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ANN KAPRANOVA, ANTON LEBEDEV, SERGEY SOLOPOV*,
ALEXANDER MELTSER**

THE APPLICATION PROCESS OF THE ORNSTEIN-ULENBEK TO THE FORMATION OF CAVITATION BUBBLES

ZASTOSOWANIE PROCESU ORNSTEIN'A-ULENBEK'A DO OPISU POWSTAWANIA PĘCHERZY KAWITACYJNYCH

A b s t r a c t

In this paper we propose the process of formation of cavitation bubbles in the pilot valve, considered as the steady and homogeneous Markov process.

Keywords: cavitation, formation of bubbles, the Fokker-Planck equation

S t r e s z c z e n i e

W pracy przedstawiono proces powstawania pęcherzy kawitacyjnych w zaworze sterującym jako stabilny i homogeniczny proces Markowa.

Słowa kluczowe: kawitacja, powstawanie pęcherzy, równanie Fokker'a-Planck'a

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* Prof. DSc. Ann B. Kapranova, Associate Professor DSc. Anton E. Lebedev, MSc Sergey A. Solopov, Mechanical engineering Department, Yaroslavl State Technical University.
** General Director Alexander M. Meltser, ZAO NPO «Regulyator», Yaroslavl.

1. Introduction

The use of regulatory bodies in piping systems allows to perform different technological operations – mixing of liquid streams, maintaining pressure and preventing reverse leakage, etc. The success of the design or modernisation of the elements of the valves depends on the effectiveness of the chosen method of combating undesirable effects of cavitation [1]. To its initial stage in the valves is the formation of bubbles filled with gas and a condensable vapour, arising due to tearing of the liquid in the case of flow in conditions of sharp falling pressure in the flow path of the device.

Application of the stochastic approach for the description of the initial stage of hydrodynamic cavitation as a rule boils down to one of three types of models of education of the nucleons (steam germ) in a liquid environment. In the absence of impurities, an equitable model of homogeneous type [2, 3], and in the presence of suspended particles on or near the surfaces with cracks – heterogeneous type [4, 5]. In addition, the known modifications of the first method of modelling the introduction of an additional coefficient for the number of Gibbs [6], which takes into account the effect of heterogeneity of the environment. However, despite known attempts, the postulation of the relevant law of the distribution of heterogeneous nuclei [7], as shown by the analysis of literary sources [8], the question of the formation of the distribution functions of the cavitation bubbles according to their sizes remains open.

2. The application of the Fokker-Planck equation for the process of formation of cavitation bubbles

In this paper, the process of formation of cavitation bubbles in the regulatory valve, considered as steady and homogeneous Markov process, is presented. In the case when the description of the probability distribution of events, the stationarity implies the possibility of shifting the origin of time, and the uniformity is dependent on time intervals. Note that the main feature of the given random process is that the transition to the subsequent state of the system is determined by its instantaneous condition.

Modelling the process of formation of cavitation bubbles is a stochastic way, including in the formalism of a process of Ornstein-Ulenbek when this bubble macro system is energetically closed from the Gibbs ensemble. Microscopic parameters of this ensemble can serve as the selected coordinates and impulses of Hamilton for each of the similar subsystems, for example, cavitation bubbles are spherical. In the process, the principle of maximum entropy runs, which is in its growth for closed macrosystem and then saving at equilibrium condition [9, 10]. For the process of bubble formation in the conditions of hydrodynamic cavitation will set in the phase space a set of variables, consisting of two variables: the radius of the spherical bubble r, and the velocity of its centre of mass v. The Fokker-Planck equation with drift and diffusion terms of the types corresponding to the process of the Ornstein-Ulenbek is recorded relative to the equilibrium distribution function condition generated spherical cavitation bubbles

$$\frac{\partial F(t,r,v)}{\partial t} = D_1 \frac{\partial^2 F}{\partial r^2} + D_2 \frac{\partial^2 F}{\partial v^2} + \varepsilon_1 \frac{\partial (rF)}{\partial r} + \varepsilon_2 \frac{\partial (vF)}{\partial v} \tag{1}$$

where in the set of phase space coordinates *r* and velocity *v*

D_1, D_2 – the diffusion coefficients,
ε_1, ε_2 – the frequency of bubble formation,
F – the equilibrium distribution function of the system condition.

