TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE
1-NP/2015

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STABILISING FEEDBACK IN MAX-PLUS LINEAR MODELS OF DISCRETE PROCESSES

STABILIZACYJNE SPRZĘŻENIE ZWROTNE W MAX-PLUS LINIOWYCH MODELACH PROCESÓW DYSKRETNYCH

A b s t r a c t

This article relates to a synthesising output feedback that is used to control a network of discrete events. The feedback stabilises the system without reducing its initial throughput and its synthesis is mainly based on the theory of residues and the Kleene operator. This article suggests some theoretical results and mathematical foundations of max-plus algebra theory, and in particularly, discusses various other aspects of controlling discrete processes and their modelling in the context of a linear max-plus system.

Keywords: *discrete processes*, *TEG*, *max-plus linear system*, *stability*, *feedback control*

S t r e s z c z e n i e

Artykuł dotyczy syntezy sprzężenia zwrotnego w sterowaniu siecią zdarzeń dyskretnych. Sprzężenie zwrotne służy do stabilizacji systemu bez zmniejszenia jego początkowej przepustowości i jego synteza opiera się głównie na wynikach teorii residuów i operatora Kleene'a. W artykule zasygnalizowano pewne wyniki teoretyczne i wprowadzono matematyczne podstawy max-plus algebry. Omówiono także inne aspekty sterowania procesami dyskretnymi oraz ich modelowanie w kategoriach liniowego systemu max-plus.

Słowa kluczowe: *procesy dyskretne, liniowy system max-plus, stabilność, sterowanie, sprzężenie zwrotne*

DOI: 10.4467/2353737XCT.15.115.4152

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1. Introduction

The name dynamic discrete event systems (DDES) applies to a group of systems, usually of a human design, exhibiting a dynamic behaviour. It includes output systems (flexible workshops, assembly lines) [8], communication networks, computer networks [24] and transportation systems (road, rail or air) [28, 34]. It is widely known that to achieve an effective design and operation techniques of different discrete event systems (DES's), methods and tools are needed to model complex material and control flows, to analyse the behaviour and interactions of manufacturing resources and to predict performance measures such as productivity, cycle times and work-in-process.

Many other discrete event dynamic systems, such as transportation networks [35] and communication networks [18], are subject to synchronisation phenomena. DDES often appear in the context of parallel computing, especially in application specific integrated circuits (ASICs) [21], project management systems [19], or telecommunication networks etc. The diversity of these systems naturally leads to different models, including ones based on finite state automata [12], Markov chains [1] and Petri nets [38].

The thesis about the importance of systems in our society leads many researchers to propose mathematical models that describe the behavior of these systems in order to evaluate their performance, optimization, control and design. Recent years have seen a quantitative growth of research on discrete systems that can be modelled as max-plus linear systems (MPLSs). Previously, the majority of literature on this class of system discussed the modelling, performance and analyses of their properties [2, 20] rather than their control.

A Petri net model playing the role of an event graph, max-plus algebra and min-plus algebra make it possible to write linear equations and model the aforementioned systems. Timed event graphs (TEG's) form a subclass of timed Petri nets and are suitable for modelling these systems. A TEG is a timed Petri net in which all places have exactly one upstream transition and one downstream transition. Its description can be transformed into a (max; +) or a (min; +) linear model and vice versa [2].

A single transition for each output of a place has the practical result that all possible conflicts concerning the use of tokens in the places have already been established. This means that there is no competition on either side of the transition but this is the unavoidable cost of maintaining the linearity of the system. Some problems must be accepted for these restrictions to be guaranteed.

More than two decades have now passed since the max-plus algebra and similar algebraic tools started to play a central role in the simulation and analysis of discrete processes. Actually, however, the use of these theories led to the creation of a linear theory for certain discrete event systems [16]. Therefore, working in the field of linear systems, max-plus uses guidelines and concepts presented by classical theories. However, the number of researchers involved in this new area of DES system theory is rather small compared to the hundreds of other scientists who contributed to the classical theory. Historically, some specific control theory for DDESs was developed including, for example, optimal feedback [26, 31, 37], open loop control [27, 29, 32] and predictive control for a perturbed system [36].

