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TORSION OF PRECAST HOLLOW CORE SLABS

SKRĘCANIE PREFABRYKOWANYCH PŁYT KANAŁOWYCH

Abstract

Pretensioned hollow core slabs are one of the most widely applied precast elements in the modern building industry all over the world. Due to the possibility of their use under various support or loading conditions, these elements repeatedly work under a complex stress state. Theoretical analysis of the phenomena of the interaction of shear and torsion in the above-mentioned HC elements is presented. The theoretical basis and design regulations of the computational model included in code PN-EN 1168+A3:2011 along with parametrical analysis of that model are demonstrated. The largest experimental research program of hollow-core slabs under shear and torsion conducted thus far is also briefly described – this study was carried out in Finland in 2004 and was the basis for the development and calibration of the numerical computational model formulated in Chalmers University of Technology in Göteborg, Sweden.

Keywords: *hollow core slab, prefabrication, pretensioned element, shear, torsion*

Streszczenie

Strunobetonowe płyty kanałowe są jednym z najczęściej stosowanych stropowych elementów prefabryko-wanych na świecie. Ze względu na możliwość funkcjonowania płyt w różnych warunkach podparcia czy obciążenia niejednokrotnie elementy te pracują w złożonym stanie naprężenia. W artykule przedstawiono analizę teoretyczną zjawiska interakcji skręcania ze ścinaniem poprzecznym w rozważanych elementach. Zaprezentowano podstawy teoretyczne i wytyczne projektowe modelu obliczeniowego zawartego w nor-mie PN-EN 1168+A3:2011 wraz z analizą parametryczną modelu. Skrótowo opisano również największe dotychczas zrealizowane badania doświadczalne ścinanych i skręcanych płyt, zrealizowane w 2004 roku w Finlandii, które posłużyły do opracowania i kalibracji numerycznego modelu obliczeniowego wykona-nego w Chalmers University of Technology w Göteborgu.

Słowa kluczowe: *płyty kanałowe, prefabrykaty, skręcanie, strunobeton, ścinanie*

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1. Introduction

Pretensioned hollow core slabs are most often designed as simply supported elements, and the computational analysis of such designs are based on the assumption of plane stress state. In most cases, it is assumed that the floor slabs are under the action of loading which is uniformly distributed over the whole surface of the slab. Hollow-core slabs, although constructed from precast elements, might be treated as floor plate with the possibility of the redistribution of loads to the adjacent precast elements. Such a performance by the floor is possibly due to monolithisation of the structure by casting the concrete in longitudinal joints between slabs or (in many structures) by casting the structural concrete topping layer [4].

The assumption of the plane stress state in the cross-section of the slab is justified when the element is subjected to uniformly distributed loading over its entire surface and supported on two parallel, relatively rigid supports, e.g. walls or beams with a large cross-section. In many cases, this assumption does not reflect the actual conditions of work and behaviour of the element. If only one of the elements of the floor structure is supported or loaded in a non-uniform way, then the distribution of the transverse force through the joint of the slabs may cause torsion of the adjacent slabs. The following slabs are examples of the occurrence and influence of torsional moments [1]:

a) elements supported on three edges, e.g. corner slab, with longer edge supported on the wall (see Fig. 1, detail: 1);
b) elements loaded with concentrated force in the longitudinal edge area, e.g. force from the trimmer beam acting as support for the neighbouring beam in the area of the large opening or recess cut for the purposes of staircases (see Fig. 1, detail: 2);
c) elements supported with one corner on the column (see Fig. 1, detail: 3);
d) slanted slabs in the support area, causing the lack of parallelism of the opposite supports (see Fig. 1, detail: 4);
e) elements supported on relatively slender beams, i.e. slim floor type [5].

