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FRICTION CURVES TRANSFORMATION OF NON-NEWTONIAN FLUIDS IN COILS

TRANSFORMACJA KRZYWYCH OPORÓW PRZEPŁYWU PŁYNÓW NIENEWTONOWSKI W WĘŻOWNICACH

A b s t r a c t

The transformation of pseudo-Newtonian dimensionless numbers: Re and f_s, describing flow of power-law non-Newtonian fluids in circularly curved tubes, has been done. It has been shown that the multi-parameter friction curves of power-law non-Newtonian fluid can be described, in new dimensionless coordinate system, with the help of single curve in the laminar as well as turbulent flow region. Moreover, the criterion of transition from laminar to turbulent region was clearly determined.

Keywords: *coil*, *non-Newtonian fluid*, *inner resistance*

S t r e s z c z e n i e

Przeprowadzono transformację pseudonewtonowskich liczb bezwymiarowych opisujących przepływ płynów nienewtonowskich, spełniających prawo potęgowe, w rurach zwiniętych kołowo. Wykazano, że wieloparametrowe krzywe oporów potęgowych cieczy nienewtonowskich mogą być opisane w nowym układzie bezwymiarowych współrzędnych za pomocą pojedynczej krzywej zarówno w zakresie laminarnym, jak i turbulentnym. Ponadto jednoznacznie określono kryterium przejścia od ruchu laminarnego do turbulentnego.

Słowa kluczowe: wężownica, płyn nienewtonowski, tarcie wewnętrzne

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Symbols

D – curvature diameter [m]
d – inner pipe diameter [m]
De – Dean number (De = Re $[d/D]^{0.5}$) [–]
De_t – characteristic Dean number (De_t = Re $[d/D]^2$) [–]
De – pseudo-Newtonian Dean number (**De** = **Re** $[d/D]^{0.5}$) [–]
$\mathbf{De}_m$ – modified Dean number (eq. (26)) [–]
F_c – pseudo-Newtonian friction factor ($\mathbf{F}_c = \mathbf{f}_c \cdot [D/d]^{0.5}$) [–]
$\mathbf{F}_{cm}$ – modified friction factor (eq. (12)) [–]
f_c – Fanning factor (eq. (14)) [–]
$\mathbf{f}_c$ – pseudo-Newtonian Fanning factor of curved pipes (eq. (13)) [m/s]
K – consistency constant of a power-law fluid [$kg \cdot s^{n-2} \cdot m^{-1}$]
L – length of a curved pipe measurement section [m]
n – power-law index [–]
Δp – pressure loss [$N \cdot m^{-2}$]
Re – pseudo-Newtonian Reynolds number (eq. (11)) [–]
Re′ – generalized Reynolds number (eq. (12)) [–]
v – mean velocity [m/s]
ρ – density of a fluid [$kg \cdot m^{-3}$]

REMARK: all bolted symbols relate to the pseudo-Newtonian fluid flow

1. Introduction

In recent years, developments in process engineering has caused liquids of various kinds to be used extensively within industry. Since many such fluids exhibit non-Newtonian flow properties, it has become increasingly important to determine the flow characteristics of non-Newtonian fluids. Curved tubes are widely used for the passage of fluids, heat exchangers and many industrial applications. For this reason, many theoretical and experimental studies on the flow of Newtonian fluids through coiled pipes have been published.

Dean [1] analytically solved the Navier-Stokes equations for the flow of a Newtonian fluid in a round curved tube under the assumption that the radius of curvature is large. He showed that a single dimensionless expression

$$\mathrm{De} = \frac{\mathrm{Re}}{\left(\frac{D}{d}\right)^{0.5}} \tag{1}$$

later called the Dean number, is the essential dynamic parameter that has an influence upon pressure losses in the flow through curved pipes in the laminar as well as turbulent region and – especially in cases where the ratio of radii D/d is small. It causes the friction factor curve of the fluid flow in the straight pipe (described in the laminar region by the Fanning formula and in the turbulent region by the Blasius formula) to be changed into a one or two-parameter cluster of curves [2–6].

