synchronous generator, total harmonic distortion

Streszczenie

W artykule przedstawiono metodę obliczania współczynnika zawartości harmonicznych, która może być stosowana dla wydatnobiegunowego generatora synchronicznego ze wzbudzeniem elektromagnetycznym lub z magnesami trwałymi. Metoda uwzględnia nasycenie obwodu magnetycznego i rzeczywisty schemat uzwojenia (łącznie z uzwojeniem o ułamkowej wartości liczby żłobków na biegun i fazę Q). Obliczenia są oparte na symulacji MES 2D, przy założeniu promieniowego przebiegu (normalnego do powierzchni stojana) składowej indukcji magnetycznej w szczelinie powietrznej. Współczynnik zawartości harmonicznych jest określony jako funkcja kształtu nabiegunnika. Analizę prowadzi się przy stałej wartości zastępczej szczeliny powietrznej. Celem jest znalezienie kształtu nabiegunnika, gwarantującego minimalną wartość współczynnika odkształcenia K_{DIST}.

Słowa kluczowe: harmoniczne napięcia, wydatnobiegunowy generator synchroniczny, współczynnik zawartości harmonicznych

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1. Introduction

Total harmonic distortion (THD) factor K_{DIST} is the measure of the line-to-line voltage wave deviation from the sinusoidal form in no-load state. The THD is limited (in accordance with GOST and IEC [1]) for generators with rated power above 100 kVA: $K'_{DIST} \leq 5\%$, for generators with rated power from 10 kVA to 100 kVA: $K'_{DIST} \leq 10\%$. This includes generators of various types: hydro generators operating in power lines, large low-speed diesel generators (10–50 MW, 100–250 rpm), working both independently and in power grid, high-speed generators with permanent magnet excitation driven by gas turbines and others.

The restriction of the THD is related to the fact that the higher harmonics in the electromotive force (EMF) generator curve cause:

– Noise in communication lines, located near power network,
– Resonance and overvoltage in power transmission lines [2],
– Additional losses in the windings and stator core of the generators,
– Vibration of the windings and the stator core; higher noise level.

Many factors affect THD, including the form of mutual magnetic flux in the air gap, the number Q of slots per pole and phase of the stator winding. To create high-speed generators and generators with very low speed, it is often necessary to design the stator winding with the number of Q, close to unity, in some cases, for $Q < 1$. This requires a thorough analysis of the higher harmonics of the EMF [3– 5]. Investigation of the distortion factor for non-salient generators is discussed in [6].

The distortion factor K_{DIST} estimation is included in the procedure of electrical generator testing in practice [1]. The line-to-line EMF of generator in no-load state is recorded in time. If the winding scheme is symmetrical, the EMF value is measured between any two line terminals; for example, between phase A and phase B terminals. The Fourier transform decomposes EMF *(time)* in a series of frequencies, which are used to calculate the total harmonic distortion factor K_{DIST} by the methods described in GOST and IEC [1].

To determine the distortion factor K_{DIST}, it is required to:

– Calculate magnetic flux (radial component) distribution in the air gap and the amplitudes of its harmonics,
– Calculate frequencies and amplitudes of the harmonics in the generator output EMF waveform.

The method presented solves both these problems using 2D FEM simulation package.

2. Calculation of the magnetic flux (radial component) distribution in the air gap and the amplitudes of its harmonics

The waveform of magnetic flux in the air gap at no-load state depends on the shape of the pole shoe (its width b_p and gap under the pole), the shape of the stator tooth zone and the saturation of the generator magnetic circuit.

2.1. Shape of the pole shoe account

The pole shoe has a circular shape. The center of the circle lays on the pole axis of the symmetry. The radius of the circle is calculated from the relation [2]:

$$R_p = D_{IN}/[2 + 8 \cdot D_{IN} \cdot (\delta_{MAX} - \delta_{MIN})/b_P^2] \tag{1}$$

where:

- D_{IN} – stator inner diameter,
- δ_{MIN}, δ_{MAX} – the minimum and maximum value of the air gap under the pole,
- b_P – pole shoe width.