To handle more complex models, it was necessary to adapt mathematical tools while keeping most of the concepts provided by earlier developments. Differential geometry, power series of noncommutative variables and differential algebra have all been used to develop such models for which essential questions concerning, for example, controllability and observability, stabilisation and feedback synthesis, etc., have been revisited. Max-plus, min-plus and other idempotent semiring structures turn out to be the right mathematical tools to at best, bring linearity back, or to at least achieve a certain suitability to the nature of the phenomena to be described in this field of DES.

This paper presents the problem of stabilising discrete processes of the system by using output feedback control. The issue of stabilising DES was considered by Commault [9] and Cohen [8]. Just as in classical models of continuous systems, stability is closely linked to the structure of the system. A TEG is structurally stable if its number of markers is restricted for all sequences of data input [2].

Commault defines a sufficient condition for the stability of the TEG which is satisfied if the network graph strongly associates events by closing the circuit from the exit to the entry. Some quantity of markers which limit the marking of the system should be added along the extra path (in the simplest method).

Furthermore, it is shown in [2] that a controllable and observable TEG can be stabilised by adding feedback output without changing its own bandwidth. Gaubert showed in [15] that the number of tokens that must be placed in the feedback to stabilize the TEG represents the optimisation of resources that can be formulated as an integer linear programming problem.

The approach presented here is based on Gaubert's work [15]. Other methods are presented as the greatest linear feedback in [30], where this represents the synthesis of dynamic feedback which minimizes the number of markers required to retain the original throughput.

Section 2 serves as a reminder of the algebraic tools necessary for synthesising feedback. Section 3 briefly explains max-plus modelling with a practical example. Section 4 shows how the existing feedback in a TEG can improve the stability of the system and how it can be implemented. Section 5 aims to present an illustrative example.

2. The mathematical basis of max-plus algebra

This section aims to concisely present formal definitions and key algebraic tools used in subsequent sections. More precisely, it reiterates Chapter 4 of [2] and [7] and certain theses [10, 17, 25, 30]. The max-plus theory is based on the lattice theory and largely reverses applications defined on ordered sets [4, 11]. Algebra max-plus use a structure of idempotent semiring algebras S.

This semiring S has two internal operations denoted by $\oplus$ and $\otimes$ which have many properties:

- operation $\oplus$ is associative, commutative and idempotent, that is $a \oplus a = a$;
- operation $\otimes$ is associative (but not necessarily commutative) and distributive on the left and on the right with respect to the $\oplus$;
- neutral elements for $\oplus$ and $\otimes$ are represented by ε and e respectively;
- ε is an absorbing element of the right $\forall a = \varepsilon \otimes a = a \otimes \varepsilon = \varepsilon$;

– just like in classical algebra, operator ⊗ will often be omitted in equations (as in classical multiplication) and $a^i = a \otimes a^{i-1}$ as well as $a^0 = e$.

$Z_{max} = \{Z, -\infty, \infty\}$ is the set of numbers endowed with the maximisation as the operator ⊕ (symbol pronounced 'o-plus') and the sum as the ⊗ operator (pronounced 'o-times') and neutral values $\varepsilon = -\infty$ and $e = 0$.