The momentum integral method was used by Ito [7] to analyse the flow of Newtonian fluid in a curved tube. He showed that the relationship between the Dean number and the dimensionless number F_c, called the friction index, can be described in the laminar flow region by the empirical formula

$$\mathrm{F}_c = \frac{344}{(1.56 + \log \mathrm{De})^{5.73}} \tag{2}$$

where

$$\mathrm{F}_c = \mathrm{f}_c \left(\frac{D}{d} \right)^{0.5} \tag{3}$$

and

$$\mathrm{f}_c = \frac{d \Delta p}{2 L \rho v^2} \tag{4}$$

In the turbulent flow region, the relationship between the non-dimensional variable,

$$\mathrm{De}_t = \mathrm{Re} \left(\frac{d}{D} \right)^2 \tag{5}$$

first introduced by Ito, and the friction index is depicted by Ito's formula

$$\mathrm{F}_c = \frac{0.079}{\mathrm{De}_t^{0.2}} \tag{6}$$

On the other hand, the flow of non-Newtonian fluids within coiled pipes has not been analysed to the same extend. The first analysis of laminar and turbulent flows of a purely viscous power-law fluid in the curved tubes was carried out by Mashelkar and Devarajan [8–10]. They assumed that in the region of high Dean numbers (De > 100), which is of practical importance, the secondary field consists of an inviscid core and a thin boundary layer adjacent to the wall. The central part of the fluid is driven towards the outer wall by the centrifugal force. Thus the fluid entering the boundary layer region is pushed back along the wall towards the inner side by a pressure gradient. It then returns to the core region and this pattern leads to the vertical motion in the cross-section of the pipe. On the whole, the axial velocity dominates the flow in the coiled tube, but it becomes comparable to the angular velocity in the boundary layer.

Despite the fact that Mashelkar and Devarajan applied the momentum integral method, they had to solve the governing equations numerically and present the numerical results in a form that was suitable for engineering design. The final correlations for the laminar and the turbulent flow regions are given as formulas (7) and (8) respectively:

$$\mathrm{f}_c = (9.069 - 9.438n + 4.37n^2) \left(\frac{d}{D} \right)^{0.5} \left[8^{(n-1)} \left(\frac{3n+1}{4n} \right)^n \mathrm{Re}' \left(\frac{d}{D} \right)^{0.5} \right]^{(0.122n - 0.768)} \tag{7}$$

$$\mathrm{f}_c = \frac{\alpha^*\left(\frac{d}{D}\right)^{0.5}}{\left[8^{(n-1)}\left(\frac{3n+1}{4n}\right)^n \mathrm{Re}'\left(\frac{d}{D}\right)^{1/(2\beta)}\right]^{\beta/(n\beta+1)}} \tag{8}$$

where α^* and β are functions of flow behaviour index.

Although Mashelkar and Devarajan stated that their results were in excellent agreement with Ito's solution for Newtonian fluids, their correlations are not equivalent to Ito's equations for $n = 1$. Similar approaches hold true for the other empirical correlations reported in the literature [11–15].

The present investigation was undertaken to study a fully developed curved pipe flow of purely viscous non-Newtonian liquids over an extensive range of Reynolds numbers.

The main objective of this work is to present a new method for predicting a pressure drop along the centre-line of a coiled pipe. A special transformation method [12] was extended and adopted to construct a pseudo-Newtonian model for the non-Newtonian flow within a curved tube.

2. Pressure losses in curved tubes

The prediction of pressure losses in the flow of non-Newtonian fluids in curved pipes is much more complicated. The reason for this is that Ito's equations (2) and (6), describing the flow of even simple Ostwald-de Waele rheological formula fluid, generate additional curves in the laminar as well as the turbulent region of the [De, F_c] co-ordinate system.

The inconvenience can be partially eliminated by the generalization of the transformation method describing flow of non-Newtonian fluids in straight pipe [16].

Consider a pseudo-Newtonian model of non-Newtonian fluid flow in the pipe. Matras and Nowak [16] defined the following dimensionless variables and expressions as follows:

$$\mathbf{f}_c = \frac{16}{\mathrm{Re}} \tag{9}$$

It was also shown [16] that the turbulent dimensionless resistance law for smooth pipes takes a form analogous to Blasius' formula

$$\mathbf{f}_c = 0.079\mathbf{Re}^{-0.25} \tag{10}$$

The modified Reynolds number **Re** is related to the generalized Metzner and Reed's Reynolds number Re′ by the equation

$$\mathbf{Re} = \mathrm{Re}'\left[\frac{2(n+1)}{3n+1}\right]^{-2.5} \tag{11}$$

where

$$\mathrm{Re}' = \frac{d^n v^{2-n} \rho}{K\left(\frac{3n+1}{4n}\right)^n 8^{(n-1)}} \tag{12}$$

