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MICHAŁ RADZIK*, TADEUSZ J. SOBCZYK**

ANALYSIS OF A SYNCHRONOUS MACHINE DRIVEN BY AN INTERNAL-COMBUSTION ENGINE

ANALIZA PRACY MASZYNY SYNCHRONICZNEJ NAPĘDZANEJ PRZEZ SILNIK SPALINOWY

A b s t r a c t

The steady state response of a synchronous machine to the torque with angle dependent pulsating component is of great practical importance for the piston drive. Determining such a response is not easy due to the necessity of solving the system of nonlinear differential equations. This paper describes an algorithm that allows directly determining the steady states of a synchronous machine driven by an internal combustion engine (e.g. diesel engine). To create such an algorithm, the harmonic balance method and the iterative Newton–Raphson procedure are used. This approach allows obtaining steady-state solutions directly in the frequency domain. Exemplary calculations are performed for synchronous generators derived from the four-stroke internal combustion engine.

Keywords: *synchronous machines*, *steady-state performance*, *harmonic balance method*

S t r e s z c z e n i e

Reakcja maszyny synchronicznej na moment napędowy zależny od kąta obrotu wirnika ma duże znaczenie praktyczne dla napędów tłokowych. Określenie rozwiązania ustalonego nie jest łatwe ze względu na konieczność rozwiązywania układu nieliniowych równań różniczkowych. W niniejszym artykule opisano algorytm pozwalający na bezpośrednie wyznaczanie stanu ustalonego w maszynie synchronicznej napędzanej przez silnik spalinowy. Algorytm ten opiera się na metodzie bilansu harmonicznych oraz wykorzystuje procedurę iteracyjną Newtona–Raphsona. Pozwala to na uzyskanie rozwiązania ustalonego bezpośrednio w dziedzinie częstotliwości w postaci szeregu Fouriera. Przykładowe obliczenia przeprowadzono dla prądnicy pracującej synchronicznie na sieć sztywną i napędzanej momentem mechanicznym pochodzącym od spalinowego silnika czterosuwowego.

Słowa kluczowe: *maszyny synchroniczne*, *stany ustalone*, *metoda bilansu harmonicznych*

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* Ph.D. Eng. Michał Radzik, Technical Institute, State Higher Vocational School in Nowy Sacz.

** Prof. D.Sc. Ph.D. Eng. Tadeusz J. Sobczyk, Institute on Electromechanical Energy Conversion, Cracow University of Technology.

1. Introduction

This paper considers the performance of synchronous generators driven by internal combustion engines, mainly by diesel engines. These are commonly used as small or medium power electric sources. The characteristic features of such a drive is the pulsating torque of the driven engine due to the pulsating nature of the forces acting on the pistons in the cylinders, these are caused by the changeable pressure of a medium in cylinders. This causes the angle depended torque, which generates the speed ripples, even if the generator is running synchronously [2]. The second independent phenomenon causing the speed ripples is a variation of the total moment of inertia of the whole piston drive with the angle of rotation [3]. The resulting fluctuations of the torque acting on the synchronous generator shaft are a function of the shaft's rotational angle. It generates oscillations in the angular velocity of the synchronous generator. In turn, it has a direct impact on the oscillations of the power angle ϑ of the synchronous generator. Also, the winding's currents reflect on those oscillations by new components. In practice, in order to counteract this, the amplitude of the alternating component in torque is reduced to a minimum, most often through multiplication of the number of cylinders and selection of an appropriate tact of work for each cylinder [4].

Determining the steady-state of a synchronous generator when the oscillation of the angular speed has an impact on the winding currents requires solving the generator electrical equations together with the equation of rotary motion. However, such a set of differential equations is nonlinear. The most popular method is a numerical integration of that equation set. As a result, the steady-state solution in the time domain could be obtained. To find useful measures of currents and torque, i.e. mainly their Fourier spectra, additional signal processing is necessary, this can be complicated because steady-state performances are quasi-periodic in such cases. The steady-state in a considered case can be directly determined in the frequency domain using the harmonic balance method for nonlinear systems presented in [5]. This allows finding a steady-state for the synchronous generator directly in the frequency domain, when the period of the alternating component of mechanical torque is known. This algorithm, after some modifications, can also be applied to determine a steady-state when the external mechanical torque is a periodic function of the rotation angle.

