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PAWEŁ SZCZUREK*, WITOLD MAZGAJ**, AGNIESZKA BANACH**

INFLUENCE OF THE MAGNETIZATION DIRECTION ON POWER LOSSES IN TRANSFORMER STEEL SHEETS

WPŁYW KIERUNKU MAGNESOWANIA NA STRATY MOCY W BLACHACH TRANSFORMATOROWYCH

A b s t r a c t

Calculation of power losses in transformer steel sheets is still a significant problem. In some places of cores of a three-phase transformer the flux density is not parallel to the rolling direction; it especially refers to the so-called T-points of the transformer cores. In these places magnetization curves differ from the curve measured for the rolling direction. The paper deals with hysteresis losses, eddy current and excess losses and their dependences on the direction of magnetization processes. Analysis is performed for several frequencies of the magnetization process. Measured power losses of the selected transformer steel sheets are compared with results obtained on the basis of the analytical formulas.

Keywords: *eddy current losses, excess losses, hysteresis losses, transformer sheets*

S t r e s z c z e n i e

Wyznaczanie strat mocy blach transformatorowych jest nadal istotnym problemem. W pewnych obszarach rdzenia transformatorów trójfazowych linie pola magnetycznego nie są równoległe do kierunku walcowania; szczególnie odnosi się to do tak zwanych T-punktów rdzeni transformatorów. W tych obszarach charakterystyki namagnesowania różnią się od charakterystyki wyznaczonej do kierunku walcowania. Artykuł dotyczy strat histerezowych i wiroprądowych oraz ich zależność od kierunku procesu magnesowania. Analizę przeprowadzono dla kilku częstotliwości procesu magnesowania. Zmierzone straty mocy wybranych blach transformatorowych porównano z wynikami uzyskanymi na podstawie wzorów analitycznych.

Słowa kluczowe: blachy transformatorowe, straty histerezowe, straty wiroprądowe nadmiarowe

* M.Sc. Eng. Paweł Szczurek, Laboratory of Magnetic Measurements, Stalprodukt SA Bochnia.

** D.Sc. Ph.D. Eng. Witold Mazgaj, M.Sc. Eng. Agnieszka Banach, Institute of Electromechanical Energy Conversion, Faculty of Electrical and Computer Engineering, Cracow University of Technology.

1. Introduction

Magnetic circuits of transformers are often made from amorphous and nanocrystalline materials, particularly for low power rating transformers. However, it seems that for a long time, magnetic cores of medium and high power rating transformers will be constructed with the use of “classical” 3% Fe-Si transformer (grain-oriented) steel sheets, whose magnetic properties are continually improved. This refers to obtaining higher values of the flux densities as well as reduction of both hysteresis and eddy current losses.

Magnetic fields that occur in the transformer cores are usually considered as one-dimensional fields, and magnetization processes are treated as the axial magnetization. Lines of the magnetic fields are parallel to the rolling direction in the most part of the transformer cores. This note applies to low power rating transformers, whose cores are made usually of dynamo steel sheets, which have almost the same properties in any magnetization direction. For this aim, the E and U sheet shapes are cut out from these sheets. Thereby, the permeability is the same in each point of the given transformer core. However, the permeabilities of the dynamo sheets have lower values with respect the transformer sheets. In the cores which are constructed using the transformer sheets, the magnetic lines are almost always parallel to the rolling direction of these sheets. However, in core corners or in T-joint points (connections between columns and yokes) of medium and high power rating transformer, the magnetic lines have different directions with respect to the rolling direction. It is necessary to stress that around the T-joint points the magnetization has rotational character of varying degrees of ellipticity [1, 2].

The transformer steel sheets are produced as grain-oriented sheets and the Goss texture is their characteristic feature. It causes that these sheets are most easily magnetized along the rolling direction [3–6]; in other directions the magnetization properties are significantly worse. Therefore, this fact should be taken into account in predicting of the power losses in the transformer cores, especially when estimation of the losses refers to the corners or T-joint points of the three-phase transformer. It is worth underlining that lines of the magnetic field may not be parallel to the rolling direction in transformer columns in overload or short-circuit states due to the occurrence of a leakage flux. Grains and thus domains of the transformer steel sheets can have an area of several square centimeters in contrary to the dynamo steel sheets whose average size is in the range from 60 to 100 μm.

