EVALUATION OF THE EFFICIENCY OF CONSTRUCTION WORKERS ON DIFFERENT DAYS OF THE WEEK IN TERMS OF FUZZY SETS

OCENA WYDAJNOŚCI PRACOWNIKÓW BUDOWLANYCH W POSZCZEGÓLNYCH DNIACH TYGODNIA ROBOCZEGO W KATEGORIACH ZBIORÓW ROZMYTYCH

Abstract
This paper presents the methodology for comparing previously designated efficiencies of teams of workers using the fuzzy set theory. Efficiency was tested using timekeeping on different days of the week in the execution of specific construction tasks by the same team. The article also presents previously tested factors in this aspect and evaluates the impact of a particular day of the week on resource efficiency.

Keywords: work efficiency, impact of the weekday on work efficiency, fuzzy logic

Streszczenie
W artykule przedstawiono metodologię porównywania wcześniej wyznaczonych wydajności brygad roboczych z wykorzystaniem teorii zbiorów rozmytych. Wydajności badano metodą chronometrażu w poszczególnych dniach tygodnia roboczego przy realizacji określonych robót budowlanych przez te same brygady. Przedstawiono również dotychczas badane w tym aspekcie czynniki i przeprowadzono ocenę wpływu konkretnego dnia tygodnia pracy na wydajność zasobów.

Słowa kluczowe: wydajność pracy, wpływ dnia tygodnia na wydajność pracy, logika rozmyta

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1. Introduction

The need for the selection of teams of workers to carry out various tasks within the scope of an entire construction project in the phase of preparation of the investment is a common and very important consideration when planning investment in a project. The reason is that the necessary workload on the execution of tasks and the number of allocated resources are often decisive for the efficient completion of the works. Assumptions relating to worker teams made by the planner during project preparation will have a significant impact on the planned completion deadline of the entire project and the ability to meet the deadline in the construction phase. In the process of executing the works, in addition to the number and qualifications of workers involved, worker efficiency is also a significant factor.

Recognising the causes affecting the efficiency of workers would increase the awareness of engineers determining the selection of worker teams in the phase of investment preparation. This would be an essential prerequisite for the development of a rational model of project management and would minimise the risk of exceeding deadlines from the point of view of resource efficiency.

The purpose of this article is to examine the influence of a particular working day (i.e. Monday, Tuesday, Wednesday, Thursday, Friday and Saturday) on the efficiency of worker teams. To achieve this goal, the rules of fuzzy logic were used, in particular, the concept of the membership function. In the latter part of the article, on the basis of the review of selected Polish and foreign literature, other factors that have thus far been taken into account in terms of examining work efficiency are presented – the day of the week was not included among them.

2. The basis of the theory of fuzzy sets

The basic concept of the theory used in this article is the concept of a fuzzy set. The definition of a fuzzy set can be most simply stated as follows: a fuzzy set is such set \( A \) whose elements \( x \) are characterised by the lack of a sharp boundary between whether \( x \) belongs or does not belong to \( A \). The degree of membership of element \( x \) to the fuzzy set \( A \) is described by the function \( \mu_A(x) \) which is called the membership function. Function \( \mu_A(x) \) assumes values in the interval \([0, 1]\), wherein:

\[\begin{align*}
\mu_A(x) = 0 & \quad \text{means no membership whatsoever of } x \text{ to } A; \\
\mu_A(x) = 1 & \quad \text{means full membership of } x \text{ to } A.
\end{align*}\]

A fuzzy set \( A \) in certain space \( X = \{x\} \), which is written as \( A \subseteq X \), is called a set of pairs [5]:

\[A = \{ (\mu_A(x), x) \}, \forall x \in X \tag{1}\]

In the literature, one can find different definitions of a fuzzy number. One of the most accurate definitions was given by Goetschel and Voxmann in [2]. A fuzzy number \( A \) is a special kind of fuzzy set defined on the set of real numbers \( X = R \), which further satisfies the following conditions:
- **it is normal**, and a set is normal if there is an argument for which the function has a value of 1;
- **it is convex**, and set $A$ is convex if
  \[ \forall x, y \in X \quad \forall \lambda \in [0;1] \quad \mu_A(\lambda \cdot x + (1 - \lambda) \cdot y) \geq \min(\mu_A(x), \mu_A(y)); \]  
  \[ (2) \]
- carries of function $\mu_A(x)$ is an interval;
- $\mu_A(x)$ is a function of continuous intervals.

The characteristic diagrams of the membership function $\mu_A(x)$ are generated assuming a trapezoidal or triangular fuzzy numbers distribution. A fuzzy number with a trapezoidal diagram is illustrated in Fig. 1.

![Trapezoidal membership function](image)

Fig. 1. Trapezoidal membership function

A description of a trapezoidal membership function is also presented in the literature using a table of values (Formula 3).

\[
\mu_A(x) = \begin{cases} 
0, & x < x_1 \\
(x - x_1) / (x_2 - x_1), & x_1 < x < x_2 \\
1, & x_2 < x < x_3 \\
(x_4 - x) / (x_4 - x_3), & x_3 < x < x_4 \\
0, & x > x_4 
\end{cases} 
\]  
\[ (3) \]

A fuzzy number with a triangular diagram is illustrated in Fig. 2.

