TECHNICAL TRANSACTIONS | CZASOPISMO TECHNICZNE
CIVIL ENGINEERING | BUDOWNICTWO

3-B/2015

MICHAŁ MUSIAŁ*

YOUNG'S MODULUS TEST IN TWO DIRECTIONS ON THE BASIS OF EIGENFREQUENCIES IN CONCRETE BEAMS

BADANIE MODUŁU YOUNGA W DWÓCH KIERUNKACH NA PODSTAWIE CZĘSTOTLIWOŚCI WŁASNYCH BELEK BETONOWYCH

Abstract

This paper covers the experimental studies of Young's modulus of concrete on the basis of eigenfrequencies. Experiments were performed on beam elements measuring 1050 × 200 × 100 mm, using operational modal analysis (OMA). Two types of concrete with different mixtures were tested. The dynamic Young's modules were calculated on the basis on the resonant frequencies measured in two directions. The values obtained were compared with static Young's modules determined on concrete cylinders in axial compression testing.

Keywords: *concrete, eigenfrequency, OMA, Young's modulus*

Streszczenie

W artykule zrelacjonowano badania eksperymentalne modułu Younga betonu na podstawie częstotliwości własnych. Eksperymenty przeprowadzono na elementach belkowych o wymiarach 1050 × 200 × 100 mm z wykorzystaniem operacyjnej analizy modalnej (OMA). Przebadano dwa betony o różnych recepturach. Dynamiczne moduły Younga obliczono na podstawie pomierzonych w dwóch kierunkach częstotliwości własnych. Uzyskane wartości porównano ze statycznymi modułami Younga określonymi na betonowych walcach w próbie osiowego ściskania.

Słowa kluczowe: *beton, częstotliwość własna, OMA, moduł Younga*

DOI: 10.4467/2353737XCT.15.168.4343

---

* Ph.D. Eng. Michał Musiał, Faculty of Civil Engineering, Wroclaw University of Technology.

1. Introduction

Young's modulus is a key deformability parameter of concrete – its value may be determined in accordance with different procedures. Normally, this involves a static load test to the point of destruction in accordance with the appropriate program and determining the value of Young's modulus for the corresponding load range (e.g. [2, 4]). Due to the high popularity of non-destructive methods [5], attempts are made to use them in testing. One of the most popular methods for the indirect determination of Young's modulus is an eigenfrequency test of the element.

There have been multiple publications concerning the described method. Furthermore, it has been regulated by a relevant standard [1]. Usually, the conducted tests compare the values obtained in the classic static axial load compression test E_{cm} with values calculated on the basis of the eigenfrequency E_d. Selected results of the conducted tests prove that these values may differ. These differences may reach 10% [6, 7]; however, in some of the experiments [10, 11], they are larger and reach approx. 30%. Mostly, the dynamic Young's modulus is a greater value. Nevertheless, results of tests are available where the trend was reversed.

So far, tests have been performed on different test elements. While in the case of the static Young's modules, these are mainly cylinders, in the case of the dynamic modulus, the tests were conducted on cylinders [7, 10] as well as on beam elements of different proportions [6, 11]. Moreover, as it turns out, the factors that affect the results of the measurements are the presence and intensity of longitudinal reinforcement in the element [6, 7]. One of the methodological differences in the existing studies was the test stand. Some of the tests were performed on simply supported beams [6], while others on the elements resting on elastic sleepers or suspended on elastic ropes [8]. A different approach was placing the element in a press and applying the axial load with varying intensity [7] – it was found that the intensity of the load can also affect the measurement results.

Our own analyses reported in this paper are the continuation of the test tasks undertaken in the literature listed above. Their goal is to systematise the existing observations and formulate guidelines for the examination of Young's modulus of concrete using the eigenfrequency measurements.

2. Laboratory tests

2.1. Test elements

Tests were performed on two series of beam elements measuring 1050 mm × 200 mm × × 100 mm. Three elements were contained in each series. The series were made from different concrete mixtures – these concrete mixtures are summarised in Table 1.

In each of the series, six additional cylinders each with a nominal diameter of 113 mm and a height of 350 mm were created. Three of the cylinders were designed to determine the mean compressive strength f_{cm}, and the other three were designed to determine the mean static Young's modulus E_{cm} for both series (concrete mixtures). The concrete was compacted using a vibrating table. All of the elements were prepared in the laboratory. Fig. 1 shows the steel moulds with the concrete mixture immediately after vibrating. Beams and cylinders were stored for 28 days in standardised curing conditions. After 28 days, all of the elements were stored under the same conditions.

