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ANALYSIS OF THE DYNAMIC CHARACTERISTICS AND VIBRATIONAL COMFORT OF SELECTED FOOTBRIDGES OVER THE S7 NATIONAL ROAD

ANALIZA CHARAKTERYSTYK DYNAMICZNYCH I KOMFORT UŻYTKOWANIA WYBRANYCH KŁADEK DLA PIESZYCH NAD DROGĄ KRAJOWĄ S7

Abstract

In this paper, the dynamic analysis of three footbridges that carry pedestrians over the S7 national road 'Zakopianka' are presented. Firstly, the dynamic characteristics of the structures were evaluated. It transpired that the lowest natural frequencies of the two selected objects coincided with the pacing frequencies of a running pedestrian. Hence, the dynamic responses to this kind of dynamic loading were calculated and the comfort criteria were assessed.

Keywords: footbridges, footbridge dynamics comfort criteria

Streszczenie

W pracy przedstawiono analizę dynamiczną trzech kładek dla pieszych zlokalizowanych nad drogą krajową S7 (tzw. Zakopianką). Po wyznaczeniu charakterystyk dynamicznych obiektów okazało się, że podstawowe częstotliwości drgań własnych pionowych dwóch obiektów pokrywają się z częstotliwościami drgań generowanych przez biegnącego pieszego. Wyznaczono więc odpowiedzi dynamiczne obiektów na tego typu wymuszenie dynamiczne i oszacowano spełnienie kryteriów komfortu wibracyjnego.

Słowa kluczowe: kładki dla pieszych, dynamika kładek dla pieszych, komfort wibracyjny

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1. Introduction

The great progress in construction techniques, building materials and calculation methods have supported architects and engineers in their tendency to design modern footbridges longer and lighter that are longer and lighter than older examples. This trend demonstrates itself in the dynamic properties of the footbridges – they have relatively lower a first natural frequency than, for example, in road or railway bridges and buildings.

In the current engineering literature, there are many cases of pedestrian bridges which have natural frequencies that are close to the critical frequencies of dynamic excitations produced by pedestrians [1–3]. Potentially, pedestrians using a footbridge may cause a resonance phenomenon through their movements. As a consequence, amplitudes of displacements and accelerations of the footbridge deck increase and this results in decreased levels of the comfort for users of the structures. Therefore, the main purpose of the dynamic analyses of footbridges is the assessment of their vibration comfort criteria [1, 2, 4].

The main objective of this study was the dynamic analysis of three footbridges that carry pedestrians over the S7 national road 'Zakopianka'. Firstly, the natural frequencies and modes of vibration of the selected structures were evaluated. It transpired that the lowest natural frequencies of vertical vibration calculated for two of the selected structures coincided with the typical pacing frequencies of a single running pedestrian. Therefore, the dynamic responses of the structures to this type of dynamic loading were evaluated. Finally, on the basis of the obtained results, the vibrational comfort criteria were assessed.

2. Footbridges – basic issues

Pedestrians are not only a source of dynamic loading while moving on footbridges, they are also the major recipients of the vibrations of the structures. Human susceptibility to vibrations is frequency dependent. To evaluate the impact of oscillation on people, an appropriate parameter should be selected. In the frequency range 1–10 Hz, perception of oscillation is proportional to acceleration, while in the range of 10–100 Hz, it is proportional to velocity. The human perception thresholds are presented in Table 1 [1].

Table 1

The limits of human perception of vertical vibration [1]

Description	Max. vibration acceleration [m/s^2] (range of frequency 1–10 Hz)	Max. vibration velocity [mm/s] (range of frequency 10–100 Hz)
Slightly perceivable	0.034	0.500
Clearly perceivable	0.100	1.300
Unpleasant	0.550	6.800
Intolerable	1.800	13.800

The main purpose of dynamic research on footbridges is the assessment of their vibration comfort criteria [1–4]. There are several standards which address the assessment of the impact of oscillations on users of walkways, such as British Standards [5] or ISO [6]. The guidelines for design practice are also provided by Eurocodes [7]. The extreme values of acceleration amplitudes in two directions, summarised in these guidelines, are presented in Table 2. The limits are appropriate for structures with natural frequencies less than 5 Hz. Other practical guidelines for the assessment of the vibrational comfort of footbridges are proposed in the SÉTRA document [8]. The acceleration ranges in vertical and horizontal direction with comfort levels assigned, suggested in this work, are presented in Table 3.