Similarly, the modified friction factor is related to the classical Fanning factor by the equation

$$\mathbf{f}_c = \mathrm{f}_c\left[\frac{2(n+1)}{3n+1}\right]^{2.5} \tag{13}$$

where

$$\mathrm{f}_c = \frac{d\Delta p}{2L\rho v^2} \tag{14}$$

This finding encouraged the author to take the modified friction factor and the modified Reynolds number, resulting from the transformation method, and use them in Ito's pseudo--Newtonian formulas to describe the pressure drops in the laminar flow

$$\mathbf{F}_c = \frac{A}{(B + \log \mathbf{De})^{\alpha}} \tag{15}$$

and the turbulent flow of non-Newtonian fluids through curved pipes

$$\mathbf{F}_c = \frac{C}{\mathbf{De}_t^a} \tag{16}$$

In both equations (1) and (16), $\mathbf{F}_c$, $\mathbf{De}$ and $\mathbf{De}_t$ are modified pseudo-Newtonian dimensionless numbers defined as follows:

$$\mathbf{De} = \mathbf{Re}\left(\frac{d}{D}\right)^{0.5} \tag{17}$$

$$\mathbf{De}_t = \mathbf{Re}\left(\frac{d}{D}\right)^{2} \tag{18}$$

$$\mathbf{F}_c = \mathbf{f}_c\left(\frac{D}{d}\right)^{0.5} \tag{19}$$

3. Experimental methods and coordidate system transformation

Experimental apparatus, in the form of a standard pipe flow facility was used to verify the utility of the proposed, generalized Ito's equations (15) and (16). Working fluids flowed into a straight or coiled pipe from a head tank. The fluid discharged through a pipe made of polyamide was recirculated or discarded. Pressure drop measurements were performed on two coils with ratio of radii d/D equal to 0.0232 and 0.0323.The geometrical details of coiled tubes are given in Figs 1 and 2. The pitch of the coiled pipe was equal to its outer diameter. Two pressure taps with a 1mm in diameter were bored in the outer wall of each pipe.

The axial pressure difference was measured by using differential pressure transducers PD1 (HBM) suitably placed to avoid the entrance and exit effects. The volumetric flow rates were measured by an electromagnetic flow meter. Experiments were conducted on aqueous solutions of low molecular methylcellulose (MC) with concentrations in the range of 0.1% to 0.4% (by wt.) and mixture of 0.3% aqueous solution of hydroxyethyl cellulose (HEC) and 0.1% water chalk suspension. Special care was taken in the preparation of the solutions. Possible effects of viscoelasticity on the rheological properties were eliminated by first preheating and then recirculating each of the studied solutions through the system for a few hours. Shear stress/shear rate data were taken by means of non-corrective capillary rheometer URK-1, [17]. All the solutions were determined to be power-law, non-Newtonian liquids in the shear rate range of 150–35,000 s^{-1} and in the flow index n range of 0.769–1.0. Shear stress/shear rate data taken for each solution just before and just after each series of runs showed no further degradation of polymer solutions, i.e., no changes in consistency K and flow-behaviour index n were observed.

A preliminary experiment was carried out to test the adequacy of the flow system. Friction indices of curved pipes for water were obtained from this experiment and these results were plotted in the form suggested by Ito's Equations. Very strong agreement was found between Ito's equation (16) and the turbulent flow data with C = 0.079 and a = 0.2. On the other hand, the data points within the laminar flow range indicate that the Newtonian friction index is slightly small in the comparison to its theoretical value calculated from Ito's original equation (2). Nevertheless, it can be well described by Ito's equation (15) with A = 321 and unchanged values of B = 1.56 and α = 5.73

Now, the generalized Ito's equation describing the pseudo-Newtonian analogue of purely viscous non-Newtonian fluid under laminar flow conditions

$$\mathbf{F}_c = \frac{321}{(1.56 + \log \mathbf{De})^{5.73}} \tag{20}$$

and equation

$$\mathbf{F}_c = \frac{0.079}{\mathbf{De}_t^{0.2}} \tag{21}$$

within the turbulent flow region can be tested.

