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OPTIMIZATION OF A CURVILINEAR THREAD PROFILE IN A PLANETARY ROLLER SCREW

OPTYMALIZACJA KRZYWOLINIOWEGO ZARYSU GWINTU W PLANETARNEJ PRZEKŁADNI ŚRUBOWEJ ROLKOWEJ

A b s t r a c t

The paper presents an application of the Finite Element Method (FEM) in stress analysis and optimization of thread profiles in cooperation of the roller with the screw in the planetary roller screw mechanism. The problem of the roller and the screw threads cooperation as well as related geometrical parameters were described. The optimization problem was formulated with the aim of obtaining a better effort of threads and lower contact pressure depending on the geometry.

Keywords: *Planetary roller screw, thread profile, finite element method, optimization*

S t r e s z c z e n i e

W artykule przedstawiono zastosowanie Metody Elementów Skończonych (MES) w analizie naprężeń i optymalizacji zarysu gwintów do współpracy rolki ze śrubą w planetarnej przekładni śrubowej rolkowej. Opisano zagadnienie współpracy gwintów śruby i rolki oraz związane z nim parametry geometryczne zarysu gwintów. Sformułowano zadanie optymalizacyjne, mające na celu poprawę wytężenia i zmniejszenie nacisków kontaktowych w zależności od geometrii gwintów.

Słowa kluczowe: *Planetarna przekładnia śrubowa rolkowa, zarys gwintu, metoda elementów skończonych, optymalizacja*

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Denotations

d_s, d_n, d_r	–	pitch diameter of the screw, the nut and the roller [mm]
n_s, n_n, n_r	–	thread starts of the screw, the nut and the roller [mm]
a_{rs}	–	distance between the axes of the roller and the screw [mm]
a_{rn}	–	distance between the axes of the roller and nut [mm]
x_1, x_2	–	dimensions of the undercut in the thread notch [mm]
R_r, R_s	–	radii of the screw and the roller thread flank [mm]
$\sigma_{\mathrm{HMH}}^{\mathrm{max}}$	–	Huber-Mises-Hencky reduced stress in the notch of thread (model without undercut) [MPa]
$\sigma_{\mathrm{HMH}}^{\mathrm{max}\_u}$	–	Huber-Mises-Hencky reduced stress in the notch of thread (model with undercut) [MPa]
$C_{\mathrm{press}}^{\mathrm{max}}$	–	maximum contact pressure on thread (model without undercut) [MPa]
$C_{\mathrm{press}}^{\mathrm{max}\_u}$	–	maximum contact pressure on thread (model with undercut) [MPa]

1. Introduction

A Planetary Roller Screw (PRS) is a low friction mechanism quite similar to a ball screw. It is used to convert the rotational motion of the screw into a linear motion of the nut or inversely (inverted planetary roller screw). Providing more contact points than the ball screws, planetary roller screws have higher load capacity with a similar efficiency (70%–90%) [5]. Previous publications on the planetary roller screw related to the thread profile problems concerning: cooperation of threads and the resulting necessity for using a curved thread profile [1], determination of the roller thread profile for a trapezoidal thread due to non-interference of cooperating threads [7], determination of the stiffness of the cooperating areas between the screw, roller and nut in order to determine the load distribution using a simplified model [2]. The values of this stiffness delivered for the specified range of axial forces were essential in [6] to determine load distribution between cooperating elements using the analytical model. In the paper [3], the procedure of a preliminary design of the planetary roller screw, including the limitation due to the carrying capacity of a thread determined based on the Hertz theory was presented.

The aim of this paper was to investigate the influence of the undercut in the notch of the thread on the stress state and contact pressure within a single pair of cooperating threads. The goal of the paper was carried out by the use of optimization.

2. Principle of operation and limitations

The PRS consists of a screw, an arbitrary number of rollers and a nut. The rollers, which spin in contact and transfer the load between the screw and the nut are additionally connected with the nut by the planetary toothed conjunction. The role of this connection is to synchronize the planetary movement of the rollers. The screw and the nut have multi-start threads whereas the rollers have a single thread.

To enable cooperation of the rollers with the nut, the axial displacement of the rollers relative to the nut must be equal to zero. This will take place when the helix angles of the nut and the rollers are the same and kinematic conditions are met [4]. Assuming the start of roller thread $n_r = 1$, the relation between pitch diameters of the screw and the rollers is obtained (eq. 1).

$$n_n = \frac{d_s + 2d_r}{d_r} = \frac{d_n}{d_r} \tag{1}$$

On the assumptions that rollers cooperate with the screw without slipping it appears that the starts of the screw and nut thread are equal $n_s = n_n$ [3].

