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NUMERICAL MODELLING OF PULSE WAVE PROPAGATION IN THE HUMAN THORACIC AORTA

MODELOWANIE NUMERYCZNE PROPAGACJI FALI TĘTNA W PIERSIOWYM ODCINKU AORTY

A b s t r a c t

In this paper, a solution to the numerical simulation of blood flow in the human thoracic aorta is presented. *In vivo* measurements of pulsewave waveforms were performed and used as the initial condition for numerical calculations. Equations resulting from the analysis of the general mass and momentum balance for blood flow were used for the description of aortal haemodynamics. The system of differential equations has been solved with the method of characteristics, which is often used in hydrodynamic problems. Numerical simulations were employed for the cases of three people of differing ages. As a result, time-dependent profiles of blood pressure and velocity as well as deformation of the arterial wall were determined.

Keywords: *blood flow; pulse wave propagation; haemodynamics*

Streszczenie

W pracy dokonano numerycznej symulacji przepływu krwi w odcinku piersiowym aorty człowieka. Jako warunek początkowy w obliczeniach numerycznych wykorzystano wzięte z pomiarów *in vivo* przebiegi fali tętna. Równania wynikające z analizy bilansu masy i pędu dla przepływu krwi oraz równowagi dla materiału ścianki zostały użyte do opisu hemodynamiki krwi odcinku aorty. Układ równań różniczkowych rozwiązany został za pomocą metody charakterystyk. Przeprowadzono symulacje numeryczne dla przypadków trzech osób w różnym wieku. W wyniku przeprowadzonych obliczeń numerycznych wyznaczono zmienne w czasie profile prędkości i ciśnienia krwi oraz deformację ścianek odcinka aorty.

Słowa kluczowe: przepływ krwi, propagacja fali tętna; hemodynamika

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1. Introduction

Blood flow in major arteries is caused mainly by cardiac function. The total energy of systole consists of the elastic potential energy required to deform the arterial walls, the kinetic energy of blood and the potential energy of position required to overcome the force of gravity [4].

In the case of heart and blood vessel haemodynamics, measurements of pressure changes and blood velocity are most commonly used and, recently, the results of numerical simulations and theoretical considerations have been also employed.

A description of many works on the numerical simulation of blood flow in the major arteries, assuming both rigid and resiliently deformable walls, is shown in [6]. The results of numerical calculation on the self-excited oscillation velocity and pressure in selected sections of the aorta, taking into account the local constriction, are shown in [8]. In [3] and [7], the flow of blood as a Newtonian fluid in the thoracic aorta has been analysed, assuming that the wall has the characteristics of linear elasticity. The known dependence of blood flow on time [3] or the corresponding pressure changes in time [7] have been used as the initial condition for numerical calculations.

The purpose of this article is to develop a non-linear, one-dimensional model of pulse wave propagation in the human thoracic section of the aorta. The model includes partial differential equations resulting from the balance of mass and momentum for the fluid area and the balance equation for the area of the vessel'swall. Pressure changes over time, measured *in vivo* using an applanation tonometer (SphygmoCor, AtCor Medical), were used as the initial condition in the calculations. In the performed numerical calculations, the average blood velocity during systole was taken into account. Based on the determined velocity fields and blood pressure, the elastic deformation of the thoracic aorta wall has also been determined.

2. Formulation of the problem

To describe the haemodynamics of the human thoracic aorta (Fig. 1a),methods known in the field of technical hydrodynamics have been used. Schematic diagrams of the considered flow are shown in Figures1a and 1b. The following assumptions have been taken into account:

- the blood vessel has the shape of a circular cylinder with varying cross-sectional area $S = S(s, t)$, constant wall thickness b, and constant length L;
- in a vessel filled with blood, the flow is unsteady and laminar under isothermal conditions;
- the vessel wall is undergoing substantial elastic deformation only in a radial direction;
- blood filling the vessel is a Newtonian fluid with a constant viscosity and a slight compressibility.

Including the above assumptions, the considered blood flow in the thoracic aorta is described by the following system of equations:

$$\frac{d\upsilon}{dt} + \frac{1}{\rho}\frac{\partial p}{\partial s} + g\frac{dz}{ds} + \frac{\lambda}{2D}\upsilon|\upsilon| = 0 \tag{1}$$

$$a^2 \frac{\partial \upsilon}{\partial s} + \frac{1}{\rho}\frac{dp}{ds} = 0 \tag{2}$$

where:

$$\frac{d\upsilon}{dt} = \frac{\partial \upsilon}{\partial t} + \upsilon \frac{\partial \upsilon}{\partial s} \tag{3}$$

$$\frac{dp}{dt} = \frac{\partial p}{\partial t} + p \frac{\partial p}{\partial s} \tag{4}$$

υ is the velocity of blood, g is the gravitational acceleration, and λ is the friction coefficient.

