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APPLICATION OF THE DIRECT SPECTRAL METHOD TO CYCLE IDENTIFICATION FOR MULTIAXIAL STRESS IN FATIGUE ANALYSIS

ZASTOSOWANIE METODY SPEKTRALNEJ BEZPOŚREDNIEJ DO IDENTYFIKACJI CYKLI DLA WIELOOSIOWEGO STANU NAPRĘŻEŃ W ANALIZIE ZMĘCZENIOWEJ

A b s t r a c t

In the article, the means of application of the direct spectral method for the identification of the stress cycles for multiaxial stress is discussed. Two cases are analyzed. The first, when components of stress tensor are in phase, and the second, when they are shifted in phase. The second case is associated with the practical application for the crane wheel.

Keywords: fatigue analysis, cycle counting, spectral method, multiaxial stress

S t r e s z c z e n i e

W artykule przedstawiono sposób zastosowania metody spektralnej bezpośredniej do identyfikacji cykli naprężeń o charakterze wieloosiowym. Rozważane są dwa przypadki. Pierwszy, gdy składowe tensora naprężeń są zgodne w fazie i drugi, gdy są one przesunięte w fazie. Drugi przypadek jest związany z praktycznym zastosowaniem dla koła suwnicy.

Słowa kluczowe: analiza zmęczeniowa, zliczanie cykli naprężeń, metoda spektralna, naprężenia wieloosiowe

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1. Introduction

In the fatigue analyses of engineering structures, it sometimes happens that vibrations of elements have the bi-modal type. Such types have been observed for one of the considered magnetic focusing machine for particles in CERN at the beginning of the 21st century or vibrations of vehicle suspension system analyzed by T.-T.Fu and D.Cebon [6]. Methods of fatigue analysis of the stress history of irregular (non-harmonic) type in time domain, which are known in literature, are in practice associated with one of the cycle counting methods (including the "rain-flow" attempt) [5, 10, 15, 16] or the spectral methods [23–25]. Due to the existence of the two harmonic components with different frequencies in the bi-modal type stress histories, there are limited numbers of articles in which the methods dedicated for this case are proposed [2, 3, 6].

Due to the geometry of the structure, during its vibrations, the generated deformations are reason of the case of multiaxial stress. Fatigue analysis of such cases have been considered for several years with the application of different theories. The authors present only the part of the important articles in this topic, not thinking about doing a review of the completely state-of the art. Reviews and comparisons of different multiaxial fatigue theories can be found e.g. in [7, 9, 26, 28, 32, 33]. The commonly used criterions are based on: empirical equivalent stress approach – Pollard [8], stress invariants – Sines [31], average stress approach – Papadopoulos et.al. [26], critical plane methods – Carpinteri and Spagnoli [4], Dang Van [1, 15], Matake [21], McDiarmid [22], Liu and Mahadevan [17], Papadopoulos [27], energy – Łagoda [18]. The simplest way for fatigue analysis of the multiaxial type stress history is the determination of the equivalent mean stress, equivalent stress amplitude and the equivalent completely reversed stress [1, 5]. In Polish literature, the fatigue analyses of the cases of stress multiaxial are discussed e.g. in [16, 18–20, 25, 28, 29, 30].

The authors proposed an original method of the fatigue analysis for the bi-modal stress history, based on the idea of reconstruction of the histories in the time domain, called the direct spectral method [13, 14]. The preliminary ideas of application of the method for multiaxial stress histories were presented in [11, 12].

The aim of the paper is to present the possibility of application of the direct spectral method for cycle counting of the bi-axial stress in-phase history (simulation) and the multiaxial stress history out-of phase associated with the realistic case of the rail wheel.

2. Basis of the spectral direct method for multiaxial stress

The bi-modal stress history can be theoretically defined in the form:

$$\begin{cases} \sigma_x(t) = A_{x,1}\sin(\omega_1 t + \varphi_{x.1}) + A_{x,2}\sin(\omega_2 t + \varphi_{x,2}) \\ \sigma_y(t) = A_{y,1}\sin(\omega_1 t + \varphi_{y,1}) + A_{y,2}\sin(\omega_2 t + \varphi_{y,2}) \\ \sigma_z(t) = A_{z,1}\sin(\omega_1 t + \varphi_{z,1}) + A_{z,2}\sin(\omega_2 t + \varphi_{z,2}) \\ \tau_{xy}(t) = A_{xy,1}\sin(\omega_1 t + \varphi_{xy,1}) + A_{xy,2}\sin(\omega_2 t + \varphi_{xy,2}) \\ \tau_{xz}(t) = A_{xz,1}\sin(\omega_1 t + \varphi_{xz,1}) + A_{xz,2}\sin(\omega_2 t + \varphi_{xz,2}) \\ \tau_{yz}(t) = A_{yz,1}\sin(\omega_1 t + \varphi_{yz,1}) + A_{yz,2}\sin(\omega_2 t + \varphi_{yz,2}) \end{cases} \tag{1}$$

where:

