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A FEW FACTS CONCERNING
THE OUTSTANDING POLISH MATHEMATICIAN
FRANCISZEK LEJA

KILKA FAKTÓW DOTYCZĄCYCH
WYBITNEGO POLSKIEGO MATEMATYKA
FRANCISZKA LEJI

Abstract

We present a collection of facts given by the outstanding Polish mathematician Franciszek Leja (1885–1979) in his unpublished memoirs, add some other information about him and list some institutions bearing his name.

Keywords: 20th century European mathematicians, Jagiellonian University, biography, Franciszek Leja

Streszczenie

Przedstawiamy pewne fakty z nieopublikowanych wspomnień wybitnego matematyka Franciszka Leji¹ (1885–1979), garść innych informacji oraz listę instytucji noszących jego imię.

Słowa kluczowe: europejscy matematycy dwudziestego wieku, Uniwersytet Jagielloński, biografia, Franciszek Leja

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¹ Franciszek Leja preferred this way of declining his name.

1. Foreword

Franciszek Leja was born on 27th January 1885 in Grodzisko and died on 11th October 1979 in Kraków. He is buried in Grodzisko Górne because he had wished it.

In the late 1970s his students asked him to write down his memoirs [40]. He admitted in the introduction that it was a difficult task since he had to rely almost entirely on his memory of the period of about 90 years. The memoirs end with the year 1958. They were handwritten, and Janina Poznańska, the secretary of the Institute of Mathematics of the Jagiellonian University, typed them. Józef Siciak had one copy. The Jagiellonian University Students' Math Society (Kóło Matematyków Studentów UJ) obtained 4 copies for its library. The state school in Grodzisko Górne has also a copy.

The author of this article is not a historian, thus this note is not a strictly historical one. Chapters 2, 3, 5 and 6 of this article are almost entirely based on [40]. They may therefore contain some errors if Leja remembered some facts incorrectly. The biographical articles about Franciszek Leja written by his students Józef Siciak ([46] and [47]) and Witold Kleiner ([10]) were also mainly based on [40]. We should emphasize that the authors of these articles are not historians either. The biographical entry on Leja in [12] seems to be based on the same materials. For more recent notes having more historical character we refer the reader to [3] and [6].

A few years ago Krzysztof Ciesielski (Institute of Mathematics, Jagiellonian University) researched the archives of the Jagiellonian University to record all doctorates in mathematics obtained at the Jagiellonian University. The list, which he prepared, is given in [50]. We used it in chapter 7. Another source of information was [51], where the promotions in the Institute of Mathematics of the Polish Academy of Sciences are listed.

2. Life [40]

Franciszek Leja was born in a small village of Grodzisko Górne near Leżajsk. His parents had a small holding of about 4 ha, and two sons and four daughters (cf. [48]). As a little boy Franciszek often played with three boys who were only a year or two older: his brother Józef, his uncle (the youngest brother of his father) Wojciech and a neighbour Jan. Of course the children had their duties too. Franciszek was herding cows and babysitting his younger sisters, he also had to look after the hens and bring pinecones and firewood from the forest.

He finished a three-class folk school in the village and then the fourth class in Leżajsk. He attended gymnasium in Jarosław in 1896–1904. He passed his secondary school-leaving examination in Jarosław in 1904.

Leja enrolled in the Faculty of Philosophy of the Lwów University. He studied until 1909, when he passed an exam which allowed him to become a teacher of mathematics and physics in secondary schools.

In 1910 Leja started to work at the Fourth Gymnasium in Kraków and in 1911 was transferred to Bochnia. He published his first article on non-Euclidean geometry in the annuals of the gymnasium in Kraków. He spent 9 months from November 1912 till June

1913 in Paris. In September 1913 he started to work in the Fifth Gymnasium in Kraków, but he was also appointed to a part-time assistant job at the Jagiellonian University.

When the First World War broke out, Leja joined the Legion of Lwów. Nine other men from Grodzisko Górne went with him. However, he resigned when the Austrian regiment demanded a pledge of allegiance to Austria.