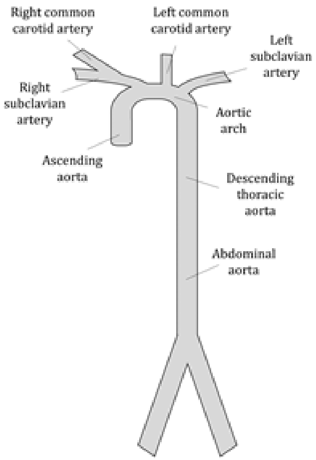

Fig. 1a. Human aorta

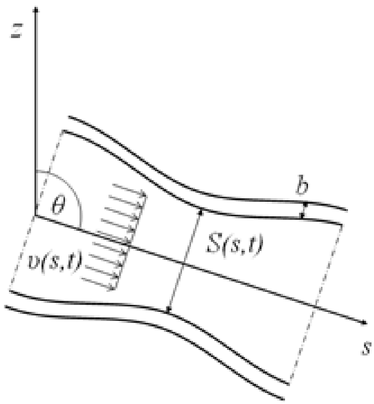

Fig. 1b. Coordinate system

Equations (1) and (2) result from the analysis of the general mass and momentum balance for blood flow in the considered region of the aorta. The equilibrium condition for the elastic vessel wall has also been included in the above equations. The velocity of the pressure wave (sound) propagation in blood within an elastic vessel (where ρ is blood density, κ is the modulus of volume elasticity, D is the vessel diameter, and b is the wall thickness) can be written as [5]:

$$a = \frac{\sqrt{\frac{\kappa}{\rho}}}{\sqrt{1 + \frac{\kappa}{E}\frac{D}{b}C}} \tag{5}$$

In Formula (5), there are the following values: modulus of circumferential elasticity E and parameter C, taking into account the wall thickness of the vessel. For a thick-walled vessel ($b/D > 1/40$), fixed at both ends, parameter C takes the form of [5]:

$$C=\frac{1}{1+\frac{b}{D}}[(1+\mu^2)+2\frac{b}{D}(1+\mu)(1+\frac{b}{D})] \tag{6}$$

Where μ is the Poisson's ratio for the vessel wall material.

To solve the system of equations (1) and (2), taking into account relationships (3 ÷ 6), the method of characteristics is commonly used in hydrodynamics (Rup & Dróżdż, 2014; Larock, Jeppson & Watters, 1999). The main idea behind the method of characteristics is to replace the partial differential equations with an equivalent system of ordinary differential equations:

$$\frac{d\upsilon}{dt}+\frac{g}{a}\frac{dH}{dt}-\frac{g}{a}\upsilon\frac{dz}{ds}+\frac{\lambda}{2D}\upsilon|\upsilon|=0 \tag{7}$$

$$\frac{d\upsilon}{dt}-\frac{g}{a}\frac{dH}{dt}+\frac{g}{a}\upsilon\frac{dz}{ds}+\frac{\lambda}{2D}\upsilon|\upsilon|=0 \tag{8}$$

It should be noted that equation (7) is valid only for the C^+ characteristic, described by the relationship:

$$\frac{ds}{dt}=\upsilon+a \tag{9}$$

In contrast, equation (8) is only valid along the C^- characteristic:

$$\frac{ds}{dt}=\upsilon-a \tag{10}$$

Where H is the hydrostatic pressure of a blood column, described by the relationship:

$$p=\rho\cdot g\cdot(H-z) \tag{11}$$

Initial and boundary conditions are based on the measurements performed *in vivo*. In particular, on measurements taken from three subjects of different ages. Using an applanation tonometer (SphygmoCor, AtCor Medical) with a piezoelectric transducer, three curves of the pressure wave in the aorta were obtained. The characteristics of the values obtained are summarised in Table 1.

Table 1

Characteristic values of the pulse wave

Age (years)	Cardiac period (s)	Number of points[a]	Maximal pressure (mmHg)	Minimal pressure (mmHg)
27	0.998	186	108.3	75.9
40	0.841	216	132.2	86.8
70	0.892	64	138.5	86.1

[a] Readout in discrete form

For the purposes of numerical calculations, pressure changes were converted according to Equation (11), obtaining in the vessel inlet section:

$$H_{P_1}(t) = \frac{p(t)}{\rho \cdot g} \tag{12}$$

In order to perform numerical integration of Equations (7) and (8) with the initial and boundary conditions listed above, numerical code in FORTRAN was designed and made compatible with the method of characteristics. The measured waveform of the pressure $p(t)$ was interpolated using the spline function [7] in the calculation process.