The total power losses in electrical steel sheets are a sum of the hysteresis losses, the “classical” eddy current losses, and the excess losses whose reason are the so-called domain eddy currents occurring in transformer sheets [5, 7–9]:

$$P_{\mathrm{tot}} = P_h + P_{ed} + P_{\mathrm{exc}} \tag{1}$$

where:

P_h – hysteresis losses,
P_{ed} – “classical” eddy current losses,
P_{exc} – excess losses.

Losses in steel sheets are determined per mass unit and they are called the specific power loss. These losses can be estimated with the use of some analytical formulas, however, these formulas do not taken into account the influence of the magnetization direction on the loss value. Despite the large number of scientific studies, the estimation of power losses in transformer sheets is not completely solved. In some cases, calculated hysteresis losses differ significantly in comparison to the measured hysteresis losses; it also concerns the total eddy current losses due to the occurrence of the excess losses caused by domain eddy currents [10–12]. Differences between measured and calculated losses may also occur when winding currents are distorted with respect to the sinusoidal shape [13, 14].

The purpose of this paper is a comprehensive discussion of the power losses occurring in the transformer steel sheets (especially their dependence on the magnetization direction) on the basis of the magnetic measurements performed by the authors. The total power losses were separated on the aforementioned loss components, and the measured losses were compared with values of the particular losses obtained using analytical formulas. The magnetic measurements were carried out for three different transformer sheets and have shown that the power losses determined for directions other the rolling direction are significantly higher than the losses in the rolling direction. Taking into account the magnetic field distribution in the core of a typical transformer, the considered directions on the sheet plane can have angles not bigger than 45° with respect to the rolling direction. The magnetic measurements and the loss analysis were carried out for following transformer sheets: M120-27S, M110-23S, and M120-30S (sheets were received from several manufacturers).

2. Hysteresis losses in transformer steel sheets

The hysteresis power losses are usually estimated using the well-known Steinmetz formula or the Richter formula [3, 15]:

$$P_h = \eta\, f\, B_m^p \tag{2}$$

where:

- η – constant whose value depends on the given electrical steel sheets,
- f – frequency of magnetic field changes,
- B_m – maximum value of the flux density during the magnetization process,
- p – exponent which is equal to 1.6 in the Steinmetz formula or 2.0 in the Richter formula.

The coefficient η depends on the given electrical steel sheet, so the same coefficient value should not be used for any transformer steel sheet. Determination of the value of the exponent p in the formula (2) is a separate problem, and the value of this exponent is usually taken between 1.6 and to 2.0.

The hysteresis losses occurring in the transformer steel sheets are usually determined for two typical value of the flux density 1.5 T and 1.7 T. The hysteresis loops measured by the authors along the rolling direction for these two values of the flux density are shown in Fig. 1 and Fig. 2. Due to occurrence of the Goss texture, the hysteresis loops measured along other directions differ significantly in comparison to the loops presented in Fig. 1. The explanation of this phenomena is widely presented in [2, 16]. In a demagnetization state, domains whose magnetization vectors are parallel to the rolling direction occur mainly in the transformer sheets. When the magnetic field strength increases in the direction perpendicular to the rolling direction, the domains which form 180-degree walls transform into domains which form the 90-degree walls. The magnetization vectors of new domains are parallel to the easy magnetization axes [010] and [001] of iron crystals; these axes are inclined to the transformer sheet plane at the angle of 45°. During this process the resultant flux density of the whole sheet sample increases relatively slowly and it begins to increase faster after the end of this process. For example, Figure 2a shows the hysteresis loops for three directions on the sheet plane measured for the transformer sheet M120-27S. These loops refer to the flux density 1.2 T, because above this flux density value the domain wall motions do not occur. Values of the hysteresis losses depend not only on the value of the flux density, but also on the direction of the magnetization process that influences the amount of these losses; this problem was considered among others in [17, 18].