The circle segment to the pole pitch τ ratio is $\alpha = b_P\ \tau$, where τ is the pole pitch. In power generators, the value of α is usually [2]: $0.55 \le \alpha \le 0.8$.

It is convenient to define new variables: the air gap ratio is $\Lambda_{GAP} = \delta_{MAX}/\delta_{MIN}$ and the value of equivalent (average) air gap δ_{EQ} under the pole [2]:

$$\delta_{EQ} = \delta_{MIN}\ [1 + (\Lambda_{GAP} - 1)/3] \tag{2}$$

The ratio Λ_{GAP} is usually lays within [2]: $1.2 \le \Lambda_{GAP} \le 2$.

Taking into account (2) the relation (1) becomes:

$$R_p = D_I/\{2 + 8 \cdot D_{IN} \cdot \delta_{EQ} \cdot (\Lambda_{GAP} - 1)/[1 + 0.333 \cdot (\Lambda_{GAP} - 1)]\ b_P^2\} \tag{3}$$

The equivalent air gap δ_{EQ} and the fundamental harmonic of the radial component of the magnetic flux provides the data for the rotor magneto-motive force (MMF) calculation in no-load state [2].

If the stator inner diameter D_{IN} and the equivalent air gap δ_{EQ} are considered to be pre-defined parameters, then the pole shoe curvature radius R_P is a function of the air gap ratio Λ_{GAP} and the pole shoe width b_P : $R_P = f(b_P, \Lambda_{GAP})$.

Therefore, the problem is to find the ratio Λ_{GAP}, which provides near sinusoidal magnetic flux distribution along the pole pitch. This problem is discussed in [7], but the saturation of the magnetic circuit is estimated approximately there, and the stator teething is represented only by the Carter factor [8, 9]. In fact, Carter factor does not reveal the teeth harmonics' influence on the distortion factor K_{DIST}. These harmonics are determined by the value Q of slots per pole and phase of the stator winding.

2.2. Stator tooth zone geometry. Generator magnetic circuit saturation level

It is a common solution for the stator tooth zone that the ratio of the slot width b_{SL} to the tooth pitch t_{ST} is: $b_{SL}/t_{ST} \approx 0.5$ (for rectangular slots). There is guidance [2] on the typical value of the magnetic flux density in stator elements. For example, for electrical steels used for powerful salient-pole machines, the magnetic flux density in the tooth usually does not exceed $B_Z \le 1.8$ T (measured at 65% of the tooth height, counting from the bottom of the slot).

2.3. Method of the magnetic flux linkage calculation at no-load state

Initial data for magnetic flux calculation are:

– Stator/rotor cross-section dimensions in a cylindrical coordinate system:

$$G = f(\rho, \theta),$$

where:

ρ – radius-vector,

θ – central angle.

– Rotor MMF at no-load state of the generator or rotor permanent magnet parameters.

The problem is to find (optimize) geometric dimensions $G = f(\rho, \theta)$, including the pole shoe radius $R_P = f(\Lambda_{GAP}, b_P)$ so that the factor K_{DIST} value is minimal (or less than specified limit).

This optimization problem is solved numerically. 2D FEM simulation software [10] is used to calculate magnetic field in generator cross section (Fig. 1). The software solves Maxwell's equations in the differential form [11] operating with vector magnetic potential **A** (**B** = curl **A**) [12].