3. Analysis of the obtained results

The numerical computations take into account the following values of parameters describing the physical properties of blood and the thoracic aorta wall at $T = 37°\text{C}$. These parameters are taken from Reference [2]:

1. Characteristics of blood
   Dynamic viscosity $\eta = \nu \cdot \rho = 0.0035\ \text{N} \cdot \text{s} \cdot \text{m}^{-2}$
   Density $\rho = 1060\ \text{kg} \cdot \text{m}^{-3}$
   Bulk modulus $\kappa = 2.15 \cdot 10^9$ Pa
2. Characteristics of the aortic wall
   Internal diameter $D = 20$ mm
   Wall thickness $b = 1.2$ mm
   Length L=160 mm
   Poisson's ratio $\mu = 0.5$

The values of the circumferential modulus of elasticity and pulse wave velocity equation (5) with regard to the age of the person are shown in Table 2. The table also contains parameter C calculated in accordance with Formula (6) and the corresponding value of time step.

Table 2

Morphometric properties of systemic circulation in human subjects

Age (years)	E [2] (Pa)	C	a (m/s)	Δt (ms)
27	$5.5 \cdot 10^5$	0.8875	5.92	0.92
40	$7.0 \cdot 10^5$	0.8875	6.68	0.83
70	$14.5 \cdot 10^5$	0.8875	9.61	0.57

The calculations take into account that the average velocity of blood in the aorta is considered to be υ_0; additionally, it is assumed that the vessel is in a vertical orientation. A section of the considered aorta was divided into twenty-nine segments with a length $\Delta s = 5.52$ mm in the calculation process.

Selected results obtained from numerical calculations during the 50th cardiac cycle from the start of the simulation, are shown in the graphs (Figures 2 ÷ 5).

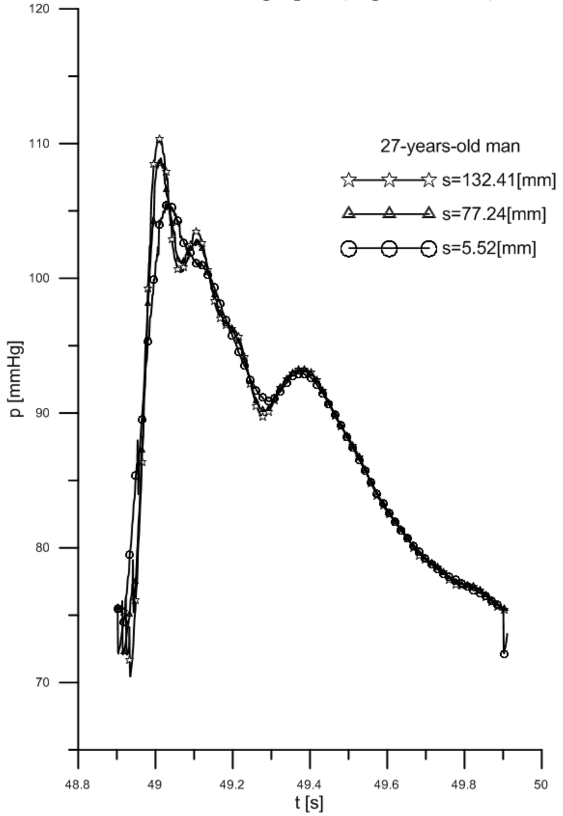

Fig. 2. Simulation of the pressure waveforms in the 27-year-old subject

Figure 2 shows three curves of the pressure wave in a selected cross-section of the 27-year old man's thoracic aorta. These cross-sections are described by the values of the *s* coordinate measured along its axis. Curves of the pulse wave at the beginning of segment 2, that is at a distance $s = 5.52$ mm from the initial cross-section, are marked with circles in Figure2.

As presented in Figure 2, the pressure wave curves illustrate an increase in the maximal pressure of flowing blood with increasing distance from the initial cross-section of the aorta segment. Thus, in the case of the pressure wave in the cross-section that is a distance of $s = 132.4$ mm from the inlet and marked with stars, the corresponding value of the maximum pressure is $p = 110.4$ mmHg. This increase in the maximum pressure is noted particularly in the initial part of each of the 50 cardiac cycles.

