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# NUMERICAL ANALYSIS OF CONSOLIDATION OF EMBANKMENT SUBSOIL REINFORCED WITH DYNAMIC REPLACEMENT STONE COLUMNS

# ANALIZA KONSOLIDACJI PODŁOŻA WZMOCNIONEGO METODĄ WYMIANY DYNAMICZNEJ POD NASYPEM DROGOWYM

A b s t r a c t

The paper presents a comparative analysis of the consolidation of soil under a road embankment reinforced with stone columns. The results were obtained from the most common analytical and numerical methods applied in driven column dimensioning. The analytical approach exploits Terzaghi's one-dimensional theory and Barron's three-dimensional theory. The numerical calculations reflected the particular stages of the embankment construction in various two- and three-dimensional systems. The geotechnical parameters that were crucial for the research were determined on the basis of geoengineering documentation, laboratory and field studies. The paper begins with a short introduction presenting the dynamic replacement method.

*Keywords: dynamic replacement, stone column, numerical analysis*

S t r e s z c z e n i e

Przedmiotem artykułu jest analiza porównawcza wyników obliczeń konsolidacji podłoża wzmocnionego wbijanymi kolumnami kamiennymi, pod nasypem drogowym, otrzymanych na podstawie najczęściej stosowanych, przy wymiarowaniu kolumn wbijanych, podejść analitycznych i numerycznych. W podejściu analitycznym zastosowano jednowymiarową teorię Terzaghiego oraz trójwymiarową teorię Barrona. W obliczeniach numerycznych zamodelowano etapową budowę nasypu w różnych układach płaskich i przestrzennych. Parametry geotechniczne, niezbędne do obliczeń, ustalone zostały na podstawie dokumentacji geologiczno-inżynierskiej, badań laboratoryjnych i polowych. Całość rozważań poprzedzona została krótką informacją na temat wymiany dynamicznej.

*Słowa kluczowe: wymiana dynamiczna, kolumny kamienne, analiza numeryczna*

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## Symbols

| | | |
|---|---|---|
| $L$ | – | spacing of columns |
| $D_{col}$ | – | diameter of a stone column |
| $D_e$ | – | diameter of the unit cell |
| $\alpha$ | – | replacement ratio |
| $x$ | – | general parameter |
| $c'$ | – | effective cohesion |
| $\phi'$ | – | effective friction angle |
| $E$ | – | resilient modulus |
| $M$ | – | constrained (oedometric) modulus |
| $\nu$ | – | Poisson's ratio |
| $\psi$ | – | dilatancy angle |
| $e$ | – | void ratio |
| $\gamma$ | – | unit weight |
| $S$ | – | saturation ratio |
| $S_r$ | – | residual saturation ratio |
| $p_w$ | – | pore water pressure |
| $k^*_{ij}$ | – | permeability tensor for unsaturated soil |
| $k_{ij}$ | – | permeability tensor for saturated soil |
| $k_r$ | – | scaling scalar function for the permeability tensor |
| $k_v$ | – | permeability coefficient in vertical direction |
| $k_h$ | – | permeability coefficient in horizontal plane |
| $m_v$ | – | coefficient of compressibility |

## 1. Introduction

When planning development of new roads passing through urban areas, it is most often impossible to bypass regions of soft foundation soil. Thus, it is necessary to strengthen the weak subsoil in order to limit the settlement of the road embankment, reduce the time of primary consolidation and to assure sufficiently high safety factor for the embankment's slopes. This can be achieved by installing stiff inclusions of granular material called stone columns. Stone columns in soft ground can be formed in two ways: by vibro replacement or dynamic replacement.

Vibro replacement (VR) columns are installed in the ground by means of a crane-suspended downhole vibrator. The vibrator is lowered, densifying and displacing the underlying stone (crushed stone, recycled concrete, gravel) and forming a column of 0.6–1.0 m in diameter ($D_{col}$). During installation the zone around a column is strongly affected by vibrations produced by working equipment (vibrator or rammer). The zone of the disturbed soil is called the smear zone.

In the dynamic replacement (DR) method, stone columns are formed in weak soil using the equipment that allows free drop of a heavy rammer from a pre-defined height. In the first phase, the rammer forms a crater (Fig. 1a) which is then refilled with coarse-grained material (Fig. 1b). Afterwards, the material is pressed into the soil forming the crater which is then refilled once again and the whole process is repeated (Fig. 1b). The column is formed until the rammer cannot penetrate the soil anymore. This may mean that the column's base have reached a stiffer soil layer (Fig. 1c) or that the energy used during the process of column formation does not allow the soil to be penetrated any more (a so-called "floating column").

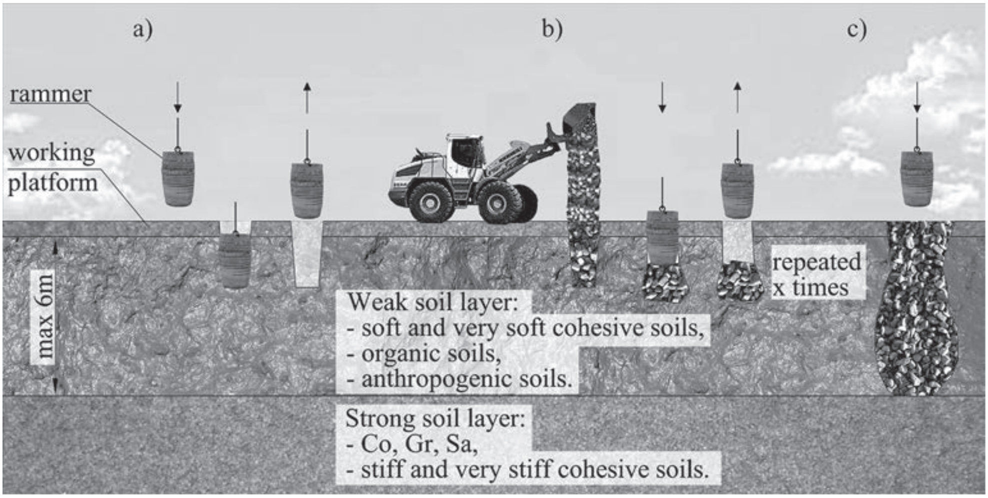


Fig. 1. Process of stone column formation

In Poland, the most common parameters applied in this technology are: rammer weight of 10–20 tonnes and drop height up to 25 m. The diameter of a column usually varies between 1.6–4.0 m [21]. In the case of a dynamic replacement column the impact of the falling rammer is so great that the whole volume of soft soil is remoulded and the smear zone cannot be distinguished. The number of rammer drops that need to be performed in order to form a column varies from 15 to 30. The ramming process results in the formation of columns of various shapes and diameters [22], which depend on soil susceptibility, applied energy and rammer shape. After the reinforcement process, the physical and mechanical parameters of the weak soil change [30].

Although analytical methods for designing vibro-column reinforcement exist [10, 24], there are no algorithms related to driven columns which would take into account the specific method of their construction [10, 11, 18]. While designing such columns, two factors are taken into consideration: the column's bearing capacity and settlement of the system consisting of the column and its surroundings [18]. Projects that take into account a drainage effect are very rare. The international literature describes this effect when it comes to vibro replacement columns [3, 4, 7], whereas for dynamic replacement columns it has not yet been examined. One of the few attempts to tackle this topic have been the model research described in [28] and [29]. The problem of the efficiency of DR column drainage in comparison to VR column has recently been discussed in [20].