Description of a triangular membership function is also presented in the literature using a table of values (Formula 4).

\[
\mu_A(x) = \begin{cases} 
0, & x < x_1 \\
(x - x_1) / (x_2 - x_1), & x_1 < x < x_2 \\
(x_3 - x) / (x_3 - x_2), & x_2 < x < x_3 \\
0, & x > x_3 
\end{cases} 
\]  
\[ (4) \]
A fuzzy number can be interpreted as a four \( \{x_1, x_2, x_3, x_4\} \). \( x_2 \) and \( x_3 \) are the interval in which the membership function reaches the value of 1. \( x_1 \) and \( x_4 \) are respectively the left and right width of the distribution of a membership function. A fuzzy number with a trapezoidal diagram can be clearly identified by setting an ordered four of numbers and is written as:

\[
X = (x_1, x_2, x_3, x_4)
\] (5)

A fuzzy number with a triangular diagram can be clearly identified by setting an ordered four of numbers and is written as:

\[
X = (x_1, x_2, x_3)
\] (6)

A fuzzy number can also be expressed in another way – this would be as a fuzzy number with a multi-point membership function or a linear membership function but always with a set of values from 0 to 1.

### 3. Factors affecting worker efficiency

In modern literature, we can find many models, e.g. WLB (work-life-balance) developed in Australia [8] and the study of factors affecting the efficiency of construction workers. Among other factors important in this regard are, rest outside the workplace [1] and atmospheric conditions; more important factors are the temperature and humidity of the working environment [6], psychophysical condition of workers [3] and the appropriate management and organisation of the workplace [4, 7]. There are no results of current research on the effects of the day of the week on the efficiency of construction workers in either Polish literature, or indeed, world literature. This factor can also significantly affect the efficiency of the work, which according to the assumption made by the authors, changes with the change of the working day.
In this context, the authors of the article carried out tests in Warsaw, Poland, using timekeeping. The observed workers were installing lining in ten staircases using prefabricated terrazzo tiling. Actions for which the efficiency of the workers was assessed, included:

– assembly of steps (c₁),
– levelling of landings and corridors (c₂),
– laying of landings and corridors (c₃),
– installation of perimeter stair plinths (c₄),
– installation of perimeter landing plinths (c₅).

For all of these tasks, efficiency was specified using the formula:

\[ w = \frac{p}{n} \]  

where:

\( w \) – efficiency,
\( p \) – amount of work performed expressed in the unit of measure,
\( n \) – workload expressed in work hours.

Many of the factors affecting the efficiency of the workers is hard to quantify and are imprecise. Fuzzy logic is well suited to solve these type of problems.

4. Analysis of the problem in fuzzy terms

In determining the efficiency of teams of workers, we use qualitative criteria in addition to quantitative criteria. Most of these criteria are quantifiable – their value is determined verbally. The interpretation of these criteria performed according to qualitative evidence leads to the categorisation of worker efficiency in a unmeasurable manner, i.e. as ‘good’, ‘satisfactory’, ‘average’, ‘mediocre’, or ‘bad’. The properties of fuzzy sets, which were chosen to be applied, make it possible to describe and compare the efficiency of workers during the performance of various tasks. In the research presented in this article, five specific finishing tasks were taken into account – these are listed in Chapter 3.

The diversity of the works and their different units of amount measurement prevents direct comparison of these works to each other. A common unit of measurement used for the installation of steps (each step is one element) is linear meters of the element (length of the elements constructed), the levelling of the surface and the laying of landings and corridors were described in square metres, while the installation of stair plinths was calculated in the number of sets (one set consist of two plinths – vertical and horizontal for each stair). Using the elements of fuzzy logic, a problem with different units found a solution. Workers efficiency in the implementation of various works with different units of measure, but expressed in a qualitative manner, can be compared.

Another aspect that led to the choice of fuzzy logic was the different ranges of values for each set of results for different tasks, for example:

– installation of steps 0.41–1.58 m/h,
– assembly of plinths 4.88–6.62 sets/h.
Although quantitative criteria are different, they can be determined in an identical manner to each other, using linguistic terms, and included in the same ranges. In order to compare worker efficiency on different days, it was necessary to determine the degree of membership of particular values to the fuzzy set of construction worker efficiency $W$, consisting of pairs according to equation (8):

$$ W = \{ (\mu W(w), w) \}, \forall w \in W $$  

(8)

### 5. Worker efficiency depending on the days of the week in fuzzy conditions

In order to be able to compare each of the five tasks ($c_1 ... c_5$), the minimum value ($x_1$) and the maximum value ($x_4$) of efficiency achieved by the workers were determined for each of them.

$$ \wedge_{w_c \in W_c} \wedge_{c \in \{1 ... 5\}}^c x_1(c) = \min(w_c) $$  

(9)

$$ \wedge_{w_c \in W_c} \wedge_{c \in \{1 ... 5\}}^c x_4(c) = \max(w_c) $$  

(10)

where:
- $c$ – task
- $w_c$ – efficiency of a single measure of tasks $c$,
- $W_c$ – set of efficiencies of all measures of tasks $c$,
- $x_1(c)$ – the value of the minimum efficiency for tasks $c$,
- $x_4(c)$ – the value of the maximum efficiency for tasks $c$.