This paper presents the steady-state analysis of a salient-pole synchronous generator running synchronously and driven by a four-stroke internal-combustion engine [2]. The basic model of synchronous machines was used assuming linearity of the magnetic circuit when the stator windings are supplied by the balanced voltage source and the field winding is supplied by a DC voltage source. The steady-state equations combining both electrical and mechanical equations were formulated by the harmonic balance method and the Newton–Raphson algorithm was used to solve them. The results are presented in the form of the Fourier spectra of all generator currents as well as the power angle.

2. Formulation of harmonic balance equations for a synchronous machine

The harmonic balance method [6] operates within the complex Fourier series. Therefore, the synchronous machine equations are presented at a specially chosen rotating coordinate system (0,+,–) [6]. For those coordinates, the three phase voltages and stator currents are represented by complex time functions and in steady-states they can be described by the complex Fourier series. Resulting equations in (0,+,-) coordinates take the forms: the electrical equations (1a) and the mechanical equation (1b), in which notations follow [6]:

$$\begin{bmatrix} u^+ \\ u^- \\ U'_f \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_s i^+ \\ R_s i^- \\ R'_f i'_f \\ R'_D i'_D \\ R'_Q i'_Q \end{bmatrix} + \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \psi^+ \\ \psi^- \\ \psi'_f \\ \psi'_D \\ \psi'_Q \end{bmatrix} + \mathrm{j}\Omega_s \begin{bmatrix} \psi^+ \\ -\psi^- \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{1a}$$

$$J\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} + D\frac{\mathrm{d}\varphi}{\mathrm{d}t} = T_{em} + T_m(\varphi);\ T_{em} = \mathrm{j}p\ (i^-\psi^+ - i^+\psi^-) \tag{1b}$$

The balanced stator voltages are represented in these coordinates by constant values:

$$u^+ = u^- = \sqrt{\tfrac{3}{2}}U \tag{2}$$

The list of unknown functions for equations (1a, b) is: $i^+(t)$; $i^-(t)$; $i'_f(t)$; $i'_D(t)$; $i'_Q(t)$; rotor rotation angle $\varphi(t)$. The harmonic balance method allows qualitatively and quantitatively predicting periodic or quasi-periodic solutions in steady-states. The rotation angle $\varphi(t)$ is not periodic in time and must be replaced by some periodic function. At a synchronous speed, the angle $\varphi(t)$ can be substituted by its fluctuations $\Delta\varphi(t)$ defined by the formula:

$$\varphi(t) = \left(\Omega_s / p\right)\cdot t + \Delta\varphi(t) \tag{3}$$

where:
Ω_s – pulsation of supplied voltages,
p – pole-pair number.

The mechanical equation (1b) takes new form:

$$J\frac{\mathrm{d}^2\Delta\varphi}{\mathrm{d}t^2} + D\frac{\mathrm{d}\Delta\varphi}{\mathrm{d}t} = T_{em} + T_m(t) - D\ \left(\Omega_s / p\right) \tag{4}$$

Combining (1a) and (4) into one set, the resulting vector equation takes the form:

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\mathbf{F}_2(\mathbf{x}) + \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{F}_1(\mathbf{x}) + \mathbf{F}_0(\mathbf{x},t) = 0 \tag{5}$$

in which the vector of unknown functions **x** has the form:

$$\mathbf{x} = \begin{bmatrix} i^+ & i^- & i'_\mathrm{f} & i'_\mathrm{D} & i'_\mathrm{Q} & \Delta\varphi \end{bmatrix}^T \tag{6}$$

And $\mathbf{F}_n(\mathbf{x})$ are vectors of functions with respect to $\mathbf{x}$.