The reduction of primary consolidation is achieved by two mechanisms: radial drainage towards columns which causes faster dissipation of pore water pressure and high column stiffness which reduces vertical load on soft soil and thereby the increase in pore water pressure. The acceleration of consolidation due to drainage was firstly recognized for vertical drains disregarding their stiffness [1, 2, 13, 15, 17], whereas the effect of column stiffness was investigated among others in [1, 5, 12, 14, 32].

In this paper the problem of consolidation of soft foundation soil reinforced with stone columns under a road embankment has been investigated by means of numerical calculations. It is a continuation of work presented previously in [19, 23]. In engineering practice a geotechnical problem like this is most frequently solved analytically by making use of the unit cell concept or as a plane-strain numerical analysis. The first approach serves only to predict the settlement of reinforced soft soil ignoring the problem of stability of the embankment. The second approach requires conversion of parameters of reinforced subsoil from axisymmetry to plane-strain conditions.

Three-dimensional analysis is the background for comparisons with the results of simplified two-dimensional computations and an analytical solution.

## 2. Soil parameters

The computational analysis presented here is related to the embankment that is part of the S7 expressway in the city of Lubień. This embankment (Fig. 2) was 12 m high. The bottom and the top of the embankment were respectively 67 m and 31 m wide. The slopes were inclined at 1:1.5. The entire embankment was formed using the method of lime stabilized soil.

In the computations, the top of the embankment was loaded with a pressure of 25 kPa [9] which substitutes the weight of the road pavement and the load induced by traffic.

Prior to the research, drillings were performed in order to examine the initial geotechnical conditions under the embankment, which allowed it to be determined that the soil was formed of soft silty clay layers locally underlain by more dense gravel with stones and/or soft rock (Fig. 2). Table 1 presents the values of soil parameters.

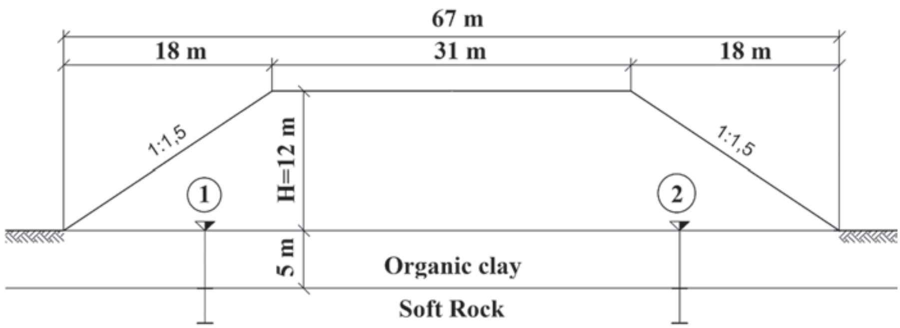


Fig. 2. Analysed embankment and foundation soil

Table 1

**Soft soil parameters**

| Layer | $\phi$ [°] | $\psi$ [°] | c [kPa] | $\nu$ [–] | E [MPa] | $e_0$ [–] | $k$ [m/s] | $\gamma$ [kN/m$^3$] |
|---|---|---|---|---|---|---|---|---|
| Organic clay | 4.6 | 0 | 31 | 0.3 | 2.65 | 0.6 | 1.00E-09 | 14.18 |

After analysing the type, condition and thickness of the soft layer, the designer decided to strengthen the soil using the dynamic replacement method. According to the designs, the main part of the embankment was supposed to be strengthened by stone columns of about 2.2 m in diameter and 5 m in length, distributed in a 3 m × 3 m grid. Columns were formed of stone aggregate of fraction 0/400 mm. The equipment used for column formation is presented in Fig. 3a, whereas Fig. 3b shows the excavated column.

a) b)

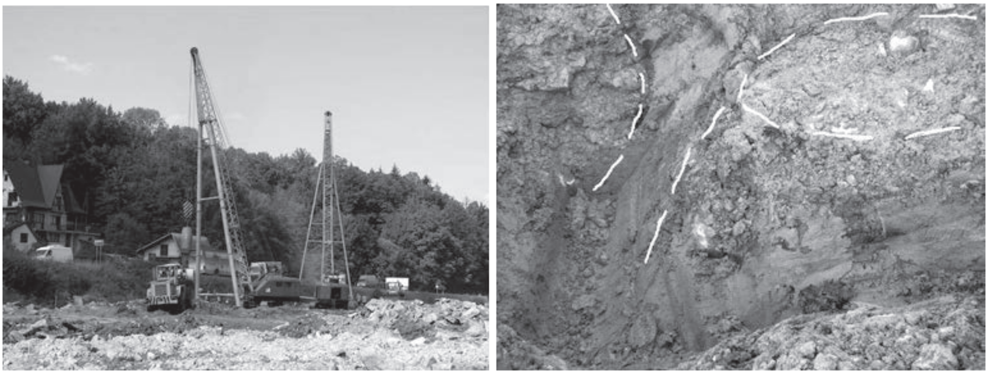

Fig. 3. a) equipment used for soil reinforcement, b) excavated column

## 3. Numerical analysis of consolidation – basic models

### 3.1. Preliminary remarks

The analysis is divided into two parts. In the first part, the authors compare the approaches applied in the projects of soil strengthening with the dynamic replacement method executed in Poland. As some of them may raise doubts, all the methods were compared to the most reliable one, i.e. the three-dimensional model. The second part of the analysis aims to present some improvements to the presented models, as well as other methods described in the literature.

### 3.2. Assumptions for the calculations

The consolidation of the soil under a road embankment reinforced using the dynamic replacement method was calculated using both analytical and numerical (FEM) methods. At the first stage of the research, five models were analysed.

Model No. I is the analytical method presented by Barron [2] and Terzaghi [33] in which the stone column is considered only as a drain that accelerates the consolidation process. The calculations of settlements were performed using Priebe's method [27] and presented in a spreadsheet, taking into account the gradually increasing load, which corresponds to the increasing height of the embankment.

It was assumed that the total settlement occurs according to the average degree of vertical ($U_v$) and horizontal ($U_R$) consolidation. The $U_v$ value is determined based on the following formula from Terzaghi's theory [33]:

$$U_V = 1 - \sum_{m=0}^{m=\infty} \frac{2}{M^2} \exp(-M^2 T_v) \tag{1}$$

where
$M$ – the constrained modulus,
$T_v$ – the time factor in vertical direction.

$U_R$ value is determined following Barron's theory [2] according to the formula:

$$U_R = 1 - e^{\frac{-8T_R}{f(n)}} \tag{2}$$

where
$T_R$ – the time factor in radial direction,
$f(n)$ = ln(n) – 0.75,
$n$ – the ratio of unit cell radius ($R_e = D_e/2$) to drain radius ($R_{col} = D_{col}/2$).

Four other calculation methods applied the finite element method (FEM). The analyses were performed using Z_Soil ver. 11.15 program [36]. Two- and three-dimensional models were analysed during the research.

In the first of the numerical analyses (model No. II), a three-dimensional model (Fig. 4) was used to present the construction of the embankment on soil reinforced with stone columns. The process of constructing the embankment was represented by adding subsequent layers of finite elements and the load induced by road surface and traffic was replaced by a unit load of 25 kPa.