An orbit, which includes the axes of the rollers is concentric with respect to the screw and the nut. The distance between the axes of the rollers and the nut and the distance between the axes of the roller and the nut must be equal (eq. 2)

$$a_{rn} = (d_n - d_r)/2, \quad a_{rs} = (d_s - d_r)/2, \quad a_{rn} = a_{rs} \tag{2}$$

From the above conditions result in the limitation on the screw and the nut thread start given by eq.3.

$$(d_s/d_r) = n_s - 2 = n_n - 2 \tag{3}$$

The smallest thread start of the screw and the nut is $n_s = n_n = 3$. The relations between pitch diameters are strictly defined (eq. 4)

$$d_n = n_n d_r = nd_r, \quad d_s = (n_n - 2)d_r = (n - 2)d_r \tag{4}$$

3. Profile of the threads

In order to ensure correct operation of the mechanism it is required to maintain constant pitch diameters of cooperating elements [1]. These diameters depend on the location of the contact points on the threads' flanks. To eliminate the fluctuations of pitch diameters, at least one of the cooperating thread's profile should be curvilinear (Fig. 1).

The most advantageous distribution of the contact pressure is obtained when one of the thread profiles is convex, whereas the other one is concave (Fig. 1c). The actual size of the contact area depends on the ratio of the curvatures' radii of the screw and the roller thread's flank. However, an assumption of too small a difference between these radii, may significantly increase the contact area and shift the contact point and worsen the conditions of the threads cooperation. A significant stress concentration appears in the notch of the screw's thread (Fig. 6). The value of this maximum stress can be reduced by a partial reliving of the screw's thread. It can be achieved by introducing an undercut in the notch of a thread (Fig. 8).

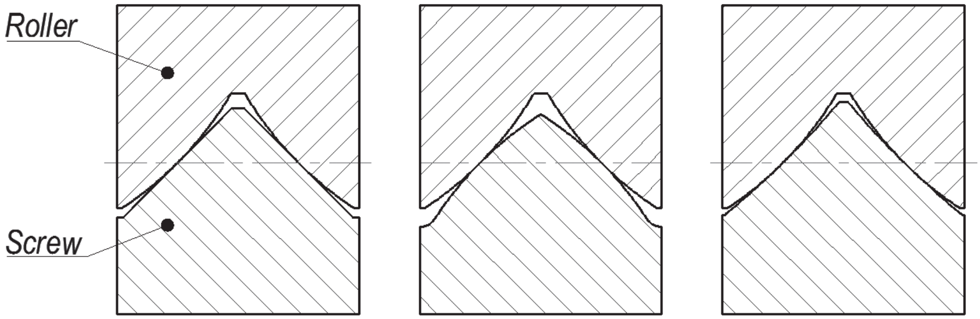

Fig. 1. Curvilinear thread profiles of the screw and the roller: a) screw – flat, roller – convex, b) screw – convex, roller – convex c) screw – concave, roller – convex

The conception of relieving the thread can be applied individually, only for one element (screw or roller).

4. Optimization of the curvilinear thread profiles with an undercut in the notch of the thread

4.1. A finite element model

To solve the optimization problem a 2D finite element model including sections of cooperating threads of the screw and roller was built (Fig. 2). The pitch dimensions of the screw and the roller as well as the thread pitch were accepted as follows: d_s = 30 mm, d_r = 10 mm, p = 2 mm. The 8-node elements PLANE82 available in the ANSYS software were used. The plane's strain was accepted. The contact was defined only for a single pair of cooperating threads. Contact elements CONTA172 and TARGET169 were accepted.

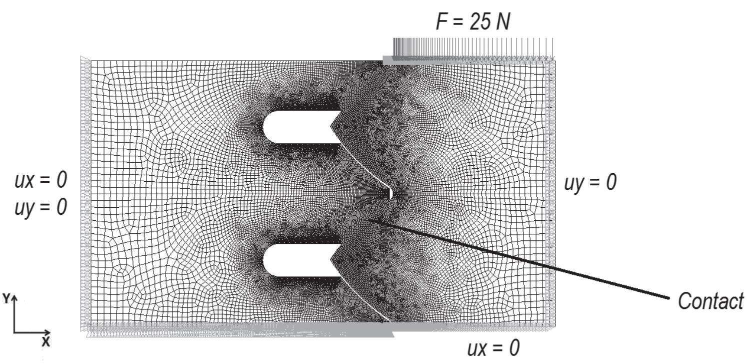

Fig. 2. Finite element model of the screw and the roller threads with boundary conditions

The coefficient of friction in the plane of the model was set to $\mu = 0.1$. the axial load $F = 25$ N was applied to the surface of the roller's core. Young modulus $E = 2.11 \cdot 10^5$ MPa and Poisson ratio $\nu = 0.3$ were accepted.