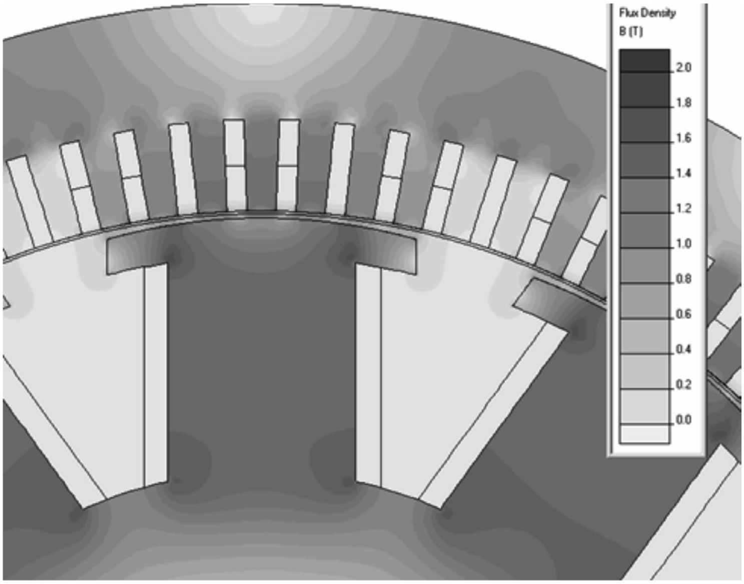

Fig. 1. Magnetic flux density distribution in the cross-section of the 2D model of the synchronous salient-pole generator (see Appendix 2) in no-load state. $\Lambda_{GAP} = 1.6$, $b_P/\tau = 0.70$

2.4. Harmonics spectrum of the magnetic flux (radial component) in the air gap

The magnetic flux radial (normal) component is used to calculate the stator winding EMF at no-load state. The flux is measured along the circular arc with radius $\rho = D_{IN} - \delta_{EQ}$. The results post-processing (from calculated magnetic field distribution to EMF value) include several stages:

- Extracting the magnetic flux density $b(\theta)$ in the air gap along the circular arc with radius $\rho = D_{IN} - \delta_{EQ}$ (fully automated in simulation package [10]) and θ is the angular coordinate.
- Calculation of the radial component the magnetic flux density $b_R(\theta)$ along the same circular arc (see Fig. 2).
- Decomposition of the $b_R(\theta)$ in the harmonic series.

For windings with integer Q (slots per pole and phase), the $b_R(\theta)$ is periodic in interval $T_{INT} = \pi \cdot (D_{IN} - 2{\cdot}\delta_{EQ})/p$. For windings with fractional number Q, the periodic interval should include the full circle length [13–18]:

$$T_{FR} = \pi{\cdot}(D_{IN} - 2{\cdot}\delta_{EQ}) \tag{4}$$

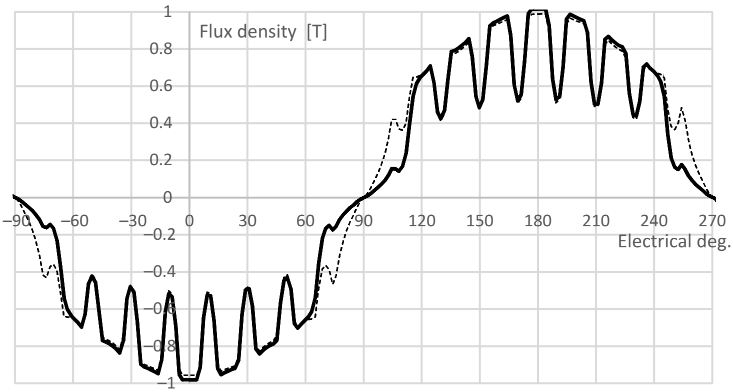

Fig. 2. Flux density distribution in the air gap of the synchronous salient-pole generator (see Appendix 2) in no-load state. $\Lambda_{GAP} = 1.6$. Solid line corresponds to the $b_p/\tau = 0.70$ case, dashed line corresponds to the $b_p/\tau = 0.80$ case

In some types of the machines (especially for slow-speed electrical machines with a large number of pole pairs), the winding scheme could be constructed as unsymmetrical on purpose: to reduce the length of the balance connectors between the phase zones [2]. In such stator windings (even with integer Q), the output EMF is not balanced between phase zones [2].

It is recommended to use the full circle length as a periodic interval T_{FR} for both types of the winding schemes: with fractional number Q (especially for $Q < 1$) and for integer Q. This allows to find not only the harmonics of the output EMF, but also its phase unbalance.

The result of the decomposition of the magnetic flux density radial component $b_R(\theta)$ in harmonics series in the interval T_{FR} can be represented as a sum of N of harmonics:

$$b_R(\theta) = \Sigma\, \underline{B}_N \cdot \exp(j \cdot \Psi_N) \tag{5}$$

where:

$\underline{B}_N = |B_N| {\cdot} \exp(j{\cdot}\Theta{\cdot}N)$ is a complex amplitude (phasor) of the Nth-harmonic,

Ψ_N – phase angle of the Nth-harmonic.