Due to periodicity of the mechanical torque generated by the combustion engine and keeping in mind the assumption that the generator runs synchronously, all variables in vector $\mathbf{x}$ will be periodic too with the period of the forced torque and consequently the functions in vectors $\mathbf{F}_1(\mathbf{x})$, $\mathbf{F}_2(\mathbf{x})$ and $\mathbf{F}_0(\mathbf{x}, t)$ are also periodic. To use the harmonic balance method for the equation (5), all of them should be presented in the form of the complex Fourier series (7) and (8) with the period $T_x = 2\pi/\Omega_x = 1/f_x$, i.e. the period of the oscillating component of the driven torque:

$$\mathbf{x} = \sum_{k=-\infty}^{\infty} \mathbf{X}_k e^{\mathrm{j}k\Omega_x t} = \sum_{k=-\infty}^{\infty} \begin{bmatrix} I_k^+ \\ I_k^- \\ I'_{f,k} \\ I'_{D,k} \\ I'_{Q,k} \\ \varphi_k \end{bmatrix} \cdot e^{\mathrm{j}k\Omega_x t} \tag{7}$$

$$\mathbf{F}_n = \sum_{k=\infty}^{\infty} \mathbf{F}_{n,k} e^{\mathrm{j}k\Omega_x t} = \sum_{k=\infty}^{\infty} \begin{bmatrix} F_{n,k}^1 \\ F_{n,k}^2 \\ F_{n,k}^3 \\ F_{n,k}^4 \\ F_{n,k}^5 \\ F_{n,k}^6 \end{bmatrix} \cdot e^{\mathrm{j}k\Omega_x t}; \quad n \in \{0,1,2\} \tag{8}$$

Substituting the series (7) and (8) into equation (5) and comparing coefficients for the same basic functions on both sides, a set of non-linear algebraic (9) is obtained. It has an infinite number of equations and an infinite number of variables, which are the Fourier coefficients of the series (7).

$$-\mathrm{diag}\begin{bmatrix} \vdots \\ \Omega^2 \\ 0 \\ \Omega^2 \\ \vdots \end{bmatrix} \cdot \begin{bmatrix} \vdots \\ \mathrm{F}_{2,1} \\ \mathrm{F}_{2,0} \\ \mathrm{F}_{2,-1} \\ \vdots \end{bmatrix} + \mathrm{diag}\begin{bmatrix} \vdots \\ \mathrm{j}\Omega \\ 0 \\ -\mathrm{j}\Omega \\ \vdots \end{bmatrix} \cdot \begin{bmatrix} \vdots \\ \mathrm{F}_{1,1} \\ \mathrm{F}_{1,0} \\ \mathrm{F}_{1,-1} \\ \vdots \end{bmatrix} + \begin{bmatrix} \vdots \\ \mathrm{F}_{0,1} \\ \mathrm{F}_{0,0} \\ \mathrm{F}_{0,-1} \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix} \tag{9}$$

The matrix $\mathbf{\Omega} = 2\pi f_x \mathbf{E}$ is created by the unity matrix $\mathbf{E}$ with dimensions [6 × 6]. Below, simplified notation is used by introducing the so-called vector and matrix representations of the complex Fourier series [6]. As a result, equation (9) takes the form:

$$-(\mathbf{\Omega})^2 \cdot \mathbf{F}_2 + \mathrm{j} \cdot \mathbf{\Omega} \cdot \mathbf{F}_1 + \mathbf{F}_0 = 0 \tag{9a}$$

3. Iterative algorithm of finding steady-state solutions for a synchronous machine

The equation (9a) constitutes an infinite set of non-linear algebraic equations with the infinite number of variables. It can only be solved numerically by using an iterative procedure when the number of equations and number of variables are reduced to finite one, i.e. reducing the number of considered harmonics in the Fourier series (7) and (8) to sufficiently high. In this paper, the Newton–Raphson iterative procedure was used given by the general formula:

$$\mathbf{x}^{i+1} = \mathbf{x}^{i} - \mathbf{J}(\mathbf{x}^{i})^{-1} \cdot \mathbf{F}(\mathbf{x}^{i}) \tag{10}$$

