TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE
2-NP/2015

PIOTR FLIN*, KATARZYNA BAJAN**, WŁODZIMIERZ GODŁOWSKI***

LUDWIK SILBERSTEIN AND THE OPERATOR CALCULUS

LUDWIK SILBERSTEIN I RACHUNEK OPERATOROWY

A b s t r a c t

In the article we outline the life of Ludwik Silberstein (1872–1948). We present his approach to the matrix calculus and its application to the operator form of relativity. We also give the list of books on the subject written by him as well as translated by him.

Keywords: *Ludwik Silberstein*, *matrix calculus*, *theory of relativity*

S t r e s z c z e n i e

W artykukle przedstawiamy zarys życiorysu Ludwika Silbersteina (1872–1948). Omawiamy podejście do rachunku macierzowego i jego zastosowanie do podania operatorowej postaci teorii względności. Prezentujemy również listę książek i tłumaczeń z różnych języków dotyczących tej tematyki.

Słowa kluczowe: *Ludwik Silberstein*, *rachunek macierzowy*, *teoria względności*

DOI: 10.4467/2353737XCT.15.213.4418

* Institute of Physics, Jan Kochanowski University, Kielce, Poland; sfflin@cyf-kr.edu.pl
** Institute of Physics, Pedagogical University, Krakow, Poland; bajanka@gmail.com
*** Institute of Physics, Opole University, Opole, Poland; godlowski@uni.opole.pl

1. Historical background

Ludwik Silberstein is regarded by historians of science as a Polish-American mathematical physicist.

Silberstein was born on May 15th, 1872 in Warsaw, Poland, and died in Rochester, USA on January 17th, 1948. His parents were Samuel Silberstein and Emilia, nee Steinkalk. The father took great care of the education of the three children. Henryk (1858–1890) obtained his PhD with Lothar Meyer 1882 in Tubingen in chemistry [4], while Ludwik's sister Adela (1874–?) obtained a doctorate in philosophy in Zurich [3].

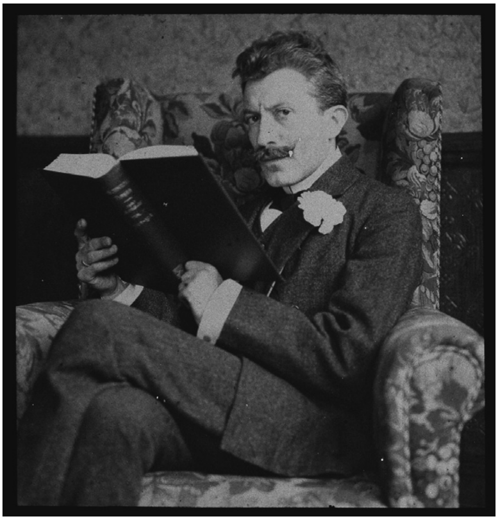
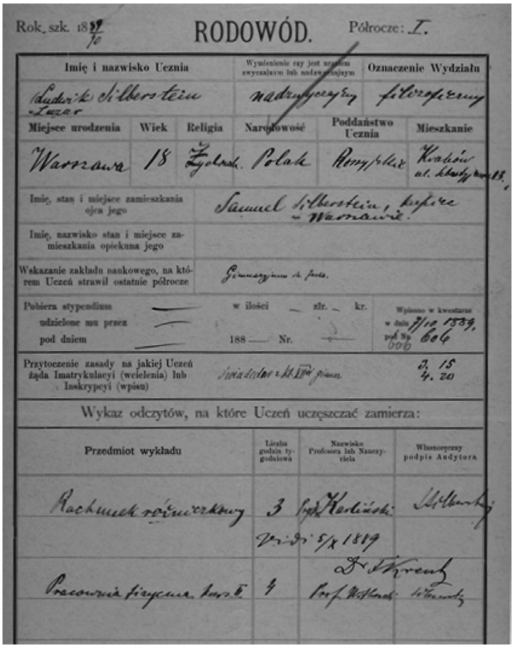


