TECHNICAL TRANSACTIONS

CZASOPISMO TECHNICZNE

FUNDAMENTAL SCIENCES

NAUKI PODSTAWOWE

ISSUE 1-NP (7)

ZESZYT 1-NP (7)

YEAR 2014 (111)

ROK 2014 (111)

WYDAWNICTWO
POLITECHNIKI
KRAKOWSKIEJ

TECHNICAL TRANSACTIONS | CZASOPISMO TECHNICZNE

FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE

ISSUE 1-NP (7) | ZESZYT 1-NP (7)
YEAR 2014 (111) | ROK 2014 (111)

Pierwotną wersją każdego Czasopisma Technicznego jest wersja on-line
www.czasopismotechniczne.pl www.technicaltransactions.com

Fundamental Sciences Series
1-NP/2014

Foreword

Central-Eastern Europe has its own specific history. For a long time its vast territories were possessed by the great Ottoman, Austro-Hungarian, or Russian empires. In XIX century, however, the empires grew weaker and their military defeats, of which the ultimate was the I World War, allowed for a restoration of old national states and birth of new ones like Bulgaria, Romania, Hungary, Czechoslovakia, Poland, Lithuania, Latvia, Estonia, Russia proper. Ambitious but backward, these states strived to reach the level of more developed countries in Central and Western Europe, then leading cultural and scientific centres.

Exact sciences – mathematics, physics, mechanics, astronomy, chemistry – seem to be an area where those struggles for excellence manifested themselves particularly intensely. By their very nature, exact sciences are international in character, but for some time the Central-Eastern Europe was on the receiving side. It was apparent both in long stays of western scientists in Central-Eastern European territories (e.g., S. Lhuillier in Poland, L. Euler and Ch.M. Bartels in Russia, O. Volk in Lithuania) and, more efficiently, in sending young people to leading centres in Western Europe and allowing for development of their talents after return (e.g., professors of Royal University in Warsaw, M.V. Ostrogradski and V.J. Bunjakowski in Russia, W. Bolyai in Hungary). Another instance of this process were voyages of Czech mathematicians to new states of the Balkans (Croatia, Slovenia, Serbia, Bosnia, Herzegovina, Bulgaria), where they served as pioneers of mathematical life there. These examples pertain to mathematics but analogous ones can be offered for other exact sciences as well, including chemistry, where such exchanges were followed by development of domestic chemical industry, e.g., in Poland.

After decades of more or less passive assimilation of ideas from the West, the general situation became ripe enough to allow for the birth of local mathematical centres of worldwide distinction. The greatest significance was achieved by two of them: Moscow mathematical school, whose most distinct leader was N. Lusin, and Polish mathematical school with Sierpiński at its head. As a result, the Central-Eastern Europe achieved in XX century the equal footing with the world science.

The main motive for organizing a conference devoted to raising the level of scientific culture in backward countries of Central-Eastern Europe was a strong conviction of historical significance of those processes both on a local scale of national cultures and on a global scale of the world science. Perhaps some specific characteristics of that development may serve as a model to adapt in other conditions.

The present volume contains lectures and posters presented at the conference "The reception of exact sciences in Central-Eastern Europe in 1850–1920", which took place in September 2013 in Cracow, along with other papers related to the subject. The Conference met with substantial interest and hopefully will be continued in the future.

Roman Duda

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE
1-NP/2014

JUOZAS BANIONIS*

THE FAMOUS MATHEMATICIAN OF LITHUANIAN UNIVERSITY OTTO THEODOR VOLK (1892–1989)

SŁAWNY MATEMATYK UNIWERSYTETU LITEWSKIEGO OTTO THEODOR VOLK (1892–1989)

A b s t r a c t

The article introduces a German mathematician Otto Theodor Volk (1892–1989), who worked as a professor at Lithuanian University in 1922–1930, and sheds light on his merits in the science of mathematics in Lithuania.

