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## HYPERELASTIC ZAHORSKI MATERIAL – NUMERICAL ANALYSIS AND SIMULATION IN ADINA SOFTWARE

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### HIPERSPRĘŻYSTY MATERIAŁ ZAHORSKIEGO – NUMERYCZNA ANALIZA I SYMULACJA W PROGRAMIE ADINA

#### Abstract

The paper discusses and presents a hyperelastic incompressible material described by Zahorski potential. The numerical example comparing effective stresses in a cylinder made of Zahorski and Mooney-Rivlin materials was included. The analysis was made in the ADINA software. Conclusions summarize numerical calculations and demonstrate the differences that suggest the use of Zahorski material for rubber and rubber-like materials subjected to large deformations.

*Keywords: hyperelastic Zahorski material, incompressible material, cylinder, FEM, Adina*

#### Streszczenie

W artykule omówiono i przedstawiono hipersprężysty nieściśliwy materiał opisany potencjałem Zahorskiego. Zamieszczono przykład numeryczny, w którym porównano naprężenia efektywne w cylindrze wykonanym z materiałów Zahorskiego oraz Mooneya-Rivlina. Analizę wykonano w programie ADINA. We wnioskach podsumowano obliczenia numeryczne i podano różnice, które wskazują na możliwość zastosowania materiału Zahorskiego dla gum i materiałów gumopodobnych poddawanych dużym odkształceniom.

*Słowa kluczowe: hipersprężysty materiał Zahorskiego, materiał nieściśliwy, cylinder, MES, Adina*

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## 1. Introduction

Analysis of non-linear hyperelastic materials can be carried out by means of numerical programs based on the finite element method (FEM). There are many softwares that have used the popular elastic potentials (e.g. ALGOR, ANSYS, ABAQUS, MARC, NASTRAN, ADINA). All of these programs support a group of selected models of materials in their libraries, including the models of non-linear hyperelastic materials. Selection of one of the models of materials allows for performing numerical analysis of the behaviour of elements of construction [1]. However, the above software does not offer possibilities for analysis of the hyperelastic Zahorski material which allows for a more precise determination of the behaviour of rubber and rubber-like materials at significantly higher deformations compared to the commonly used materials such as Mooney-Rivlin or neo-Hookean materials [2]. The proposed solution will translate into a reduction of material consumption and will contribute to a more effective management of the production of rubber products. It can thus reduce the cost of materials, which will have a positive impact on the economics of enterprises which implement orders for many industries [3].

## 2. Hyperelastic materials - constitutive equations

It can be assumed that constitutive equations that describe the relationships between deformations and energy or between deformations and stresses for hyperelastic materials are obtained based on the equations of mechanical energy balance. In terms of the theory of elasticity and in the widely understood mechanical problems, including continuum mechanics, elastic bodies are considered as continuum of material with or without internal bonds. For the elastic bodies without bonds, the properties of such a medium are given if the function  $W$  can be defined. Function  $W$  is typically defined as a function of deformation energy and for any deformation  $d$  of medium, it determines the corresponding elastic energy  $W=W(d)$  accumulated in the unit of volume with respect to the reference configuration  $\mathbf{B}_R$ .

For uniform isotropic elastic bodies, the constitutive equations can be written as:

$$W = W(I_1, I_2, I_3) \quad (1)$$

where:

$I_1, I_2, I_3$  – are deformation tensor invariants.

An elastic body with imposed internal bonds cannot be subjected to any deformations. The only acceptable deformations with regard to incompressible bodies are deformations which do not change its volume (isochoric). A condition for acceptable deformations  $I_3 = 1$  must be met. This is the cause why  $I_3$  does not occur as an argument of the deformation energy function which, for the incompressible body, represents a function of only two other invariants. This can be re-written in an analogous form to the Eq. 1:

$$W = W(I_1, I_2) \quad (2)$$

The above Eq. 2 define the constitutive relations for incompressible material.