Leja obtained his Ph.D. at the Jagiellonian University in Kraków in June 1916. The title of his thesis was *Własność niezmiennicza równań różniczkowych zwyczajnych ze względu na przekształcenia styczościowe* (Polish) [Invariant property of ordinary differential equations with respect to contiguous transformations]. According to [6], the paper [14] contains this thesis.

In 1919 he was among the 16 mathematicians who founded Mathematical Society in Kraków and he became its secretary for the period of 1919–1921 (cf. [39]). This organization was transformed into the Polish Mathematical Society in 1920. In 1922 Leja passed his examination to obtain the title of the docent (the associate professor) of mathematics. [15] was his habilitation thesis (cf. [6]).

In 1923 he received two offers: he could obtain the chair of mathematics either at the Poznań University or at the Warsaw Technical University. He chose Warsaw and obtained the chair of mathematics in the Faculty of Chemistry. In 1924 he married Janina Mizerska. In the same year his habilitation from the Jagiellonian University was accepted by the University of Warsaw. Thus, even though he had the chair at the Warsaw Technical University, he gave also lectures at the University of Warsaw.



Fig. 1. Photo from 1930 by Jadwiga Lanczewska (source: Library of the Institute of Mathematics, Polish Academy of Sciences)

Professor Leja planned to stay in Warsaw for good and even started to build a house there. But when he was the Dean of the Faculty of Chemistry, two professors from the Faculty intercepted a letter addressed to the Dean and answered it in his name without telling him about it. When Leja found out about it, he asked the Rector to call them to account and to punish them. However, the authorities of the Technical University decided to hush

the situation up. Leja did not like it and decided that he did not want to stay in Warsaw any longer.

In 1936 he returned to Kraków. He obtained a chair of mathematics at the Jagiellonian University, namely the one vacated by Stanisław Zaremba (1863–1942), who had retired. In 1937–1938 two well known mathematicians visited Kraków: Henri Lebesgue (1875–1941) from France and Mauro Picone (1885–1977) from Italy. They gave lectures at the Jagiellonian University. Franciszek Leja was invited to go to France and Italy to give talks in his turn. Together with his wife he went to Paris in the first half of 1939. After his lecture, he was told that nobody expected him to arrive. They were sure that the war would start soon. The travel to Italy turned out to be impossible.

On the 6th November 1939 he was among the 183 academics arrested by gestapo and taken to Sachsenhausen concentration camp. After the international interventions the older professors were freed in Spring 1940. Although Leja was not in the first group freed in March 1940, he was back in Kraków in May 1940. He was allowed then to go to Grodzisko Górne. He worked in his garden and prepared his textbook on Calculus during the remainder of the Second World War.

In 1947 his first textbook for students [27], written during the war, was published. It was a very popular book and its multiple editions always disappeared quickly from the bookshops. Later other textbooks: [32, 36, 38] were published.

In 1957 Leja was invited to a conference on analytic functions in Helsinki. He participated in the International Congress of Mathematicians held in Edinburgh from 14 to 21 August 1958 and gave a lecture entitled *Sur les moyennes arithmétiques, géométriques et harmoniques des distances mutuelles des points d'un ensemble* there.

Józef Siciak recalls that professor Leja was very active even after his retirement in 1960 (when he was 75 years old). The Institute of Mathematics was located on the fifth floor of the university building on Reymonta 4 and Franciszek Leja regularly went there never using the lift, even when he was 80 or 90 years old. In 1963 the University of Łódź awarded him its honorary doctorate.

3. Scholarships and studies ([40]; for a wider background on the education in that area and that time see [2])

Now it is quite easy to obtain a secondary or even higher education. It was not so for Franciszek Leja. He could attend the three-class folk school in Grodzisko Górne, but when he finished it, it seemed that he would not be able to go any further. His parents could not afford longer education for all of their children. They decided to send only their oldest son Józef to the secondary school. But Józef did not want to leave the village and especially the horses he loved and suggested that Franciszek should go instead. Franciszek went willingly.