4.2. Design variables, state variables and objective function

The geometry of threads was parameterized as shown in Fig. 3. As the design variables for optimization problem, the dimensions of undercut x_1, x_2 and the radius of the roller thread flank's curvature R_r were assumed. The radius of the screw thread flank's curvature was accepted as $R_s = k \cdot R$, where $k = 1.5$. The coefficient k has been selected so that the segment of the threads being in contact is not bigger than 20% of the thread flank's length. Contact pressure on the thread flank was accepted as a state variable. Restrictions assumed for design and state variables as well as starting points are summarized in Table 1.

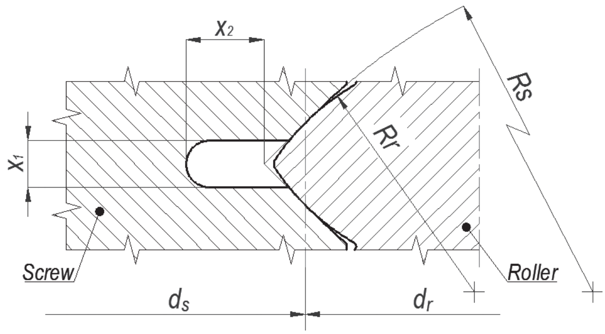

Fig. 3. Design variables and dimensions of the screw and the roller thread

Table 1

Design and state variables with restrictions and optimization starting points

Design variables	Sp1	Sp2	Sp3	Sp4	Restrictions
x_1	0.1	1	1	0.1	$0.1 < x_1 < 1$
x_2	0.5	3	3	0.1	$0.1 < x_2 < 3$
R_r	2	2	10	20	$2 < R_r < 20$
State variables					
R_s	$1.5 \cdot R_r$	$1.5 \cdot R_r$	$1.5 \cdot R_r$	$1.5 \cdot R_r$	–
$C_{\text{press}}^{\max\_u}$	–		–		$C_{\text{press}}^{\max\_u} < 0.6 C_{\text{press}}^{\max}$

As an objective function Huber-Mises-Hencky reduced stress in the notch of the screw's thread was accepted. The aim of the optimization was to determine the minimum of the function (eq. 5).

$$Q(x_1, x_2, R_r) = \sigma_{\mathrm{HMH}}^{\max\_u} \tag{5}$$

$$Q \to \min \tag{6}$$

where:

Q	–	objective function,
$\sigma_{\mathrm{HMH}}^{\max\_u}$	–	Huber-Mises-Hencky reduced stress in the thread undercut,
x_1, x_2, R_r	–	design variables.

4.3. Results

Table 2 summarized the results obtained for the following starting points. Fig. 4 and Fig. 5 present the values of the maximum Huber-Mises-Hencky reduced stress in the thread's undercut and the maximum contact pressure on the thread obtained from optimization. The results were compared with two reference geometry without any undercut in the thread notch. First (**Ref 1**) with a radius of $R_r = 2$ mm, and the second (**Ref 2**) with $R_r = 20$ mm.

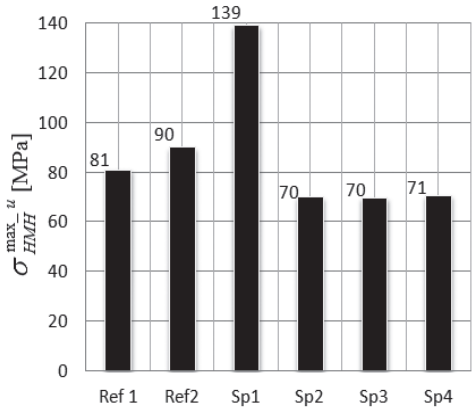

Fig. 4. Maximum Huber-Mises-Hencky reduced stress in the thread undercut

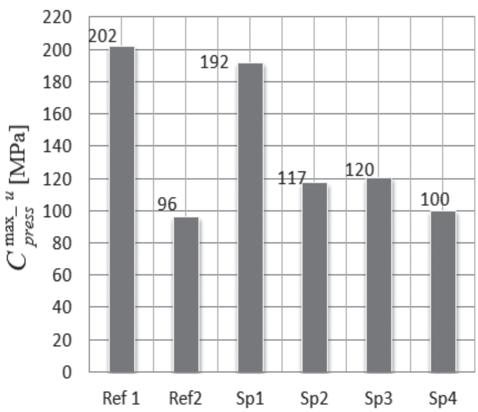

Fig. 5. Maximum contact pressure on thread

Table 2

Design and state variables for the best optimization results

Design variables	Sp1	Sp2	Sp3	Sp4
x_1	0.11	0.83	0.85	0.78
x_1	0.18	0.1	0.1	0.1
R_r	2.00	12.77	11.59	19.91
State variables				
R_s	3.00	19.16	17.39	29.87
$C_{\mathrm{press}}^{\max\_u}$	191.9	117.1	120.4	99.87
$\sigma_{\mathrm{press}}^{\max\_u}$	139.2	69.9	69.7	70.58