Table 2

Acceleration criteria for footbridges (EN 1990:2002/Al:2005) [7]

Direction of vibration	Acceleration [m/s^2]
Vertical	0.7
Horizontal in normal use	0.2
Horizontal for exceptionally crowded conditions	0.4

Table 3

The ranges of comfort of use (SÉTRA 2006) [8]

Comfort level	Ranges of comfort [m/s^2]	
	Vertical	Horizontal
Maximum comfort	0.0–0.5	0.00–0.15
Mean comfort	0.5–1.0	0.15–0.30
Minimum comfort	1.0–2.5	0.30–0.80
Uncomfortable	> 2.5	> 0.8

The results of experimental investigations of natural frequencies carried out for 67 pedestrian bridges are shown in Fig. 1 [4]. The typical frequency range generating by a single pedestrian walking normally (from 1.7 to 2.2 Hz) is also presented in Fig. 1. It transpires that pacing frequencies typical for both walking and running pedestrians may coincide with the lowest natural frequencies typical of footbridges. Hence, the normal usage of the bridge may lead to resonant behaviour of a structure and cause reductions in the comfort of pedestrians.

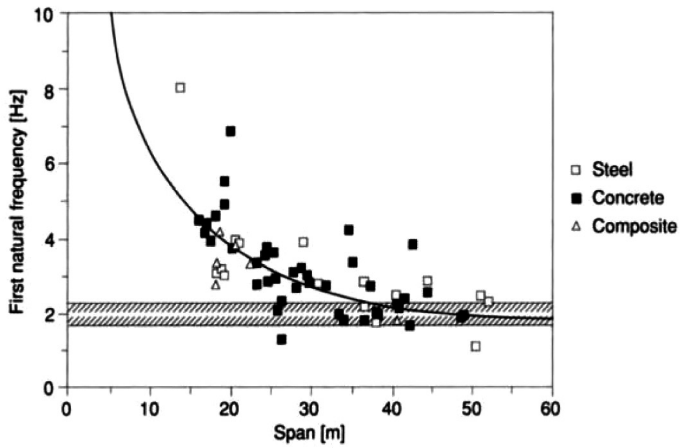

Fig. 1. The basic natural frequencies of footbridges as a function of the span length with the typical range of frequencies generated by the movement of a single pedestrian (walking or running) [4]

3. Dynamic loading generated by the movement of a single pedestrian

Dynamic loading normally generated by people on footbridges are as a result of walking and running. These type of loading are periodic and change in time and space. The ranges of frequency typical of different human movement types are summarised in Table 4.

Table 4

Typical pacing frequencies for walking and running [1, 2]

Type of movement	The range of frequencies [Hz]			
	Total range	Slow	Normal	Fast
Walking	1.40–2.40	1.40–1.70	1.70–2.20	2.20–2.40
Running	1.90–3.30	1.90–2.20	2.20–2.70	2.70–3.30

The force generated by a single walking or running pedestrian is modelled as a sum of dynamic and static components by the equation [3]:

$$F(t) = G\left[1 + \sum_{i=1}^{n} A_i \sin(2i\pi f_s t - \varphi_i)\right] \qquad i = 1, 2, \ldots, n \tag{1}$$

where:

G – weight of pedestrian;
f_s – fundamental loading frequency;
A_i,φ_i – amplitude and the phase angel of the i-th harmonic, respectively.

The Fourier coefficients in equation (1) for different types of human motion are summarised in Table 5.

Table 5

Coefficients of Fourier decomposition [3]

Type of movement	A_1	φ_1(rad)	A_2	φ_2(rad)	A_3	φ_3(rad)
Walking	0.4	0	0.1	1.57	0.1	1.57
Running	1.6	0	0.7	0.00	0.2	0.00

The model expressed by equation (1) is sufficient in the case of walking because each step overlaps with the previous one thus making the force continuous. However, in the case of running this assumption seems to be inappropriate, since the loading generated by a running pedestrian has a discontinuous nature. In this case, a half-sine model [1] appears to be better. Forces generated by a pedestrian running are calculated from the following formula:

$$F_z(t) = \begin{cases} k_z G \sin\left(\pi \dfrac{t}{t_c}\right) & \text{for} \quad t \le t_c \\ 0 & \text{for} \quad t_c < t < t_u \end{cases} \tag{2}$$

where:

k_z – impact factor ($k_z = F_{max}/G$);
F_{max} – maximum dynamic load of running;
G – weight of pedestrian;
t_c – period of contact;
t_u – period of running ($t_u = 1/f_u$);
f_u – frequency of running.