3. Description of stochastic cavitation energy sphere

It is believed that when cavities formed after the rupture of liquids under conditions of rapid pressure drop are filled with gas and a condensable vapour, there is an internal vortex motion in the cavitation bubbles. Then, according to [10], the element dΩ of the phase space is defined as

$$\mathrm{d}\Omega = v\,\mathrm{d}v\,\mathrm{d}r \tag{2}$$

Description of stochastic cavitation bubble energy *E* with respect to its radius *r* and the speed of the centre of mass of *v* implies taking into account the kinetic energies of motion of the bubble in the liquid flow and the internal motion of the system of gas-steam, the energies of formation of the free surface and cavity fill energy due to the condensation of vapour, energy of the hydrodynamic interaction of bubble and liquid. Consequently, introducing coefficients c_1 and c_2

$$c_1 = q_1 \cdot \bar{r}^3 + q_2 \cdot \bar{r}^{-1} \tag{3}$$

$$c_2 = \frac{5 \cdot q_1 \cdot M^2 \cdot \bar{r}^{-7}}{8} + q_3 \cdot \bar{r} + q_4 \tag{4}$$

depending on the structural and regime parameters of flow area control valve and physico-mechanical characteristics of the transported medium, the simulated stochastic energy takes the form

$$E = c_1 \cdot r^2 + c_2 \cdot v^2 \tag{5}$$

where

$\bar{r}$ – the average value of the radius of the bubble,
M – the random component of angular momentum.

Expression (3) and (4) contain the constants q_i, $i = \overline{1,4}$ when asked by the formulas

$$q_1 = \frac{2 \cdot \pi \cdot (\alpha_g \cdot \rho_g + \alpha_s \cdot \rho_s)}{3} \tag{6}$$

$$q_2 = \frac{k_\varsigma \cdot \varsigma_{12}\rho_1}{4}, \quad q_3 = \frac{8 \cdot \pi \cdot P_s}{3} \tag{7}$$

$$q_4 = 4 \cdot \pi \cdot \sigma \tag{8}$$

where

α_g, α_s – the volume fraction of filling bladder with gas and steam,

ρ_g, ρ_s, ρ_l – the density of the gas, steam and liquids,

k_ζ – the parameter of proportionality in the expression for the energy of hydrodynamic interaction $E_{hi} = k_\zeta \cdot \Delta P/(2 \cdot \mathrm{r})$ when calculating the pressure drop in the valve by the formula of Weisbach $\Delta P = \zeta_{12} \cdot \rho_i \cdot \nu^2/2$,

ζ_{12} – the coefficient of hydraulic resistance of the transition zone for movement of liquid medium, which corresponds to the limit of variation of the Reynolds number $10 < Re < 10^4$, according to [1],

P_s – the saturated vapour pressure,

σ – e surface tension of the liquid.

Note that the value of ζ_{12} is determined by the sum of the coefficients of hydraulic resistance for two zones of laminar and turbulent flows of fluids in the flow part of the valve, which are calculated according to the principle of superpositions of local losses taking into account design and operating parameters of the regulating device and the physical properties of the surfaces of the channels. For example, such parameters include: the diameters of conditional passes and various sections of the spool, the length of the straight channels and the throttle passages, a square cross-section entrance of the (conditional, before and after sudden expansion), gap width of the throttle passage, the degree of roughness of the internal surfaces of the valve, etc.

4. Representation of the kinetic equation of the equilibrium process in the formalism of stochastic energy of the cavitation bubble

The representation (1) of the kinetic equation for $F\,(t, r, v)$ is the sought distribution function over the States relative to one set of variables (time, radius, velocity the centre of mass of the bubble) to the Fokker-Planck equation with a different set of (stochastic time and energy bubble from the expression (5)) when $\tilde{F}\,(t, E)$

$$\frac{\partial \tilde{F}(t,E)}{\partial t} = \frac{\mathrm{d}E_0}{\mathrm{d}t} \cdot \left(E \cdot \frac{\partial^2 \tilde{F}}{\partial E^2} + \frac{\partial \tilde{F}}{\partial E} \right) + \frac{1}{E_0} \cdot \frac{\mathrm{d}E_0}{\mathrm{d}t} \cdot \left(E \cdot \frac{\partial \tilde{F}}{\partial E} + \tilde{F} \right) \tag{9}$$

where

E_0 – the energy of the system at the time of stochastic according to [9, 10].

In particular, during the transition from the representation (1) to the form (9) for a given kinetic equation the equilibrium state of the system, it is assumed that the ratios are

$$c_1 \cdot D_1 = c_2 \cdot D_2 \tag{10}$$

$$\varepsilon_1 = \varepsilon_2 \tag{11}$$

where

c_1, c_2 – the coefficients of the expressions (3).