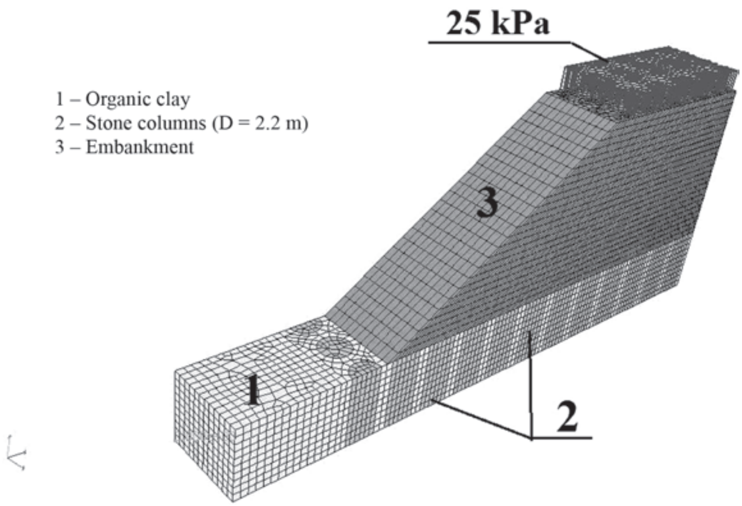


Fig. 4. Model No. II

Model No. III uses a unit cell (Fig. 5) determined on the basis of the method described in [10]. It simulates the work of a large system of repetitive cells placed under a vast construction, in which the columns are distributed in a square system with $L$ spacing. The diameter of the unit cell was determined to be $D_e$ = 1,13 × $L$. The unit cell is modelled as an axisymmetric case.

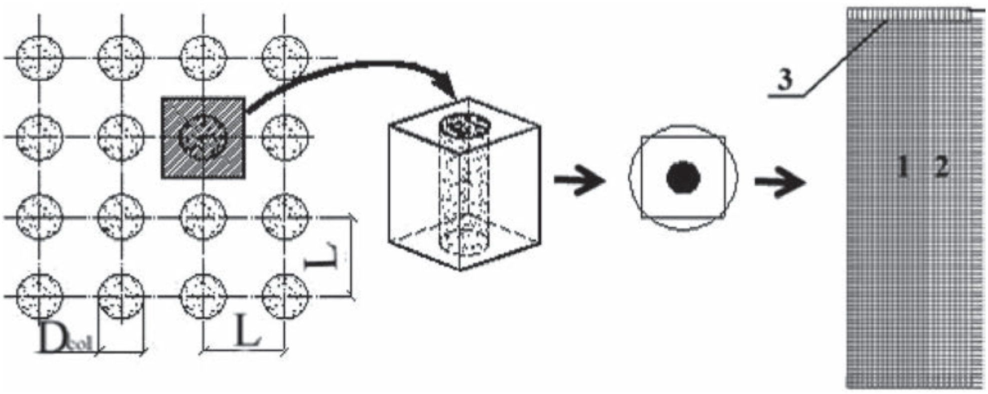


Fig. 5. Model No. III

Model No. IV presents the simulation of the construction of embankment in plane-strain, situated on homogenous soil which is characterized by averaged parameters (Fig. 6). The average value of each parameter was determined on the basis of the percentage of column material in the volume of the unit cell, i.e. using the replacement ratio α as weight coefficient. α is defined as the ratio of the surface of column's cross section to the surface of the unit cell:

$$\alpha = \frac{A_{col}}{A_e} = \left(\frac{D_{col}}{D_e}\right)^2 \tag{3}$$

Composite values of unit weight γ, cohesion c, friction angle ϕ, resilient modulus E and permeability coefficient k were determined by the formulae proposed by Dimaggio [8] and reported in [6]. Its general form with respect to any parameter $x$ is given by the Eq. (4)

$$x = x_{col}\alpha + x_s\left(1-\alpha\right) \tag{4}$$

where:
$x$ – average value,
$x_{col}$, $x_s$ – represent a parameter for a column and soft soil respectively.

Model No. V is the simulation of the embankment in plane-strain conditions with columns represented by strip elements of width equal to the actual column's diameter (Fig. 7). Parameters for soft soil and stone column material remain unmodified in any way.

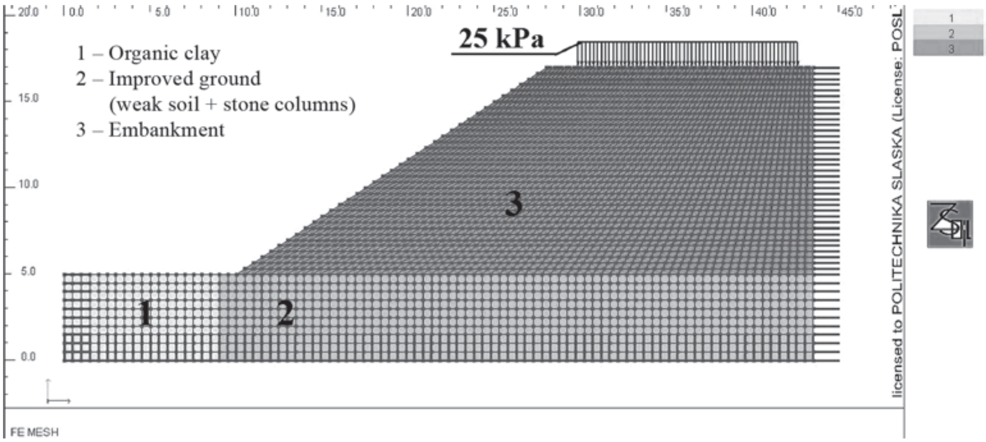


Fig. 6. Model No. IV

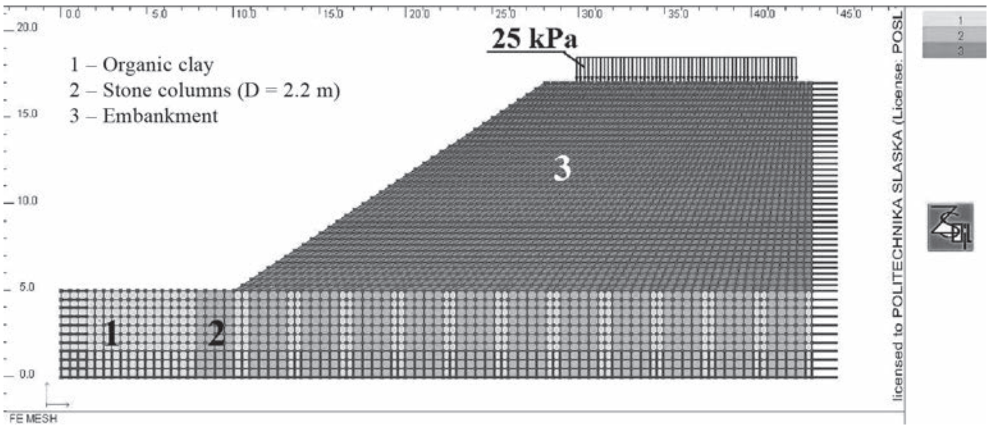


Fig. 7. Model No. V

The presented models IV and V are strongly simplified, however. In model No. IV, the average value of permeability coefficient was also determined based on equation (4), which may not be fully reliable.