The matrix $\mathbf{J}(\mathbf{x})$ and the vector $\mathbf{F}(\mathbf{x})$ are the respective matrix and vector representation and they are determined by the formulas below:

$$\mathbf{F}(\mathbf{x}) = -(\boldsymbol{\Omega})^2 \cdot \mathbf{F}_2(\mathbf{x}) + \mathrm{j} \cdot \boldsymbol{\Omega} \cdot \mathbf{F}_1(\mathbf{x}) + \mathbf{F}_0(\mathbf{x}) \tag{11}$$

$$\begin{aligned} \mathbf{J}(\mathbf{x}) &= \frac{\partial \mathbf{F}(\mathbf{x})}{\partial \mathbf{x}} = -(\boldsymbol{\Omega})^2 \cdot \frac{\partial \mathbf{F}_2(\mathbf{x})}{\partial \mathbf{x}} + \mathrm{j} \cdot \boldsymbol{\Omega} \cdot \frac{\partial \mathbf{F}_1(\mathbf{x})}{\partial \mathbf{x}} + \frac{\partial \mathbf{F}_0(\mathbf{x})}{\partial \mathbf{x}} = \\ &= -(\boldsymbol{\Omega})^2 \cdot \mathbf{F}_{d,2}(\mathbf{x}) + \mathrm{j} \cdot \boldsymbol{\Omega} \cdot \mathbf{F}_{d,1}(\mathbf{x}) + \mathbf{F}_{d,0}(\mathbf{x}) \end{aligned} \tag{12}$$

The matrices $\mathbf{F}_{d,k}(\mathbf{x})$ (for $k = 1, 2, 3$) are calculated based on the functions in the vectors $\mathbf{F}_1(\mathbf{x})$, $\mathbf{F}_2(\mathbf{x})$ and $\mathbf{F}_0(\mathbf{x}, t)$.

$$\mathbf{F}_{d,n}(\mathbf{x}) = \frac{\partial \mathbf{F}_n(\mathbf{x})}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial f_{n,1}}{\partial i^+} & \frac{\partial f_{n,1}}{\partial i^-} & \frac{\partial f_{n,1}}{\partial i_f^{'}} & \frac{\partial f_{n,1}}{\partial i_D^{'}} & \frac{\partial f_{n,1}}{\partial i_Q^{'}} & \frac{\partial f_{n,1}}{\partial \Delta\varphi} \\ \frac{\partial f_{n,2}}{\partial i^+} & \frac{\partial f_{n,2}}{\partial i^-} & \frac{\partial f_{n,2}}{\partial i_f^{'}} & \frac{\partial f_{n,2}}{\partial i_D^{'}} & \frac{\partial f_{n,2}}{\partial i_Q^{'}} & \frac{\partial f_{n,2}}{\partial i\Delta\varphi} \\ \frac{\partial f_{n,3}}{\partial i^+} & \frac{\partial f_{n,3}}{\partial i^-} & \frac{\partial f_{n,3}}{\partial i_f^{'}} & \frac{\partial f_{n,3}}{\partial i_D^{'}} & \frac{\partial f_{n,3}}{\partial i_Q^{'}} & \frac{\partial f_{n,3}}{\partial \Delta\varphi} \\ \frac{\partial f_{n,4}}{\partial i^+} & \frac{\partial f_{n,4}}{\partial i^-} & \frac{\partial f_{n,4}}{\partial i_f^{'}} & \frac{\partial f_{n,4}}{\partial i_D^{'}} & \frac{\partial f_{n,4}}{\partial i_Q^{'}} & \frac{\partial f_{n,4}}{\partial \Delta\varphi} \\ \frac{\partial f_{n,5}}{\partial i^+} & \frac{\partial f_{n,5}}{\partial i^-} & \frac{\partial f_{n,5}}{\partial i_f^{'}} & \frac{\partial f_{n,5}}{\partial i_D^{'}} & \frac{\partial f_{n,5}}{\partial i_Q^{'}} & \frac{\partial f_{n,5}}{\partial \Delta\varphi} \\ \frac{\partial f_{n,6}}{\partial i^+} & \frac{\partial f_{n,6}}{\partial i^-} & \frac{\partial f_{n,6}}{\partial i_f^{'}} & \frac{\partial f_{n,6}}{\partial i_D^{'}} & \frac{\partial f_{n,6}}{\partial i_Q^{'}} & \frac{\partial f_{n,6}}{\partial \Delta\varphi} \end{bmatrix}; \quad n \in \{0,1,2\} \tag{13}$$