### 3. Mooney-Rivlin and Zahorski incompressible materials

Non-linear theory of elasticity causes the necessity to renew the definition of constitutive relations so that it matches the problem analysed. With large deformations, each of the rubber-like materials behaves in a specific manner. Therefore, for each individual case of experimental procedure, it is necessary to determine a model of constitutive equation.

#### 3.1. Mooney-Rivlin and neo-Hookean materials

The Mooney-Rivlin and neo-Hookean models are the only popular models used for the description of incompressible rubber and rubber-like materials. It is a peculiar case that results from the general form of elastic energy function defined by Rivlin and Saunders [4] and the empirical form of deformation function proposed by Mooney [5]. The model of Mooney-Rivlin material was defined with the following equation:

$$W = W(I_1, I_2) = \frac{\mu}{2}(f(I_1 - 3) + (1 - f)(I_2 - 3)) \quad (3)$$

and represents the most general theoretical model of behaviour of elastic rubber materials.

The neo-Hookean material is a particular case of the Mooney-Rivlin material. The model of neo-Hookean material is defined by the following energy equation:

$$W = W(I_1) = \frac{\mu}{2}f(I_1 - 3) \quad (4)$$

According to the literature [6, 7], the neo-Hookean model defines elastic behaviour of homogeneous rubbers for small and moderate deformations.

#### 3.2. Zahorski material

The model of Zahorski material is described by the equation with non-linear dependency on the invariants of the deformation tensor [8]:

$$W = W(I_1, I_2) = C_1(I_1 - 3) + C_2(I_2 - 3) + C_3(I_1^2 - 9) \quad (5)$$

where:

$C_1, C_2, C_3$  – are material constants. The values of constants for three types of rubber were given in a study [6].

The above constitutive equation allows for a more comprehensive analysis of the wave phenomena propagating in elastic incompressible materials. A description that suits the behaviour of rubber for the main elongation was obtained even for  $\lambda = 3$ , whereas for the neo-Hookean and Mooney-Rivlin materials, the acceptable results are observed for  $\lambda = 1.4$  [9].

The Eq. 5 models the effects of the dynamic behaviour of materials and is used for the analysis of wave phenomena that concerns propagation of disturbance in the form of shock waves, travelling waves and soliton waves ([10–12] et al.). In the study [13] it was

demonstrated that the constitutive equation with non-linear dependency on the invariants of deformation tensor defines more precisely the behaviour of rubber at much higher deformations than in the case of Mooney-Rivlin or neo-Hookean materials.

#### 4. Comparison of the Mooney-Rivlin and Zahorski materials

Differences in the stress-strain function between the Mooney-Rivlin and Zahorski materials for rubber “A” were shown in the paper [14]. Stress-strain diagrams presented by the authors have clear differences in the shape of curves. The diagrams for Zahorski material were generated from the ADINA software, after modifications introduced into material libraries [2].

Table 1 presents elastic constants for rubber “A”. The values presented in the table are based on the study [6].

Table 1

Constants  $C_1$ ,  $C_2$ ,  $C_3$

Constants	$C_1$	$C_2$	$C_3$
Rubber “A” [Pa]	$6.278 \cdot 10^4$	$8.829 \cdot 10^3$	$6.867 \cdot 10^3$

#### 5. Numerical example

For comparative computation, a cylinder of 12 cm in height and 4 cm in diameter was adopted. Load in the object was obtained by means of a linear displacement of the upper surface of the cylinder applied in the  $z$  direction. The size of the displacement was determined by displacement  $\lambda = 2$ , according to the study [6], assuming 20 steps of time. On the opposite side surface of the cylinder (i.e., on the surface of the base), bonds were added to prevent displacement in  $x$ ,  $y$  and  $z$  directions (in accordance with the adopted Cartesian coordinate system).

The aim of numerical calculations by means of FEM was to compare the distribution of stress in the Mooney-Rivlin and Zahorski material at the declared load and boundary conditions. This was obtained through a demonstration of differences in the distribution of effective stress in both materials. In Fig. 1a the distributions of stress in the Mooney-Rivlin material, whereas in Fig. 1b the distributions for Zahorski material are presented. Both materials were obtained for rubber “A”. It can be observed that the deformation assumed (including the assumed boundary conditions) yields the expected results.