For example:

$$y = 14 \otimes 3^2 \oplus 3 \otimes 5^8 = \max((14 + 2 \times 3), 3 + 8 \times 5) = 43$$

The implicit equation $x = a \otimes x \oplus b$ determines $a = a^* \otimes b$ where the Kleene star operator:

$$a^* = \bigoplus_{i=0}^{\infty} a^i \tag{1}$$

For matrices if $\mathbf{A}, \mathbf{B} \in Z_{\max}^{m \times n}$ and $\mathbf{C} \in Z_{\max}^{m \times p}$, then:

$$(\mathbf{A} \oplus \mathbf{B})_{ij} = a_{ij} \oplus b_{ij} = \max(a_{ij}, b_{ij})$$

$$(\mathbf{A} \otimes \mathbf{C})_{ij} = \bigoplus_{k=1}^{n} a_{ik} \otimes c_{kj} = \max_{k=1 \ldots n}(a_{ik} \otimes c_{ki}) \text{ for all } i, j.$$

The Kleene star operator can also be applied to matrices:

$$\mathbf{A}^* = \bigoplus_{i=0}^{\infty} \mathbf{A}^i \quad \text{with} \quad \mathbf{A}^{i+1} = \mathbf{A} \otimes \mathbf{A}^i \quad \text{and} \quad \mathbf{A}^0 = \mathbf{I} \tag{2}$$

where the identity matrix **I** that is $(\mathbf{I})_{ij} = \begin{cases} e & \text{if} \quad i = j \\ \varepsilon & \text{if} \quad i \neq j \end{cases}$

Equation (2) which has a nilpotent matrix, achieves convergence (all coefficients ε).

3. Timed Event Graph as Max-Plus Linear System

A Petri net is called an event graph if all arcs have the weight 1 and each place has exactly one input and one output transition, that is, $\left|{}^*p_i\right| = \left|p_i^*\right| = 1, \ \forall p_i \in P.$ P is the (finite) set of places but $\left|{}^*p_i\right|, \left|p_i^*\right|$ is the quantity of the input and output transition, respectively of, place p_i. In general (standard) Petri nets, and consequently also event graphs, only the logical behaviour and the precise ordering of the events, that is, the possible sequences of firings of transitions, are modelled. However, in many applications, the timing of the events plays an essential role and specific firing times or the earliest possible firing times of transitions are of particular interest. Consequently, event graphs have been equipped with timing information. Time can be associated either with transitions (representing transition delays) or with places (representing holding times) and provides the TEG.

In TEGs, transition delays can always be converted into holding times (by simply shifting each transition delay to all input places of the corresponding transition). However, it is not possible to convert every TEG with holding times into a TEG with transition delays. In a TEG with holding times, a token entering place has to spend some time units before it can contribute to the firing of its output transition. The example of a TEG with the determined holding time of 2 units in place P1 is given in Fig. 1 [5].

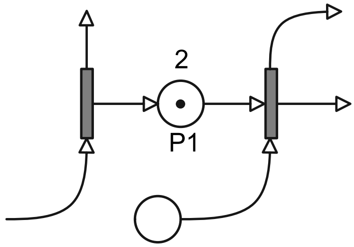

Fig. 1. Graphical representation of a TEG

State-space descriptions in the max-plus algebra for a certain class of discrete-event--systems become linear representations which are similar to state-space equations in the traditional modern control theory [36]. Generally speaking, for any TEG system, one obtains the following kind of equations as a MPLS [6]:

$$\mathbf{x}(k) = \bigoplus_{i=0}^{M} \mathbf{A}_i \mathbf{x}(k-i) \oplus \mathbf{B}_i \mathbf{u}(k-i) \tag{3.1}$$

$$y(k) = \bigoplus_{i=0}^{M} \mathbf{C}_i \mathbf{x}(k-i) \tag{3.2}$$

where **x, u,** and **y** are vectors of dimensions equal to the numbers of internal, input and output transitions, respectively. $\mathbf{A}_i$, $\mathbf{B}_i$, and $\mathbf{C}_i$ are matrices of the appropriate dimensions with entries in the max-plus algebra, and M is the maximal number of tokens in the initial marking. The variables of (3) are time instances and the represented events occur at k-times. Entries of $\mathbf{A}_0$ correspond to places with no tokens and (3) has an implicit form.