The necessity to guarantee the safety of the HC slabs, which are widely applied in the modern building industry all over the world, brings an urgent need for the detailed analysis of the behaviour of these elements, including the influence of torsion, and the application of suitable design procedures.
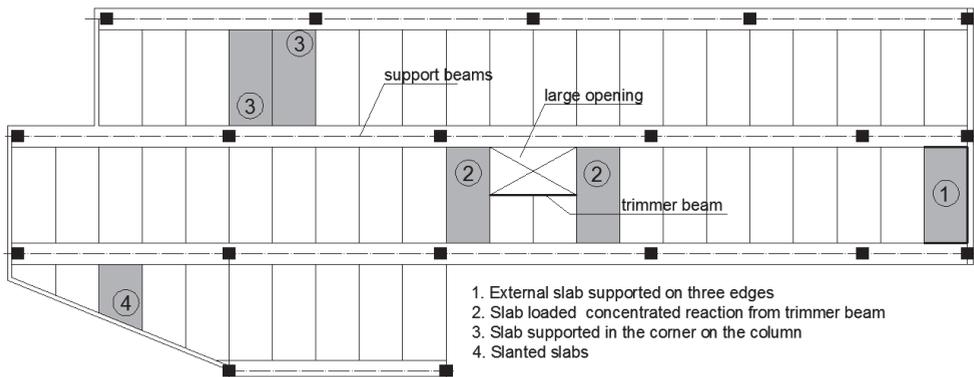

Fig. 1. Specific support and loading situations causing the torsion of the slab

2. Calculation of hollow core slabs under torsion and shear

Both transverse force V_{Ed}, and torsional moment T_{Ed}, occurring in the cross-section of the HC slab, generate tangent shear stresses. Tangent stresses τ_V, due to the transverse shear action, obtain the largest value at the level of the centroid of the cross-section and values of zero at the horizontal edges. In the so-called free, non-constrained shear, in case of full freedom of axial displacements, tangent stresses τ_T due to the torsional moment are distributed linearly along the width of the element, obtaining the highest value at the outline and a value of zero at the center line of the cross-section. The thinner the walls of the cross-section under torsion, the smaller its torsion resistance, thus at the relatively small values of the torsional moment T_{Ed} the maximum tangent stresses τ_T reach significant values.

In case of the constrained torsion, i.e. in the cross-sections where the interaction of the different internal forces appear (M_{Ed}, T_{Ed}, V_{Ed}, N_{Ed}), at the height of the cross-section develop additional tangent stresses, due to the transverse forces. These stresses are different from zero in the centre of the cross-section, but the strains they cause are negligibly small [10]. In the case of the assumption of torsion and shear interaction in the element with a non-solid cross-section (e.g. hollow core slab with empty cores, not filled with concrete), one outermost web is subjected to the combined action of the stresses $\tau_V + \tau_T$, the value of which is definitely higher than the stresses acting on the remaining webs (Fig. 2).
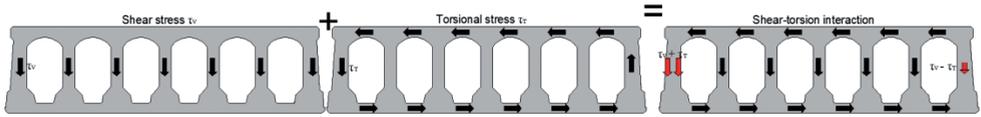

Fig. 2. Distribution of the tangent stresses in a cross-section of the hollow core element

A significant increase of the tangent stresses due to the interaction of shear and torsion may cause the value of the principal tensile stress σ_1 to overrun the tensile concrete strength. As a result, cracks start to form in the concrete, and they develop in the direction implicated by the trajectory of the principal tensile stresses. In calculations, it is assumed that in the areas subjected to major shear actions, the development of cracks on the vertical parts of the element (e.g. concrete webs of the slab) have a slanted course, and the angle of deviation from the longitudinal axis of the element remains in the range from 22° to 45°. With the effect of the additional torsion, the cracks may propagate helically at the level of the element surface at an angle of about 45°. The failure of the element without transverse reinforcement takes place at the moment of the occurrence of the first helical circumferential crack [9].