Details of calculating the individual components of expressions (11), (12) and (13) after each iteration are described in [7]. The problem of selecting the starting point for the iterative algorithm is also discussed in that paper.

4. The results of numerical calculations

The iterative algorithm was implemented in MATLAB. Calculations were provided for the synchronous machine with rated data $P_N = 1250$ kW, $U_N = 6$ kV, $\cos\varphi_N = 0.9$, $n_N = 750$ rpm, $J_s = 250$ kg · m^2. It was assumed that $I'_f = 0.95 I'_{fN}$ and that the mean value of the torque was equal to the rated torque. The spectra of the phase stator current and load angle were selected as the most representative measures of the generator properties.

Numerical tests were done to study the influence of the moment of inertia and the number of combustion engine pistons on the performance of the synchronous generator. As a reference, the case when a torque performance versus rotary angle is as in Fig. 1, i.e. represents the torque of a four-stroke internal combustion engine [2].

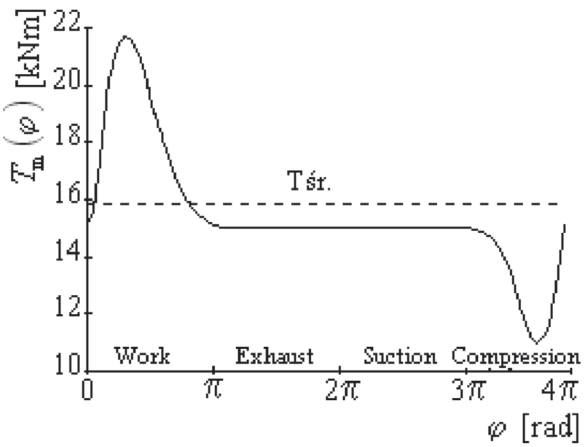

Fig. 1. The torque of the four-stroke internal combustion engine [2]

Figures 2 and 3 present the influence of the moment of inertia on the spectra of the stator phase current and the load angle. The amplitudes of the individual components are done in decibels due to significant differences in their values. The spectrum of the phase stator current $i_a(t)$ given by the formula:

$$i_a(t) = \sum_{k=-N}^{N} I_k \cdot \cos\left[\left(\Omega_s + k \cdot (\Omega_s / 2p)\right)t + \alpha_k\right] \tag{14}$$

is shown in Fig. 2. The component I_0 with network frequency $f_0 = 50$ Hz dominates in this spectrum. Frequencies of other harmonics take the values according to relationships $f_s = f_s + k \cdot (f_s/2p)$, in which p is the generator pole-pair number. From Fig. 2, it follows that if the moment of inertia grows twice, the spectrum of the stator current changes significantly and only the component with network frequency f_s do not change. The first additional components I_{-1} and I_1 are almost 20 dB smaller to the main harmonic I_0 for basic case and are reducing by 10 dB when the moment of inertia is growing twice. For sequent components I_{-2} and I_2, the same tendencies can be observed and they achieve the levels of 2% and 1% of I_0, respectively.

Generally, the growing moment of inertia twice additional components in the stator currents are reducing more than twice and only the basic harmonic I_0 does not change. So, fluctuations of the stator current are reduced, which could be expected.