4. The numerical models of the selected footbridges

The first analysed structure is located in Gaj, the second is situated in Jawornik and the third, which at the time of writing is planned but not yet built, is also to be located in Jawornik. The numerical models of the structures were prepared in ABAQUS [9]. In the numerical models of all selected structures, beam elements were applied for girders and crossbars. Decks were discretised with shell elements and solid elements were used for the foundations. Fixed boundary conditions, reflecting the high rigidity of subsoils, were applied in all cases. The hangers were modelled as truss elements with the ‘no compression’ option [9] in order to guarantee that compressive stresses would not be generated during dynamic analysis. However, when such elements are used, instability of the model usually occurs. This difficulty was overcome by overlaying each truss element which had no compression stiffness with another element which had a small degree of compression stiffness. This technique enables the creation of non-zero stiffness that stabilises the model. Hence, for the model stabilization beam elements were introduced to model hangers: the level of 5% of the hangers' stiffness was implemented for the stabilizing elements. The *Tie* constrains provided by ABAQUS [9] were applied to guarantee identical kinematic behaviour of the truss and the beam elements.

4.1. The footbridge in Gaj

The first structure selected for analysis was the suspended footbridge located in Gaj. The length of the footbridge is 50.0 m, the entire width of the footbridge is 4.50 m and the heights of the pylons are 15.5 m and 17.5 m. The footbridge in Gaj is shown in Fig. 2.

Fig. 2. The footbridge over the S7 national road 'Zakopianka' located in Gaj

The primary structural system of the footbridge consists of two reinforced concrete girders (cross-section 85 x 40cm). The girders are connected by two concrete crossbars (cross-section 90 x 50cm) at both ends and by ten steel crossbars (diameter 244 mm, thickness 16mm) along the span. The thickness of the reinforced concrete deck is 0.22 m. The superstructure is suspended by twenty trusses (diameter 56mm) from steel pylons. The trusses are hinged to the pylons and to the deck. The structure is supported by elastomeric bearings (cross-section 30×30 cm, height 20 cm). The numerical model of the footbridge with points of the deck selected for dynamic analysis is presented in Fig. 3. The material parameters are summarised in Table 6.

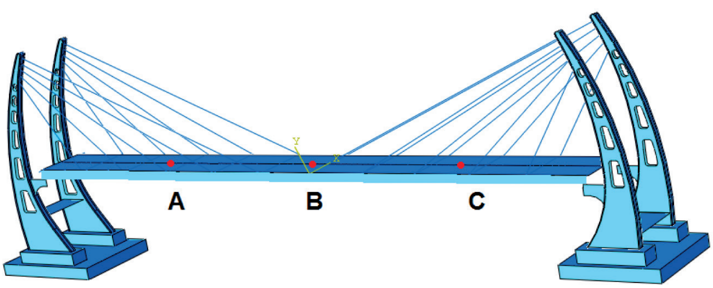

Fig. 3. Numerical model of the footbridge located in Gaj with points of the deck selected for dynamic analysis

Table 6

Parameters of structural materials of the footbridge in Gaj

	Elasticity modulus [GPa]	Poisson's ratio [-]	Mass density [kg/m^3]
Concrete	32	0.2	2500
Structural steel	210	0.3	7800
Steel trusses	200	0.3	7800

4.2. The footbridge located in Jawornik

The second selected footbridge is a steel-frame structure with a suspended reinforced concrete deck located in Jawornik. The length of the footbridge is 57.15 m, the entire width is 5.0m and the height of the frame is 1.08 m. The footbridge in Jawornik is shown in Fig. 4.

The primary structural system of the footbridge consists of two steel frames (rectangular cross-section 180×50 cm). The superstructure of the footbridge consists of two reinforced reinforced concrete girders (cross-section 55×25 cm) and the reinforced concrete deck is 3.4 m wide. The deck is suspended from the frames by twenty hangers (27 mm in diameter) located at a distance of 2.5 m from each other. The deck is also equipped with elastomeric bearings (cross-section 30×30 cm, height 20 cm) placed on steel cantilevers mounted on the frame columns. The thickness of the deck varies from 14 to 18cm. In further analysis, this footbridge is referred to as 'Jawornik I'. A numerical model of the Jawornik I footbridge with points of the deck selected for dynamic analysis is presented in Fig. 5. The material parameters of the Jawornik I footbridge are summarised in Table 7.