These values are stored in Table 1 and used for the harmonics calculation of the stator phase winding output EMF. The decomposition in harmonics is an automated simulation package [10] and is not discussed in this paper.

Table 1

Harmonics of the magnetic flux density radial component: complex amplitudes (phasor) $\underline{\boldsymbol{B}}_N$ and phase angles $\boldsymbol{\Psi}_N$. The periodic interval is chosen according to (4)

$\underline{\boldsymbol{B}}_N$	$\underline{B}_1$	…	$\underline{B}_R$	$\underline{B}_S$	$\underline{B}_U$	…	$\underline{B}_W$	$\underline{B}_K$	…	$\underline{B}_M$
$\boldsymbol{\Psi}_N$	Ψ_1	…	Ψ_R	Ψ_S	Ψ_U	…	Ψ_W	Ψ_K	…	Ψ_M

Indices of 1… *R*, *S*, *U*... *W*, *K*... *M* in Table 1 denote the harmonics numbers. Following groups of harmonics could be distinguished:
- from 1 to *R* are harmonics of "low" order,
- $S = p$ is fundamental harmonic,
- from *U* to *W* are "high" harmonics (with order below than tooth harmonics order),
- from *K* to *M* are "high" harmonics (with order higher than tooth harmonics order).

3. Harmonics of the generator output EMF

The generator stator winding EMF calculation based on the results of harmonics calculation of the magnetic flux radial component (Table 1) is the second stage of the distortion factor K_{DIST} estimation.

First, the EMF harmonics related to the stator teething are calculated. Then, the equations are derived for EMF harmonics calculation of any particular order *N*, and finally, the distortion factor is evaluated.

3.1. EMF "tooth" harmonics

3.1.1. EMF "tooth" harmonics of the winding with integer value of *Q* (slots per pole and phase)

Typically, salient-pole generators with torque value below 20 tonne-force·meter (for example, a series of high-speed diesel generators with rated power of 0.5–10 MW and speed of 500–1500 rpm) are usually constructed with windings with integer value of *Q* [2, 7–11].

EMF tooth harmonics' order that has maximum amplitude is calculated from the relation [13] for windings with an integer value of *Q* as:

$$N_1 = 2 \cdot m_{\text{PH}} \cdot Q - 1, \quad N_2 = 2 \cdot m_{\text{PH}} \cdot Q + 1 \tag{6}$$

where:

m_{PH} – is the number of phases of the generator.

3.1.2. EMF harmonics of the winding with fractional value of *Q*

Typically, powerful salient-pole generators (such as hydro and low speed diesel generators) with torque value above 20 tonne-force·meter (tf·m) are constructed with windings with fractional value of *Q*.

Such windings feature higher tooth harmonics order (and therefore lower amplitude [8, [13). EMF tooth harmonics' order that has maximum amplitude is calculated from the relation [16] for windings with the fractional value of Q as:

$$N_1 = 2 \cdot m_{PH} \cdot Q \cdot F - 1, \quad N_2 = 2 \cdot m_{PH} \cdot Q \cdot F + 1 \tag{7}$$

where:

$F > 1$ – the smallest integer that makes the term $(2 \cdot m_{PH} \cdot Q \cdot F)$ an even number.

Example

For $m_{PH} = 3$, $Q = 2$, according to (2) the tooth harmonics orders are: $N_1 = 11$, $N_2 = 13$.

For $m_{PH} = 3$, $Q = 2.5$, according to (4) at $F = 2$, the tooth harmonics orders are: $N_1 = 29$, $N_2 = 31$.