Based on the general assumptions of the concrete mechanics, both the values of the tangent stresses τ_V and τ_T, and the principal tensile stresses in the concrete σ_1 can be described with the following expressions (1)–(3):

$$\tau_V = \frac{V_{Ed} \cdot S_{cs}}{b_w \cdot I_{cs}} \tag{1}$$

$$\tau_T = \frac{T_{Ed}}{W_T} \tag{2}$$

$$\sigma_1 = \frac{\sigma_{cp}}{2} - \sqrt{\left(\frac{\sigma_{cp}}{2}\right)^2 + \tau^2} \le f_{ctd} \tag{3}$$

where:

V_{Ed} – transverse force acting in the considered cross-section;
T_{Ed} – torsional moment acting in the considered cross-section;
S_{cs} – static moment of the part of the cross-section above the centroid horizontal axis of the cross-section;
I_{cs} – moment of inertia of the cross-section;
b_w – the sum of the widths of the webs at the level of the centroid of the cross-section;
W_T – sectional modulus of the cross-section under torsion;
σ_{cp} – the mean compressive stress, measured positive, in the concrete due to the design axial force (including the prestressing force) and bending moment;
f_{ctd} – design tensile concrete strength, $f_{ctd} = f_{ctk}/\gamma_c$.

When analysing pretensioned hollow core slabs, the possibility of the occurrence of cracks due to simultaneous torsion and shear actions is a substantial problem because, for technological reasons, hollow-core slabs do not have transverse reinforcement. The shear resistance of such a slab, assuming the absence of cracks due to bending, is limited by the tensile strength of the concrete ribs. Slant crack forming in the webs propagate in the direction of top and bottom flanges. The failure usually appears in the support zone, where the full transmission length has not been reached, and the value of the prestressing force remains incomplete. Introducing the compressive force from prestressing into the cross-section causes the natural increase of crack resistance of the concrete element subjected to shear and torsion. It is the effect of the two-axial state of stresses resulting from shear and eccentric compression, delaying the forming of the cracks [9].

The lowest value of the pure shear resistance usually appears at the level of the smallest width of the web, i.e. in the half-height of the cross-section. The general condition of the shear strength for pretensioned hollow-core slabs is described by analogical expressions included in EN 1168+A3:2011 [6], Model Code 2010 [8] and Eurocode 2 [7]:

$$V_{Rd.c} = \varphi \cdot \frac{I_{cs} \cdot b_w}{S_{cs}} \cdot \sqrt{f^2_{ctd} + \beta \cdot \alpha \cdot \sigma_{cp} \cdot f_{ctd}} \tag{4}$$

where:

α – l_x/l_{pt2} for prestressing strands, max. 1.0;
l_x – the distance from the concerned cross-section to the front of the element;
l_{pt2} – upper design value of transmission lengths (1.2 l_{pt});
σ_{cp} – ompressive stress from the prestressing in the concrete;

φ – empirical reduction factor (φ = 0.8 by EN 1168 and MC2010; φ = 1.0 by
     Eurocode 2);

β – reduction factor (β = 0.9 by EN 1168; β = 1.0 by Eurocode 2 and MC2010).

The engineering evaluation of tangent stress from torsion action τ_T in the cross-section of the hollow core slab requires simplification of the cross-section geometry by reducing to thin-walled rectangular cross-section. The guidelines in the latest version of Model Code 2010 [8] and EC2 [7] concerning the determination of the effective width of the separated shell, refer to the solid reinforced cross-sections or box sections with reinforced walls. There are no unequivocal regulations describing how to assume a simplified cross-section for hollow-core slabs, which are practically without reinforcement, especially that in common practice various shapes of the cross-sections of HC units are being applied – the most popular examples of cross-sections are shown in Fig. 3.
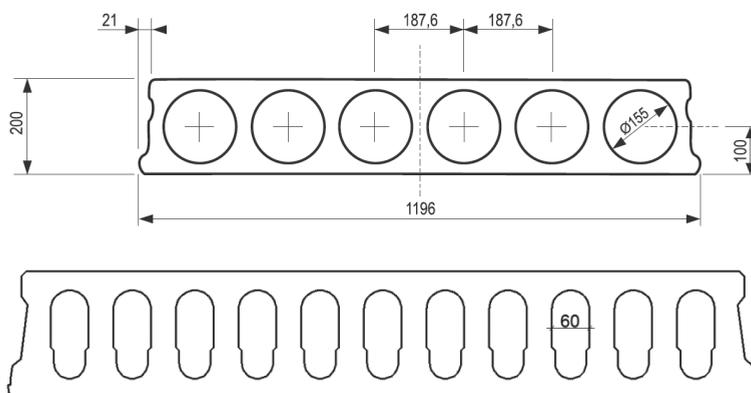