Then taking into account (5) the energy of the system at the time of stochastic E_0 and its flow $\mathrm{d}E_0/\mathrm{d}t$ when $j = \overline{1,2}$ can be specified in the form

$$E_0 = (2 \cdot \varepsilon_j)^{-1} \frac{\mathrm{d}E_0}{\mathrm{d}t} \tag{12}$$

$$\frac{\mathrm{d}E_0}{\mathrm{d}t} = 4 \cdot c_j \cdot D_j \tag{13}$$

Therefore, taking into account the expressions (3) and (12), we obtain

$$\varepsilon_1 = \varepsilon_2 = (2 \cdot E_0)^{-1} \frac{\mathrm{d}E_0}{\mathrm{d}t} \tag{14}$$

$$D_1 = \left[4 \cdot \left(q_1 \cdot \bar{r}^{3} + q_2 \cdot \bar{r}^{-1}\right)\right]^{-1} \frac{\mathrm{d}E_0}{\mathrm{d}t} \tag{15}$$

$$D_2 = \left[4 \cdot \left(\frac{5 \cdot q_1 \cdot M^2 \cdot \bar{r}^{-7}}{8} + q_3 \cdot \bar{r} + q_4\right)\right]^{-1} \frac{\mathrm{d}E_0}{\mathrm{d}t} \tag{16}$$

where

q_i – constants defined by formulas (6 ÷ 8) when $i = \overline{1,4}$.

The Fokker-Planck equation (6) has a reduced representation [9]

$$\frac{\partial \tilde{F}(t,E)}{\partial t} = \frac{\mathrm{d}E_0}{\mathrm{d}t} \left[\frac{\partial}{\partial E} \left(E \frac{\partial \tilde{F}}{\partial E} \right) + \frac{1}{E_0} \frac{\partial (E\tilde{F})}{\partial E} \right] \tag{17}$$

with the solution according to the described way of modelling a stochastic energy of the cavitation bubble (5) in the form of the representation of the equilibrium distribution function on the system condition

$$\tilde{F}(t, E(t,r,v)) = A \cdot \exp\left[-\frac{E(t,r,v)}{E_0} \right] \tag{18}$$

where

A – normalisation constant, defined with an input element of the phase space $\mathrm{d}\Omega$ according to (2) from the relation

$$\int_{\Omega} \tilde{F}(t, E(t, r, v))\mathrm{d}\Omega = 1 \tag{19}$$

5. The main results and conclusions

Thus, this operation of transition from the representation of the kinetic equation (1) to the form (9) for the equilibrium state of the system leads to the relations (14 ÷ 16) for connection between the diffusion coefficients D_1, D_2, the frequency of formation of cavitation bubbles ε_1, ε_2 and the energy of the system at the time of randomisation E_0.

Note that the General form of the obtained expressions (12) for the energy characteristics of the process of bubble formation coincides with the given parameters for the process of disintegration of the liquid jet from the work [10]. The equilibrium distribution function for the states of the system in the form (18) with normalisation of the ratio (19) and formed of stochastic energy of the cavitation bubble (5) can be used to determine the differential distribution function of bubbles on radiuses.

References

[1] Arzumanov E. S., *Cavitation in local hydraulic resistances*, Energy, Moscow, 1978.

[2] Frenkel J. I., *Kinetic theory of liquids*, Nauka, Leningrad 1959.

[3] Lienhard J H., Karimi A., *Homogeneous nucleation and the spinodal line*, Journal of Heat Transfer, vol. 103(1), 1981, 61-64.

[4] Shin T .S., Jones O. C., *Nucleation and flashing in nozzles-1. A distributed nucleation model*, Int. J. Multiphase Flow, vol. 19(6), 1993, 943-964.

[5] Hsu Y. Y., *On the size range of active nucleation cavities on a heating surface*, Journal of Heat Transfer, vol. 94, 1962, 207–212.

[6] Ellas E., Chambre P. L., *Bubble transport in flashing flow*, Int J. Multiphase Flow, vol. 26, 2000, 191-206.

[7] Kumzerova E. J., Schmidt A. A., *Numerical modeling of nucleation and bubble dynamics with the rapid loss of fluid pressure*, Journal of Engineering. Physics, vol. 2(7), 2002, 36-40.

[8] Kapranova A. B., Lebedev A. E., Melzer A. M., Nekludov S. V., Serov E. M., *On the methods of modeling the main stages of development of hydrodynamic cavitation*, Fundamental Research, vol. 3(2), 2016, 268-273.

[9] Klimontovich Y. L., *Turbulent motion and structure of Chaos: A new approach to the statistical theory of open systems*, LENAND, Moscow 2014.

[10] Zaitsev A. I., Bytev D. O., *Impact processes in the dispersion-film systems*, Chemistry, Moscow 1994.