Table 1

Concrete mixtures

Ingredient	B1 series	B2 series
Cement 42.5 R [kg/m^3]	300	350
Sand 02 [kg/m^3]	639	639
Aggregate 28 [kg/m^3]	639	639
Aggregate 816 [kg/m^3]	639	639
Water [kg/m^3]	143	143
W/C [–]	0.48	0.41
Plasticizer [kg/m^3]	–	3.15

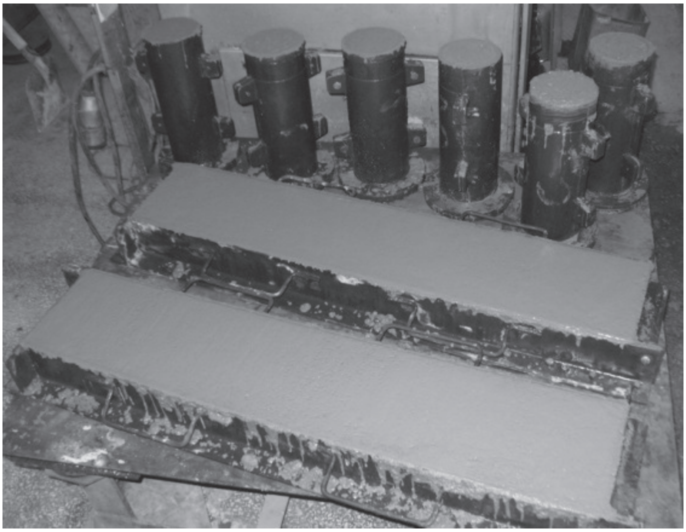

Fig. 1. Execution of test elements

2.2. Strength and deformability properties

Strength and deformability properties were determined on cylinders on the eve of testing the beams. On the three cylinders, the mean compressive strength f_{cm} for both concrete mixtures was determined. The other cylinders (three in each series) were used to determine the mean static Young's modulus E_{cm}.

Young's modulus was determined according to the internal procedure worked out in the Accredited Laboratory of the Department of Civil Engineering of Wroclaw University of

Technology. The cylinders were loaded in the strength press in accordance with a program set out in Fig. 2. The initial cycles were aimed at stabilising the deformation. Young's modulus was determined as a secant during the last cycle in the range of $0.1f_{cm} – 0.3f_{cm}$. Compression strength values for the load program were taken on the basis of the previously examined cylinders.

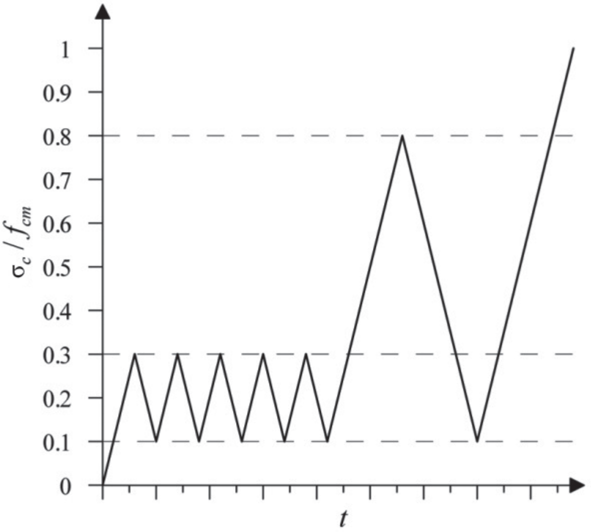

Fig. 2. The load program for determining the static Young's modulus

Fig. 3. Young's modulus testing

Deformation was measured using electrical resistance strain gauges with a 50 mm measuring base. On each specimen, three strain gauges were attached evenly and distributed around the circumference at half height. The specimen placed in the press is shown in Fig. 3. In Figures 4 and 5 are sample diagrams showing an example of deformation recorded for the specimens from the two measurement series.

The results of the compressive strength measurement and Young's modules are summarised in Table 2, the values were averaged for each series. For the mean compressive strength, six specimens were used – three specimens tested to determine the load program and three specimens destroyed after testing Young's modulus.

Measurements were performed using the B&K Pulse system. On each beam, 8 piezoelectric accelerometers were placed at a spacing of 150 mm – these recorded the response of the test element to random force generated by the environment.