Keywords: *Lithuanian University*, *mathematics*, *differential geometry*, *theory of functions*, *history of mathematics*, *philosophy of mathematics*

S t r e s z c z e n i e

W artykule przedstawiono niemieckiego matematyka Otto Theodora Volka (1892–1989), który był profesorem Uniwersytetu Litewskiego w latach 1922–1930. Zaprezentowano również jego zasługi dla rozwoju matematyki na Litwie.

Słowa kluczowe: *Uniwersytet Litewski*, *matematyka*, *geometria różniczkowa*, *teoria funkcji*, *historia matematyki*, *filozofia matematyki*

* Juozas Banionis, Lithuanian University of Educational Sciences.

1. Introduction

In the independent Republic of Lithuania, Otto Theodor Volk became the most prominent scholar in mathematics among professors of Lithuanian University.

On 16 February 1922, when Lithuanian University was established, the core of the Faculty of Mathematics and Natural Sciences of the university was created. The rest of the research staff of the faculty was to be formed on a competitive basis. Vacant positions at the faculty were announced in the daily newspaper *Lietuva* on 22 July 1922. The announcement was also addressed to Universities in Berlin, Munich, Königsberg, Vienna, Tartu, Riga, Zürich, and Helsinki. Among the candidates who applied for the position of a professor of differential equations and higher algebra there was an associate professor doctor O. T. Volk from Munich University[1].

2. Beginning of the story of O.T. Volk's life

On 13 July 1892, a fifth child was born to the family of the Volks who lived in a town of Neuhauzen on the Filder plateau south of Stuttgart, in the Baden-Württenberg Land[2]. The child was given a Christian name Otto and brought up following catholic traditions. O.T. Volk studied at gymnasiums in Rottenburg and Ehingen. After passing school--leaving examinations, he studied in Tübingen University, Munich Technical Higher School and Munich University. Besides mathematics, O.T. Volk took courses in astronomy, history and philosophy. He attended the lectures of such famous scientists as O. Perron, K.O.H. Liebman, C.L.F. von Lindemann, A. Voss, A. Pringsheim.

O.T. Volk finished studies in 1917 and passed examinations for teacher's qualification in Stuttgart. In 1919, under the supervision of a specialist in differential geometry K. Liebman, he wrote a research work *A Study of Potential Theory: The Problem of the boundary Values* and defended the work in Munich Technical Higher School acquiring Doctor's degree in Engineering. After working as teacher for a short time in Swabia, in 1919 O.T. Volk moved to Munich University to work as an assistant of C. Lindemann, who was a well--known specialist in number theory and algebraic geometry. A year later, O.T. Volk presented another scientific work *Expansion of Complex Functions of one Variable in terms of Elliptic Cylinder Functions*. He was awarded Doctor's degree in Philosophy (PhD) for this work. This scientific work enabled O.T. Volk as a young scientist to highlight the importance of computational methods.

O.T. Volk was awarded Doctor's degrees in engineering and philosophy for scientific works in the field of mathematics[3]. In subsequent research he concentrated on special functions and extension of the calculations of differentials by fundamental functions. On 4 March 1922, O.T. Volk completed the habilitation procedure and acquired the rank of Associate Professor in Munich University. O.T. Volk worked at Munich University until his invitation to Lithuania.

---

[1] J. Banionis, *Matematikos mokslo raida Lietuvoje 1920–1940*, Vilnius 1994, p. 27.

[2] W. Barthel, *Zum 85. Geburtstag von Otto Volk*, Würzburg,, 1977, p. 1.

[3] O.T. Volk, *Gesammelte Abhandlungen*, Würzburg 1990, p. 687.

3. Invitation to Kaunas

On 24 September 1922, the Board of the Faculty of Mathematics and Natural Sciences of Lithuanian University considered the applications of the candidates for the university's vacant teaching positions and approved O.T. Volk's application, inviting him to teach at Lithuanian University[4]. The newly established Lithuanian University had great expectations from O.T. Volk. These expectations were fulfilled, especially in the field of mathematics. What were the key factors which shaped the decision to invite O.T. Volk to Lithuanian University? First of all, O.T. Volk was a graduate of Munich University, which had deep mathematical traditions. Secondly, since the candidate would also have to teach at the Technical Faculty, Doctor's degree and education in engineering would strengthen O.T. Volk's position. Besides, O.T. Volk had published research works in a number of journals in Germany and some books on complex number theories. Positive references provided by famous professors A. Voss and C. Lindeman were also very important.