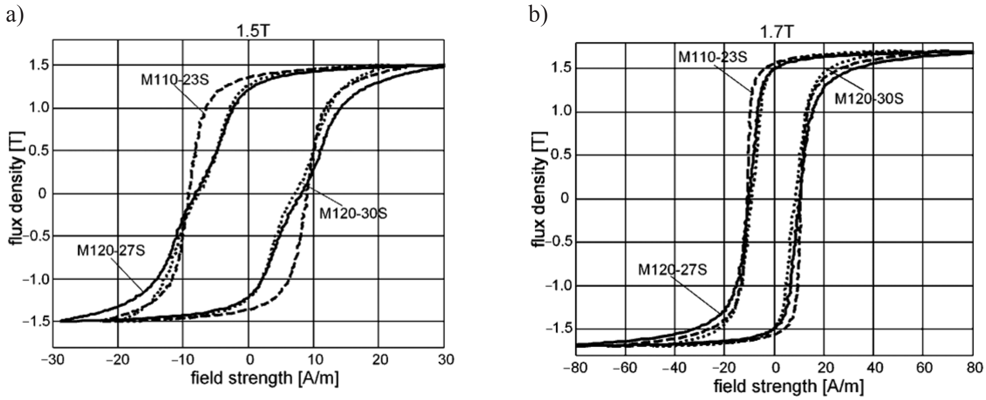

Fig. 1. Hysteresis loops along the rolling direction for the maximum flux density: a) 1.5 T, b) 1.7 T

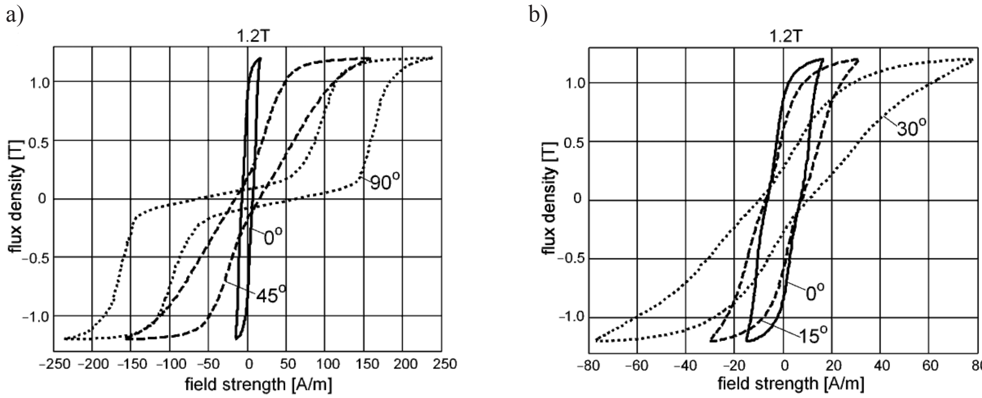

Fig. 2. Hysteresis loops of the transformer sheets M120-27S for the maximum flux density 1.2 T: a) 0° (rolling direction), 45° and 90°, b) 0°,15° and 30°

As it was earlier mentioned, this is caused by occurrence of the Goss textures in the transformer sheets. The hysteresis losses for an assumed maximum value of the flux density is equal to the surface of the measured hysteresis loop. Magnetic measurements carried out by means of the Epstein frame have shown that the hysteresis power losses depend significantly on the magnetization direction.

The hysteresis losses were determined for each selected magnetization direction on the basis of the static hysteresis loops. These losses measured for angles greater than 15° with respect to the rolling direction can be higher even several times than analogous losses estimated for the rolling direction. For example, Fig. 3a presents hysteresis specific power losses for the sheet M120-27S determined for selected directions on the sheet plane. In turn, the specific power losses of the sheet M110-23S measured for the rolling direction and for three angles 15°, 30° and 45° are included in Fig. 3b. The hysteresis losses for flux densities greater than 1.5 T are not shown in Fig. 3a due to significant saturations of tested transformer sheets and possible measured errors during magnetization along the angles greater than 30° with respect to the rolling direction. It is worth underlining that the Steinmetz formula does not take into account the angle of the magnetization in the estimation of the hysteresis losses. For comparison, values of the hysteresis specific power loss determined on the basis of Steinmetz formula (2) are shown in Fig. 3b. Calculations were performed with the assumption that the parameter η was equal to 0.0032 and the exponent p was equal to 1.6 for the flux density in the range from 0.2 T to 1.3 T and $p = 1.8$ for the flux density higher than 1.3 T.

a) b)