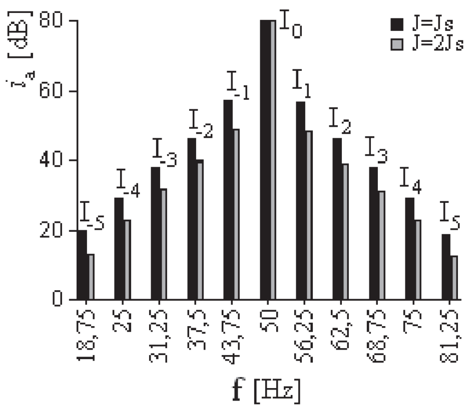

Fig. 2. Spectrum of phase stator current for two different moments of inertia

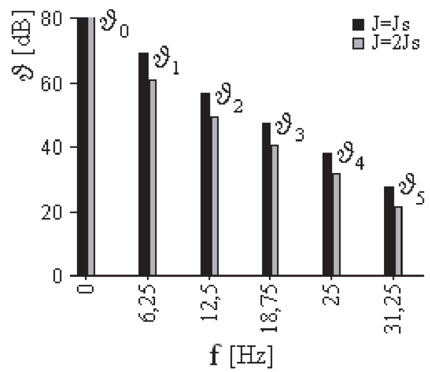

Fig. 3. Spectrum of the power angle for two different values of the moment of inertia

The spectrum of the power angle in Fig. 3, which is done by the formula:

$$\vartheta(t) = \vartheta_0 + \sum_{k=1}^{N} \vartheta_k \cdot \cos\left[\left(k \cdot \left(\Omega_s / 2p\right)\right)t + \beta_k\right] \tag{15}$$

contains components with frequencies $f_k = k \cdot (f_s/2p)$. The component ϑ_0 represents the mean value of the power angle and the amplitudes of sequent components decrease but not as fast as in the stator current spectrum. When increasing the moment of inertia twice, the mean value of the power angle is not changed, whereas the amplitudes of the other harmonics are reduced by over 50%. This means that fluctuations of the angular velocity of the synchronous generator driven by the combustion engine can be reduced by increasing the total moment of inertia, which is in line with expectations.

Improving the uniformity of the mechanical torque of an internal combustion piston engine is commonly achieved by increasing the number of cylinders. For the confirmation of the validity of such a statement, the performance of the mechanical torque versus angle was changed as is shown in Fig. 4. It was found from the curve in Fig. 1 adding two curves respectively shifted, but keeping the mean value. The differences to the mean value decrease and change the Fourier spectrum of the torque. It allows showing the influence of a number of cylinders on the steady-state. Results of calculations are shown in Fig. 5 and Fig. 6.

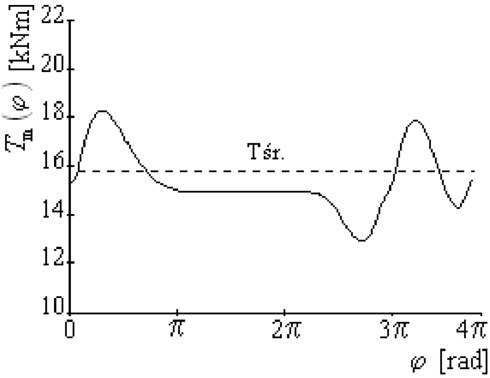

Fig. 4. Mechanical torque of the four-stroke internal combustion engine with double the number ○of the cylinders

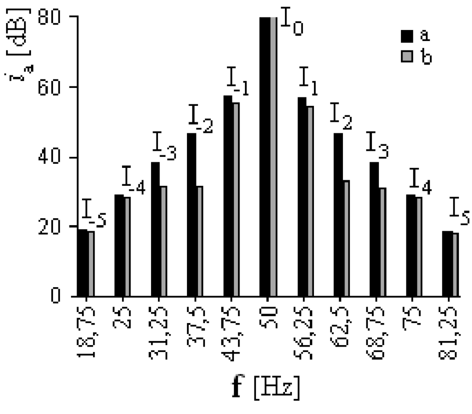

Fig. 5. Spectrum of phase stator currents for: a) four-stroke internal combustion engine, b) four-stroke internal combustion engine with doubled number of cylinders

The spectrum of the phase stator current in Fig. 5 confirms that all higher harmonics are more or less reduced. The most limited are components I_2 and I_{-2} with frequencies of 62.5 Hz and 37.5 Hz respectively, which dropped nearly five times. This means that the stator currents become really close to being sinusoidal.