Figure 3a shows blood velocity changes calculated for 50th cardiac cycle in the same cross-sectionsas above. The maximum blood velocities in the vessel cross-sections are: $\circ - \upsilon_{max}(s = 5.52\text{ mm}) = 0.660$ m/s, $\Delta - \upsilon_{max}(s = 77.24\text{ mm})$ 0.522 m/s, $* - \upsilon_{max}(s = 132.41\text{ mm}) = 0.368$ m/s.

The corresponding, real blood velocity waveform in the 27-year-old subject's aorta cross-section, measured by the Doppler ultrasound method, is shown in Figure 3b.

a) b)

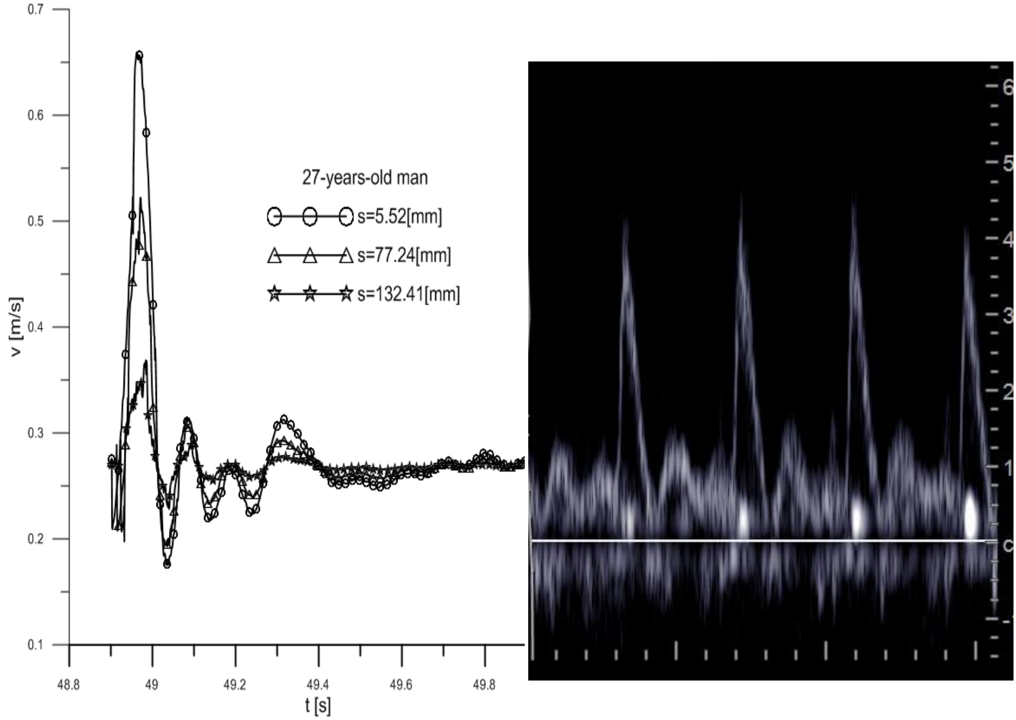

Fig. 3a, b. Simulated and measured blood velocity waveforms

Corresponding changes of blood velocity for people aged 40 are shown in Figure 4. Velocity changes over time in the particular sections of the aorta are characterised by decreasing extreme values with increases in the person's age.

It should be noted that the areas of blood velocity disorders extend over a greater length of the aorta in the elderly.

On the basis of the blood pressure and velocity changes in the thoracic aorta, relative elastic deformation of the vessel wall was determined. Taking into account the influence of a pulsatile blood flow on the elastic vessel wall, the following relationship was introduced:

$$\frac{\delta D}{D}(s,t)=\frac{D}{2b}\cdot\frac{C}{E}\left[p(s,t)-\rho\cdot a\left(\upsilon(s,t)-\upsilon_0\right)\cdot\left(1+\upsilon(s,t)/a\right)\right] \tag{13}$$

The variables used in equation (13) have been previously described in the text. The relative elastic deformation of the aorta wall according to equation (13) is shown in Fig. 5.