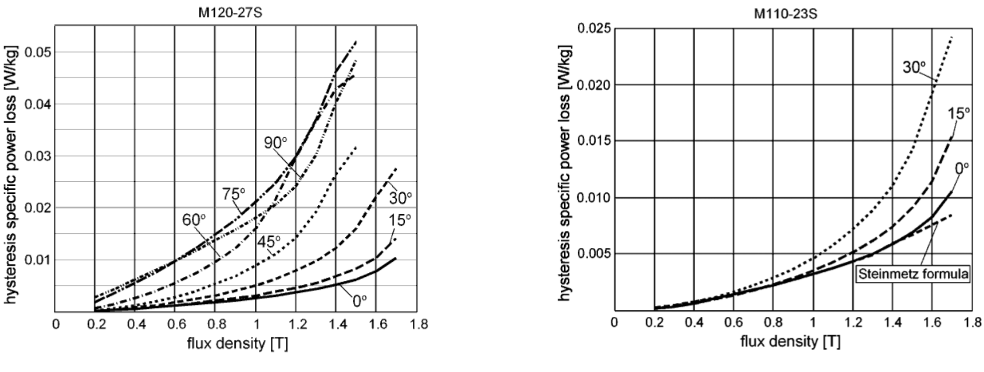

Fig. 3. Hysteresis specific power loss of the transformer sheets: a) M120-27S, b) M110-23S

Dependences of the hysteresis losses on the flux density and the magnetization angle for other two transformer sheets are similar as for the sheet M120-27S. The bigger the angle between the magnetization and the rolling direction is, the faster increases the value of the hysteresis losses. Due to only four measurement points, these dependences were approximated by means of the third degree polynomials; it allows the estimation of the hysteresis power losses for any angle between 0° (the rolling direction) and 45°.

The coercive forces, remanences, characteristic values of the field strength and specific hysteresis power losses of three considered transformer sheets are given in Table 1. These values are presented for four magnetization direction on the sheet plane. The hysteresis losses determined for other direction than the rolling direction are significantly higher. Thus, the estimation of these losses requires additional magnetic measurements. In some cases. it may be troublesome and even impossible when the width of the tested transformer sheet is smaller than 280 mm in view of tests utilizing the Epstein frame.

Table 1

Magnetic parameters and specific hysteresis loss

Type of sheet		**M120-27S**			**M110-23S**			**M120-30S**		
Flux density [T]		1.0	1.5	1.7	1.0	1.5	1.7	1.0	1.5	1.7
Coercive force [A/m]	0°	6	8	10	7	9	11	6	7	9
	15°	6	9	12	7	10	12	6	8	10
	30°	9	13	14	8	11	13	7	11	13
	45°	13	17	19	12	16	17	11	16	–
Remanence [T]	0°	0.64	1.21	1.49	0.85	1.36	1.56	0.72	1.24	1.50
	15°	0.44	0.89	1.20	0.50	1.03	1.27	0.51	0.92	1.17
	30°	0.21	0.50	0.65	0.29	0.88	0.98	0.17	0.38	0.61
	45°	0.13	0.24	0.27	0.24	0.43	0.50	0.15	0.30	–
Field strength [A/m]	0°	13	30	90	11	24	69	11	21	50
	15°	23	59	186	23	63	228	21	52	129
	30°	58	225	2500	44	189	2767	54	164	–
	45°	106	3729	12133	114	4025	12312	116	300	–
Hysteresis loss [W/kg]	0°	0.0027	0.0062	0.0104	0.0032	0.0069	0.0106	0.0027	0.0056	0.0083
	15°	0.0032	0.0083	0.0141	0.0035	0.0091	0.0154	0.0032	0.0076	0.0120
	30°	0.0051	0.0161	0.0276	0.0047	0.0142	0.0242	0.0046	0.0139	0.0250
	45°	0.0089	0.0317	0.0283	0.0086	0.0280	0.0234	0.0082	0.0294	–

The magnetization directions in individual points of the T-joint areas are various; so it would be desirable to perform for the given transformer sheet appropriate magnetic measurements, for example, every 5 degrees. Figure 4 shows the ratio (determined by the authors) between the hysteresis losses measured along the assumed magnetization direction and the hysteresis losses determined for the rolling direction (0°) for the transformer sheets M120-27S and M110-23S. Dependences of this ratio on the magnetization angle for the third transformer sheets M120-30S are similar to the curves presented in Figure 4a. It is necessary to stress that studies have been performed only for three transformer sheets which have different thickness. It seems that research carried out for several transformer sheets with the same thickness would be desirable in further studies on the magnetization processes in these sheets.