"Since the university is primarily the place for pure spiritual sciences and the place were sciences are cherished, our first duty is to promote and develop pure science for its own sake. The highest aim of science is the triumph of spirit and our main objective is to carry out research and push science forward"[5]. It was the credo of Lithuanian University, which was then taking only its first steps. The credo was outlined by a young 30-year-old professor who finished his work in Munich and started professorship at the Department of Mathematical Analysis in Kaunas on 1 April 1923. As the head of the department, O.T. Volk put a lot of effort in taking science of mathematics to a higher level. He also made contacts with scientists from Germany, Sweden and other countries. This endeavour was aimed at making Lithuanian University equal to other European universities.

4. Working at Lithuanian University

From the very first years of working at Lithuanian University, O.T. Volk energetically started his activity. At first, he offered courses in higher algebra, differential equations, and function theory. Since autumn 1925, he began running courses in analytical mechanics and number theory. In spring 1928, O.T. Volk introduced Fourier series, theory of complex functions and elliptic functions. At the beginning, these courses were offered in German. In three years the professor could already teach students in Lithuanian.

In the process of forming the Department of Mathematics and following the ideas of German universities, O. Volk initiated establishing of Mathematics Seminar and a mathematics library. O.T. Volk suggested purchasing the library of the Munich University professor Aurelius Voss, who was O.T. Volk's teacher, as the basis for the collection of the library. A. Voss's library contained around 2000 volumes and 4500 offprints and brochures on algebra, geometry, and mechanics. One could find here classical works of C.F. Gauss, P.G.L. Dirichlet, I L. Fuchs, J. Steiner, and 96 volumes of the famous Encyclopaedia of Mathematical

---

4 Faculty of Mathematics and Natural Sciences Board, minutes of meetings in 1922., LCVA (Central State Archive of Lithuania), F.631, Ap.1, B.118, L.179

5 O. Folkas, *Matematika ir pritaikomieji mokslai*, Kosmos, No. 4, 1924, p. 313.

Sciences[6]. The library also contained thoroughly arranged volumes of worldwide-known long-established journals "Mathematische Annalen" (109 volumes since 1869), "Journal für die reine und angewandte Mathematik" (Crelles Journal, 60 volumes since 1863), "Archiv der Mathematik" (since 1841), "Jahrbuch über die Fortschritte der Mathematik" (since 1871) and other issues. In 1924, the whole library was purchased for a modest sum of money, i.e., $ 2000 (approximately 20 000 Litas), and moved to Kaunas. A year later, O.T. Volk helped to enrich the library with books bought from a German scientist Carl Neumann[7].

Newly established Seminar of Mathematics had a great role in development of research in the field of mathematics. O.T. Volk was elected the head of Seminar of Mathematics on 24 January 1925. It was the place for mathematicians to get acquainted regularly with the original research carried out in Kaunas, as well as to explore the heart of scientific works of Europe and other parts of the world.

Seeing the shortage of Lithuanian textbooks for higher schools and understanding the importance of studies in the native language, O. T. Volk found the ways to overcome the drawback. Thus he became the author of the first mathematics textbook for higher schools which was published in "Spindulys" publishing house in 1929. As the Board of Mathematics pointed out, in the textbook "Lectures on Theory of Ordinary and Partial Differential Equations" "(…) everywhere, attention is paid to geometric interpolation of integral curves, most importantly in the parametric form"[8]. The statement acknowledged that the textbook conformed to the modern science of mathematics. Since the textbook was essential for students, it was decided to publish 1000 copies at the faculty's expense. The textbook was illustrated with examples and mathematical problems from different areas of mathematics. It also included a long list of supplementary literature.