The experimental values of the friction indices obtained in this work have been compared with these relationships in Fig. 1 for polymer solutions of different concentrations. The different data symbols refer to different values of curvature ratios d/D. The data tested in this work correspond to the modified Dean number's $\mathbf{De}_t$ range of approximately 0.01 to 80. A statistical analysis performed to test the strength of the fit indicated that all the data on purely viscous fluids were correlated with a standard deviation of approximately 3%. Such a strong agreement between the theory and the experimental results confirms the validity of the hypothetical pseudo-Newtonian model for the laminar curved pipe flow.

The non-Newtonian power-law index n does not have an influence on the shape and location of the curve (20) in the laminar flow region. However, the curve described by the mentioned equation is no longer valid above some critical values of Dean number $\mathbf{De}_*$. The value of $\mathbf{De}_*$ depends on d/D ratio.

The turbulent flow data reported in Fig. 1 was plotted in the form suggested by the generalized Ito's equation (21). The scattering of data about the curve (21) in the turbulent flow range does not exceed ±2% and does not depend on either the curvature ratio d/D or the flow-behaviour index n.

Representative experimental data of the aqueous solutions flow of MC and HEC in curved pipes presented in the [De_t, $\mathbf{F}_c$] co-ordinate system (Fig. 1) overlap each other, according to expression (21), along a single curve in the turbulent region. On the other hand, the one-parameter series of curves is observed within the laminar flow region. There is no knowledge of the upper limit of the validity of expression (21). $\mathbf{De}_*$ is unknown function of the ratio (d/D).

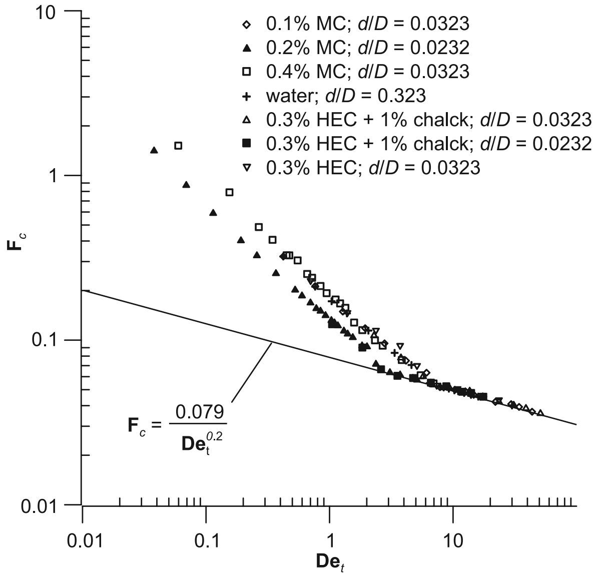

Fig. 1. $\mathbf{F}_c$ vs. $\mathbf{De}_t$ for aqueous polymer solutions

It is desirable that co-ordinates should be transformed in that way to make experimental data in Fig. 1 lie on a single modified curve in a new co-ordinate system.

Assume that there is a non-dimensional co-ordinate system to address the problem. Then, the critical value of the adequately transformed Dean number $\mathbf{De}_{t*}$ should statistically meet a constant value regardless of the pipe curvature and power-law index value – n.

Transform the equation (20) into the following form

$$\mathbf{F}_c = \frac{321}{\left\{1.56 + \log[\mathbf{De}_t]\left(\dfrac{d}{D}\right)^{-1.5}\right\}^{5.73}} \tag{22}$$

Making the problem somewhat easier one can assume that the intersection of the curve (21) and any other, followed by the expression (20), obtain the point **($\mathbf{De}_{t*}$, $\mathbf{F}_{c*}$)** as a place of transition from the laminar flow region to the turbulent one. Such an assumption is quite justified by the experimental data analysis shown in Fig. 1, which proves the violent nature of the mentioned transition. If we eliminate the friction index $\mathbf{F}_{c*}$ out of equations (21) and (22), we obtain

$$1 = \frac{\mathbf{De}_{t*}^{0.2}}{\left\{1.56 + \log[\mathbf{De}_{t*}]\left(\frac{d}{D}\right)^{-1.5}\right\}^{5.73}} \tag{23}$$