Fig. 3. Examples of cross-sections of HC slabs

In most cases, the simplification of the hollow-core slab cross-section consist on assumption of rectangular box section with the walls thickness equal the smallest width of the outermost webs of the real slab (Fig. 4). The equivalent width of the wall of the reduced cross-section t should not exceed the ratio A/U, where:

A – area of the cross-section, including the area of the cores;

U – perimeter of the reduced cross-section according to Fig. 4 [2].

For the simplified box section, the torsional sectional modulus W_T is expressed by the equation:

$$W_T = 2t \cdot \left[h - \frac{(t_{top} + t_{bottom})}{2} \right] \cdot (b_{sl} - b_{w.out}) \tag{5}$$

where $t = t_{top}$ (or $b_{w.out}$) for the calculation of stresses in the top or bottom flange (or in the outermost web).

Other geometrical denotations for the purpose of the W_T calculation are given in Fig. 4.
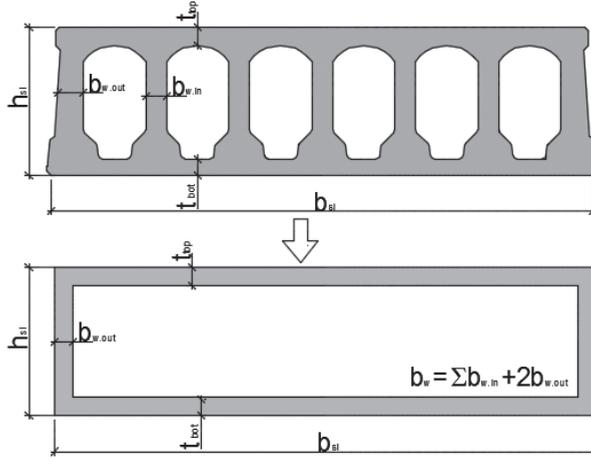
Fig. 4. The method of transformation of the real HC slab cross-section into
a reduced thin-walled cross-section

Pure torsion resistance in the pretensioned hollow-core cross-section, which is reached in the moment of the appearance of the first crack, should be determined separately for horizontal flanges of the cross-section $T_{Rd.top}$ and outermost webs $T_{Rd.web}$, according to the following equations:

$$T_{Rd.top} = W_{t.top} \sqrt{f_{ct}^2 + \sigma_{cp} f_{ctd}} \tag{6}$$

$$T_{Rd.web} = W_{t.web} \sqrt{f_{ct}^2 + \alpha \sigma_{cp} f_{ctd}} \tag{7}$$

The analytical computational model of the shear resistance in the case of simultaneous actions of transverse forces and torsional moment, is based on the assumption that the tangent stresses from shear and torsion could be super-positioned. As a result, shear strength $V_{Rd.c}$ is reduced by the design value of the transverse force caused by torsion V_T:

$$V_{Ed.n} = V_{Rd.c} - V_T \tag{8}$$

where $V_{Ed.n}$ is a 'net' value of the transverse force, due to transverse shear only.

The value of the transverse force from torsion actions V_T can be evaluated in the following way [1]:

$$V_T = \frac{T_{Ed} \cdot b_w \cdot I_{cs}}{2b_{w.out} \cdot (b_{sl} - b_{w.out}) \cdot [h_{sl} - (t_{top} + t_{bot}) \cdot 0.5] \cdot S_{cs}} \tag{9}$$

Under the assumption $I_{cs}/S_{cs} \approx d$ and $[h_{sl} - (t_{top} + t_{bot}) \cdot 0.5] \approx d$, where d denotes the effective height of the cross-section, the expression defining V_T force can be written as:

$$V_T = \frac{T_{Ed} \cdot b_w}{2b_{w.out} \cdot (b_{sl} - b_{w.out})}$$

(10)

The abovementioned equation was accepted in standard [6] concerning the designing of the hollow-core slabs.