The comparative analysis of the states of effective stresses for the adopted model revealed differences in the distributions of effective stresses obtained in the Mooney-Rivlin and Zahorski materials (see also [14]).

Fig. 1 illustrates the distributions of stresses in rubber “A” for the cylinder. Substantial difference in the obtained levels of stresses and their distribution on the cylinder can be observed. The maximum value of stress in the Mooney-Rivlin material equals  $\sim 540$  kPa (Fig. 1a), whereas this value for the Zahorski material amounts to  $\sim 780$  kPa (Fig. 1b).

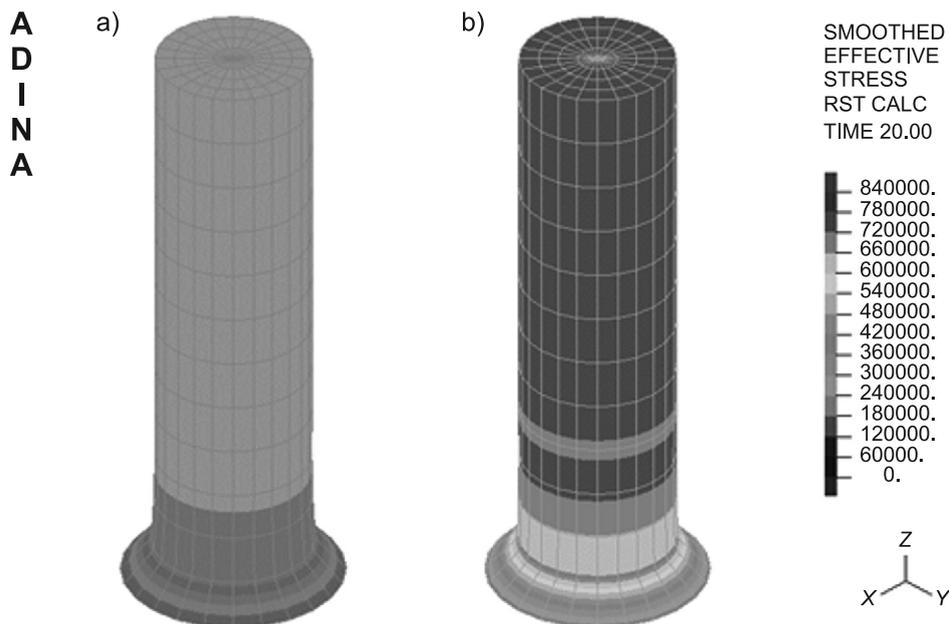


Fig. 1. Distribution of stress [Pa], step 20: a) Mooney-Rivlin material, b) Zahorski material

The comparison of obtained distributions provides information about the differences in the distribution of effective stress.

## 6. Summary

The study presented and discussed a hyperelastic incompressible material described by Zahorski potential. Distinct differences were obtained for the distribution of effective stresses for the geometrical object modeled with two different hyperelastic materials i.e. the Mooney-Rivlin material and Zahorski material for the 3D model studied. The results obtained from model-based studies show clear qualitative and quantitative differences in the distributions of stresses between the Mooney-Rivlin and Zahorski materials since small differences occur between elastic potentials for both materials.

The method of modification of the material library in order to allow for numerical tests in the ADINA software has considerable values that can be used in technological applications in many scientific fields. With the popularity of hyperelastic incompressible materials (used in different fields of science and technology), the Zahorski potential allows for supplementation of the results of calculations obtained previously for the Mooney-Rivlin material. The non-linear term  $C_3(I_7^2 - 9)$  in the Zahorski potential supports a more precise analysis and allows for obtaining other qualitative elements in numerical analyses of rubber and rubber-like materials with respect to non-linear and incompressible hyperelastic materials [15, 16].

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