Fig. 1. Ludwik Silberstein in his home about 1915 (source: Courtesy of George Eastman House, International Museum of Photography and Film, Rochester, USA)

Ludwik went to a high school in Warsaw, and later passed the examination to attend the sixth class of the St. Hyacinth Imperial-Royal Gymnasium (c.k. Gimnazjum św. Jacka) in Cracow. His given name was Lazar, which he used until the end of high school. In 1890, he studied at Cracow's Jagiellonian University, even before completing his secondary education. He was attending calculus and laboratory classes conducted by professors August Witkowski (1854–1913) and Franciszek Karliński (1830–1906) respectively, and later always listied them among his most important teachers. There, at the Jagiellonian University, the name `Ludwik' appeared for the first time. The name 'Lazar' is seen in the matriculation document, because all school documents contained this name; however, afterwards the name 'Lazar' never appeared. From the document it follows that he regarded himself as a Pole (Fig. 2).

After a short period of study in Cracow, Silberstein spent one semester in Heidelberg and several years in Berlin, where he obtained his doctorate at the faculty of philosophy (where Max Planck was the dean) at Berlin's Friedrich-Wilhelms-University. Von Helmholtz wrote an opinion of Silberstein's dissertation *Ueber die mechanische Auffassung*


Rok szk. 1889/90 RODOWÓD. Półrocze: I.

Imię i nazwisko Ucznia: Ludwik Silberstein Lazar
Wymienienie czy jest zwyczajnym lub nadzwyczajnym: nadzwyczajny
Oznaczenie Wydziału: filozoficzny

Miejsce urodzenia	Wiek	Religia	Narodowość	Poddaństwo Ucznia	Mieszkanie
Warszawa	18	Żydowsk.	Polak	Rosyjskie	Kraków

Imię, stan i miejsce zamieszkania ojca jego: Samuel Silberstein, kupiec w Warszawie.
Imię, nazwisko stan i miejsce zamieszkania opiekuna jego:
Wskazanie zakładu naukowego, na którem Uczeń strawił ostatnie półrocze:
Pobiera stypendium — w ilości — złr. — kr.
udzielone mu przez —
pod dniem — 188 — Nr.
Wpisano w kwesturze w dniu 7/10 1889, pod Nr. 606
Przytoczenie zasady na jakiej Uczeń żąda Imatrykulacyi (wcielenia) lub Inskrypcyi (wpisu):
3. 15
4. 20

Wykaz odczytów, na które Uczeń uczęszczać zamierza:

Przedmiot wykładu	Liczba godzin tygodniowa	Nazwisko Profesora lub Nauczyciela	Własnoręczny podpis Audytora
Rachunek różniczkowy	3		
Pracownia fizyczna	4		

Fig. 2. Matriculation document of Ludwik Silberstein Wpis Ludwika Silbersteina na studia w Uniwersytecie Jagiellońskim (source: Jagiellonian University Archive, Cracow, Poland)

elektromagnetischer Erscheinungen in Isolatoren und Halbleitern [5], which circulated in the faculty from December 18, 1893.

Von Helmholtz wrote: "Mr Silberstein's work shows clear understanding of difficult and abstract problems and aptness for mathematical treatment. I propose its assessment as: "sollertiae et ingenii specimen laudabile". Below this document is a short note by the dean: "I agree in all points" and the signature of Max Planck. On July 5th, the PhD examination took place – the examiners were Max Planck in physics, Karl Hermann Schwarz in mathematics, Hans Landolt in chemistry, and Carl Stumpf in philosophy.