The implicit part can be eliminated by successive substitutions or by using the Kleene star (2). Finally, the MPLS has an explicit form:

$$\mathbf{x}(k) = \mathbf{A}\,\mathbf{x}(k-1) \oplus \mathbf{B}\,\mathbf{u}(k) \tag{4.1}$$

$$\mathbf{y}(k) = \mathbf{C}\,\mathbf{x}(k) \tag{4.2}$$

where:

$\mathbf{A} \in Z_{\max}^{m\times n}$ – state-transition (or system) matrix,

$\mathbf{B} \in Z_{\max}^{m\times p}$ – input matrix,

$\mathbf{C} \in Z_{\max}^{m\times n}$ – output matrix,

$\mathbf{x} \in Z_{\max}^{n}$ – state vector,

$\mathbf{u} \in Z_{\max}^{p}$ – input (control) vector,

$\mathbf{y} \in Z_{\max}^{m}$ – output vector.

As there are transforms similar to the Laplace and Z-transforms (used to represent continuous and discrete time trajectories in the classical theory), the input-output behaviour of a TEG can be represented by another transfer relation.

For trajectories $x(k)$, this can be the γ-transform $X(\gamma) = \bigoplus_{k \in Z} x(k)\gamma^{k}$ where γ can be considered as the backward shift operator i.e. $\gamma x(k) = x(k-1)$.

Below, the model of the system is represented by (γ, δ) – transform – $M_{in}^{ax}[[\gamma, \delta]]$ a set of formal power series for two variables γ and δ. A finite series of $M_{in}^{ax}[[\gamma, \delta]]$ is a polynomial and is used to code a set of information concerning the transition of a TEG. Monomial $\gamma^{k}\delta^{t}$ may be interpreted as the k-th event occurring at least at time t.

4. Synthesis of a feedback system

The feedback stabilisation requires a certain quantity of starting markers in the feedback, e.g. when the TEG describes a production system with markers for the initial resource, which may be any transport or recycling equipment (robots, pallets or raw materials). For computational processes, extra resources can represent some units of memory. Consequently, it is of particular importance to limit their quantity as far as possible. What is being considered is the problem of minimising the marking for both restrictions (stabilise and preserve throughput). The problem of optimising resources is solved by the (min, +) algebra in [14] and also by other authors who solved the problem, possibly without defining an approach (max, +). The TEG system consists of m inputs and p outputs, as well as some arcs of places between outputs and inputs to make the system strongly consistent.

The problem is solved using an the algebraic structure $M_{in}^{ax}[[\gamma, \delta]]$ of the open system

$$\mathbf{y} = \mathbf{H}\,\mathbf{u} \tag{5}$$

where:

$\mathbf{H} \in M_{in}^{ax}[[\gamma, \delta]]_{p \times m}$.

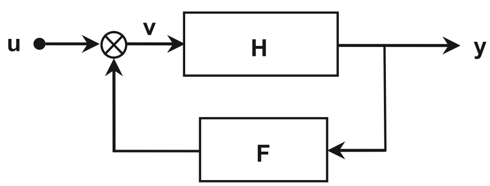

Fig. 2. Block diagram of a system with feedback

By applying (2), the closed system (Fig. 2) is

$$\mathbf{y} = \mathbf{H}(\mathbf{FH})^{*}\mathbf{u} \tag{6}$$

where:

$\mathbf{F} \in M_{in}^{ax}[[\gamma, \delta]]_{p \times m}$ – output feedback

The feedback system is a matrix transfer (transmittance) with elements $F_{ij} = \gamma^{q_{ij}}$ where arcs containing the q_{ij} markers connect the output y_j with the input signal u_i at the initial state. If $F_{ij} = \varepsilon$, then there is no arc. In practice, it is not always necessary to connect all outputs to all inputs for a strong relationship.

The stabilisation problem refers to calculating and minimizing $q = \{q_{ij}\}$ in a closed-loop system to maintain the same throughput as an open loop has. Authors of [14] have shown that this problem can be solved as an integer linear programming task with a linear cost function

$$J(q) = \sum_{i=1}^{i=m} \sum_{j=1}^{j=p} \alpha_{ij} q_{ij} \tag{7}$$

where:

α_{ij} – weighting factor associated with the price of each resource to maintain the same throughput with a restrictive condition

$$\lambda(q) \geq \lambda_p \tag{8}$$

where:

λ_p – throughput for the open loop system,
$\lambda(q)$ – throughput for the feedback system.