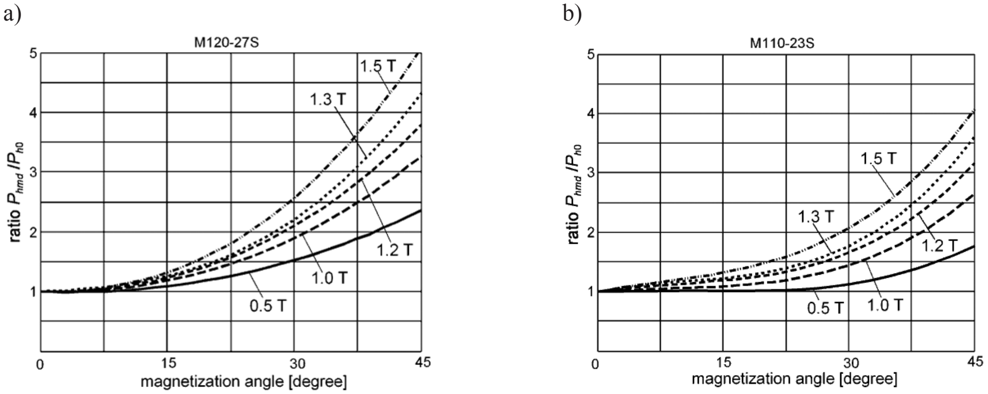

Fig. 4. Ratio between the hysteresis losses P_{hmd} measured along the assumed magnetization direction and the hysteresis losses P_{h0} measured for the rolling direction (0°): a) M120-27S, b) M110-23S

3. Power losses caused by eddy currents

It is well known that measured eddy current losses of the electrical steel sheets are significantly higher than the value of these losses calculated with the use of the analytical formulas. As it is proposed in [11], the measured eddy currents losses for the assumed magnetization direction are differences between the total power losses and the hysteresis losses for the particular frequencies at which magnetic measurements were carried out. Values of these losses for the chosen conditions of the magnetization process are presented in Table 2.

The eddy current losses are usually estimated using the formula [3, 9]:

$$P_{ed} = \frac{\pi^2 \, \sigma \, d^2 \, f^2 \, B_m^2}{6} \tag{3}$$

where:

σ – conductivity of the transformer sheet,

d – thickness of the given transformer sheet.

This relationship was formulated with the assumption that the magnetic permeability has a constant value in a wide range of the flux density changes. However, this assumption can be acceptable when the flux density is not higher than 1.5 T for the rolling direction and not higher than about 1.2 T, when the angle of the magnetization direction is lesser than 45°. It is worth underlining that the value of the eddy current losses calculated with the use of the formula (3) depends also on the accuracy of the parameter determination; this remark refers especially to the value of the conductivity of the tested transformer steel sheet. The conductivities of three tested transformer sheets were determined with the use of the Thomson bridge and by means of the technical methods. Average values of these conductivities are presented in Table 2. Similarly as the Steinmetz formula (2) the relationship (3) does not take into account the angle between magnetization direction and the rolling direction.

As earlier mentioned, the measured eddy current losses are bigger than the eddy current losses calculated with the use of the formula (3). The difference between these losses is treated as the excess losses which are caused by eddy microcurrents occurring around moving domain walls in the transformer sheets. The excess losses occur also in dynamo steel sheets [10, 19], but the share of these losses is relatively smaller in comparison with the transformer sheets.

Measurements performed by the authors for three tested transformer sheets have shown that the excess losses depend significantly on the magnetization direction. The bigger the angle between the magnetization direction and the rolling direction is, the higher are these losses. Figure 5 shows dependences of these losses on the frequency and the magnetization direction for the transformer sheet M120-77S; analogous dependences for two other sheets have similar character.