From 1895–1897 Silberstein was an assistant at the Politechnika Lwowska (Lviv Technical School) where Prof. Olearski was the head of the physics department. The scientific work and collaboration with Prof. Olearski were fruitful, but it was impossible for Silberstein to continue his work in Lviv. At that time Poland did not exist, having been partitioned by three empires: Russian, Prussian and Austro-Hungarian. As he was born in Warsaw, Silberstein was a Russian subject, while both Cracow and Lvov (Lviv, Lemberg) were situated in the Austro-Hungarian Empire. The authorities did not want to employ foreigners. His best-known paper from that period was probably *O tworzeniu się wirów w płynie doskonałym* [7], which allowed Bjerknes to formulate his theorem on turbulent motion, which formed the foundation of meteorological investigations and weather forecasting based on numerical calculations (1904) (application to eddy currents in the atmosphere).

In 1899 Silberstein took the position of lecturer (*libero docente*) in mathematical physics in Bologna, in 1904 moving to La Sapienza University in Rome. He was affiliated with Rome University until 1920, even though in the period 1912–1913 he was a lecturer in general relativity at London's University College, where the textbook *General Relativity* was written and published (1914). In 1920 he left Europe and went to the USA, where he remained until the end of his life, living in Rochester and collaborating with the Eastman Kodak Laboratory. He published several books, with the phrase "former lecturer at Rome University" appearing below his name, which shows he was proud of this work.

On 29 June 1905 he married Rose Eisenman from Warsaw, and they had three children (Georg P., Hedwig and Hannah).

He maintained close connections with Poland practically until the end of his stay in Europe. He corresponded with several physicists, was the organizer of the summer school of theoretical physics in Zakopane in 1904, and was very helpful in the organization of Polish science. In a letter to Banachiewicz, Sierpiński (1917) advised him to ask Silberstein in order to obtain books published in Europe.

2. Silberstein and matrix calculus

Silberstein translated several books dealing with methodology and methods of scientific investigations, written in several languages: Polish, German, French and English (see appendix A). Also, throughout his life he wrote textbooks on vectors and operators (see appendix B).

Matrix calculus was his most prominent subject of study, and he was one of the first to notice the advantages of using this calculus in physics. There are several papers written as early as 1901 and 1902. Interestingly, he proposed the quaternion form of general relativity [10]. As we know, almost twenty years later this calculus found its main application in quantum mechanics. The novelty and importance of these papers was noted by as great a specialist as Jammer [1].

In his book The conceptual development of quantum mechanics [1] Max Jammer wrote:

> "In concluding our brief survey on the early development of operators in mathematical physics, we should like to draw the reader's attention to a rather unknown paper by Ludwik Silberstein [9], which anticipated to some extend the formal aspects of the operational approach in modern quantum mechanics. The study of symbolic integrals of the equations of the electromagnetic field suggested to Silberstein [8], who is known mostly for his writings on the theory of relativity, a theory of "physical operators", in terms of which he attempted to give a unified representation of such disparate phenomena as mechanical oscillations, heat conduction, and electrodynamic process. Defining the "state" of a physical system by a time-dependent function $\psi(t)$, Silberstein introduced what ha called "chrono-operators" $\begin{Bmatrix} H \\ t \end{Bmatrix}$ by means of

which the state at time $t = t$ can be determined from the knowledge of the state at time $t = 0$,in accordance with the equation $\psi(t) = \begin{Bmatrix} H \\ t \end{Bmatrix} \psi(0)$. He defined the inverse operator $\begin{Bmatrix} H \\ t \end{Bmatrix}^{-1}$ by $\begin{Bmatrix} H \\ -t \end{Bmatrix}$, showed that $\begin{Bmatrix} H \\ nt \end{Bmatrix} = \begin{Bmatrix} H \\ t \end{Bmatrix}^n$ and that $\begin{Bmatrix} H \\ t \end{Bmatrix} = \{e^{Ft}\}$, where F is another operator, connected with H in a definite manner, but independent of t. Silberstein even spoke of the superposition of states and mentioned numerous other details which, including the notation, made their appearance in the operational formalism of quantum mechanics some twenty-five years later. "Every class of physical phenomena", declared Silberstein in 1901, "or at least those amenable to quantitative treatment, will be characterized by corresponding physical operators; and the scientific study of natural phenomena will proceed by detailed examinations of the properties of these operators, based on observation and nurtured by experiment".