O.T. Volk's doctorate student Petras Katilius (1903–1995) helped the professor to carry out the gigantic work of writing the textbook. O.T. Volk and P. Katilius translated O. Volk's German textbook. Lithuanian mathematics terminology was adapted by O. Volk's friend, an Honorary Doctor of mathematics at Lithuanian University, the prelate Aleksandras Jakštas-Dambrauskas (1860–1938). The originality of the work was confirmed by the fact that the textbook was quoted in the famous E. I. H. Kamke's textbook *Differential Equations*. Students printed two more books, i.e. *High Algebra* (1925) and *Analytical Mechanics* (1929) taking O. Volk's lecture notes as the main material for these books. O.T. Volk mentioned one more textbook in Lithuanian related to Kaunas period – *Theory of Functions*. However, this textbook or even its manuscript cannot be found in the collections of Lithuanian libraries.

It is necessary to emphasize that these textbooks were prepared and published in the Lithuanian language. Material in the textbooks broadened, consolidated and complemented knowledge of higher mathematics in Lithuania. The textbooks also were

[6] Faculty of Mathematics and Natural Sciences letter on naming A.Fosui the mathematics honorary professor, LCVA (Central State Archive of Lithuania), F.631, Ap.1, B.98, L.59.

[7] Faculty of Mathematics and Natural Sciences commission letter, 20/11/1924, LCVA (Central State Archive of Lithuania), F.631, Ap.1, B.138, L.171.

[8] Decision of Faculty of Mathematics and Natural Sciences Board, 22/11/1927, LCVA (Central State Archive of Lithuania), F.631, Ap.1, B.137, L.74.

the basis for creating and standardizing Lithuanian terminology of mathematics. Thus, all the textbooks contributed to the foundation of science of mathematics in Lithuania.

The works carried out by O.T. Volk embraced a wide spectrum of areas of mathematics. A. Voss fostered O.T. Volk's interest in differential geometry. O.T. Volk's research of the nets of non-isometric isogonal curves was significant, too. He also wrote about K. Liebman remarks concerning J.G. Darboux's equation when he worked on surfaces with the nets of rhombic, triangular, circular and other curves.

Another important part of O.T. Volk's scientific interests was related to theory of functions. Works dedicated to that aspect focused on the expansion of analytic functions in series of Hermite and. Laguerre functions, generalisation of the conformal image to a complex algebraic function in two variables, Lame function series and C.L.F. Lindemann result. O.T. Volk's works on boundary value problems in potential theory should be mentioned, too. O.T. Volk published his works in journals of Bavarian Science Academy, Heidelberg Science Academy and Lithuanian University.

Since O.T. Volk was familiar with pure and applied mathematics theory very well and thoroughly used references, his merits in the history of mathematics are evident. His articles published in the third decade of the 20th century in philosophy and natural sciences journals "Logos" and "Kosmos" were very significant, too.

In 1924 O.T. Volk published an article about B. Pascal as a mathematician and physicist, highlighting his merits not only in "abstract mathematics" but in creating a calculating machine. In O.T. Volk's considerations about J. Kepler's "Mysterium cosmographicum" one can find that the author philosophically described J. Kepler's work as a phenomenon of astronomy, theology, physics and mathematics.

In 1925 O.T. Volk published an article *Kant and Mathematics*, in which he formulated some problems. In the article he made some comments concerning the deceased Munich astronomer H. von Seeliger and a well-known mathematician F. Ch. Klein. Merits of the latter are mentioned in the following areas: theories of algebraic equations and elliptic functions, reform of teaching of higher mathematics, publishing of "Encyclopaedia of Mathematical Sciences". O.T. Volk also discussed the questions of mathematical physics which were studied by a Russian mathematician V. Steklov.

To commemorate 200th anniversary of I. Newton's death, O.T. Volk published a comprehensive article in which he reviewed works of I. Newton's predecessors and also discussed correlation of A. Einstein's relativity theory with I. Newton's mechanics. Another article was dedicated to commemoration of C. Lindemann's 75th jubilee. C. Lindemann was O.T. Volk's teacher and a scholar who studied Ludolph's number π. The great mathematician was presented not only as a researcher of the problem of the quadrature of the circle but as a personality, too.