$A_{x,1}$, $A_{x,2}$, $A_{y,1}$, $A_{y,2}$, $A_{z,1}$, $A_{z,2}$, $A_{xy,1}$, $A_{xy,2}$, $A_{xz,1}$, $A_{xz,2}$, $A_{yz,1}$, $A_{yz,2}$ – stress amplitudes of the harmonic components for the stress tensor components,

ω_1, ω_2 – angular frequencies of the harmonic components,

$\varphi_{x,1}$, $\varphi_{x,2}$, $\varphi_{y,1}$, $\varphi_{y,2}$, $\varphi_{z,1}$, $\varphi_{z,2}$, $\varphi_{xy,1}$, $\varphi_{xy,2}$, $\varphi_{xz,1}$, $\varphi_{xz,2}$, $\varphi_{yz,1}$, $\varphi_{yz,2}$ – phases of the harmonic components for the stress tensor components.

With such a formulation, the component frequencies f_1 and f_2 and corresponding periods T_1 and T_2 can be obtained using the equations (2) and (3).

$$f_1 = \frac{\omega_1}{2\pi}, \quad f_2 = \frac{\omega_2}{2\pi} \tag{2}$$

$$T_1 = \frac{1}{f_1} = \frac{2\pi}{\omega_1}, \quad T_2 = \frac{1}{f_2} = \frac{2\pi}{\omega_2} \tag{3}$$

The basic assumptions and form of the application of the direct spectral method for bi-modal waveforms for multiaxial stress can be described as follows:

- Based on the values of periods T_1 and T_2, the so-called block of stress is determined, of which length (time range) T_B depends on the ratio T_1/T_2. It is the smallest integer number of period T_1, for which the ratio T_B/T_2 is an integer. In practical applications, this condition is satisfied approximately, hence assuming the value of T_B, is an arbitrary decision. It depends on the precision of determination of T_1 and T_2, usually by the identification of frequencies f_1 and f_2.
- The primary stress cycle – only one present within the block – has the stress amplitude equal to $A_{k,1} + A_{k,2}$ for each stress component (where $k = x, y, z, xy, xz, yz$), and if not stated otherwise (e.g. constant value present in FFT function of stress' signals, static assembly stress or thermal stress), the average stress value is equal to zero. This assumption is the basis for calculating the equivalent stress amplitude e.g. by the application of the Huber-Mises-Hencky (von Mises) formula (5) [3] and then the equivalent complety reversed stress, e.g. Morrow's type (6) [5].
- The amplitudes of secondary stress cycles vary depending on the value $A_{k,2}$ and the leading waveform of frequency f_1 for each component of stress tensor. Some of the identified cycles are not taken into account, when they do not have the full stress-cycle form. For the slow-changing waveforms of frequency f_1 when comparing to frequency f_2, the amplitudes of the secondary cycles are approximately equal to $A_{k,2}$. The acquired values are the basis to obtain the equivalent amplitude, e.g. by the application of the Huber-Mises-Hencky (von Mises) formula (5) [3], equivalent mean stress value, e.g. in the form of Sines stress (4) [3] and then to obtain the equivalent a completely reversed stress e.g. of Morrow type (6) [3].
- The obtained data, which describes the identified stress cycles for a given waveform, are the basis for fatigue analysis using the chosen stress cumulative hypothesis, e.g. Palmgreen-Miner's (7) [5].