Thus, the utilization of the winding with a fractional value $Q = 2.5$ (instead of $Q = 2$) increases the tooth harmonics order/ frequency (N_1, N_2) more than two times. Respectively, with an increase of the harmonics order, the harmonics amplitude decreases. Unfortunately, the windings with fractional Q not only reduce the higher harmonics, but also at the same time, induce the lower harmonics in the output EMF. If a generator operates in load state, the lower harmonics cause additional vibration of stator core and housing. In some cases, the vibration force exceeds the permissible level, despite the fact that the EMF harmonics amplitudes are significantly less than the fundamental harmonic amplitude [17]. To reduce the vibration level, it is usually necessary to increase the size and weight of the housing (this way to increase its rigidity). That solution is often implemented in average power generators (20 tf·m $< M <$ 200 tf·m, where M is the torque on the shaft).

3.1.3. Ways of reducing the stator winding EMF tooth harmonics amplitude and the distortion factor K_{DIST}

There are several ways to reduce the EMF harmonics amplitude and the distortion factor K_{DIST} [8, 9, 15]. These solutions could be applied for the windings with integer Q. For the windings with fractional Q, it is recommended to try to adjust the Q value.

Generators with outer stator diameter of 990 mm or less [2] are often designed with the stator slot skewing. The stator slots skewing is difficult to construct for the generators of larger size, the rotor poles skewing [2, 7] in the axial direction or rotor poles shift in tangential direction is then used. You can find the basic equations of the EMF tooth harmonics calculation in [15], including the case of the stator slot skewing by the tooth pitch value t_{ST}. The effects of possible technological deviations of the skewing value on the K_{DIST} were studied. Specific cases $0.5 \cdot t_{ST}$, $0.75 \cdot t_{ST}$, $1.1 \cdot t_{ST}$ were analyzed (see Table 3) for the generator featuring winding with integer Q. Generator data are listed in Appendix 2.

In case Q is a fractional number, the harmonics have low magnitude (see p. 3.1.2) and the slot skewing is not required. Nevertheless, it still could be applied to mitigate winding asymmetry or rotor eccentricity effects.

3.1.4. EMF tooth harmonics calculation

In [15] the EMF amplitudes ratio of the tooth harmonic (order $N \neq p$, according to (6) or (7)) to the fundamental harmonic (order $N = p$) is presented. For example, for the phase A the equation is:

$$E_{A,(N\neq p)} = (2/\pi) \cdot [\underline{B}_{(N\neq p)}/\underline{B}_{(N=p)}] \cdot E_{A,(N=p)} \tag{8}$$

where:

$\underline{B}_{(N\neq p)}, \underline{B}_{(N=p)}$ – are magnetic flux harmonics amplitudes shown in the Table 1.

The equation for the EMF $E_{A,(N=p)}$ calculation is presented in the section 3.2.

For phases *B*, *C*, there will be similar equations. Sometimes, the first factor is replaced with 1 yielding a bit higher tooth harmonics magnitude (with positive margin).

3.2. EMF harmonics of arbitrary order calculation

3.2.1. Representation of the stator winding (bars or coils)

The EMF of the phase is the sum of the EMF of phase' bars (coils) that are distributed in Q_s slots. In section 2.3.3, it is mentioned that the windings are not always constructed to be symmetrical [13, 14, 16]. For example, the phase zone A may include a few bars (coils) from the nearest phase zone, e.g. *C*. It is done to reduce the length of the balance connectors between the phase zones, and eventually this leads to phase EMF unbalance. The degree of asymmetry can be estimated if EMF of bars (coils) located in all Q_s stator slots of the winding is calculated. This also allows to find possible errors in the winding scheme design, which is important for the windings of multi-pole machines, and for the windings with a fractional value of Q (including $Q < 1$).

Appendix 1 contains the scheme of bar (coil) connections of the stator winding. It provides the sequence of slots for each phase of each of the six zones of the three-phase windings. For phase zone *A*, it has the form: $A'_1, A'_2, A'_3, A'_4, ..., A'_L$. For example, consider the winding of the generator with $Q_s = 66$, $p = 4$. For this winding, the phase zone corresponding sequence of bars (coils) is [13, 14, 16]: 10, 11, 26, 27, 28, 42, 43, 44, 59, 60, 61. The difference between any two slots numbers determines the phase angle between the EMF of their bars (coils). The order of the slots in the sequence is arbitrary, for example, this would be a correct sequence, too: 28, 11, 43, 27, 10, 61, 26, 44, 59, 60, 42.