In 1927, after the death of two mathematicians, i.e. "the father of applied mathematics" C.D.T. Runge and a Swedish mathematician, the founder of "Acta Mathematica" M.G. Mittag-Leffler, O.T. Volk published two more articles in the "series of commemoration". In the article he mentioned the Swedish mathematician's, who was a specialist in differential equations, positive attitude towards Lithuanian University. It is worth mentioning, that M.G. Mittag-Leffler sold to Lithuanian University 26 issues of the journal, whose editor he was, at the lowest possible price.

Prof. O.T. Volk was also interested in philosophy. O.T. Volk's philosophical ideas are reflected in three original articles, two of which we mention here. In the first article *Mathematics and Applied Sciences* O.T. Volk tried to define the place of mathematics among other natural sciences supporting his own considerations by ideas of great thinkers.

The problem of mathematics and faith is analyzed in the article *Mathematics and Worldview*. O.T. Volk had deep understanding of theological problems. In 1915 he was ordained a catholic priest. O.T. Volk admitted that "faith, i.e. transcendental thesis of mind, without which the whole science is dead and totally irrelevant, does not begin only with God, liberty and immortality"[9] . In the other article *On Mathematical Cognition* O.T. Volk discussed evolution of science and its significance[10].

O.T. Volk's articles on history of mathematics familiarised Lithuanian readers with world-famous personalities. He was acquainted with a number of mathematicians whom he mentioned in his articles and exchanged correspondence with some of them. That is why these memoir-like articles are so ingenious.

O.T. Volk knew Latin, Greek, and French. It explains why his articles are full of quotations from original classical works of such famous scientists as C.F. Gauss, C. Jacobi, G. Galilei, K.Th.W. Weierstrass and others. O.T. Volk's works will never lose their value since they are deep and broad.

The professor was faithful to theoretical mathematics. In the dispute about foundations of mathematics he declared himself an advocate of Hilbert. Advocates of Hilbert's view on mathematics neglected "intuitive mathematics". They did not take into consideration the meaning of the content and gave priority to mathematical formulae. Würzburg University professor W. Barthel characterized O. T. Volk as a pure analyst. W. Barthel claimed that "Clear and independent analysis rather than exploration of possibilities were means with which O. T. Volk proved theorems in geometry"[11].

5. O.T. Volk's legacy to Lithuanian mathematics

At the beginning of his career at Lithuanian University, O.T. Volk proclaimed credo of his teaching: to deliver lectures "not only for benefit, i.e., their possible application in practice", but to try to develop in students the "spirit of pure erudition", to prepare them "for science for its own sake". Deliberately, the very first diploma works of mathematics students were supervised by O.T. Volk. 31 topics for diploma works from favourite areas of mathematics – theories of differential equations, special functions and functions of complex variable – were assigned to students by O.T. Volk during 7 years of his work at Lithuanian University in Kaunas. The professor supervised future famous mathematicians P. Katilius, M. Gotleras, O.Stanaitis, A.J. Gliksonas R. Lakovskis[12].

For 6 years in turn O.T. Volk represented Lithuanian science of mathematics at international scientific events in Innsbruck, Munich, Münster, Düsseldorf, Königsberg

[9] O. Folkis, *Matematika ir pasaulėžiūra*, *op. cit*., p. 67.
[10] O. Folkis, *Apie matematišką pažinimą*, Logos, 1925, No. 2, p. 85-114.
[11] Barthel W., *op. cit.*, p. 3.
[12] J. Banionis, *op. cit.*, p. 37.

and Boulogne where he initiated, maintained and strengthened relations between Lithuanian and European mathematicians. As he pointed out, these relations showed "great affinity towards Lithuanians"[13].

In 1929, after the death of Würzburg University professor E. Hilb, O.T. Volk was invited to work at this university (being the disciple of E. Hilb). The invitation was accepted and on 20 January 1930 O.T. Volk, the only professor of mathematics at Lithuanian University, handed in the resignation starting 1 of May 1930. The life and activity of O.T. Volk (until 1988) was centered at Würzburg University, where he worked as a professor of mathematics and astronomy.