The estimation of the excess losses is still a valid problem. The first proposal of the estimation of the excess losses in the transformer sheets was proposed by Pry and Bean [12]. In turn, G. Bertotti has proposed a certain statistical method allowing the estimation the excess losses [1, 2, 20]. He has assumed that domain wall movements during the magnetization process consists of random jumps in iron crystals of the transformer sheet. He has treated parts of these walls as *n* certain magnetic objects which are active simultaneously and behaviour of these objects is associated with the occurrence of a certain magnetic strength which is defined as:

$$H_{\mathrm{exc}} = \frac{P_{\mathrm{exc}}}{4fB_m} \tag{4}$$

Determining of the number of the magnetic objects is made according to [11] for the rolling direction and for two frequencies 50 Hz and 100 Hz. The dependences $n = f(H_{\mathrm{exc}})$ determined for the rolling direction are presented in Figure 6. Different ranges of field strength changes are the result of various values of the excess losses in the assumed magnetization conditions.

The dependences $n = f(H_{\mathrm{exc}})$ are more complex and the number *n* of the magnetic objects decreases (especially for higher values of the field strength H_{exc}) when the magnetization direction forms an angle greater than about 15° with respect to the rolling direction; it significantly complicates the estimation of the excess losses based on the Bertotti's approach. It is worth underlining that the number *n* of the magnetic objects depends significantly on the thickness of the given transformer sheet.

For field strengths higher than about 3 A/m for 50 Hz and about 6 A/m for 100 Hz, what corresponds to the flux density value of about 0.5 T, it can be assumed that the amount of *n* magnetic objects can be approximated as follows [10, 11]:

$$n = n_0 + \frac{H_{\mathrm{exc}}}{V_0} \tag{5}$$

where:

n_0, V_0 – parameters characteristic for the given transformer sheet which are estimated on the basis of dependences $n = f(H_{\mathrm{exc}})$ determined for the assumed frequency.

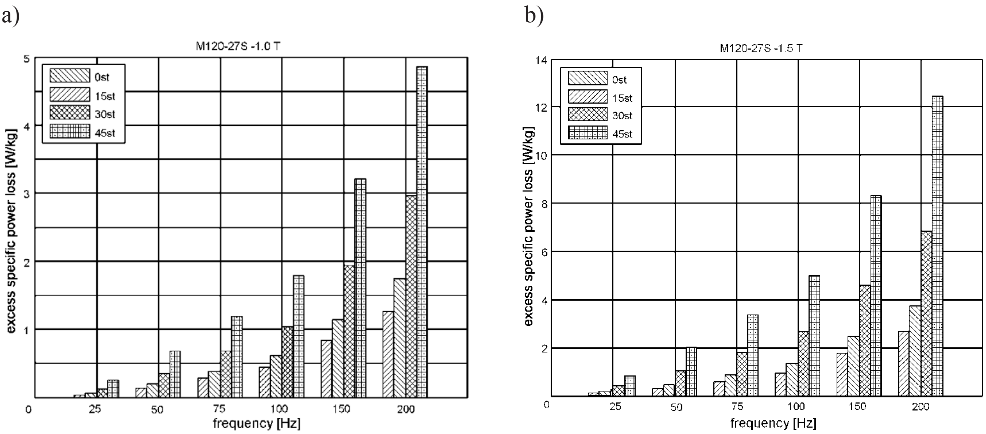

Fig. 5. Excess power losses of the sheet M120-27S measured for the maximum flux density:
a) 1.0 T, b) 1.5 T

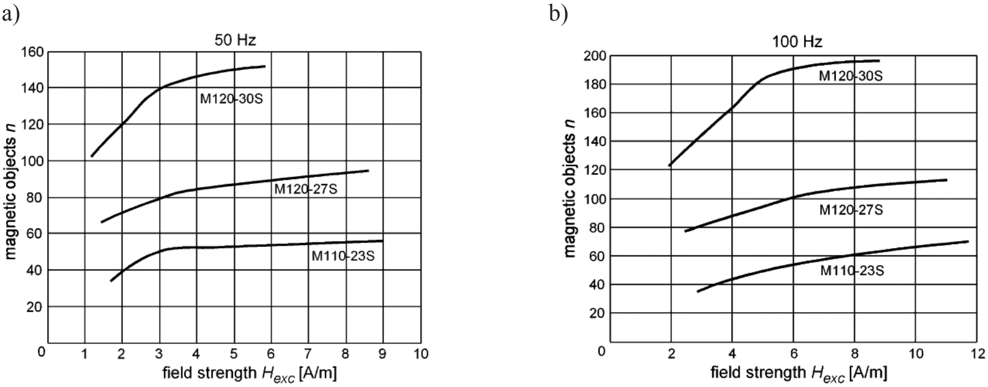

Fig. 6. Number of magnetic objects as dependences on the field strength H_{exc}: a) 50 Hz, b) 100 Hz