3. Experimental tests and numerical analysis of HC slabs under torsional actions

The issue of the influence of torsion in hollow-core slabs was the subject of an extensive experimental program carried out at full scale, in the VTT Research Centrum of Finland in 2004 [11–14]. This is the world's leading institution engaged in the problem of hollow-core floor slab performance. Research program assumed testing the influence of pure torsion and interaction of torsion and shear on single elements and panel floors consisting of the assembly of four hollow-core slabs.

In the first stage of the experiment [11], individual elements loaded with two alternating concentrated forces (see Fig. 5 [L]) were tested. One force was located in the corner zone of passive support, the other in the corner zone of active support (with the possibility of displacement). This static scheme, with concentrated forces applied to the element eccentrically relative to the axis of the element, allowed for the induction of pure torsion action.

In all slabs, failure occurred by the cracking of the top surface of the precast element, which developed at the angle of 45° to the longitudinal axis of the element. For slabs with a height of 400 mm, the value of the torsional rigidity determined from experimental investigations was close to the value determined analytically; however, for slabs with a height of 200 mm, the calculated value of stiffness was 30% smaller than that obtained from tests. Design torsional resistance was respectively 60% and 70% of the strength acquired from the experimental testing for slabs with heights of 200 and 400 mm.

In the second stage [12], fifteen simply supported hollow-core elements were tested. In those tests, the effect of shear was acquired due to the application of one or two concentrated forces in the support zone; additionally, the effect of torsion was induced by applying two concentrated forces in two alternating corners of the slab (see Fig. 5 [P]).

In ten out of fifteen tests, slabs were supported on neoprene pads and the cores remained empty (as opposed to being filled with concrete). For those slabs, the transverse bending of the element cross-section caused lateral cracking and accelerating failure which took place at a load value lower than the design strength. In the remaining five cases, the cores were filled with concrete in the support zone and the slabs were supported directly on the non-deformable beam. Change of the support conditions had a major influence on the bearing capacity of the slabs – this reached values comparable or higher than the analytical calculation results.

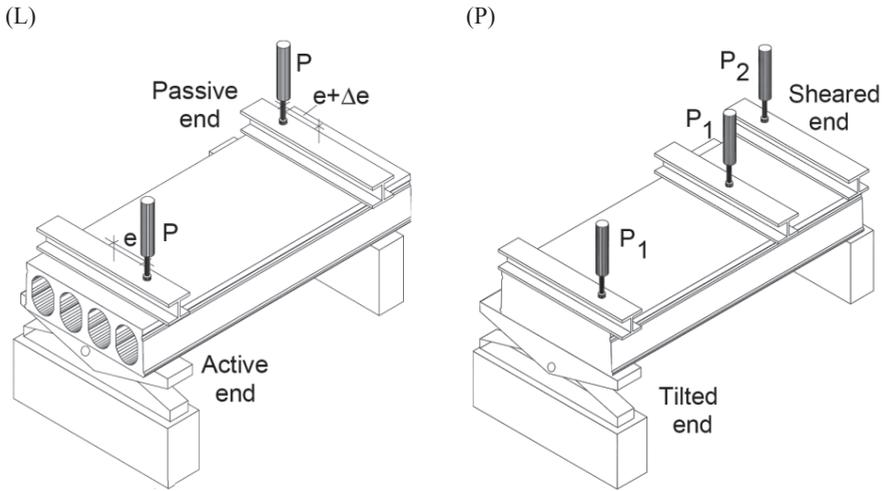(L)   (P)