Fig. 4. The footbridge located in Jawornik

The third structure selected for analysis is a beam footbridge which is planned to be built in Jawornik. The length of the planned footbridge is 51.4 m and the entire width is 5.1 m. A visualisation of the footbridge in the planned location is shown in Fig. 6.

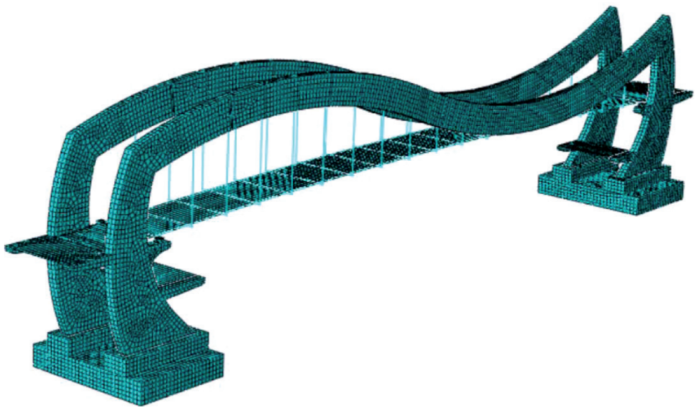

Fig. 5. Numerical model of the footbridge located in Jawornik with points of the deck selected for dynamic analysis

Table 7

Parameters of structural materials of the Jawornik I footbridge

	Elasticity modulus [GPa]	Poisson’s ratio [–]	Mass density [kg/m^3]
Concrete	35	0.2	2500
Structural steel	210	0.3	7850
Steel trusses	205	0.3	7850

4.3. The footbridge planned to be built in Jawornik

The primary structural system of the footbridge consists of two steel arch-shaped girders located at a distance of 5.1 m from each other. The length of the beams is 47.0 m, and the height of the beams is 1.6 m. The thicknesses of the beams’ upper flange, bottom flange and web are 600×40 mm, 600×40 mm and 1530×20 mm, respectively. The main beams are connected by steel crossbars made of HEB 200. The reinforced concrete deck of the bridge is 5.1 m wide and 0.2 m thick. A system of four reinforced concrete columns connected by crossbars constitutes the support for the main girders. The structure is supported by elastomeric bearings (cross-section 30×30 cm, height 20 cm). The fixed boundary conditions reflected the high rigidity of the foundation soil. In further analysis, this footbridge is referred to as ‘Jawornik II’. A numerical model of the Jawornik II footbridge with points of the deck selected for dynamic analysis is presented in Fig. 7. The material parameters of the Jawornik II footbridge are summarised in Table 8.

Fig. 6. A visualization of the Jawornik II footbridge in the planned location

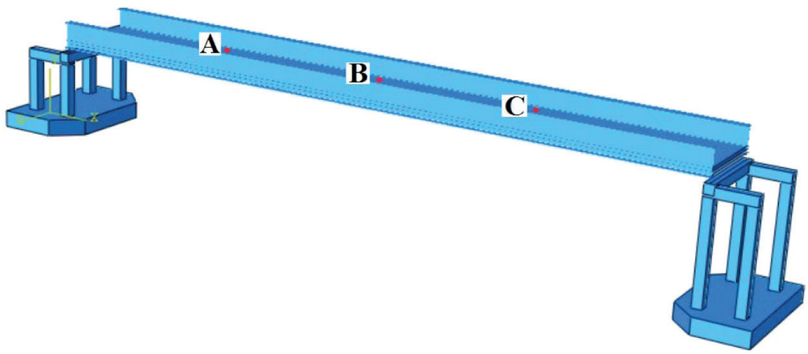

Fig. 7. A numerical model of the footbridge planned to be built in Jawornik with points of the deck selected for dynamic analysis

Table 8

Parameters of structural materials of the footbridge planned to be built in Jawornik

	Elasticity modulus [GPa]	Poisson's ratio [-]	Mass density [kg/m^3]
Concrete	35	0.2	2450
Structural steel	210	0.3	7850

5. The dynamic characteristics of the selected footbridges

In the first step of the numerical investigations, the natural frequencies and modes of vibrations of the selected footbridges were evaluated. The first modes of vertical vibration of the analysed footbridges are illustrated in Fig. 8.