For a two-layer winding, it is sufficient to present a sequence of bars (coils) only for one layer, e.g., the upper. The sequence for the second layer can be obtained by shifting on the chording (β). The value of (β) should be specified in the Appendix 1.

3.2.2. EMF harmonics calculation

a) EMF of the winding phase calculation.

The complex amplitude of the electromotive force' *N*th-harmonic for the phase zone *A* is:

$$E_{A,N}^{\mathrm{PH}} = \left\{ b_{R,N} \cdot \left[\begin{array}{c} \exp\left(j \cdot A_1 \cdot \Theta_{\mathrm{ST}} \cdot N\right) + \exp\left(j \cdot A_2 \cdot \Theta_{\mathrm{ST}} \cdot N\right) + \\ + \exp\left(j \cdot A_3 \cdot \Theta_{\mathrm{ST}} \cdot N\right) + \ldots + \\ + \exp\left(j \cdot A_Q \cdot \Theta_{\mathrm{ST}} \cdot N\right) \end{array} \right] \right\} \cdot \Pi = \tag{9}$$
$$= \left|E_{A,N}\right| \cdot \exp\left(j \cdot \varphi_{A,N}\right)$$

where:

$b_{R,N} = \left|B_N\right| \cdot \exp(j \cdot \psi_N)$ – according to (5) and Table 1,

$\Theta_{\mathrm{ST}} = 2 \cdot \pi / Q_s$ – angle between slots,

$\Pi = \frac{1}{2} \cdot \omega_1 \cdot \left(\frac{2}{\pi} \cdot \tau \cdot L \cdot S \right) \cdot K_{\mathrm{SK}}$ – coefficient that takes into account stator dimensions and slot zone parameters (see equation 9a),

ω_1 – EMF circular frequency of network,

L – the length of the core,

S – number of effective conductors of the coil,

K_{SK} – skew factor [2].

Notes:

1. The meaning of the Π coefficient could be derived from the known expression for the phase EMF of the stator winding:

$$E^{\mathrm{PH}}_{A,N} = \omega_1 \cdot \Phi \cdot w \cdot K_W \cdot K_{SK} = \omega_1 \cdot \left(\frac{2}{\pi} \cdot \tau \cdot L \cdot S \cdot \underline{B}_N \right) \cdot w \cdot K_W \cdot K_{SK} \qquad (9a)$$

where:

w – number of turns of the phase zone winding,

K_w – winding factor [8].

2. In the general case, the phase zone EMF is calculated using equation (9). In case of symmetrical winding construction, the more simple equation (9a) can be used. The difference in results calculated by (9) and (9a) for symmetrical winding is less than 1%. For the phase zone A', the equation of EMF *Nth*-harmonic is as follows:

$$E^{\mathrm{PH}}_{A',N} = \left\{ b_{R,N} \cdot \left[\begin{array}{c} \exp\left(j \cdot A'_1 \cdot \Theta_{\mathrm{ST}} \cdot N\right) + \exp\left(j \cdot A'_2 \cdot \Theta_{\mathrm{ST}} \cdot N\right) + \\ + \exp\left(j \cdot A'_3 \cdot \Theta_{\mathrm{ST}} \cdot N\right) + \ldots + \\ + \exp\left(j \cdot A'_L \cdot \Theta_{\mathrm{ST}} \cdot N\right) \end{array} \right] \right\} \cdot \Pi = \qquad (9')$$

$$= \left| E_{A',N} \right| \cdot \exp\left(j \cdot \varphi'_{A,N}\right)$$

In case of the two-layer winding ($S_{LR} = 2$, Appendix 1), the value of $E_{A,N}$ and $E_{A',N}$ should be multiplied by the value:

$$T = 1 - \exp(j \cdot \beta \cdot \pi \cdot N/p),$$

where:

β – chording.

The equations for the complex amplitudes (phasor) of the EMF for the phase zones B, B', C and C' would be similar to the (9), (9') and are not presented here.

b) Line-to-line EMF calculation.