Although all the other parameters for both weak soil and stone columns are of the same order, the difference between the permeability coefficient for both materials is huge.

In model No. V, the percentage content of the column increases significantly in the geometrical model, which leads to an increase in the replacement ratio. As a result, the settlements of the system may be underestimated.

## 3.3. Applied constitutive models and their parameters

All the data in the numerical analyses were described using the elastic-perfectly plastic model with Mohr-Coulomb surface modified by Menetrey and Willam [36]. For the abovementioned model, five parameters need to be known: resilient modulus E, Poisson's ratio ν, friction angle ϕ, cohesion *c* and dilatancy angle ψ.

As previously mentioned, the embankment was formed with lime stabilized cohesive soil. The parameters of this model for the material forming the embankment were determined on the basis of the analyses in direct shear apparatus and oedometric tests [19]. Poisson's ratio was determined to ν = 0.3 and the dilatancy angle ψ = 0.

The parameters of the stone column and deposit materials used in each model were determined on the basis of back analysis, i.e. matching the results of laboratory and field tests to the results of the numerical analysis of the boundary value problem.

Triaxial compression allowed the values of friction angle ϕ and cohesion c to be determines and oedometric tests – the virgin and reloading constrained modulus. This research was presented in [18]. Poisson's ratio for weak soil was determined on the basis of the body of literature [35] and its dilatancy angle was equal to 0.

As the columns were formed from coarse-grained material of fraction up to 400 mm, it was impossible to analyse the material in laboratory tests. However, test loading of the completed column was performed using a plate located on the column's head, which induced a pressure of 0–1373 kPa. The results of the test loading were used to calibrate the numerical system "stone column – weak soil surrounding" [18]. The similarity of the "loading – settlement" curve obtained from the numerical models and as the result of test loading was considered to be the criterion of adequacy for the numerical model itself. Column parameters were heuristically determined on the basis of the numerical analyses. According to these, the coefficient of determination $R^2$ was found to be equal to 0.9982 [18]. In the analysis performed, the non-associated flow rule (ψ = 0) was used.

Values of the permeability coefficient for a column and soft soil were taken from the literature. Materials were assumed to be isotropic in terms of permeability. In analyses hydro-mechanical coupling (consolidation) is described by the Van Genuchten model [34], in which flow in a non-saturated medium depends essentially on two hydrodynamic characteristics: water retention and permeability. To determine the saturation ratio *S*, which is the function of pore water pressure $p_w$, the model makes use of two constants: the residual saturation ratio $S_r$ and parameter β that describes the curvature of a water retention curve:

$$S = S(p_w) = \begin{cases} 1 & \text{if } p_w \le 0 \\ S_r + \dfrac{1 - S_r}{\left[1 + \left(\beta p_w / \gamma_w\right)^2\right]^{0.5}} & \text{if } p_w > 0 \end{cases} \tag{5}$$

The permeability tensor $k^*_{ij}$ is obtained by scaling the $k_{ij}$ tensor for a fully saturated medium by a scalar valued function $k_r$ dependent on the saturation ratio *S*:

$$k^*_{ij} = k_r(S)\,k_{ij} \tag{6}$$

$$k_r(S) = \begin{cases} 1 & \text{if } S = 1 \\ \dfrac{(S - S_r)^3}{(1 - S_r)^3} & \text{if } S < 1 \end{cases} \tag{7}$$

Also the initial void ratio $e_0$ is required for complete specification of the model.
All parameters taken for analyses are presented in Tables 2 and 3.

Table 2

**Mechanical parameters and soil weight**

| Model No. | Material No. | $E$ [kPa] | ν [−] | γ [kN/m$^3$] | φ' [°] | $c$' [kPa] |
|---|---|---|---|---|---|---|
| I | 1 | 2 650 | 0.30 | – | – | 31 |
| II, III, IV,V | 1 | 2 650 | 0.30 | 14.18 | 4.6 | 31 |
| I | 2 | 76 000 | 0.20 | 16.53 | 42.0 | – |
| II, III, V | 2 | 76 000 | 0.20 | 16.53 | 43.5 | 5 |
| IV | 2 | 33 620 | 0.26 | 15.16 | 21.0 | 20 |
| I, III | 3 | – | – | 20.00 | – | – |
| II, IV,V | 3 | 45 000 | 0.30 | 20.00 | 35.0 | 45 |

Table 3

**Hydraulic parameters**

| Model No. | Material No. | $e_0$ [–] | $k_h$ [m/s] | $k_v$ [m/s] | $S_r$ [–] | α [−] |
|---|---|---|---|---|---|---|
| I | 1 | – | 1.00E-09 | 1.00E-09 | – | – |
| II, III, IV,V | 1 | 0.60 | 1.00E-09 | 1.00E-09 | 0.20 | 0.5 |
| I | 2 | 0.84 | – | – | – | – |
| II, III, V | 2 | 0.84 | 1.00E-01 | 1.00E-01 | 0.00 | 10.0 |
| IV | 2 | 0.84 | 4.22E-02 | 4.22E-02 | 0.10 | 4.5 |
| I, III | 3 | 0.40 | – | – | – | – |
| II, IV,V | 3 | 0.40 | 1.00E-07 | 1.00E-07 | 0.20 | 1.0 |

## 3.4. Discussion of the results of the first stage analyses

All the results presented below were determined at a point located just under the foundation soil surface in the middle of the embankment. The point was decided to be representative of the comparison of the calculation results.

The results of the first stage analyses – depending on the applied model – show various values of soil settlements (Fig. 8) and consolidation time (Fig. 9, 10).

All the two-dimensional models underestimate the final settlements, in comparison to the three-dimensional model (model No. II). The differences are most significant for the plane-strain models, i.e. the one with the homogenized parameters (model No. IV) and the other with the stripes of columns of width equal to the column's actual diameter (model No. V). The settlements predicted on the basis of these models are over 60% smaller than those obtained from the 3D model. In the case of the unit cell (model No. III), as well as in the case of analytical calculations, the settlements are smaller by slightly over 20%.

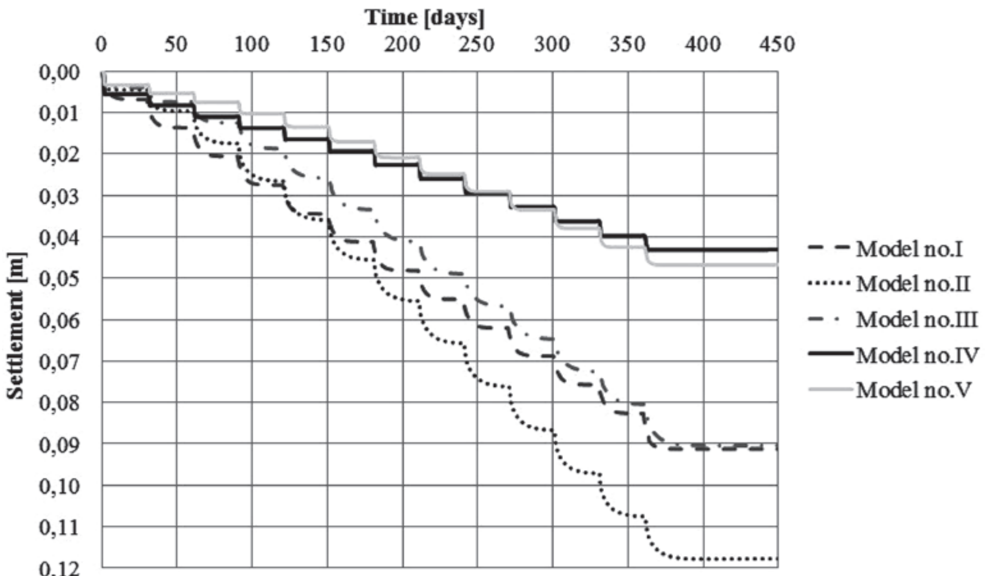


Fig. 8. Settlement predicted by models No. I-V

Figures 9 and 10 represent the diagrams of pore water pressure changes predicted in numerical models No. II–V.