For the best optimization result (**SP4**), in reference to **Ref 1**, the decrease of the stress concentration in the thread notch was about 12 % and the decrease of the contact pressure was about 50%whereas in comparison with t**Ref 2**, about 21% decrease of the stress concentration in the thread notch and a slight increase (about 4%) of the contact pressure was obtained.

Fig. 6 and Fig. 7 present Huber-Mises-Hencky reduced stress and contact pressure on the thread for the reference geometry (Ref 2), whereas Fig. 8 and Fig. 9 for the best result of the optimization (Sp4).

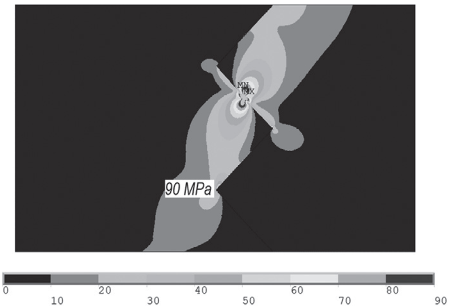

Fig. 6. Huber-Mises-Hencky reduced stress for the reference geometry (Ref 2) ($\sigma_{\text{press}}^{\max}$ = 90 MPa)

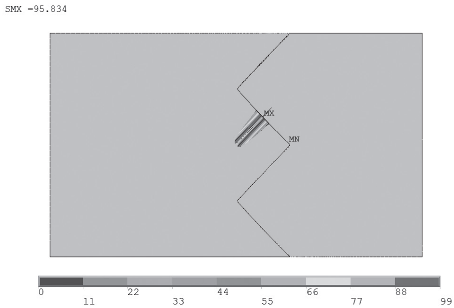

Fig. 7. Contact pressure on the thread for the reference geometry (Ref 2) ($C_{\text{press}}^{\max}$ = 96 MPa)

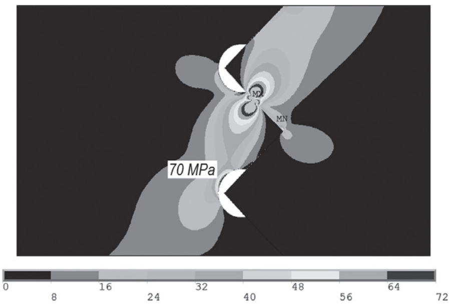

Fig. 8. Huber-Mises-Hencky reduced stress for the best result of the optimization (Sp4) ($\sigma_{\text{press}}^{\max\_u}$ = 70 MPa)

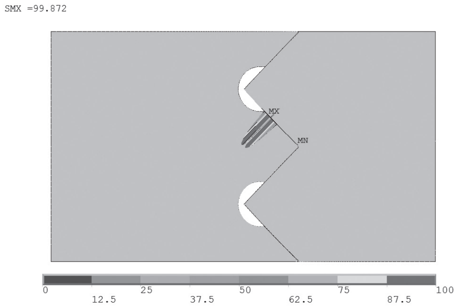

Fig. 9. Contact pressure on the thread for he best optimization result (Sp4) ($C_{\text{press}}^{\max\_u}$ = 100 MPa)

5. Conclusions

The result obtained from the finite element analysis with the optimization procedure showed that the use of the undercut in the thread notch has a positive effect on the reduction of stress concentration. Concerning an effect on a single cooperating pair of threads, it can be concluded that it is recommended to use the values of the optimization parameters based on the proportion referred to the thread pitch as follows:

$$x_1/p \approx 0.4; \quad x_2/p \approx 0.05; \quad R_r/\text{p} \approx 10$$

However, it should be taken into account, that too wide an undercut can undermine the contact zone. At the same time, the results obtained for the reference models (without undercut), revealed that the increase in the radii of the roller and the screw thread's flank curvature results in a significant reduction in the contact pressure. However, at the same time it slightly increases stress concentration in the thread notch.

Such an undercut could be also advantageous due to lubrication. It can act as a place for lubricate accumulation. From the manufacturing point of view, it is advantageous to make undercuts in the thread notches of the rollers.

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