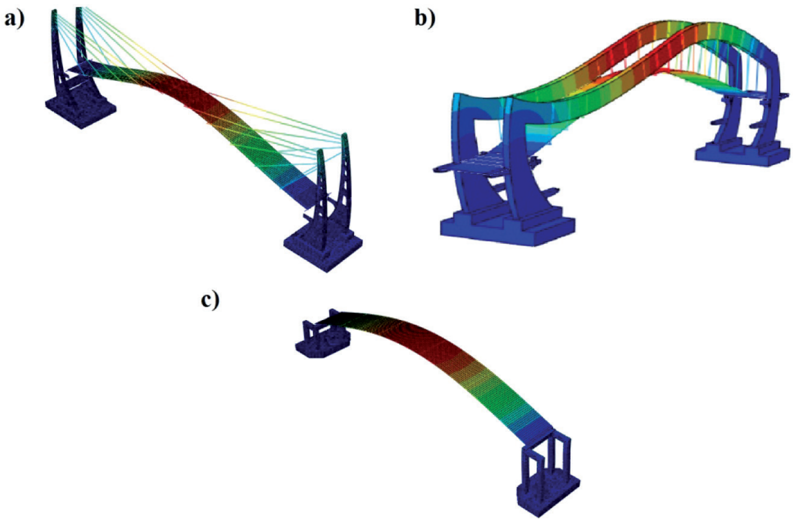

Fig. 8. First vertical modes of vibrations of: a) Gaj footbridge (frequency 1.92 Hz), b) Jawornik I footbridge (frequency 3.4 Hz), c) Jawornik II footbridge (frequency 1.90 Hz)

The lowest natural frequency of vertical vibrations of the Gaj and Jawornik II footbridges was about 1.90 Hz; this falls within the range of 1.40 to 2.40 Hz which represents the typical pacing frequency of a walking and running pedestrian (see Table 3). Hence, to assess the comfort criteria, the dynamic responses of these footbridges to a single running pedestrian should be calculated.

The lowest natural frequency of vertical vibrations of the Jawornik I footbridge was 3.4 Hz; this falls beyond the normal frequency range for walking or running. Therefore, it was not necessary to assess comfort criteria for this structure and further dynamic analysis was not performed.

6. Comfort criteria assessment and dynamic response of the Gaj and Jawornik II footbridges to pedestrian loading

In the second step of the dynamic analysis, the dynamic responses of the Gaj and Jawornik II footbridges to pedestrian movements were evaluated. The modal dynamic analysis was carried out, during which, the twenty lowest modes of vibration were considered with a damping ratio of 2% for each mode. The model of forces generated by a single running pedestrian, presented by equation (1), was used. The following values were assumed in the analysis: pedestrian weight – $G = 800$ N; impact factor – $k_z = 3$ (according to [2]); first natural frequency of footbridge – $f_s = 1.92$ Hz; frequency of running – $f_u = 1.92$ Hz; period of running – ($t_u = 1/f_u$) $t_u = 0.52$ sec; period of contact – $t_c = 0.26$ s. The function of forces generated by a single running pedestrian is shown in Fig. 9.

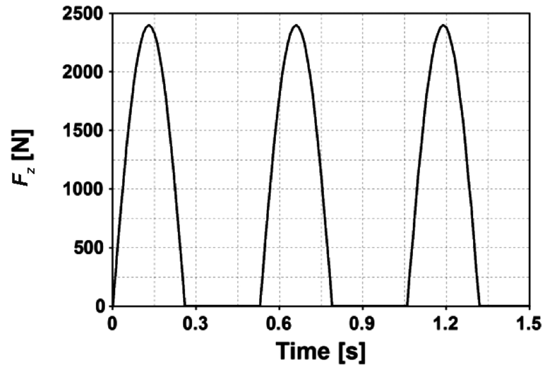

Fig. 9. Periodic function F_z of forces generated by a single pedestrian or running

The time histories of accelerations occurring due to a single running pedestrian, calculated for points B located on the Gaj footbridge (see Fig. 3), are presented in Fig. 10. The maximum values of accelerations that occurred during the whole pedestrian passage along the footbridge at points A, B and C are summarised in Table 9. The maximum acceleration was 0.245 m/s^2 and took place at point B located at the midpoint of the footbridge span.