The complex amplitude (phasor) of the *Nth*-harmonic of line-to-line EMF *A-B* is:

$$E_{A,B(N)} = \left(E^{\mathrm{PH}}_{A,N} - E^{\mathrm{PH}}_{B,N} \right) = \left| E_{A,B(N)} \right| \cdot \exp\left(j \cdot \varphi_{A,B(N)}\right) \qquad (10)$$

This equation is based on the results obtained in (9), (9'). The equations for the complex amplitudes of the line-to-line EMF *A-C* and *B-C* would be similar to the (10) and are not presented here.

4. Stator windings asymmetry

The line-to-line terminal voltage asymmetry tolerances are regulated by standards GOST, IEC [1]. The voltage asymmetry could be caused by the errors in winding connection scheme or by the rotor eccentricity. The developed algorithm allows to take into account both these factors. The winding asymmetry is dealt with in p. 3.2.1, the rotor eccentricity is dealt with by the simulation package [10].

5. Total harmonic distortion factor K_{DIST}

The calculation is based on the results shown in the Table 1. It is assumed (see Section 2.3.3) that the harmonic order $S = p$ – is fundamental, the harmonic orders of 1… R – are lower, and orders of U... W, K… M – are higher. Also see Table 1.

For the line-to-line EMF A-B, the total harmonic distortion factor K_{DIST} can be calculated as:

$$K_{\mathrm{DIST}} = \sqrt{\begin{array}{l} \left|\dfrac{E_{A,B(1)}}{E_{A,B(S)}}\right|^2 + \cdots + \left|\dfrac{E_{A,B(R)}}{E_{A,B(S)}}\right|^2 + \\ + \left|\dfrac{E_{A,B(U)}}{E_{A,B(S)}}\right|^2 + \cdots + \left|\dfrac{E_{A,B(W)}}{E_{A,B(S)}}\right|^2 + \\ + \left|\dfrac{E_{A,B(K)}}{E_{A,B(S)}}\right|^2 + \cdots + \left|\dfrac{E_{A,B(M)}}{E_{A,B(S)}}\right|^2 \end{array}} \tag{11}$$

The equations of distortion factor K_{DIST} for the line-to-line EMF A-C and for the line-to-line EMF B-C would be similar to the (11) and are not presented here.

6. Example

Table 2 shows the result of the distortion factor calculation $K_{\mathrm{DIST}} = f(\Lambda_{\mathrm{GAP}}, b_{\mathrm{P}})$ for the generator (see Appendix 2), which have winding with integer number Q of slots per pole and phase. The factor K_{DIST} is calculated for the two cases $\alpha = b_{\mathrm{P}}/\tau$: when $\alpha = 0.70$ (*) and $\alpha = 0.8$ (**), with skew factor $K_{sk} = 1$ (the skew is done by one tooth-pitch).

These results indicate that for the $1.2 \le \Lambda_{\mathrm{GAP}} \le 2.0$ the distortion factor K_{DIST} satisfies GOST [1] requirements.

Variation of $\alpha = b_{\mathrm{P}}/\tau$ in more than 5 – 7% drastically changes the magnetic flux density in the air gap. The air gap value δ_{EQ} and the rotor MMF should be adjusted then. This could lead to the full re-design of the geometry of the active part of the machine and requires new electromagnetic analysis to be carried out.

Table 2

Harmonic distortion factor K_{DIST} as a function of Λ_{GAP} (keeping the equivalent air gap δ_{EQ} a constant value) for generator with integer number Q and $K_{sk} = 1$

Λ_{GAP}	1.2	1.4	1.6	1.8	2.0
K_{DIST} (*)	0.0148	0.0140	0.0135	0.0131	0.0131
K_{DIST} (**)	0.0108	0.0109	0.0112	0.0114	0.0116

(*) – at $b_p/\tau = 0.70$, (**) – at $b_p/\tau = 0.8$

It is important to know the tolerances on the skew step value t. The dependency $K_{DIST}(t)$ is calculated for the case of $\Lambda_{GAP} = 1.6$ and $b_p/\tau = 0.7$. Table 3 contains the results for the skew step t equal to stator tooth pitch t_{st} (nominal value, 100%) and for the t equal to 50%, 75% and 110% of the t_{st}.