In the analysis, the following parameters are used: equivalent mean stress value (Sines stress) (4), equivalent stress amplitude (according to the von Mises equivalent stress) (5), equivalent completely reversed stress (Morrow stress) (6).

$$\bar{\sigma}_m = \sigma_{1,m} + \sigma_{2,m} + \sigma_{3,m} \quad (4)$$

$$\bar{\sigma}_a = \frac{1}{\sqrt{2}}\sqrt{\left[\sigma_{x,a} - \sigma_{y,a}\right]^2 + \left[\sigma_{x,a} - \sigma_{z,a}\right]^2 + \left[\sigma_{y,a} - \sigma_{z,a}\right]^2 + 6\left[\tau^2_{xy,a} + \tau^2_{xz,a} + \tau^2_{yz,a}\right]} \quad (5)$$

$$\sigma_{\mathrm{EQV}} = \begin{cases} \dfrac{\bar{\sigma}_a}{1 - \dfrac{\bar{\sigma}_m}{\sigma_u}} & \text{for } \bar{\sigma}_m > 0 \\ \bar{\sigma}_a & \text{for } \bar{\sigma}_m \le 0 \end{cases} \quad (6)$$

$$B \cdot \sum_{i=1}^{N} \frac{N_i}{\left(N_f\right)_i} = 1 \quad (7)$$

$$T = B \cdot T_B \quad (8)$$

where:

- $\bar{\sigma}_m$ – equivalent mean stress,
- $\sigma_{1,m}, \sigma_{2,m}, \sigma_{3,m}$ – main values of mean stress,
- $\bar{\sigma}_a$ – equivalent stress amplitude,
- $\sigma_{x,a}, \sigma_{y,a}, \sigma_{z,a}, \tau_{xy,a}, \tau_{xz,a}, \tau_{yz,a}$ – amplitude stress components,
- σ_{ar} – equivalent completely reversed stress,
- σ_u – ultimate stress,
- N – total number of cycles identified in a block,
- N_i – number of cycles with amplitude σ_i identified in a block,
- $(N_f)_i$ – number of cycles to damage for stress with amplitude σ_i (S-N curve),
- B – number of blocks,
- T_B – time length of block,
- T – estimated lifetime.

3. Examples of cycle identification by the direct spectral method

3.1. Simulation of the bi-axial in-phase stress history

Let us consider the case when the bi-axial stress' components $\sigma_x(t)$ and $\tau_{xy}(t)$ are of the bi-modal type (9). The time histories are shown in Fig. 1. There are two active frequencies f_1 = 10 Hz and f_2 = 50 Hz. Hence the block length is equal to $T_B = T_1 = 0.1$ s. The amplitudes of normal stress are equal to $A_{x,1}$ = 310 MPa and $A_{x,2}$ = 155 MPa. The amplitudes of shear stress are 10% of the suitable normal ones, hence $A_{xy,1}$ = 31 MPa and $A_{xy,2}$ = 15.5 MPa. It is assumed that material has ultimate stress equal to σ_u=625 MPa. The identified equivalent completely reversed stress cycles after application of the spectral direct method are given in Table 1.

$$\begin{cases} \sigma_x(t) = 310\sin\left(2\pi 10t\right) + 155\sin(2\pi 50t) \\ \tau_{xy}(t) = 31\sin\left(2\pi 10t\right) + 15.5\sin(2\pi 50t) \end{cases} \quad (9)$$

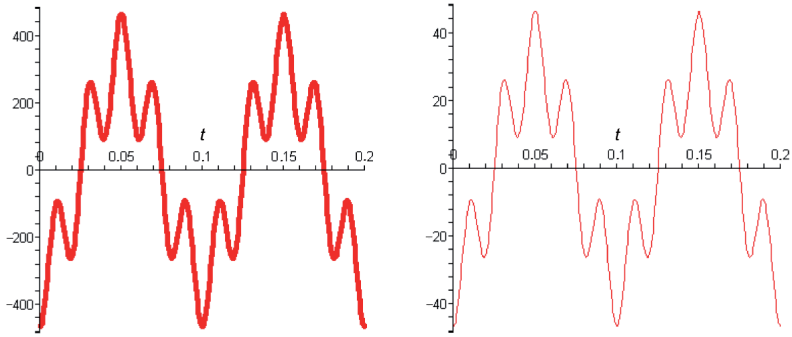

Fig. 1. Time histories of normal stress $\sigma_x(t)$ – left and shear stress $\tau_{xy}(t)$ – right

Table 1

Identified cycles for one block of the analyzed bi-axial and bi-modal stress histories