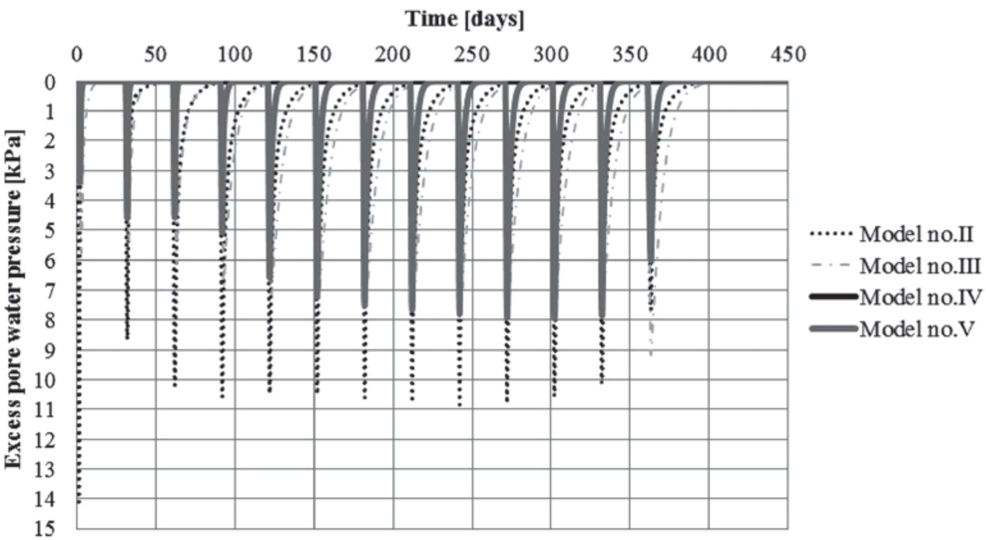


Fig. 9. Excess pore water pressure predicted by models II–V

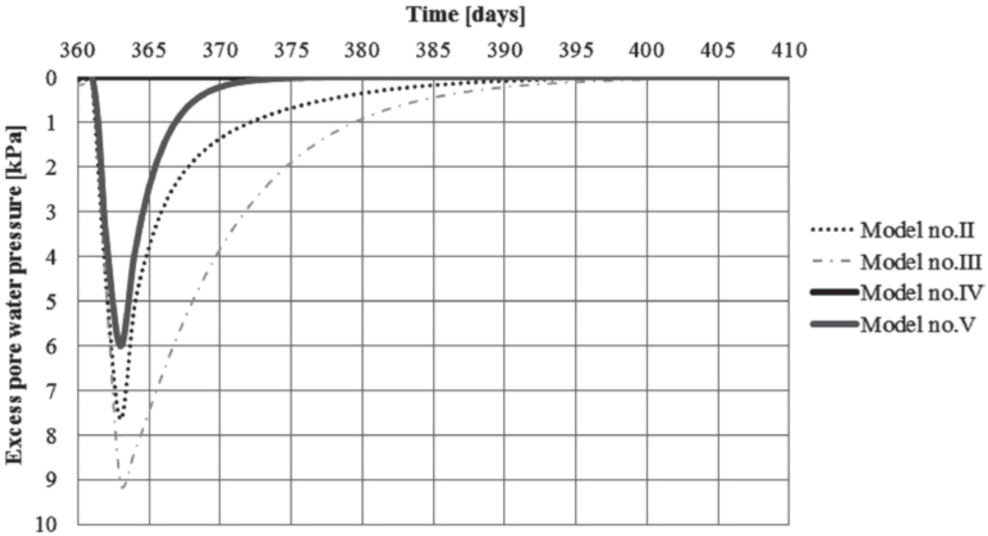


Fig. 10. Excess of pore water pressure in last steps of analyses predicted by models II-V

The excess pore water pressure and consolidation time of the reinforced soil are significantly underestimated by the plane-strain models (No. IV and V) compared with the predictions of the 3D model. When soft soil and columns are treated as a homogenized layer, the calculations show almost no increase in pore pressure resulting from the construction of the embankment. The pressure is immediately dissipated due to the high value of the permeability coefficient *k*. The consolidation time for model No. V is 60% shorter than the 3D model (model No. II). Only the estimations of the unit cell model (model No. III) indicate the time of water pressure dissipation longer by about 30%.

The conducted analysis revealed that the proposed methods IV and V, widely applied in engineering calculations, show significant discrepancies in the estimations of consolidation time of soil reinforced by stone columns. Therefore they should not be used for this purpose. For this reason, the analyses presented in the following part of this paper will try to propose modifications of the abovementioned models.

## 4. Computational analysis of the consolidation of reinforced soil under road embankment – other models

### 4.1. Assumptions for the calculations and parameters of the examined models

In the second, additional stage of the analysis, the authors propose some modifications to the 2D models (IV and V) and offer another model (No. VI).

In models IVa and IVb, as in model No. IV (Fig. 7) described previously, soft soil parameters were averaged. The difference between the models consists in the way the permeability coefficient and the friction angle were determined.

In models IVa and IVb, the averaged friction angle of soil was calculated in a different way than in model IV. The formula (4) was substituted with the following:

$$\tan\phi = \alpha \cdot \tan\phi_{col} + (1-\alpha)\cdot \tan\phi_s \tag{8}$$

The same approach was presented in e.g. [6] and [8].

Another essential modification introduced in both models was the method of calculating the average permeability coefficient in horizontal plane. The formula presented below was used in model IVa:

$$\frac{1}{k_h} = \frac{1}{k_{h\,col}}\alpha + \frac{1}{k_{h\,s}}(1-\alpha) \tag{9}$$

where:

$k_{h\,col}$, $k_{h\,s}$ – permeability coefficients in the horizontal plane for column and soft soil respectively.

Model IVb made use of the simplified formula presented in [26]. The value of $k_h$ coefficient was calculated as follows:

$$k_h = K_{\text{composite}} \cdot k_s \cdot 10^4 \text{ [m/day]} \tag{10}$$

where $K_{\text{composite}}$ was drawn from the calibration graph presented in [25], prepared on the basis of the numerical calculations using the finite element method.

In both Va and Vb models, the diameters of the stone columns were the same as in V (Fig. 8); however, the parameters of columns and weak soil were changed. Column parameters were averaged due to their periodical structure, according to the approach presented by Dimaggio [6].

In model Va, the following formulae were used in order to calculate the permeability coefficient in horizontal and vertical plane [33] for column strip elements:

$$k_v = \frac{1}{L}\left[k_{col}D_{col} + k_s\left(L - D_{col}\right)\right] \tag{11}$$

$$k_h = \frac{L}{D_{col}/k_{col} + \left(L - D_{col}\right)/k_s} \tag{12}$$

Equations (11) and (12) were derived for water flow in parallel and perpendicular directions to layered subsoil by Terzaghi.