The embroiled form of expression (23) causes it to be of little use in predicting the lower validity limit of equation (21). However, it can be approximated with a high degree of accuracy by the expression

$$\mathbf{De}_{t*} = \left[114 + 24568\left(\frac{d}{D}\right)\right]\left(\frac{d}{D}\right)^{1.5} \tag{24}$$

The corresponding value of the critical friction index is as follows

$$\mathbf{F}_{c*} = \frac{0.079}{\mathbf{De}_{t*}^{0.2}} \tag{25}$$

Transform the [$\mathbf{De}_t$, $\mathbf{F}_c$] dimensionless co-ordinate system to the new one [$\mathbf{De}_{tm}$, $\mathbf{F}_{cm}$] in the way, that one moves each point laying on the curve (21) (adequate to the critical value of Dean number (24)), to the point of the ($\mathbf{De}_{t0}$, $\mathbf{F}_{c0}$) co-ordinates laying also on the curve (21). All the points which create the curve described by the equation (22) then move to the single constant curve $\mathbf{F}_{cm} = \mathrm{f}\,(\mathbf{De}_{tm})$ that crosses the curve (21) at the point of ($\mathbf{De}_{t0}$, $\mathbf{F}_{c0}$) co-ordinates.

Consider the new modified criterion numbers $\mathbf{De}_{tm}$ and $\mathbf{F}_{cm}$ defined as follows

$$\mathbf{De}_{tm} = \mathbf{De}_t \frac{\mathbf{De}_{t0}}{\mathbf{De}_{t*}} \tag{26}$$

and

$$\mathbf{F}_{cm} = \mathbf{F}_c \frac{\mathbf{F}_{c0}}{\mathbf{F}_{c*}} \tag{27}$$

where $\mathbf{De}_{t*}$ is defined by means of equation (24) and $\mathbf{F}_{cm*}$ as follows in (24) and (25), i.e.

$$\mathbf{F}_{c*} = \frac{0.079}{\left\{\left(114 + 24568\frac{d}{D}\right)\left(\frac{d}{D}\right)^{1.5}\right\}^{0.2}} \tag{28}$$

Let the critical value of Dean number $\mathbf{De}_{t0}=1$. Then, the equations

$$\mathbf{F}_{cm}=\frac{321}{(4.26+\log\mathbf{De}_{tm})^{5.73}} \tag{29}$$

and

$$\mathbf{F}_{cm}=\frac{0.079}{\mathbf{De}_{tm}^{0.2}} \tag{30}$$

should respectively describe the single curve of hydraulic losses in the laminar as well as the turbulent flow region.

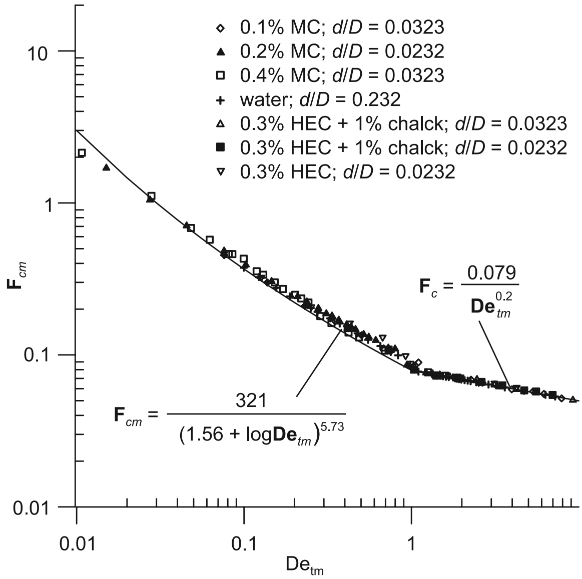

Fig. 2. Modified resistance curve of the non-Newtonian fluids flow in curved pipes

This proposal was confirmed by the experimental data shown in Fig. 2. Marked points represent the same experimental data as shown in Fig. 1, but they had been transformed according to formula (26) and (27). The generalized hydraulic loses curve was drawn as a solid line described by equations (29) and (30).

A strong agreement between equations (20) and (21) with experimental data fully confirm the hydrodynamic analogy of the laminar and turbulent flow of the non-Newtonian fluid in curved pipes to the Newtonian fluid.

Regardless of the power-law index n, the single curve correlations (29) and (30), can be applied in ranges $0.01 < d/D < 0.1$ and $0.01 < \mathbf{De}_{tm} = 1 < 100$.

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