He finished the fourth class in Leżajsk first and then the classical gymnasium in Jarosław thanks to a private scholarship founded by the late Rev. Czesław Kaczorowski and to money he earned by tutoring. During those years he was constantly undernourished. For a long time he was the only person from Grodzisko Górne, who attended a secondary school. In the second class he had trouble with the subject of proportions. The teacher seemed not to be able

to explain it clearly. Later, however, Franciszek had no more problems with mathematics. While he was in the sixth class, a pupil from the eighth class gave him mathematical tasks which Franciszek solved. During his secondary school-leaving exam he wrote the solutions of the test in mathematics for half of his class. The other half obtained the solutions from another boy. The whole class passed the exam.

His parents wanted him to become a priest and were disappointed when he refused. He intended to study at the Lwów University, but his family could not help him financially. Fortunately, their parson Feliks Świerczyński helped Franciszek. He talked with his parents and succeeded in soothing them. He also provided a further scholarship (from the same fund left by Rev. Kaczorowski): 20 koronas per month. Franciszek needed at least 80 for his boarding, living and studies. He earned the missing amount accepting various jobs: he was a tutor, a land surveyor, an accountant, an assistant manager in a small cigarette tube factory and he was also employed at collecting documents in libraries about the history of the Church (his employer then was Rev. Adam Sapięha (1867–1951) – later a bishop and a cardinal).



Fig. 2. The credit book of Franciszek Leja (provided by Józef Siciak)

Leja wrote later about his studies at the Faculty of Philosophy of Lwów University in his memoirs [40]: “I was interested not only in mathematics and physics, but in philosophy and psychology too. The lectures in psychology were really good, but unfortunately I could listen to only a few of them because of my jobs which I needed to earn my living. On the other hand, the “metaphysics” disappointed me, even though I attended every lecture. When I finished this course, I began to have much more respect for mathematics and its deductive methods than for philosophy”.

Cursus V. Per semestre <i>hivernali</i>				anni scholastici 1906/7 <i>1892</i>		
Nomen Magistri	Index scholarum	Classis horae	Didacticum notatum aut immutatum datum testatur Quaesitor	Receptum nomen m. p. testatur Magister	Scholae frequentatae	Adnotata
1. Puzyna	Geometria obliquo	3.		<i>Puzyna</i>	<i>Puzyna</i>	
2. Puzyna	Trigonometria	2.		<i>Puzyna</i>	<i>Puzyna</i>	
3. Puzyna	Algebra	2.		<i>Puzyna</i>	<i>Puzyna</i>	
4. Smoluchowski	Optica	1.				
5. Smoluchowski	Acoustica	1.				
6. Smoluchowski	Electricitas	4.		<i>Smoluchowski</i>	<i>Smoluchowski</i>	
7. Smoluchowski	Electromagnetismus	5.		<i>Smoluchowski</i>	<i>Smoluchowski</i>	
8. Smoluchowski	Optica	2.		<i>Smoluchowski</i>	<i>Smoluchowski</i>	
9. Smoluchowski	Acoustica	10.		<i>Smoluchowski</i>	<i>Smoluchowski</i>	
10. Smoluchowski	Electricitas	4.		<i>Smoluchowski</i>	<i>Smoluchowski</i>	

Fig. 3. From the credit book of Franciszek Leja. Some courses given by Puzyna, others by Smoluchowski

During his studies Leja attended lectures given e.g. by Józef Puzyna (1856–1919) in mathematics and by Marian Smoluchowski (1872–1917) in physics.