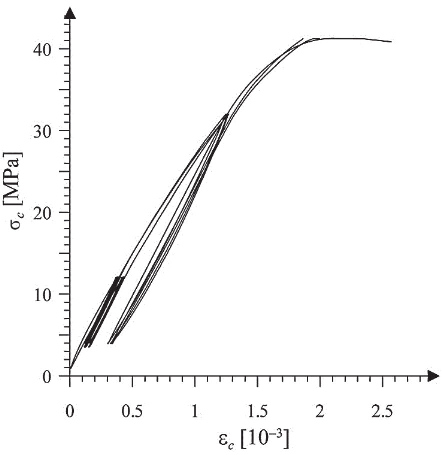

Fig. 4. Deformation for the B1 series specimen

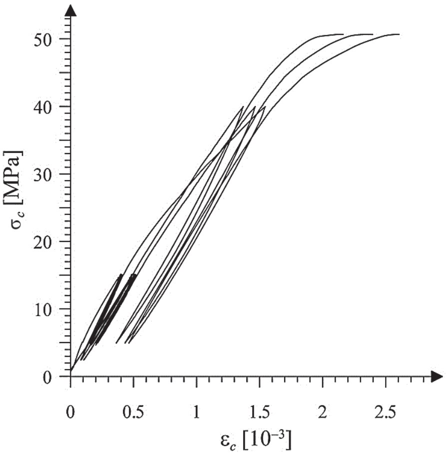

Fig. 5. Deformation for the B2 series specimen

Table 2

Concrete properties

Series	Mean compressive strength f_{cm} [MPa]	Mean Young's modulus E_{cm} [GPa]
B1	38.17	29.59
B2	50.05	32.50

The Operational Modal Analysis (OMA) [3] used in the experiment allowed the determination of the dynamic characteristics of the system (modal damping, frequency and eigenforms) without the need to record excitation as in the experimental modal analysis (EMA). The described method is also widely used in studies on real engineering structures in the natural scale [12].

2.3. The measurement of eigenfrequencies

Because of the simplicity of the implementation of the experiment, it was decided to conduct measurements on beams suspended on elastic ropes. The numerical model of the suspended beam is a simple bar without kinematic boundary conditions. This testing method provides a very accurate reflection of the theoretical model in laboratory conditions. The author's experience in this field [9] shows that the construction of another test bench (e.g. a simply supported beam) posed considerable difficulties. Effective setting of the appropriate boundary conditions in the laboratory model is problematic. At high eigenfrequencies and small vibration amplitudes during the measurements, locking the displacements at points of support is virtually impossible. The accuracy of the applied model is evidenced by the recorded eigenforms, where differences between the theoretical and measured values do not exceed 5%. Examples of the primary eigenforms recorded for the B1 series together with the theoretical forms are shown in Fig. 6.

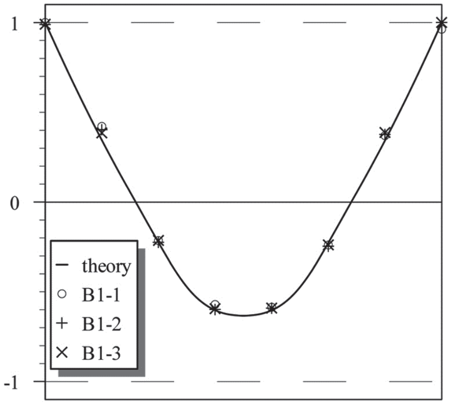

Fig. 6. B1 series primary eigenforms

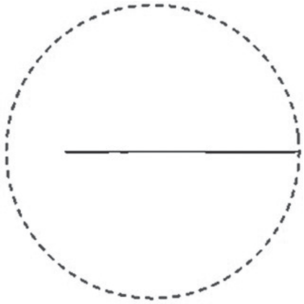

Fig. 7. Sample Nyquist’s graph
for the B1-1 beam

Nyquist’s graphs are an additional confirmation of the good quality of the identification of the eigenforms. A sample graph is shown in Fig. 7. The circle shows the greatest amplitude, while the segments are the amplitudes of individual points (1 to 8) including the phases. If all the points oscillate in the consistent or opposite phase, the obtained segments are collinear. If there are inaccuracies, the sections ‘scatter’.

Tests were conducted in 60-second cycles. Elements were tested in two positions. First, the elements were positioned so that they obtained a greater moment of inertia (Fig. 8). They were then rotated by 90° (Fig. 9). The measurement results are summarised in Table 3. The results were given for individual beams and the mean values were given for the B1 and B2 series.