In model No. Vb the permeability of the soft soil was modified. Modification concerns only permeability coefficient in horizontal direction. Coefficient of permeability in vertical direction is assumed to be the same for axisymmetric and plane-strain conditions.

The calculations of the permeability coefficient in the horizontal plane were carried out on the basis of the method proposed in [15] and [32]. The method gives the possibility to calculate the permeability coefficient in the horizontal plane in a plane-strain model ($k_{h,pl}$) on the basis of the values of this coefficient in an axisymmetric model ($k_{h,ax}$). The method is based on the assumption that the flow path length normal to the column perimeter corresponds to the flow path length in the plane-strain conditions and the radius of drainage zone $R_e$ is taken equal to the equivalent plane-strain width $B$ of the strip of columns:

$$\frac{k_{h,pl}}{k_{h,ax}} = \frac{F(N)_{pl}}{F(N)_{ax}} \left[ \frac{m_{vs} m_{vc} (1-\alpha)}{m_{vc}(1-\alpha) + m_{vs}\alpha} \right]_{pl} \left[ \frac{m_{vc}(1-\alpha) + m_{vs}\alpha}{m_{vs} m_{vc} (1-\alpha)} \right]_{ax} \frac{B^2}{R_e^2} \tag{13}$$

where:

$\alpha$ – replacement ratio,
$R_e$ – radius of a unit cell,
$B$ – plane-strain half width equivalent to $R_e$,
$m_{vc}$ i $m_{vs}$ – coefficients of compressibility for column and soft soil respectively,

$$F(N) = \left[ N^2 \big/ \left( N^2 - 1 \right) \right] \ln(N) - \left( 3N^2 - 1 \right) \big/ \left( 4N^2 \right) \tag{14}$$

$N = D_e/D_{col}$ – for axisymmetric conditions,
$N = B/b_{col}$ – for plane-strain conditions,
$b_{col}$ – half width of a column in plane-strain conditions.

It should be noted that assuming that the width of a stone column in plane-strain conditions is equal to the diameter in the axisymmetric case may result in excessive enlargement of the area replacement ratio. In model No. VI (Fig. 11), the diameter of a stone column was reduced so that in plane-strain conditions the replacement ratio was maintained. All the mechanical parameters remained unmodified in any way. The permeability coefficient for soft soil in the horizontal direction was calculated using the formula (13) as in model No. Vb.

The simplified assumption was made in the calculations: the parameters for soft soil under the embankment and next to it were the same. In real conditions, the values of the permeability coefficient for soil adjacent to the embankment is different from those for the soil among the columns (under the embankment).

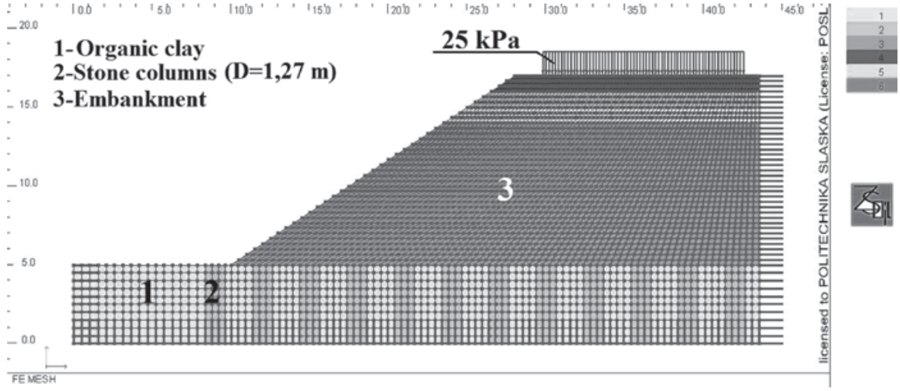


Fig. 11. Model No. VI

Tables 4 and 5 contain the mechanical and hydraulic parameters calculated according to the abovementioned assumptions.

Table 4

**Mechanical parameters and soil weight**

| Model No. | Material No. | $E$ [kPa] | ν [−] | γ [kN/m$^3$] | φ' [°] | $c'$ [kPa] |
|---|---|---|---|---|---|---|
| IVa, Ivb, Va, VI | 1 | 2 650 | 0.30 | 14.18 | 4.6 | 31 |
| Iva, IVb | 2 | 33 620 | 0.26 | 15.16 | 24.1 | 20 |
| Va, Vb | 2 | 56 440 | 0.23 | 15.90 | 35.2 | 12 |
| VI | 2 | 76 000 | 0.20 | 16.53 | 43.5 | 5 |
| IVa, Ivb, Va, VI | 3 | 45 000 | 0.30 | 20.00 | 35.0 | 45 |

Table 5

**Hydraulic parameters**

| Model No. | Material No. | $e_0$ [–] | $k_h$ [m/s] | $k_v$ [m/s] | Sr [–] | α [−] |
|---|---|---|---|---|---|---|
| IVa, IVb, Va | 1 | 0.84 | 1.00E-09 | 1.00E-09 | 0.20 | 0.5 |
| Vb | 1 | 0.84 | 1.17E–10 | 1.00E-09 | 0.20 | 0.5 |
| VI | 1 | 0.84 | 2.62E-09 | 1.00E-09 | 0.20 | 0.5 |
| IVa | 2 | 0.74 | 3.46E-09 | 4.22E-02 | 0.12 | 4.5 |
| IVb | 2 | 0.74 | 1.50E-06 | 4.22E-02 | 0.12 | 4.5 |
| Va | 2 | 0.66 | 3.75E–10 | 7.00E-04 | 0.05 | 7.5 |
| Vb | 2 | 0.66 | 1.00E-01 | 1.00E-01 | 0.05 | 7.5 |
| VI | 2 | 0.60 | 1.00E-01 | 1.00E-01 | 0.00 | 10.0 |
| IVa, IVb, Va, VI | 3 | 0.40 | 1.00E-07 | 1.00E-07 | 0.20 | 1.0 |

### 4.2. Results of computations and their discussion

Fig. 12 presents the values of the settlements predicted in 2D calculations and those obtained by models No. IV and V during the first and the second stage of the research. The total settlement obtained in the 3D model (No. II) is equal to 11.8 cm and for clarity is not presented in the diagram.

The comparison of the settlement values predicted by the model of homogenous subsoil with the averaged parameters of the column-weak soil system (model No. IV) shows that replacing the average value of the friction angle determined by formula (6) with the value calculated by formula (7) resulted in smaller settlement values. This shows that the predicted values obtained on the basis of models IVa and IVb differ greatly from those determined by the 3D model (No. II) due to the reduction of the friction angle value.