Throughput $\lambda(q)$ is derived from the formula

$$\lambda(q) = \min_{c} \frac{w_{Nc}(q)}{w_{Tc}} \tag{9}$$

where:

$w_{Nc}(q)$ – sum of markers in circuit c,
$w_{Tc}(q)$ – sum of holding times in circuit c.

From (8) and (9), the following constraints are satisfied for each cycle:

$$w_{Nc}(q) \geq \lambda_p w_{Tc} \tag{10}$$

To summarise, the stabilisation method of solving the problem with limited resources consist of the following steps:

– determine the transform of **H** for an open system (5);
– formulate transform feedback **F** (6);
– solve the problem of integer programming (7, 9) and get the values of **F**;
– verify the results (e.g., by a simulation).

Feedback **F** stability provides a closed system retaining the same throughput and minimising the cost function.

5. Example

An example of the practical application of the above described theoretical considerations is presented here. Consider the controllable and structurally observable marked TEG in Fig 3. Its matrix transfer as $M_{in}^{ax}[[\gamma,\delta]]$ is obtained as follows:

$$\mathbf{H} = \begin{bmatrix} \delta^{10} + \gamma^2\delta^{12}(\gamma\delta)^* & \delta^8(\gamma^2\delta^4)^* & \delta^5(\gamma^2\delta^3)^* \\ \varepsilon & (\delta^{11} + \gamma\delta^{12})(\gamma^2\delta^4)^* & \varepsilon \end{bmatrix} \tag{11}$$

This TEG represents a system process consisting of 5 units P1 to P5. Due to the difference in the throughput of the units, it is noted that the model is not a stable because is accumulated markers (T2, P53) before of the P3 (Fig 4). The stability of this system can be obtained by adding a feedback output and this is enough to make the TEG strongly consistent and achieve stability. In this particular case, it becomes highly coherent by adding TEG feedback in the form of:

$$\mathbf{F} = \begin{bmatrix} \gamma^{q_{11}} & \varepsilon \\ \gamma^{q_{21}} & \gamma^{q_{22}} \\ \gamma^{q_{31}} & \varepsilon \end{bmatrix} \tag{12}$$

The problem here can be considered as the optimisation of resources in order to minimise the following cost function (9)

$$\lambda(q) = \min\left(\frac{q_{11}}{10}, \frac{q_{21}}{8}, \frac{q_{22}}{11}, \frac{q_{31}}{5}\right) \tag{13}$$

This problem can be solved by taking into account the sum of expression tokens and times for each elementary circuit determining the rate of the TEG process.

A naive algorithm for calculating the elementary circuits is simpler than writing a linear program which is unpractical for large graphs i.e. for full graph vertices, the complexity is $O((n - i)!)$. Gaubert's approach [12] allows only the n^2 inequality to be considered.

The feedback $\mathbf{F}_R$ (12) which stabilises the system while maintaining its original performance and minimising the amount of resources (marked in Fig. 3 as $q_{i,j}$ places) is:

$$\mathbf{F} = \begin{bmatrix} \gamma^5 & \varepsilon \\ \gamma^4 & \gamma^6 \\ \gamma^3 & \varepsilon \end{bmatrix}$$

Subsequent figures present some results for the considered example (interpolated to make them more visual): time evolution for $u_1 = u_2 = u_3 = \mu$ in an open system Fig. 4 and closed system Fig. 5 and Fig. 6.