Cursus III. Per semestre <i>hivernali</i>				anni scholastici 1905/6 <i>1892</i>		
Nomen Magistri	Index scholarum	Classis horae	Didacticum notatum aut immutatum datum testatur Quaesitor	Receptum nomen m. p. testatur Magister	Scholae frequentatae	Adnotata
1. Haeftenberg	Matematyka	4.		<i>Haeftenberg</i>	<i>Haeftenberg</i>	
2. Rajewski	Matematyka	2.				
3. Puzyna	Trigonometria	2.		<i>Puzyna</i>	<i>Puzyna</i>	
4. Rajewski	Matematyka	2.				
5. Rajewski	Matematyka	2.				
6. Rajewski	Matematyka	2.				
7. Rajewski	Matematyka	2.				
8. Rajewski	Matematyka	2.				
9. Rajewski	Matematyka	2.				
10. Rajewski	Matematyka	2.				
11. Rajewski	Matematyka	2.				
12. Rajewski	Matematyka	2.				
13. Rajewski	Matematyka	2.				
14. Rajewski	Matematyka	2.				
15. Rajewski	Matematyka	2.				
16. Rajewski	Matematyka	2.				
17. Rajewski	Matematyka	2.				
18. Rajewski	Matematyka	2.				
19. Rajewski	Matematyka	2.				
20. Rajewski	Matematyka	2.				

Fig. 4. From the credit book of Franciszek Leja Some courses given by Rajewski are deleted in the second year

Unfortunately, Puzyna was the only mathematician at the university apart from Jan Rajewski (1857–1906), whose classes Leja could attend only in his first year of studies. Thus there were not enough mathematical courses (cf. [6]). That is why Leja said in [40]:

“I was not satisfied with my studies in Lwów and I told my friends who congratulated me on my final exam: Alas, I do not know this mathematics.

At the university, I had only a chance to learn that there is a huge building of higher mathematics, but I had no opportunity to get to know anything from this building because there were not enough lectures and courses. That is why I decided to try to continue my studies while teaching at a high school”.

Leja planned to study further and that is why he refused the first assignment to a post in a school in Drohobycz. He wanted to work in an academic center, close to a university. Fortunately, he obtained another post in Kraków.

In 1911 Kazimierz Żorawski (1866–1953), a professor of mathematics at the Jagiellonian University in Kraków, noticed the first article published by Leja. He liked the paper so much that he sought out its author and offered him a subsidy from the fund left by late Władysław Kretkowski (1840–1910) (cf. the CV of Leja from 1916 in [2]). This money sufficed for one year of studies abroad. Leja accepted with pleasure. He could choose the Sorbonne in Paris or Göttingen in Germany. He spoke German better than French, so he decided to choose France to have an opportunity to learn its language better. And thus he went to Paris for the academic year 1912/1913. He learnt French, studied mathematics at the Sorbonne and attended high teas at Władysław Mickiewicz’ house on Thursdays there. He was also able to visit London.

Kazimierz Żorawski was later the advisor of Leja’s Ph.D. thesis. And when Leja started to work part-time at the Jagiellonian University in 1913, his wages were paid from some private funds distributed by the Academy of Learning.

4. Adoption [4]

The only child of Franciszek Leja and his wife died in infancy. On the other hand, Józef Leja, the oldest brother of Franciszek, who married Aniela Pawlik in 1908, had six children. He was not able to educate all of them, he was too poor, just like his parents were before. Franciszek saw that and offered in 1927 that he might adopt Józef’s son Jan Leja (1918–2009) and give him home and education. Hence, starting from 1927, Jan lived with his uncle, first in Warsaw then in Kraków. Jan did not know why he was chosen for adoption in preference to one of his brothers. Perhaps the reason was his willingness to run errands for his uncle when Franciszek came to Grodzisko Górne.

In 1935, after a skiing trip from Zakopane to Kraków, Jan had to lie in bed for five months with rheumatic fever. His uncle provided him with a series of mathematical Calculus problems. In this way he tried to keep the boy entertained. By the time Jan recovered, he had completed a first year course in Calculus. As we already mentioned, after World War II Franciszek Leja published a very popular textbook on Calculus [27].

In 1937 Jan started to study at the Mining Academy in Kraków (which later became the AGH University of Science and Technology).