According to the Bertotti's approach, the excess losses in the transformer steel sheets can be estimated using the following formula:

$$P_{\mathrm{exc}} = 2B_m f\left(\sqrt{16\sigma\, G\, S\, V_0 B_m f + \left(n_0 V_0\right)^2} - n_0 V_0\right) \tag{6}$$

where:

- G – constant value which is equal to 0.1356,
- S – area of the cross-section of the tested sheet sample.

Values of the excess losses estimated for the frequency 50 Hz and 100 Hz and for three values of the flux density are included in Table 2. In most cases, the relative error of the estimated excess losses is lesser than 10 percent with respect to the measured values.

Table 2

Conductivities and specific eddy current loss in W/kg

Type of sheet				**M120-27S**			**M110-23S**			**M120-30S**		
Conductivity $[(\Omega \cdot m)^{-1}] \cdot 10^6$				2.13			1.64			2.38		
Flux density [T]				1.0	1.5	1.7	1.0	1.5	1.7	1.0	1.5	1.7
Eddy current losses 100 Hz 0.360			50 Hz	0.090	0.202	0.259	0.041	0.092	0.118	0.115	0.259	0.333
			0.808	1.036	0.164	0.368	0.472	0.460	1.036	1.332		
Excess losses	0°	meas.	50 Hz	0.140	0.295	0.385	0.136	0.288	0.409	0.104	0.222	0.260
		estim.	50 Hz	0.139	0.296	0.373	0.144	0.310	0.392	0.106	0.226	0.285
		meas.	100 Hz	0.448	0.948	1.159	0.464	0.950	1.229	0.310	0.664	0.775
		estim.	100 Hz	0.410	0.852	1.066	0.499	0.982	1.207	0.326	0.695	0.876
	15° measured 100 Hz		50 Hz	0.203	0.474	0.681	0.168	0.456	0.734	0.187	0.424	0.613
			0.624	1.362	1.862	0.548	1.359	2.002	0.551	1.188	1.635	
	30° measured 100 Hz		50 Hz	0.365	1.049	1.349	0.252	0.816	1.361	0.310	0.889	1.585
			1.051	2.663	4.094	0.783	2.205	3.501	0.899	2.317	3.906	
	45° measured 100 Hz		50 Hz	0.678	2.039	1.924	0.554	1.831	1.401	0.638	2.088	1.998
			1.796	5.011	4.989	1.536	4.555	3.676	1.671	4.954	4.304	

4. Conclusions

It has been known that the power losses in the transformer steel sheets depend on the magnetization direction but research carried out for three different transformer sheets has allowed us to assess the quantitative differences between the losses for any magnetization direction and the power losses determined for the rolling direction. This remark refers to both the hysteresis losses and the eddy current losses. The reason of the hysteresis loss increase with the angle between the magnetization direction and the rolling direction is the presence of both the Goss texture and the quite complex process of the domain structure transformation for directions that significantly differ with respect to the rolling direction. It should be stressed once again that the Steinmetz formula does not take into account other magnetization directions than the rolling direction. It is desirable in future research to try to determine the parameters occurring in the Steinmetz formula as functions of the magnetization direction. However, this requires to carry out appropriate magnetic measurements for higher number of tested transformer sheets.

The dependence of the total eddy current losses on the magnetization direction is qualitatively similar to the case of the hysteresis losses. However, the “classical” eddy current losses are calculated without taking into account the microcurrents flowing around the moving domain walls, and these losses do not depend on the magnetization direction. Thus, it can be assumed that the excess losses have significant meaning and their values depend not only on the frequency and the flux density but the magnetization direction influences the excess losses significantly. It is worth underlining that the Bertotti’s approach allows us to estimate the excess losses but the use of this method requires performance of the appropriate magnetic measurements of the given transformer sheet. It can be concluded that in further research the emphasis should be placed on methods allowing the estimation of the excess losses.

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