Fig. 5. Testing stand for pure torsion (L) and torsion-shear interaction (P) [11, 12]
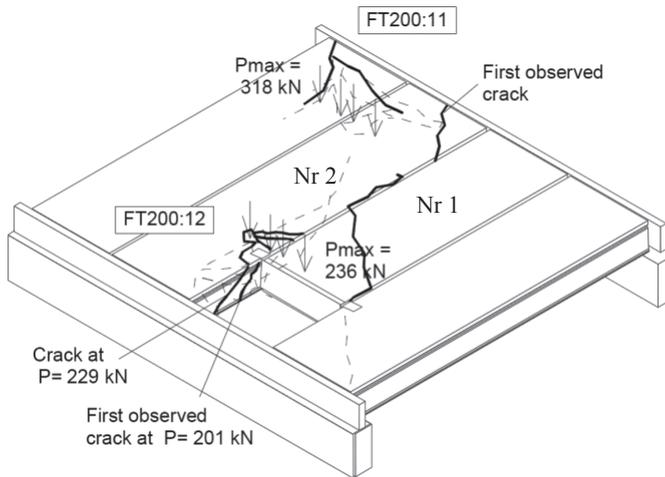

Fig. 6. Testing stand for the torsion-shear interaction for HC200 with the recess [13]

In the third stage [13], twelve floor slabs consisting of four hollow-core elements with heights of 200 mm were put to the tests of interaction of torsion and shear (see Fig. 6), thus being the tests of confined torsion. In the area of the support zone of one of the middle elements, a recess was made and shortened slab No1 was supported on the two adjacent slabs by the steel trimmer beam. The loading was applied as two linear arrays, each consisting of four concentrated forces localised in the zone of the trimmer-beam support and the opposite corner of slab No. 2. The force causing failure of the floor slab was significantly larger than that for the single element from the second stage of the testing program. It was also noted that the trimmer-beam in the recessed area carries only a minor proportion of the support reaction

from the variable loading applied to slab No1, and the major proportion of it is carried by the slab joints filled with the concrete.

In the tests of the torsion and shear interaction [14] of the hollow-core slabs with a height of 400 mm, fifteen floor slabs were tested. In the first twelve tests, the value of the transverse force was limited to the maximum value concerning the serviceability limit state, and there were no distinct symptoms of failure observed. The remaining three testing elements were loaded up to failure, and the damaging force was from 25% to 97% larger than that in the tests of single slabs with the same support conditions.

Since major differences between the test results of strength and torsional rigidity and the results of the earlier analytical calculations were observed, the need for development of more accurate computational models was necessary. The conducted experimental tests served the purpose of the development and calibration of the numerical computational model utilising FEM analysis, executed by the Holcotors team at Chalmers University of Technology in Göteborg.

The results of the calculations of torsion and shear interaction on the basis of FEM analysis differ from the results of analytical calculations according to the code EN1168. They give much higher values of capacity, independently from the loading scheme with transverse force and torsional moment. However, for the case of pure torsion, FEM calculations provided lower values of strength than analytical calculations. This is associated with a different model of failure, and as a consequence, the possibility of cracking of the top flange of the slab – this is not taken into consideration by the code EN 1168.

The authors of the Swedish model are of the opinion that diagrams created on the basis of the numerical analysis could be the most convenient and at the same time, the most accurate method of designing hollow core slabs under those types of actions [2]. The exemplary diagrams of torsion-shear interaction for the prestressed slab with a height of 200 mm and a transmission length in the range of the width of the shear zone equal to 0.8 m (black continuous line) and 0.5 m (grey continuous line) have been presented. The straight grey line represents an interaction curve determined on the basis of the standard method EN1168. The map of cracking based on the FEM analysis is presented in Fig. 7, pictures (a) – (j) [3].