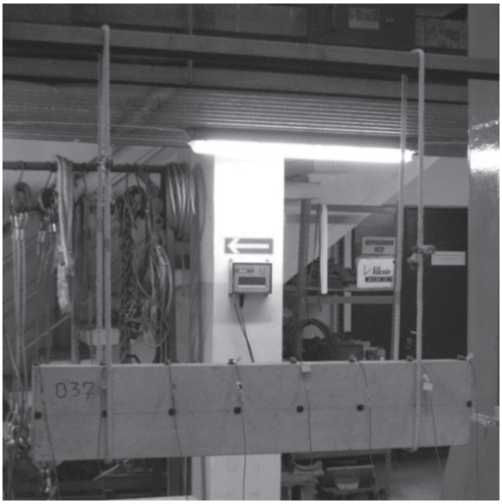

Fig. 8. B2-1 beam in position 1

Fig. 9. B2-1 beam in position 2

Table 3

The measured eigenfrequencies

Position of the beam	Eigenfrequency [Hz]							
	B1-1	B1-2	B1-3	**B1**	B2-1	B2-2	B2-3	**B2**
Vertical	671	687	669	**676**	672	690	680	**681**
Horizontal	365	373	370	**369**	372	372	364	**369**

When measuring the eigenfrequency of the beams in the vertical position, higher values were obtained for the B2 series. As expected, this is a consequence of the higher Young's modulus. On the other hand, this regularity was not found in case of testing the beams in the horizontal position. The obtained mean values of the eigenfrequencies, as tested in this position, are approximately equal to each other.

2.4. Dynamic Young's modulus

To determine the dynamic Young's modulus E_d, the relationship (1) taken from the elementary dynamics of the building was applied. In the illustrated form, this relationship allows us to calculate the frequencies of the bar with continuous weight distribution.

$$f_i = \gamma_i \cdot \sqrt{\frac{E_d \cdot I}{m \cdot l^4}} \tag{1}$$

where:

f_i – i^{th} natural frequency (in tests $i = 1$),
γ_i – coefficient for the i^{th} eigenform depending on the element's scheme,
I – moment of inertia of the cross-section,
m – rod's mass per unit length,
l – total length of the bar.

The coefficient γ_1 needs to be discussed – for the simple bar model, this is approx. 3.56. This value, however, applies to a one-dimensional element. Typically, a beam having a ratio of $l/h \geq 10$ is considered to be a one-dimensional element. Due to the proportions of the tested beams (in particular, in the vertical position, where $l/h = 5.25$) coefficients were calculated using the finite element method for three-dimensional volume structures. The values of the coefficients with corresponding eigenforms are shown in Fig. 10. The results of the calculations of the dynamic Young's modules are summarised in Table 4. For comparison, the values of the static Young's modulus for both concrete mixtures are given.

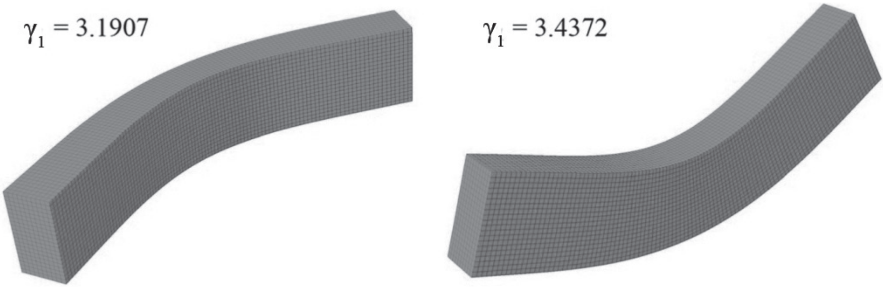

Fig. 10. Eigenforms and their corresponding coefficients

Table 4

Results of measurements and calculations

Beam/Series		B1-1	B1-2	B1-3	**B1**	B2-1	B2-2	B2-3	**B2**
E_{cm} [GPa]		29.59				32.50			
E_d [GPa]	Vertical position	35.62	37.33	35.50	**36.15**	35.10	37.67	37.71	**36.83**
	Horizontal position	36.15	37.87	37.37	**37.13**	37.20	37.31	36.63	**37.05**

2.5. Comparison of the results

Based on the tests, it was found that the dynamic Young's modulus was slightly higher in the case of the test performed in the horizontal position. The beam operated then in the direction of concrete casting. As mentioned above, these differences are insignificant and do not exceed 3%. This observation applies to both test series and is purely qualitative; its quantitative impact on the issues of structural engineering is, however, negligible.