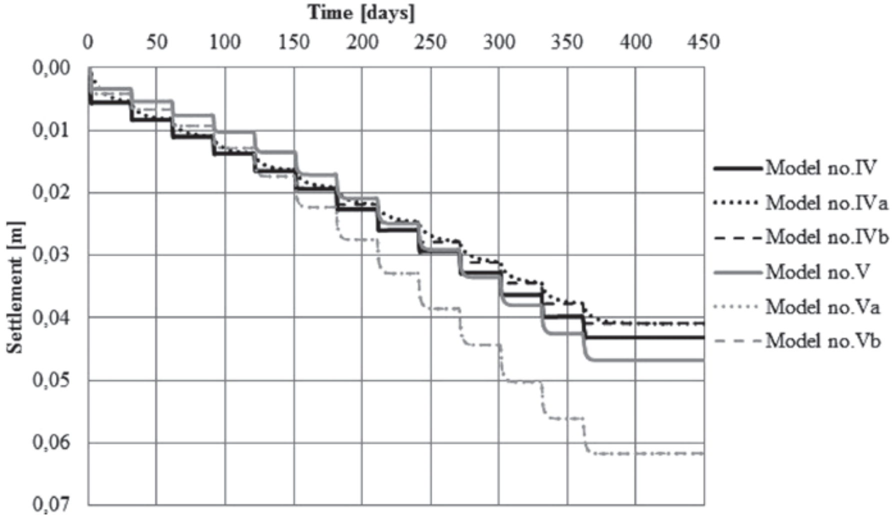


Fig. 12. Settlements predicted by the analysed 2D models

For models in which the width of the column is equal to the actual column diameter (Va and Vb), the modification of column parameters caused the results to be more similar to the results obtained from the 3D model (total settlement 11.8 cm). However, the settlement values obtained in the plane-strain models were underestimated. The graphs for models Va and Vb are almost identical.

Fig. 13 demonstrates the comparison of settlement values for the base 3D model and model No. VI, in which the real value of the replacement ratio was maintained by the proper choice of column width.

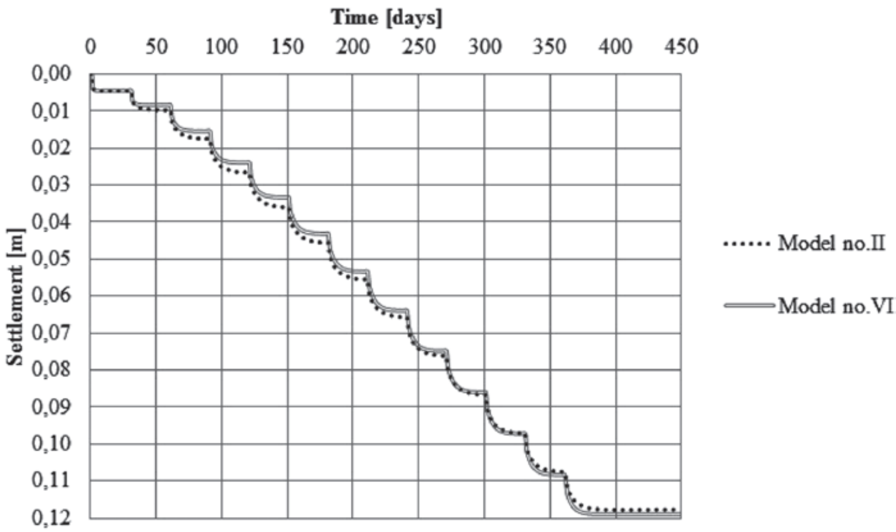


Fig. 13. Comparison of settlements predicted by models No. II and VI

The settlement values obtained from model No. VI are closest to the results from 3D model No. II. No modifications were introduced to the parameters of the column and soft soil in model No. VI but the strips representing columns were narrower. The total settlements and the load-settlement curves are almost identical.

Fig. 14–17 represent the excess pore water pressure calculated using the models examined in the second phase of the research and compare it to the values obtained from the base 3D model (No. II).

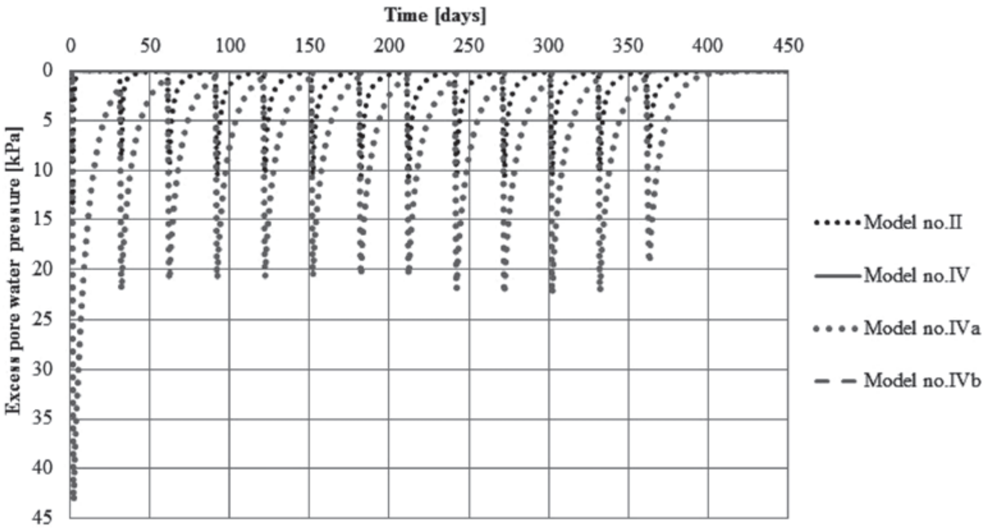


Fig. 14. Comparison of excess pore water pressure predicted by models No. II, IV, IVa, IVb

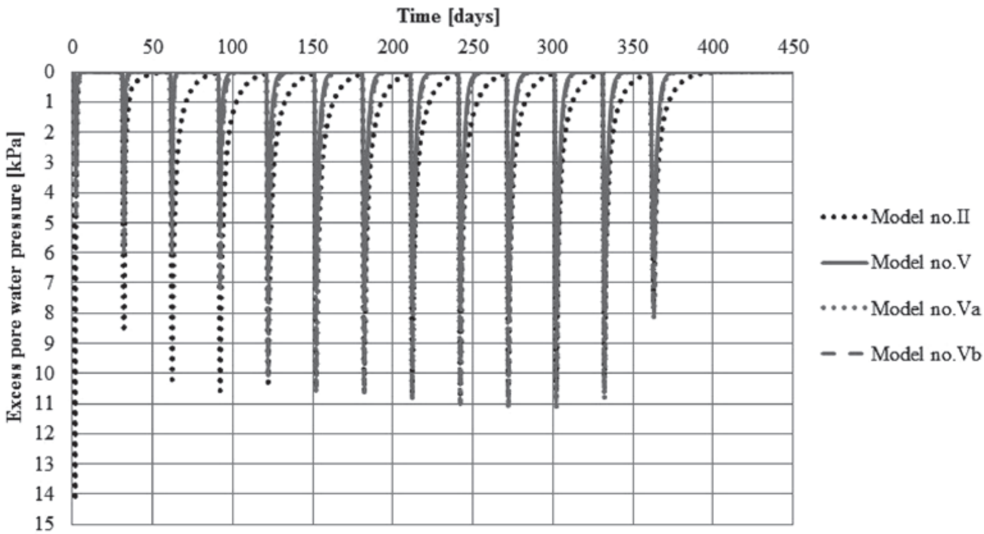


Fig. 15. Comparison of excess pore water pressure predicted by models No. II, V, Va, Vb