The spectra of the power angle for those two cases are presented in Fig. 6. All harmonics are decreasing too when the number of cylinders is doubled, beside the mean value. The second harmonics and higher are reduced significantly – this means that the power angle fluctuations are practically mono-harmonic.

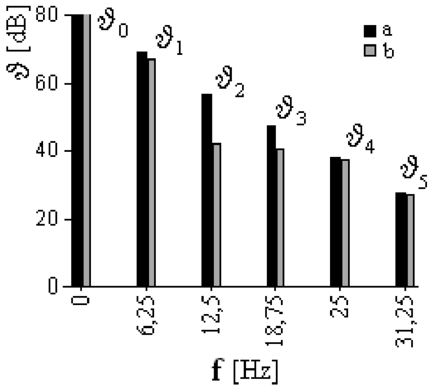

Fig. 6. Spectrum of the power angle for: a) four-stroke internal combustion engine, b) four-stroke internal combustion engine with double the number of cylinders

The developed algorithm also determines the spectra of the field current, of currents of equivalent damping windings and also of the angular velocity but these results are not presented in this paper.

5. Conclusions

The algorithm presented in this paper allows the direct steady-state analysis of synchronous generators driven by a piston engine, which is a source of angle dependent torque. Such an electromechanical system is described by a set of nonlinear differential equations and direct steady-state analysis is extremely complicated. In this paper, the steady-states equations have been created by the harmonic balance method. They have a form of an infinite set of algebraic equations with an infinite number of unknowns. To solve them, the Newton–Raphson iterative algorithm was developed for those equations, when limited to finite dimensions.

Numerical tests show that the presented algorithm is rather effective and allows directly determining the Fourier spectra of generator currents and mechanical variables. This ability of the algorithm can also be applied for determining the spectra in cases of internal faults in generators or faults in the mechanical part of a drive when the angle dependent distortions of the torque are generated. Detail properties of the Fourier spectra in faulty states can be useful for diagnostic purposes using motor current signature analysis.

References

[1] Laudyn D., Pawlik M., Strzelczyk F., *Elektrownie*, WNT, Warszawa 2000.

[2] Anuszczyk J., *Maszyny elektryczne w energetyce. Zagadnienia wybrane*, WNT, Warszawa 2005.

[3] Jagiełło A., *Systemy elektromechaniczne dla elektryków*, Wydawnictwo Politechniki Krakowskiej, Kraków 2008.
[4] Laible Th., *Die Theorie der Synchronmaschine im nichtstationären Betrieb*, Springer--Verlag, Berlin–Göttingen–Heidenberg 1952.
[5] Sobczyk T.J., Radzik M., *Algorytm bezpośredniego określania stanów ustalonych w maszynach synchronicznych z uwzględnieniem równania ruchu metodą bilansu harmonicznych*, Zeszyty Problemowe – Maszyny Elektryczne, 2009, nr 83, BOBRME Komel, pp. 83–88.
[6] Sobczyk T.J., *Metodyczne aspekty modelowania matematycznego maszyn indukcyjnych*, WNT, Warszawa 2004.
[7] Radzik M., *Algorytm bezpośredniego określania stanów ustalonych w maszynach synchronicznych z uwzględnieniem równania ruchu*, praca doktorska, Cracow University of Technology, Cracow 2011.
[8] Radzik M., Sobczyk T.J., *Analiza pracy maszyny synchronicznej napędzanej silnikiem tłokowym*, Zeszyty Problemowe – Maszyny Elektryczne, 2014, nr 103(3), Wyd. Instytut Maszyn i Napędów Elektrycznych (KOMEL), pp. 217–222 (SME 2014).