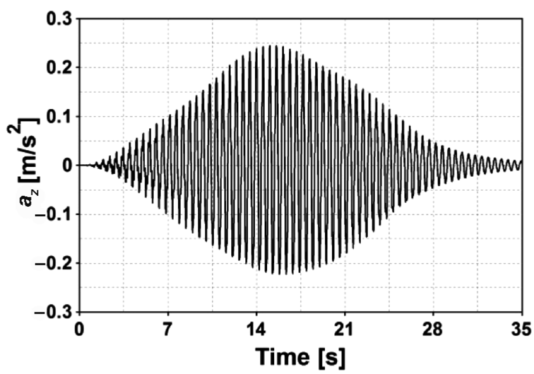

Fig. 10. Time histories of vertical accelerations occurring due to a single running pedestrian calculated for the footbridge in Gaj at point B

Table 9

The maximum vertical accelerations at points A, B and C on the footbridge in Gaj

Point	Maximum value of acceleration [m/s^2]
A	0.235
B	0.245
C	0.170

The time histories of accelerations occurring due to a single running pedestrian, calculated for points B located on the Jawornik II footbridge (see Fig. 3) are presented in Fig. 11. The maximum values of accelerations that occurred during the whole pedestrian passage along the footbridge at points A, B and C are summarised in Table 10. The maximum acceleration was 0.245 m/s² and took place at point B located at the midpoint of the footbridge span.

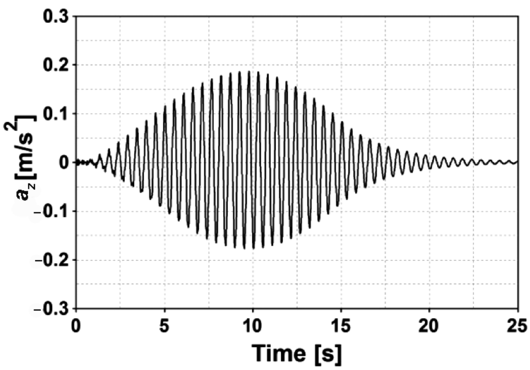

Fig. 11. Time histories of vertical accelerations occurring due to a single running pedestrian calculated for the Jawornik II footbridge at point B

Table 10

The maximum vertical accelerations at points A, B and C on the Jawornik II footbridge

Point	Maximum value of acceleration [m/s²]
A	0.160
B	0.190
C	0.170

On the basis of the results presented in Tables 9 and 10, it can be observed that in the case of a single running pedestrian, the requirements of comfort criteria for vertical vibrations, recommended by Eurocode (see Table 2), are fulfilled for both footbridges. The level of vibrations does not exceed 0.7 m/s². Additionally, according to SÉTRA recommendations (see Table 3), the 'maximum comfort' level is guaranteed for both of the analysed structures. The level of 0.5 m/s² is not exceeded. However, according to CEB 1991 recommendations (see Table 1), the level of vibrations exceed the 'clearly perceivable' thresholds which is 0.100 m/s².

7. Conclusions

The dynamic characteristics, specifically, natural frequencies and modes of vibration, of three footbridges over the S7 national road 'Zakopianka' were evaluated. The dynamic responses of the footbridges to a single running pedestrian were also calculated. On the basis of the obtained results, the vibrational comfort criteria for the analysed footbridges were also assessed. The following final remarks can be formulated on the basis of the performed dynamic analysis:

- The results of modal analysis revealed that the natural frequencies of two structures, the footbridge in Gaj and the footbridge planned to be erected in Jawornik (Jawornik II), coincide with the frequency of steps of a single pedestrian running. Due to the fact that this may result in the resonance phenomenon, the dynamic responses of both structures to this type of pedestrian activity had to be evaluated;
- It transpired that the requirements of comfort criteria for vertical vibrations, recommended by Eurocode, were fulfilled for both footbridges and 'maximum comfort', according to SETRA recommendations, was guaranteed for both structures;
- The vibrations were 'clearly perceivable' by pedestrians using both footbridges, according to CEB recommendations;
- The first vertical frequency of the other footbridge located in Jawornik (Jawornik I) fell beyond the normal exploitation frequency range (walking or running). Therefore, it was not necessary to assess comfort criteria for this object.

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