In September 1939 Jan sent his uncle to Grodzisko Górne and joined the Polish Army. His unit was intercepted by Russians. Fortunately Jan succeeded to escape and to get to Lwów where he met his older brother Stanisław Leja (1912–2000), who in 1938 had finished his studies in mathematics and since then had worked at the University of Lwów (cf. [5]). Stanisław had also joined the Polish Army, his unit had also been intercepted by Russians and he had also succeeded to flee. Both brothers wanted to join the resistance, but each of them was imprisoned by Russians, in different circumstances. Jan was sent to a slave labour camp Vorkuta Gulag, Stanisław was deported to Sverdlovsk Oblast by Ural (he had to work as a woodcutter there). Both managed to survive the terrible conditions and to join the Polish Army formed under Władysław Anders (1892–1970) in 1941. It was really a miracle that they met in Persia, when each of them was in a different unit of the Army.

The story of Jan's life is related in [4], a good account of Polish history written by the brother-in-law of Jan Leja.



Fig. 5. Jan Leja, photo from 1966 (source: University of British Columbia Archives, [UBC 5.1/1731])

Eventually, Jan became a professor of mining and mineral process engineering at the University of British Columbia, Vancouver, in Canada, and his brother Stanisław a professor of mathematics at the Western Michigan University, Kalamazoo, in the USA. In 1976 the Maria Curie-Skłodowska University in Lublin awarded Jan Leja its honorary doctorate. Jan had six children.

Just after the Second World War Franciszek Leja took another young boy to his household – it was Roman Danak (1935–1994) (cf. also [48]), a grandson of Franciszek's sister Katarzyna Danak. The father of the boy was killed by Germans. In the late 1950s Roman Danak studied oriental philology at the Jagiellonian University and later became a journalist and a science fiction writer [7].

In [40] Leja referred only once to Jan, saying that while they lived in Warsaw, he and his wife took care of his nephew who attended gymnasium. He said nothing more about the adoption or any other help he gave other young people.

5. Benefits to Grodzisko Górne [40]

Franciszek Leja never forgot his family home. He usually spent his holidays in Grodzisko Górne, his birthplace. He was also a benefactor of this village.

Already in his early youth, when he attended gymnasium, he organized, together with his friends, lectures, theater performances and fêtes in the village. The collected money was used for buying books for the library of the Association of the Folk School.

In 1910 he and his uncle Wojciech helped to found the Dairy Cooperative. In 1912 he was one of the founders of the Peasant Orchestra in Grodzisko Górne (cf. [4]).

6. Hunger [40]

When Franciszek Leja was 90 years old, he said “I am wondering why I have lived so long. Probably because I have often been hungry” (see [10]).

When he was a child, the villages in Galicia were really poor and everybody was there quite often undernourished. Later it was so too. Jan Leja said that as a child in Grodzisko Górne he had enough food only on Christmas Eve during the traditional celebratory meal (cf. [4]).

When Franciszek attended gymnasium, he ate in a canteen. The portions were very meagre and never satisfying. He remembers a discussion between hungry pupils there, on the best method of consumption: whether quick or slow eating made the eater more sated. An older boy finally decided that it was better to swallow very fast, because in this way the food, not arranged, took more place, so the stomach seemed to be more filled.

Franciszek Leja was hungry in Paris too, at least at the beginning of his stay. Namely, he did not want to take all his money with himself on the journey. He took only a part of it and his uncle had the rest and was to send an amount to Paris each month. Almost just after leaving Poland, Franciszek lent most of his money to an acquaintance, who seemingly needed it only for a few days. Unfortunately, the man never contacted Franciszek again; thus Leja lost the money and had to live on very little till the next part of money arrived.

In the beginning of his period in Warsaw the conditions were difficult too. It was not easy to have enough food. And of course Leja was on the verge of starvation quite often during the Second World War.