After determining the maximum and minimum efficiency determined during the test, a function was defined, evaluating the membership of the result obtained to the set of high efficiencies or low efficiencies. The range of results (efficiency achieved by the workers) is divided into three parts, of which all efficiencies found in the first 25% of the range were considered as inefficient and given a value of 0. The top 25% of the range of the results was identified as high efficiency and the value of the function was specified as 1. The middle range of the results was determined using a linear relationship with the values belonging to the interval (0, 1). To determine the characteristic points of the monotonic change of the function, two designations ($x_2$ and $x_3$) were introduced.

![Graph](image.png)

**Fig. 3.** Graph of the membership function for the set of high efficiencies $\mu W(w)$
\[ x_2 = x_1 + 0.25(x_4 - x_1) \quad (11) \]
\[ x_3 = x_1 + 0.75(x_4 - x_1) \quad (12) \]
\[ \mu W(w) = \begin{cases} 
0, & w \leq x_2 \\
(w-x_2)/(x_3-x_2), & x_2 < w < x_3 \\
1, & w \geq x_3 
\end{cases} \quad (13) \]

After the calculations, all the characteristic values for each of the works were obtained.

**Table 1**

Characteristic values of \( x_1, x_2, x_3, x_4 \) determined for each task based on the relationship, respectively (9)‒(12)

<table>
<thead>
<tr>
<th>Unit of efficiency</th>
<th>Installation of steps ([m/h])</th>
<th>Levelling of landings ([m^2/h])</th>
<th>Laying of landings ([m^2/h])</th>
<th>Installation of stair plinths ([set/h])</th>
<th>Installation of landing plinths ([m/h])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determination of tasks</td>
<td>(c_1)</td>
<td>(c_2)</td>
<td>(c_3)</td>
<td>(c_4)</td>
<td>(c_5)</td>
</tr>
<tr>
<td>Value of (x_1)</td>
<td>0.41</td>
<td>1.13</td>
<td>0.79</td>
<td>4.88</td>
<td>3.39</td>
</tr>
<tr>
<td>Value of (x_2)</td>
<td>0.7</td>
<td>1.44</td>
<td>0.95</td>
<td>5.32</td>
<td>3.84</td>
</tr>
<tr>
<td>Value of (x_3)</td>
<td>1.29</td>
<td>2.05</td>
<td>1.26</td>
<td>6.19</td>
<td>4.74</td>
</tr>
<tr>
<td>Value of (x_4)</td>
<td>1.58</td>
<td>2.36</td>
<td>1.42</td>
<td>6.62</td>
<td>5.19</td>
</tr>
</tbody>
</table>

Fig. 4. Average value of the membership function for the results for the set of high efficiencies on different days of the week.
After determining all the characteristic values, each measurement result for each task was assigned the value of the function $\mu_W(w)$ based on equation (13). The collected results of calculations were ranked by days of the week. For example, on Monday, for all the tasks ($c_1 \ldots c_5$), the following results for the membership function for the set of high efficiencies were obtained: 0.01; 0; 0.67; 0.51; 0.8; 0.22; 0; 0.75; 0.64; 0.78; 0.26; 1; 0.28; 0.7; 1; 0.84; 0.8; 0.99; 0.18; 1; 0. Then, the arithmetic mean of the combined results was calculated (0.53). Similar calculations were made for each day of the week, and the resulting averages are presented in the chart.

6. Conclusions

The paper presents a scientifically valid methodology for assessing the impact of the day of the week on the efficiency of teams of workers in their performance on five specific construction works. The method is based on the theory of fuzzy sets and uses the concept of a membership function. Despite the use of fuzzy logic, operations performed on previously collected timekeeping data are simple; thus, they do not raise difficulties in their practical application. This gives a greater potential for its use in construction, where the efficiency of workers has a very large and direct impact on the duration of the works.

The results show that there is a relationship between labour efficiency and the day of the week. The smallest efficiency was achieved on Saturday (0.3), while the greatest was on Tuesday (0.71). The rest of the results are comparable: the results for Monday and Friday are very similar, and for Wednesday and Thursday exactly the same (0.61). This information may be useful in planning and the detailed scheduling of works. It shows that there is a relationship between labour efficiency and the day of the week, and the resulting ratio expresses its size.

In further work on this topic, I plan to expand the study to other factors and to create a model supporting the planning of works in practice. Analyses will continue to be conducted in terms of labour efficiency of teams of workers.

References