It has been found that the higher static Young's modulus in the B2 series is not reflected in the dynamic Young's modulus. When tested in the vertical position for the B2 series, the value of the dynamic Young's modulus only increased by less than 2%. This difference, when tested in the horizontal position, was even lower – almost negligible.

The mean values of the dynamic Young's modules for both testing directions were 36.64 and 36.94 GPa, respectively, for the B1 and B2 series. On the basis of these values, the relationship E_d/E_{cm}, which is the most widely used indicator in the literature that allows the evaluation of the difference between the analysed values, was determined. It amounted to 1.24 and 1.14.

3. Conclusions

Based on the conducted tests, it can be concluded that the implemented test stand fully reflects the adopted theoretical model of the free bar. When conducting dynamic analyses, the recording of the eigenforms that allow its verification seems necessary.

In the dynamic testing, similar values of Young's modulus were obtained regardless of the direction of the vibration of the analysed sections. The applied method can be used for testing concrete homogeneity.

Despite various mixtures that allow the obtaining of concrete with different static Young's modules, the examined dynamic Young's modules were similar. It was found that in the case of the B1 series, the dynamic Young's modulus was 24% higher than the static modulus, and by 14% for the B2 series. On the basis of the tested elements, it can therefore be concluded that this difference depends on the concrete compressive strength and it decreases with the increase of the strength. This observation, however, should be confirmed in further studies for other concretes.

Higher values of the dynamic Young's modules are caused, for example, by the fact that the investigated beams are minimally stressed – only under their own weight. The static Young's modules were determined for the effort of 10–30%, after six load-unload cycles.

The sensitivity of the Young's modulus calculated according to the transformed equation (1) is extremely important to the accuracy of the frequency measurement. For example, if the eigenfrequency is measured with a 10% error, the Young's modulus will be encumbered with an error of approx. 20%. In the case of the inverse analysis, this problem is not so important, because a 10% error in the case of the Young's modulus translates into only a 5% error of the calculated frequency – this should be kept in mind when planning tests with the use of the described technology.

The author wishes to thank Mr. Jacek Grosel, PhD of the Faculty of Civil Engineering of Wroclaw University of Technology for his assistance in carrying out laboratory tests and his extremely valuable guidance.

References

[1] ASTM C215 – 14, Standard Test Method for Fundamental, Longitudinal, and Torsional Resonant Frequencies of concrete Specimens, ASTM Standard.

[2] ASTM C469/C469M – 14, Standard Test Method for Static Modulus of Elasticity and Poisson's Ratio of Concrete in Compression, ASTM Standard.

[3] Batel M., *Operational Modal Analysis – Another Way of Doing Modal Testing*, Sound and Vibration, August, 2002, 22–27.

[4] EN 12390-13:2014, Testing Hardened Concrete – Part 13: Determination of Secant Modulus of Elasticity in Compression.

[5] Hoła J., Schabowicz K., *State-of-the-art non-destructive methods for diagnostic testing of building structures – anticipated development trends*, Archives of Civil and Mechanical Engineering, Vol. 10(3), 2010, 5–18.

[6] Jerath S., Shibani M.M., *Dynamic Modulus for Reinforced Concrete Beams*, Journal of Structural Engineering, Vol. 110(6), 1984, 1405–1410.

[7] Malaikah A., Al-Saif K., Al-Zaid R., *Prediction of the Dynamic Modulus of Elasticity of Concrete Under Different Loading Conditions*, International Conference On Concrete Engineering and Technology, Kuala Lumpur, 19–21 April 2004, 32–39.

[8] Musiał M., *Static and dynamic stiffness of reinforced concrete beams*, Archives of Civil and Mechanical Engineering, Vol. 12(2), 2012, 186–191.

[9] Musiał M., Grosel J., *Identyfikacja schematu stanowiska badawczego z wykorzystaniem operacyjnej analizy modalnej*, Materiały Budowlane, 3, 2013, 42–44.

[10] Newtson C.M., Johnson G.P., Enomoto B.T., *Fundamental Frequency Testing of Reinforced Concrete Beams*, Journal of Performance of Constructed Facilities, Vol. 20(2), 2006, 196–200.

[11] Plachy T., Padevet P., Polak M., *Comparison of Two Experimental Techniques for Determination of Young's Modulus of Concrete Specimens*, Recent Advances in Applied and Theoretical Mechanics, Puerto De La Cruz, 14–16 December 2009, 68–71.

[12] Wójcicki Z., Grosel J., Sawicki W., *Eksperymentalne badania dynamiczne budowli*, DWE, Wrocław 2015.