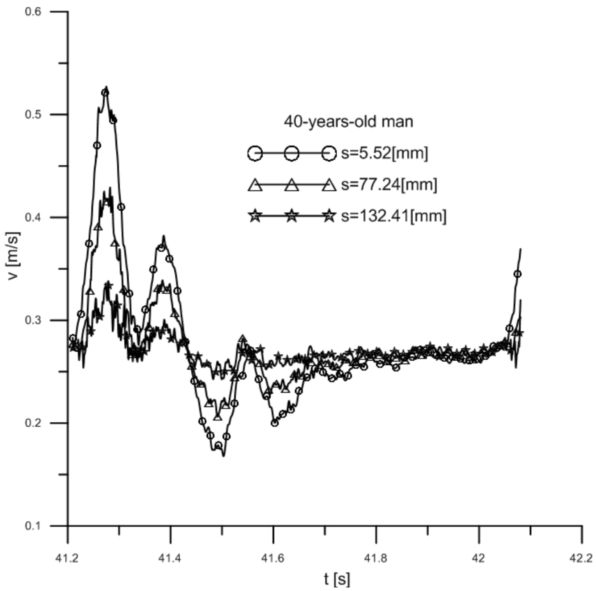

Fig. 4. Simulated velocity waveforms in the 40-year-old subject

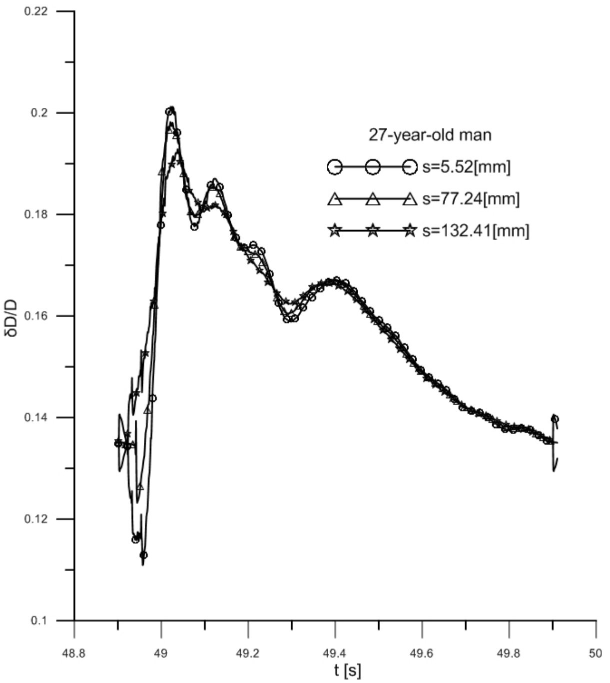

Fig. 5. Simulated relative deformation in the 27-year-old subject

Extreme values of the relative deformation of the vessel wall are observed in Figure 5 in the initial part of each of the fifty cardiac cycles. In the case of the younger age groups, in which blood vesselshave greater elasticity ($E = 5.5 \cdot 10^5$ Pa), local relative deformation reaches an extreme value of $\delta D/D(s = 5.52$ mm, $t = 49.0308$ s$) = 0.200$.

4. Final remarks and conclusions

This paper is an attempt to develop a non-linear, one-dimensional model of blood flow in the human thoracic aorta. The developed model includes measurements of the blood pressure changes performed *in vivo*. Furthermore, in the considered model,pulsatile blood flow caused by elastic deformations of the vessel wall is included.

The model includes partial differential equations resulting from the mass and momentum balance recorded for blood flow and the power balance condition for the area of the vessel wall.

The method of characteristics was used for integration of the equations. The analysis obtained from numerical calculations is limited to fifty cardiac cycles. It is worth noting that in each of the fifty cardiac cycles, as the initial condition for pressure, pulse waveforms measured by an applanation tonometer were used.

The analysis shows a high degree of result stability obtained throughout the whole considered temporal and spatial area.

The numerical results describing the velocity and pressure of blood flow are characterised by high intensity of changes at the beginning of each cardiac cycle. These changes are manifested by a build-up of their peak values with increasing distance of the segment from the initial section of the vessel. The relative increase in peak blood pressure is 4.5% at $s = 132.4$ mm from the inlet section for a 27-year-old person. In the case of results obtained for the blood velocity, the intensity of changes is more intense in the initial part of the considered part of the aorta – the relative maximum velocity increase is 144% for this person.

Elastic deformations of the considered vessel section are associated with the propagation of the pressure wave. Changes in blood velocity are, in turn, coupled to pressure changes. Consequently, the relative intensity of the elastic deformation of the wall is similar to changes in blood velocity. The elastic deformation of the wall takes extreme values in the initial part of the vessel and in the initial phase of systole. The maximum relative strain is about 0.200 in the 27-year-old person.

The numerical results for the velocity, blood pressure and deformation of the vessel wall obtained for 40- and 70-year-old subjects are characterised by lower intensity of changes, as compared with the corresponding results for a younger person.

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