3. Quaternion form of general relativity

Quaternions are quadrupels of real numbers. Quaternions can be written in different ways:

1. The matrix form

$$\begin{bmatrix} z & w \\ -\bar{w} & \bar{z} \end{bmatrix} = \begin{bmatrix} a+bi & c+di \\ -c+di & a-bi \end{bmatrix}$$

where $z = a + bi$, $w = c + di$.

2. The algebraic form

$$i = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}, \quad j = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad k = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$$

with a unit matrix 2 × 2 for real r. $r = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}$. Each quaternion can be explicity write as $q = a + bi + cj + dk$, where a, b, c, d are real.

Quaternions satisfy the following relations:

$$i\hat{\ }2 = j\hat{\ }2 = k\hat{\ }2 = ijk = -1$$

$$ij = -ji = k$$

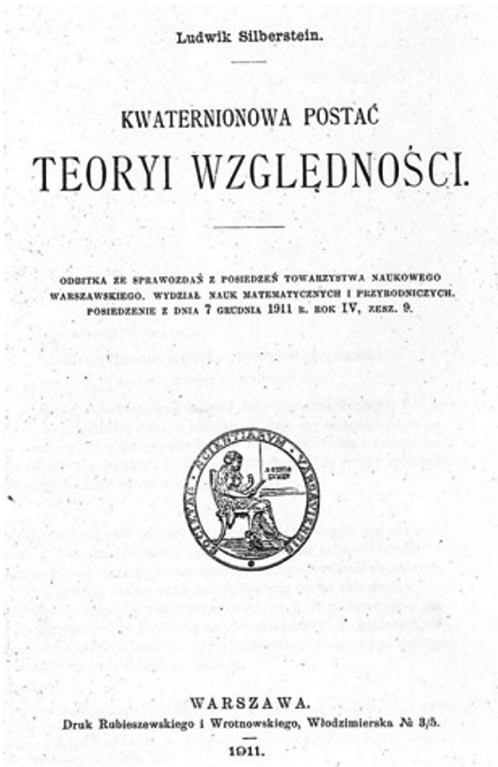
$$jk = -kj = i$$

$$ki = -ik = j$$

$$1q = q1 = q \text{ for each } q$$

$$rq = qr \text{ if } r \in \mathbb{R},\ q \text{ a quaternion}$$

$$q\bar{q} = |q|^2$$


Ludwik Silberstein.

KWATERNIONOWA POSTAĆ

TEORYI WZGLĘDNOŚCI.

ODBITKA ZE SPRAWOZDAŃ Z POSIEDZEŃ TOWARZYSTWA NAUKOWEGO WARSZAWSKIEGO. WYDZIAŁ NAUK MATEMATYCZNYCH I PRZYRODNICZYCH. POSIEDZENIE Z DNIA 7 GRUDNIA 1911 R. ROK IV, ZESZ. 9.

WARSZAWA.
Druk Rubieszewskiego i Wrotnowskiego, Włodzimierska № 3/5.
1911.

Fig. 3. The cover of the book *Kwaternionowa Postać Teoryi Względności*

We thank several archives for giving us access to the documents. This work started with the collaboration of dr Hilmar Duerbeck (1948–2014), whose absence is very sad and unfortunate.

References

[1] Jammer M., *The Conceptual Development of Quantum Mechanics*, McGraw-Hill, 1966; 2nd ed., American Institute of Physics, New York 1989.

[2] Sierpiński W., letter to T. Banachiewicz from Moscow on 7th Nov. 1917.

[3] Silberstein A., *Leibniz' Apriorismus im Verhältnis zu seiner Metaphysik, Inaugural-Dissertation*, Weimar: R. Wagner Sohn, Zürich 1902.

[4] Silberstein H., *Uber Diazoderivate des symmetrischen Tribromanilins und deren Umsetzungsproducte. Inaugural-Dissertation*, T¨ubingen, B.G. Teubner, Dresden 1882.