Table 3

Harmonic distortion factor K_{DIST} as a function of skew step t, calculated for $\Lambda_{GAP} = 1.6$ and $b_p/\tau = 0.70$ for generator with integer number Q

Stator slot skew step, t_1/t_{st}	50%	75%	100%	110%
K_{DIST}	0.13	0.064	0.0135	0.02

Table 3 data indicates that skew step tolerances are very strict.

7. Implementation of the distortion factor calculation method in a general design process

It is possible to implement the discussed calculation method in the generator design process. The generator design process consists of several stages. In the first stage, the geometrical dimensions of the active part and winding machine data are determined, which would satisfy basic requirements (generator weight, efficiency, winding temperature) of the Technical Specification and Russian State Standards. The results obtained at the first stage of calculation do not often satisfy additional requirements. So in the second stage, the special requirements are examined. In order to comply with special requirements, the related basic parameters could be corrected as well.

One of these special requirements could be the limitation of the distortion factor K_{DIST}. It depends on basic parameters (3): b_p, Λ_{GAP} at δ_{EQ} = const, and Q (number of slots per pole and phase).

8. Conclusions

1. The proposed method of synchronous salient-pole generator no-load state line-to-line EMF THD calculation is distinguished by its ability to take into account: actual magnetic field distribution (2D FEM simulation [10]), harmonic spectrum of radial

(normal) magnetic flux density distribution in the air gap, saturation of ferromagnetic materials, winding scheme (both with integer or fractional number Q of slots per pole and phase). The method could also be used to find the dependency of the THD factor on the different factors, for example – pole shoe shape and Q. It is found that the pole shoe radius is a function of three parameters: $R_P = f(D_{IN}, \Lambda_{GAP}, b_P)$.

2. In case of the winding constructing with fractional value Q of slots per pole and phase, the Fourier decomposition periodic interval should be taken according to (4). In this case, the fundamental harmonic of the magnetic flux distribution in the air gap and of the winding electromagnetic force is of the order $N = p$, and we get not only higher, but also lower harmonics. The possible errors in the winding scheme could be revealed using harmonic analysis (which is actual for multi-pole machines with the fractional value Q). Equation (9) supports EMF calculation for non-symmetrical winding as well. The analysis allows to determine the unbalance of phase and line-to-line EMF also.
3. The method could answer some questions arising in the manufacturing practices of synchronous generators with electromagnetic excitation:
   - What are the tolerance limits on the slot skew angle?
   - Is the stator slot skew necessary?

Appendix 1. Input data sheet

1. The scheme of three-phase six-zone winding of the stator (in this example: Q – integer).

A_1	A_2	A_3	…	A_Q	C'_1	C'_2	C'_3	…	C'_Q	B_1	B_2	B_3	…	B_Q
|	|	|	|	|	|	|	|	|	|	|	|	|	|	|
A'_1	A'_2	A'_3	…	A'_Q	C_1	C_2	C_3	…	C_Q	B'_1	B'_2	B'_3	…	B'_Q

2. The geometrical dimensions.

D_{IN}, D_A – the stator inner and outside diameters, β – chording,
b_{SL}, h_{SL} – the width and height of stator slot, Q_s – the number of stator slots,
b_M, h_M – the width and height of pole core, D_0 – the rotor inner diameter,
S_{LR} – the type of stator winding: for one-layered winding $S_{LR} = 1$, for two-layered winding $S_{LR} = 2$,
δ_{MAX}, δ_{MIN}, p, b_P, h_P – see description in the text.

3. F_{EX} – MMF of the rotor winding at no load state.

Appendix 2. The synchronous generator data

Power rating	1250 kW
Rated line voltage	6300 V
Number of rotor poles	10
Frequency	50 Hz
Stator outer diameter	1195 mm
Stator inner diameter	922 mm
Stator core length	600 mm
Steel grade	2412
Equivalent air gap	4.5 mm
Slot dimensions	13.1 × 60 mm
Number of effective conductors in the slot	12
Number of parallel paths	1
Chording	0.78
Phase connection type	star

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