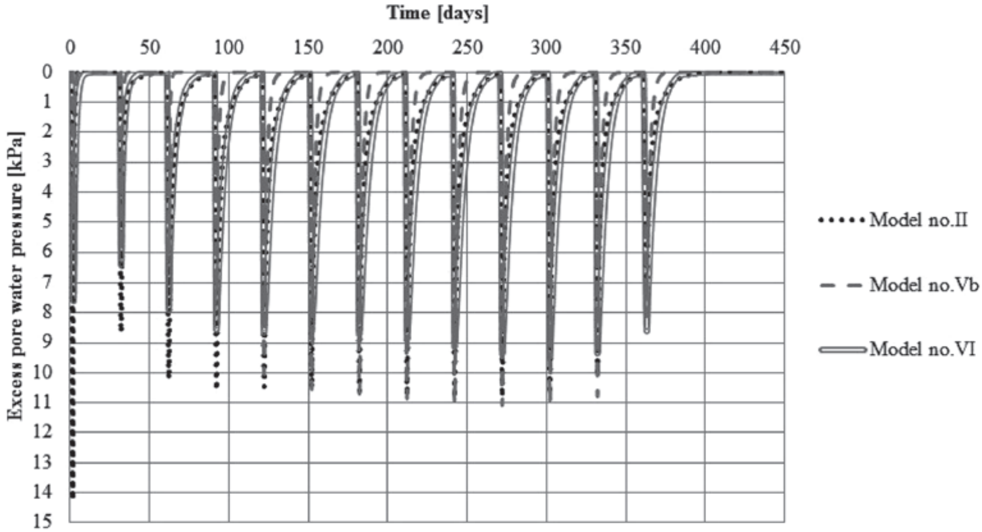


Fig. 16. Comparison of excess pore water pressure predicted by models No. II, Vb, VI

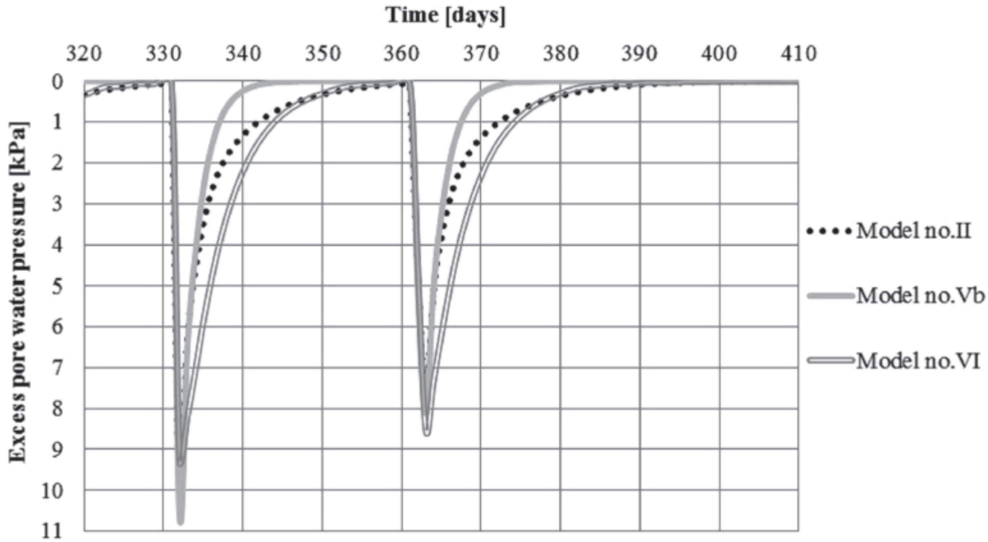


Fig. 17. Comparison of excess pore water pressure predicted by models No. II, Vb, VI in the last increment of loading

The discrepancy between results in the models with the averaged parameters and the 3D base model is the also largest for predictions of the consolidation time. Plane-strain models predict that the excess pore water pressure is dissipated almost immediately or on the contrary, the dissipation time is 60% longer (IVa) than for the 3D model. For 2D models with distinguished column strips characterized by averaged parameters and by the width the same as in case of the real column (Va and Vb), consolidation time is about 60% shorter.

The plane-strain models with the permeability coefficient in the horizontal plane converted from axisymmetric to plane-strain conditions by virtue of Eqn (13) [32] predict dissipation of pore water pressure in time in the way that is in the best agreement with the 3D consolidation results (Fig. 17). Although from the very close inspection of the diagram in Fig. 17 it follows that model No.VI overestimates the time of dissipation of excess pore water pressure by 60% with respect to the 3D model, it must be noted that the pore water pressure dissipation curve fits the best 3D results. At the end of the 3D consolidation model No. VI predicts only remaining 2% of the initial excess pore water pressure.

## 5. Summary

The paper has presented research on various attempts at calculating settlements and pore water pressure dissipation during construction of an embankment on soft soil reinforced with dynamic replacement stone columns. The authors sought a reliable plane-strain model which could be applied instead of a spatial analysis.

The results obtained by various simplified models were compared to the results obtained by the base 3D model marked as model No. II. The following computational attempts were checked.

Model No. I is an analytical method in which the stone column is considered only as a drain that accelerates the consolidation process. Priebe's analytical method for calculating settlement was used. Model No. III is an axisymmetric FEM model of a unit cell. Model No. IV is a plain-strain model where a zone of columns and soft soil under the embankment is homogenized. The average value of each parameter was determined using the replacement ratio α as weight coefficient. Model No. V is a plane-strain model with columns represented by strips of width equal to the actual column's diameter. Parameters for soft soil and stone column material remain unmodified in any way.

Models IVa and IVb are based on model IV but both employ the value of the friction angle for the homogenized zone calculated in a different way to that in model No. IV. Furthermore, these models assume anisotropic permeability of the homogenized zone. In model No. IVa the averaged coefficient of permeability in horizontal plane $k_h$ was calculated by Eqn (9) whereas in model No. IVb the method for calculating $k_h$ proposed in [25] was used.

In models Va and Vb, the diameters of stone columns were the same as in V but mechanical parameters for the columns were taken as average according to Dimaggio [6]. The permeability of column strip elements in model No. Va was modified according to Terzaghi's formulae to give coefficients of permeability in horizontal ($k_h$) and vertical ($k_v$) directions. In model No. Vb the permeability of the soft soil was modified according to the manner proposed in [15] and [32]. The coefficient of soil permeability in the horizontal direction in plane-strain conditions was calculated by matching the permeability in plane-strain and axisymmetric conditions. The coefficient of soil permeability in the vertical direction for soft soil remained unchanged.

In model No. VI the diameter of a stone column was reduced to obtain an equal replacement ratio in axisymmetric and plane-strain conditions. All mechanical parameters for soft soil and columns were not modified. Coefficients of permeability were assumed as in model No. Vb.

The computational analysis conducted showed that it is possible to obtain good agreement between results from a simplified plane-strain model and full 3D consolidation numerical analysis. This was achieved in model No. VI. The assumptions made in this model have been tested in the past but in combination with other geometrical and material configurations.

It has been found that to obtain satisfactory settlement the same replacement ratio must be preserved in the transition from spatial to plane-strain conditions. The permeability coefficient in the horizontal direction must be properly calculated since it has the essential influence on agreement between dissipation of pore water pressure in 3D and plane-strain conditions.

Although the mechanical parameters employed in the analysis were determined in laboratory and field tests for real soils the results obtained in the numerical calculations need to be verified by comparison with results of monitoring of the full scale embankment construction.

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