4. Parametric analytical studies

For the purpose of presenting the scale of the torsion effect in hollow-core slabs, the computational example on the basis of analytical model presented above was conducted. For the benefit of calculations, hollow-core slabs made from concrete class C50/60 were assumed with three different cross-section heights of 200 mm, 320 mm, 500 mm and effective spans of 8 m, 12 m and 16 m. Slabs were assumed to be simply supported, loaded with their own weight with an additional static load of $\Delta g_d = 1$ kN/m² and uniformly distributed live load of $q_d = 5$ kN/m². In the analysis of torsion, the value of the applied design torsional moment was assumed to be within the range 13–36 kNm depending on the height of the slab. This value was determined in the exact same way as for the case when the considered element acts as a support for the trimmer-beam of the adjacent slab with the recess. Torsional strength of the flange was determined from equation (6), and the outermost web from equation (7). The sectional modulus was calculated from equation (5).
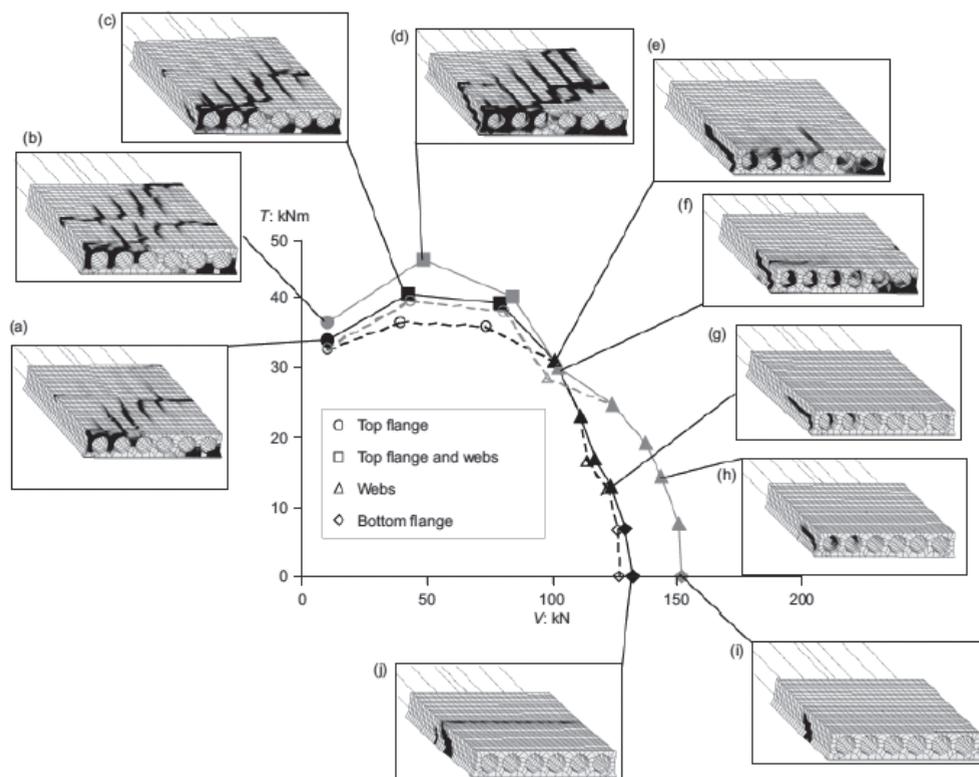
Fig. 7. Torsion and shear interaction for HC200 slab, according to the Holcotors numerical model

The subject of the first analysis was a strength comparison of hollow-core slabs with different cross-sections depending on the production technology. The cross-sections of elements produced using slip-forming technology (approximately rectangular cores) – type 1, and extruded elements (circular cores) – type 2, were compared. The results of the calculations of transverse force due to torsion in the external web V_T (also as a percentage of the total shear load capacity $V_{Rd.c}$) and the 'net' value of the transverse force $V_{Ed.n}$ compared to the value of shear force due to vertical load V_{Ed} were presented in Table 1.

The results of the conducted analysis proved that with the assumption of a highly probable design value of the torsional moment (in practice, cases with bigger contribution of torsional moment e.g. due to the additional concentrated loads or additional torsion caused by flexibility of the supports, may happen) the contribution of torsion in overall bearing capacity of the slab's web equals around 22%–37%. Obviously, most susceptible to torsion are slabs with small cross-section heights – in those elements, with unfavourable loading configurations, design deficiency of the shear capacity may arise.

The influence of the structural topping concrete layer on changes in maximal tangent stresses and transverse forces in the slab's web due to the interaction of torsion and shear, were also subjected to the analysis. Calculations were carried out for a slab with a height of 320 mm and a span of 12.0 m loaded with variable loading q_d = 5 kN/m² and a design

torsional moment of 22 kNm. A layer of reinforced concrete topping was assumed with three different thicknesses: 50 mm, 65 mm and 80 mm. Full composite action between the precast floor element and the topping layer was considered. The results of the calculations are presented in Table 2.