Number of cycles	Normal stress		Shear stress		Equivalent mean stress	Equivalent stress amplitude	Equivalent completely reversed uniaxial stress
	Mean	Amplitude	Mean	Amplitude			
1	0	465	0	46.5	0	471.9	471.9
1	176.5	85.3	17.7	8.5	176.5	86.6	120.7
1	278.1	186.9	27.8	18.7	278.1	189.7	341.7
1	–176.5	85.3	–17.7	8.5	–176.5	86.6	86.6
1	–278.1	186.9	–27.8	18.7	–278.1	189.7	189.7

3.2. Analysis of the multiaxial out-of-phase stress history

After P. Romanowicz and B. Szybiński [30], let us consider the interesting case when stress existing in a crane wheel, which is in contact with a rail, is of the multiaxial type, and the stress components are shifted in phase during contact. The shear stress $\tau_{yz}(t)$ are shifted in phase in comparison with the normal stress components $\sigma_x(t)$, $\sigma_y(t)$ and $\sigma_z(t)$ – see Fig. 2. The same effect can be observed for the ball bearings [29]. The direct spectral method can be applied for finding the equivalent completely reversed stress for the one stress block. By one stress block, the variation of stresses during one rotation of wheel is understood (Fig. 2). For one stress block, two cycles are identified:

– for maximal values of normal stresses ($x/a = 0$) – zero-to-tension;
– for maximal values of shear stresses ($x/a = 1$) – completely reversed.

The detailed values of stress components used for the identification of completely reversed stress by the direct spectral method are given in Table 2.

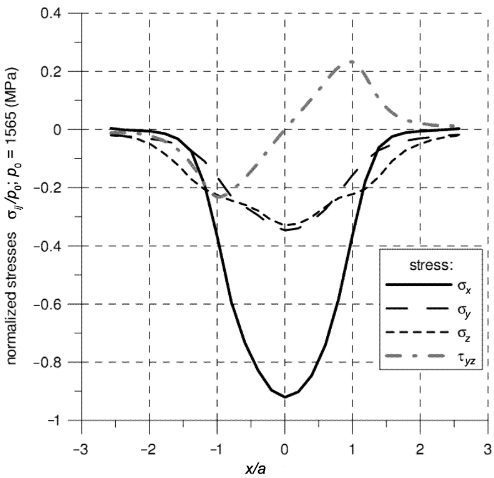

Fig. 2. Subsurface stress distribution on radius of Palmgren-Lundberg points for crane wheel; *a* – semi-axis of contact ellipse [30]

Table 2

Identified cycles for a crane wheel

Number of cycles	Normal stress [MPa]			Shear stress [MPa]			Equivalent mean stress [MPa]	Equivalent stress amplitude [MPa]	Equivalent completely reversed stress [MPa]
	Comp.	Mean	Ampl.	Comp.	Mean	Ampl.			
1	σ_x σ_y σ_z	–360 –133 –125	360 133 125	τ_{xy} τ_{yz} τ_{xz}	0 0 0	0 0 0	–618	231	231
1	σ_x σ_y σ_z	–579 –235 –344	0 0 0	τ_{xy} τ_{yz} τ_{xz}	0 0 0	0 376 0	–1158	651	651

The application of the other theories for the determination of the equivalent completely reversed stress leads to the values of equivalent completely reversed shear stress given in Tab. 3. After application of the von Mises relationship, the equivalent completely reversed normal stress given in Tab. 3 is estimated. The values estimated by the proposed direct spectral method are not far from those obtained by Papadopoulos 2, Crossland and energy methods, assuming that the ultimate stress for the material is equal to σ_u = 1250 MPa, and elasticity limit is equal to R_e = 1050 MPa.

Table 3

Comparison of equivalent completely reversed stress

Theory	Equivalent completely reversed shear stress [MPa]	Equivalent completely reversed (normal) stress [MPa]
Papadopoulos 1 [30]	471	816
Papadopoulos 2 [30]	373	646
Crossland [30]	386	669
energy [30]	376	651
direct spectral	–	651/231

4. Conclusions

Fatigue analysis of engineering problems, when stresses are of the multiaxial type, is not easy and there is no representative fatigue hypothesis to estimate it. Moreover, the results of analyses are different for the application some of theories.

The cases that were analyzed in this article make it possible to formulate the following conclusions:

– The direct spectral method seems to be an alternative approach for counting the stress cycles of the multiaxial type.
– The direct spectral method can be formulated for the cases when components of the stresses are in-phase or out-of phase.
– The natural applications of the direct spectral method in fatigue analysis are the cases of the single-modal (harmonic) and the bi-modal stress process in the frequency domain.

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