Results have been calculated using a self-developed software package based on

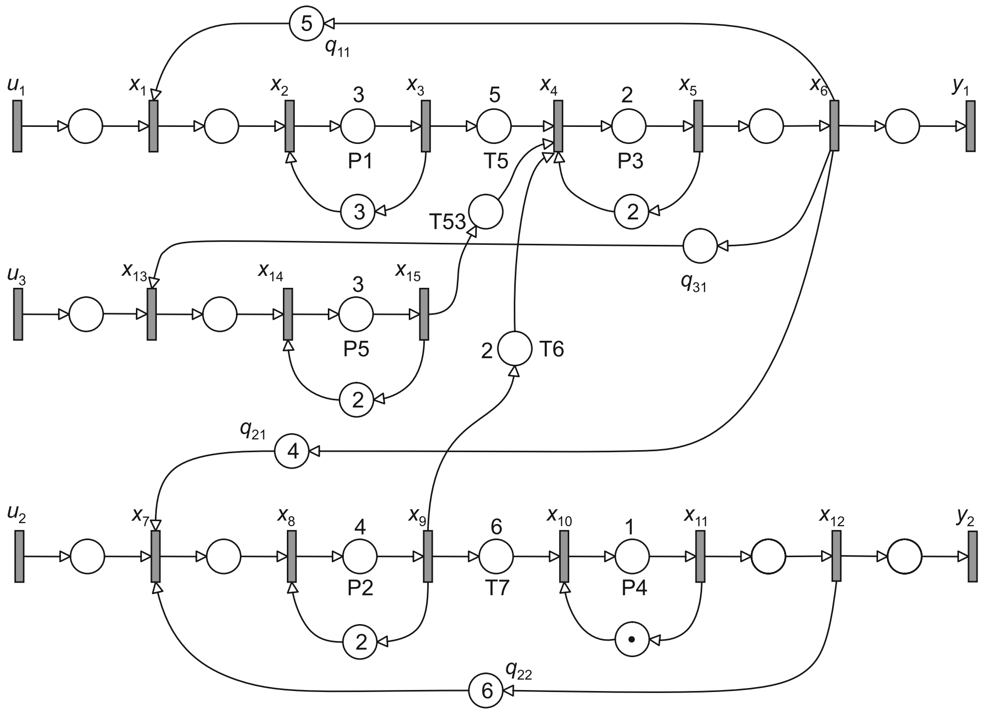

Fig. 3. TEG of the system with feedback

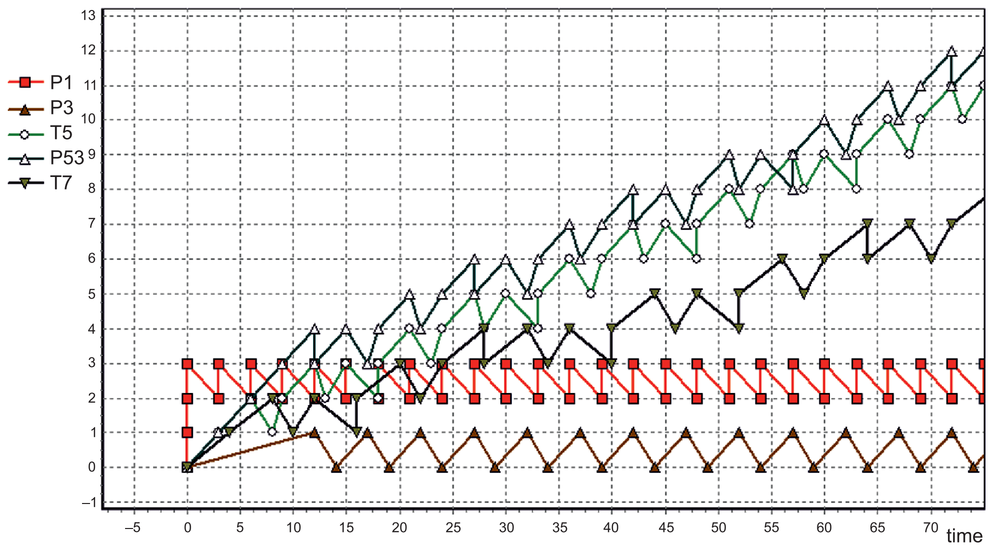

Fig. 4. Time evolution states of markers in open system

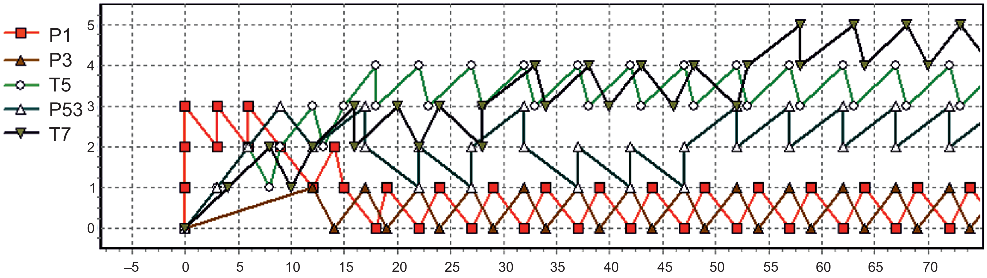

Fig. 5. Time evolution states of markers in closed system

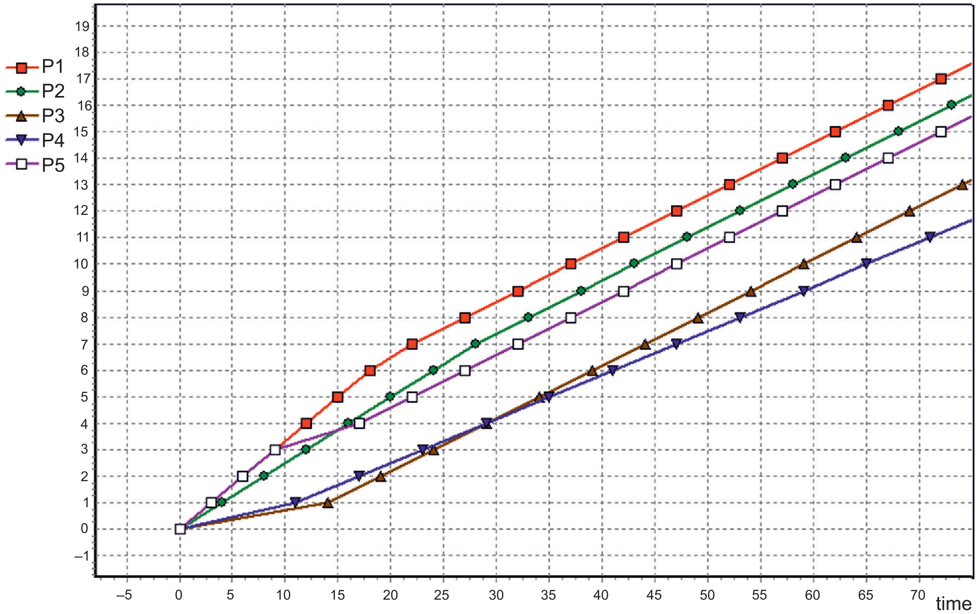

Fig. 6. Time evolution count of activity processes in closed system

Borland libraries TeeChart, Component and Software Tools for Manipulating Periodic Series: http://www.istia.univ-angers.fr/hardouin/outils.html and Toolbox TINA http://projects.laas.fr/tina/home.php.

6. Conclusions

This paper presents the synthesis of stabilisation feedback control for limited resources. It describes the synthesis of feedback control to stabilise the quantity of resources. This is only an initial example of the application of the max-plus systems theory and further research will include applying the system to larger practical system optimisation procedures and synthesising process control with state feedback as well as using models for predictive and adaptive control. More than a decade ago, the authors [6] concluded that: "By comparison with classical linear system theory, there are areas which are practically untouched, mostly because the corresponding mathematical tools are yet to be fabricated". Nowadays, problems of using max-plus systems are still a reality [3, 13, 15, 22, 23, 25, 33].

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