The presence of the topping concrete layer changes the geometry of the cross-section, and thus influences the magnitude of tangent stresses in the outermost web of the slab. As a result of increasing the thickness of the topping concrete, a constant increase of the transverse force from torsion V_T (see eq. [7]) is obtained. Due to the fact that simultaneously, the increase of the topping concrete thickness is followed by the increase of transverse shear strength $V_{Rd.c}$ (because the height of the cross-section increases), the final influence of the topping concrete on torsion-shear bearing capacity can be assumed negligible.

T a b l e 1

Limit transverse force in outermost rib under torsion-shear interaction

			Cross-section type 1	Cross-section type 2
HC200	$V_{Rd.c}$	[kN]	139	94
	V_T	[kN]	51	30
	$V_T/V_{Rd.c}$	[%]	37	32
	$V_{Ed.n} = V_{Rd.c} - V_T$	[kN]	88	64
	V_{Ed}	[kN]	61	56
HC320	$V_{Rd.c}$	[kN]	156	158
	V_T	[kN]	45	46
	$V_T/V_{Rd.c}$	[%]	29	29
	$V_{Ed.n} = V_{Rd.c} - V_T$	[kN]	111	112
	V_{Ed}	[kN]	101	110
HC500	$V_{Rd.c}$	[kN]	289	280
	V_T	[kN]	84	62
	$V_T/V_{Rd.c}$	[%]	29	22
	$V_{Ed.n} = V_{Rd.c} - V_T$	[kN]	205	218
	V_{Ed}	[kN]	156	157

**The influence of the thickness of the topping concrete on changes
in tangent stress and transverse forces in the slab's web due to the interaction
of torsion and shear**

		Thickness of the concrete topping layer [mm]			
		0	50	65	80
$V_{Rd.c}$	[kN]	156	174	180	186
V_T	[kN]	45	55	60	65
$V_T/V_{Rd.c}$	[%]	29	32	33	35
$V_{Ed.n} = V_{Rd.c} - V_T$	[kN]	111	119	120	121
V_{Ed}	[kN]	101	112	117	121

5. Conclusions

In the case of designing precast, pretensioned hollow-core floors, with complicated support conditions (e.g. support on three edges; on non-parallel supports; direct support on the columns; cases of slim floor structures) or non-uniformly distributed loading (e.g. linear loading and loading with concentrated forces), it is necessary to take into consideration simultaneous actions of torsional moment and transverse force. Both torsional moment and transverse force generate tangent stress. In the case of compound actions of torsion and shear in the hollow core slab, one outermost web is subjected to the joined influence of tangent stress with a considerably higher value than the remaining webs, which may lead to cracking and loss of bearing capacity.

The computational method of taking into account this phenomenon presented in the code PN-EN 1168+A3:2011 is based on the same principal assumptions as EC2 and Model Code 2010 but introduces several simplifications. The conducted computational analysis proved that the contribution of torsion actions in the bearing capacity of the outermost web of the hollow core element could be significant, and in the case of small height slabs (e.g. 150 mm up to 200 mm), endangered by the presence of significant torsional moments, design deficiency of the shear capacity may arise. Experimental tests and advanced numerical analyses conducted in leading research centres around the world demonstrated that the results of capacity calculations according to the standards, for the case of torsion and shear interaction, are significantly lower than values obtained from experimental tests or precise numerical calculations, which lead to acquiring a larger safety margin. However, it should be noted that in case of pure torsion, overestimated values of design resistance are obtained – this is caused by the occurrence of a different failure model than that assumed by the standards, i.e. torsional cracking of the upper surface of the slab, instead of cracking of the outermost webs.

The layer of the structural concrete topping cast on the precast elements is usually regarded as a method of increasing the capacity of the slab, but based on the conducted analysis, it can be concluded that in the case of torsion-shear interaction, this effect is insignificant.

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