Finally, Leja recalled another difficult situation, which happened while he was abroad after the war. He was invited, together with Mieczysław Biernacki (1891–1959) and Zygmund Charzyński (1914–2001), to a conference on analytic functions in Helsinki in 1957. All three of them should have obtained 60 U.S. dollars from the Polish authorities for their expenses. But they were given only bonds of the Polish Bank, and outside of Poland those were only worth one third of the sum which they theoretically represented. The conference lasted 8 days and the talks of the Polish mathematicians were planned for the second half of the conference. Therefore, Leja and the others could not leave earlier and had to eat very sparingly.

7. Teaching

As we said before, Franciszek Leja was the author of very popular textbooks [27, 32, 36, 38]. He prepared parts of the texts testing them on his students during the lectures (cf. [48]).

Leja's students after the Second World War had a nickname for him: they called him *Grandpa* (cf. [48]). He was always very elegant and seemed to be working all the time. The exams he gave were really difficult but he always cared for his students and was proud of their achievements.

Professor Leja was the advisor of at least 9 Ph.D. theses. His students were: Jan Leśniak (1947; later a professor at the Higher College of Teacher Training in Kraków), Jerzy Górski (1950; later a professor of the University of Silesia in Katowice), Witold Kleiner (1954; now a professor emeritus at the Jagiellonian University), Franciszek Bierski (1959; later a professor at the AGH University of Science and Technology), Andrzej Szybiak (also 1959; later e.g. a professor at the University of Tlemcen, in Algeria), Józef Siciak (1960; now a professor emeritus at the Jagiellonian University), Władysław Bach (also 1960; lived only for 35 years but succeeded in obtaining his habilitation at the Jagiellonian University in 1965), Czesław Loster (1961; later worked at the Technical University of Kraków), and finally Bolesław Szafirski (1963; now a professor emeritus at the Jagiellonian University too). Seven of the promotions took place at the Jagiellonian University ([50]) and two: those of Witold Kleiner and Andrzej Szybiak, in the Institute of Mathematics of the Polish Academy of Sciences ([51]). According to Bolesław Szafirski, the youngest on the list (who was exactly half a century younger than Leja), starting from 1950 there were no other doctorates advised by Leja than those listed here. We do not know whether Leja advised any other Ph.D. thesis (apart from that one of Jan Leśniak) before that year.

The author of this article is a scientific 'great granddaughter' of Franciszek Leja. Namely, Leja was the advisor of the Ph.D. thesis of Józef Siciak, who was the advisor of the one of Wiesław Pleśniak and he in his turn was the advisor of the Ph.D. thesis of Marta Kosek.

8. Scientific achievements. For the whole list of publications see [10]

Franciszek Leja's research interests started with differential equations (see e.g. [13–15]) and he had some results in this domain later too (see e.g. [29]).

In [42] Pierre Dugac, Beno Eckmann, Jean Mawhin and Jean-Paul Pier give a list of the main mathematical achievements in the first half of the twentieth century, so called *Guidelines 1900-1950*. There, under the year number 1925, one can find: *Leja F.; Schreier O., Topological groups*. Namely, Leja and Schreier introduced the notion of an abstract topological group independently. In the references of the book [42] two papers by Leja: [19, 20] and two papers by Schreier are listed. Let us cite here one more article on this subject, namely [21].

Leja worked on (power) series (see e.g. [18, 22, 23]) and interpolation theory (see e.g. [25, 26, 28]). His research on the theory of analytic functions (in particular those

of several variables) (see e.g. [16, 17, 30]) and extremal problems (see e.g. [31, 33, 34]) was taken up by his students and their students and is now continued by a strong group at the Jagiellonian University.

The importance of the work of Leja and his students was underlined in the Preface of [44] (as Józef Siciak noticed in [47]): “The external field problem has its origins in the work of C.F. Gauss, and is sometimes referred to as the” Gauss variation problem. “O. Frostman investigated the problem and the Polish school headed by F. Leja made important contributions during the period 1935–1960 that have greatly influenced the present work. A rebirth of interest in the Gauss variational problem occurred in the 1980’s when E.A. Rakhmanov and, independently, collaborators H. N. Mhaskar and E. B. Saff used potentials with external fields to study orthogonal polynomials with respect to exponential weights on the real line”.