[5] Silberstein L., *Ueber die mechanische Auffassung elektromagnetischer Erscheinungen in Isolatoren und Halbleitern. Inaugural-Dissertation*, C. Vogts Buchdruckerei, Berlin 1894.
[6] Silberstein L., *The Theory of Relativity*, MacMillan and Co., Limited, London 1914.
[7] Silberstein L., *O tworzeniu się wirów w płynie doskonałym*, Rozpr. Wydz. Mat-Przyr. Akad Umiejętności, Kraków, t. XXXI, 1896.
[8] Silberstein L., *Symbolische Integrale der elektromagnetischen Gleichungen, aus den Andeutungen zu einer allgemainen Theorie der physicslischeen Operatoren*, 25 p, Wiedmann (Drude) Ann. Phys.6, 1901.
[9] Silberstein L., *Versuch einer Theorie der physikalischen Operatoren*, Przegl. Filozof. (in Polish), Oswald Ann d. Naturphilosophie 2 (in German), 1903.
[10] Silberstein L. *Kwaternionowa postać teoryi względności*, Wydz. Mat-przyr. Warszawskie Tow. Naukowe, 1911.

Appendix A

[1] Clifford W.K. (1845–1879), *Szkice filozoficzne*, tł. Adela Silberstein, Ludwik Silberstein, Warszawa: [s.n.], druk K. Kowalewskiego, Warszawa 1899.
[2] Young J.W. (1879–1932), *Dwanaście wykładów o zasadniczych pojęciach algebry i geometryi*, nakł. Księgarni E. Wende i Sp.; Lwów: H. Altenberg, G. Seyfarth, E. Wende i Ska, Warszawa:1918, 4, 172.
[3] Clifford W.K., *The aims and instruments of scientific thought*, Ueber die Ziele und Werkzeuge des wissenschaftlichen Denkens; Beteiligt: Georg Schmidt; Ludwig Silberstein Ausgabe: Deutsche autoris. Ausg. v. Georg Schmidt, kgl. preuss. Premierlt. i.R., u. Dr. Ludwig Silberstein, Assist... in Lemberg, (Dr. v. B. Heller; Berlin: Speyer & Peters), München 1896.
[4] Helmholtz H., *Zaelen und Messen, erkenntnistheoretisch bearbeitet* (Liczenie i mierzenie z punktu widzenia teorii poznania), druk J. Sikorskiego, Wyd. „Przeglądu Filozoficznego", Warszawa 1901.

Appendix B

[1] *Teorja operatorów fizycznych: (związek zjawisk w czasie)*, druk J. Sikorskiego, Warszawa 1903, [2], 51 p. Odb. Przegląd Filozoficzny.
[2] *Krótki zarys mechaniki w języku wektorów*, Skład główny Księgarnia G. Centnerszwera i S-ki, (Warszawa: Druk. Rubieszewskiego i Wrotnowskiego), IV, 120 p.: rys., err. Odb.: Przegląd Techniczny, Warszawa: 1908.
[3] *Elektryczność i magnetyzm: wykład teoretyczny, poprzedzony wstępem o algebrze i analizie wektorów*, T. 1 nakł. Księgarni E. Wende i S-ka, Warszawa 1908, [2], VIII, 366.
[4] *Vectorial mechanics*, MacMillan, London 1913, 206 p.
[5] *Elements of Vector Algebra*, Longmans, Green and Co. NY1919 42 p.
[6] *Elements d'algebre vectorielle et d'analyse vectorielle*, tł. G. Matisse, Gauthier-Villars et Cie, Editeurs, Libraires de Bureau des Longitudes, de l'Ecole Polytechnique, 125 p. Paris 1921.
[7] *Projective Vector Algebra; An Algebra of Vectors Independent of the Axioms of Congruence And of Parallels vii +78 p.*, London 1919.
[8] *Vectorial mechanics*, Ed. 2. x, 205 p., MacMillan, London 1926.