Paragraph III.5 of this book is entitled *The function of Leja and Siciak* and it starts with the following sentence: “In this section we introduce and investigate a function – due to F. Leja and J. Siciak – that gives the smallest upper bound for polynomials majorized by a weight on a set Σ ”. In this way the authors introduce the Leja-Siciak extremal function. They refer to 13 papers and one book by Leja (i.a. [24, 33, 36]) and many other articles by his students: Jerzy Górski (10 papers), Witold Kleiner (10), Józef Siciak (8).

Let us also mention another very important book, cited in [44] too. Maciej Klimek, a scientific ‘grandson’ of Franciszek Leja, namely a student of Józef Siciak, now a professor in Uppsala University, wrote the monograph [11] in 1991. The book gives a comprehensive study of plurisubharmonic functions. In Kraków this subject started with analytic functions of multiple variables studied by Leja. Many results obtained by the group working at the Jagiellonian University (built of the students of Leja and their students in turn) are discussed in this book, even though of course they do not exhaust the whole material. The author cites his own results but also refers to papers written by Leja (to [24, 37] and to book [36]), by his student Józef Siciak, by students of Siciak: Wiesław Pleśniak, Marek Jarnicki, Sławomir Kołodziej and finally to a few ones written by one scientific ‘great grandson’ of Leja, namely Mirosław Baran, a student of Pleśniak.

It is not the aim of this paper to discuss in detail the scientific achievements of Leja and their impact on the mathematics. For a whole account we refer the reader to [46, 10, 47] (cf. also [6]). We chose only to mention here two objects which are named after Leja. Let us start with a famous strong result, namely the Polynomial Lemma of Leja introduced in [24]. We will present only its simplest form (cf. [36]).

Polynomial Lemma *Let E be a connected compact set (consisting of at least 2 points) in the complex plane, M be a positive number. For every $\varepsilon > 0$ there exist positive numbers δ and N such that:*

if (P_n) is a sequence of polynomials where P_n is of degree at most n and $|P_n(z)| \leq M$ for all $z \in E$, $n \in \{1, 2, 3, \dots\}$, then for every $w \in E$

$$|P_n(z)| \leq M(1 + \varepsilon)^n, \quad \text{whenever} \quad |z - w| < \delta, \quad n > N.$$

For a generalization of this lemma and some information on its impact see e.g. [41, 43, 45] (cf. Leja polynomial condition in [11] too).

An important notion is that of a Leja sequence (also known under the name of Leja points), defined by Leja in [35], independently from Edrei [8]. Let us recall the definition. Let E be a nonempty compact set in the complex plane and (a_n) be a sequence of points in E . We say that (a_n) is a *Leja sequence* if

$$|(a_n - a_0) \dots (a_n - a_{n-1})| = \max_{z \in E} |(z - a_0) \dots (z - a_{n-1})|, \quad n \in \{1, 2, 3, \dots\}.$$

For some generalizations of this notion and results on it see e.g. [9, 44, 49] and the references given there. Recently, pseudo Leja sequences were defined and turned out to be a useful tool in the numerical optimization (see [1]).

9. The foundation, the school, streets and a lecture hall

Leja was thankful for the scholarship which had enabled him to study. In his turn he tried to help other people (as we had seen already). In 1977 he gave 200 000 złoty (it was a professor's yearly salary then) to the Institute of Mathematics of the Jagiellonian University for scholarships (awards) for outstanding students and young mathematicians. The first three of the beneficiaries were Piotr Jakóbczak in 1978, Piotr Tworzewski in 1979 and Maciej Klimek in 1980. Later the Leja Foundation was established and thus the scholarships are still granted. The whole list of the beneficiaries is given in [52] together with other informations about the scholarship. We present here photos of two diplomas.

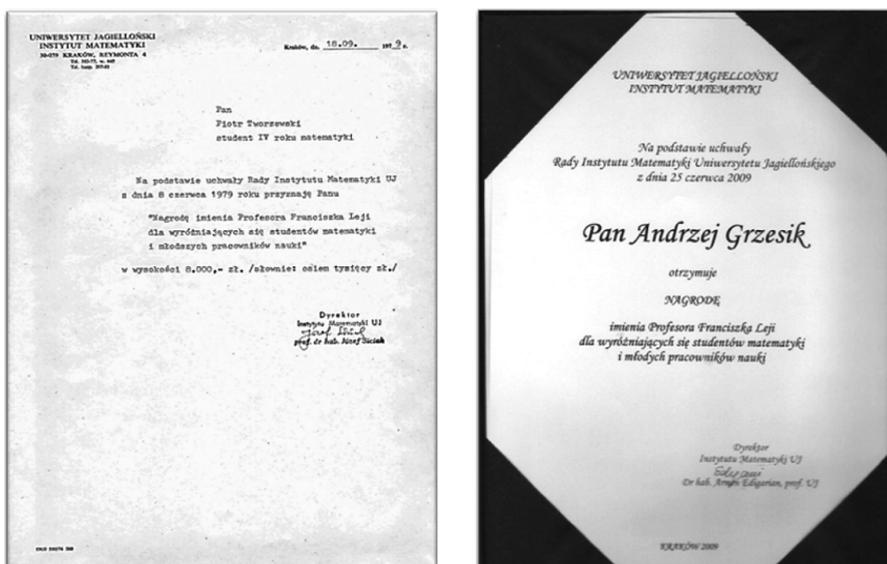


Fig. 6. The beneficiaries allowed the use of their diplomas

In 1998 the primary school in Grodzisko Górne celebrated its hundredth anniversary. There was a big fête and the school was then named after Franciszek Leja.

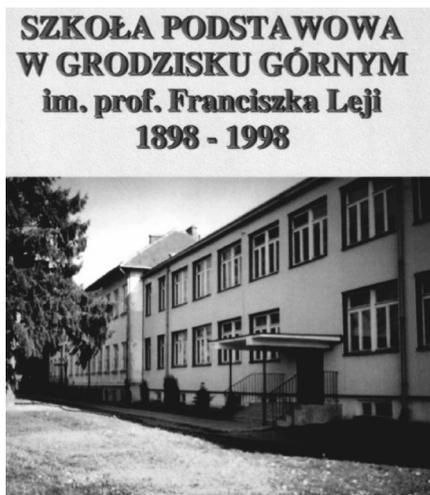


Fig. 7. The cover of [48]

In 2002 the gymnasium in Grodzisko Górne joined the primary school. Now it is a Complex of Schools named after Franciszek Leja: the primary school and the gymnasium together (cf. [53]). The schools make the name of Franciszek Leja known in some circles other than those of mathematicians. In [54] the competition scores from the Global Conference on Educational Robotics in Albuquerque in 2015 are given. One can read there that Franciszek Leja State School in Grodzisko Górne, Poland, won the first place in the International Botball Double Elimination, the fourth place in the International Botball Seeding and the Overall Judges' Choice Award there. More than 50 teams participated in the competitions and other awarded teams came from the USA (most of them), Austria, China and Kuwait. Polish television related some information about earlier achievements of the school in this area ([55, 56]). Unfortunately, the reporters spoke only about the school in Grodzisko Górne and did not mention its patron.

At least two streets are named after Franciszek Leja: one in Kraków and one in Jarosław.



Fig. 8. Streets in Kraków and Jarosław (photos by Marta Kosek and Jerzy Szczepański)

The biggest lecture theater in the new building of the Faculty of Mathematics and Informatics of the Jagiellonian University is now called the Franciszek Leja lecture hall.



Fig. 9. Lecture theater in the new building of the Faculty of Mathematics and Informatics of the Jagiellonian University (photos by Marta Kosek)

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