Before leaving Kaunas, O.T. Volk had prepared his successors for the Faculty of Mathematics and Natural Sciences of Lithuanian University (later renamed Vytautas Magnus University). Three of O.T. Volk's former students – Petras Katilius, Paulius Slavėnas (1901–1991) and Otonas Stanaitis (1905–1988) – were admitted to doctoral studies at Heidelberg, Yale and Würzburg Universities with the mediation of the professor. Afterwards, these mathematicians became associate professors at the Faculty of Mathematics and Natural Sciences of Vytautas Magnus University.

O.T. Volk's contemporaries and students characterised him as a strict, honourable and dignified man who constantly inspired others to search for the truth. In 1931, senior students of Lithuanian University sent the telegram to O. T. Volk in which they wrote: "For the rest of our life you will stay in the hearts of the Lithuanian youth".

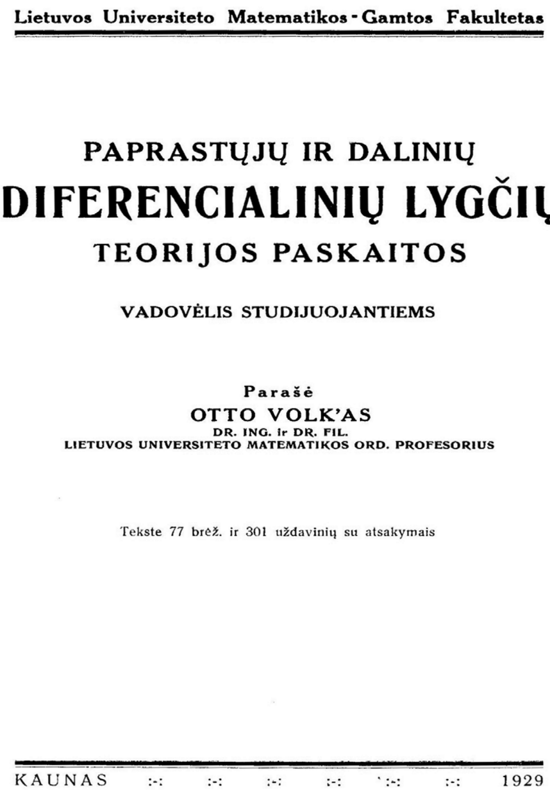
We have to admit that Prof. W. Barthel was right when he said that O.T. Volk's activity "(…) in the university of the temporary capital of Lithuania carried personal risk and tremendous commitment". In spite of that, O. T. Volk himself once claimed that "The years spent in Kaunas are among the best years of my life"[14]. Today, when we overview the development of science of mathematics in Lithuania and observe the achievements in mathematics, we are proud that Professor O.T. Volk was at the origins of our science, significantly contributing to the development of science of mathematics and culture in Lithuania.

Translated into English by Linas Selmistraitis

Fig. 1. Otto Theodor VOLK (1892–1989)

[13] O. Volk's report, Innsbruck Congress of mathematicians, 1924. LCVA (Central State Archive of Lithuania), F.631, Ap.1, B. 137, L. 52.

[14] Barthel W., Zum 85. Geburtstag von Otto Volk, Würzburg, 1977, p. 3.



Lietuvos Universiteto Matematikos - Gamtos Fakultetas

PAPRASTŲJŲ IR DALINIŲ

DIFERENCIALINIŲ LYGČIŲ

TEORIJOS PASKAITOS

VADOVĖLIS STUDIJUOJANTIEMS

Parašė

OTTO VOLK'AS

DR. ING. ir DR. FIL.

LIETUVOS UNIVERSITETO MATEMATIKOS ORD. PROFESORIUS

Tekste 77 brėž. ir 301 uždavinių su atsakymais

KAUNAS :-: :-: :-: :-: :-: :-: 1929

Fig. 2. O. Volk "*Lectures on Theory of Simple and Partial Differential Equations*" – the first mathematics textbook for higher schools in Lithuanian

References

[1] J. Banionis, *Matematikos mokslo raida Lietuvoje 1920–1940*, Vilnius 1994.
[2] W. Barthel, *Zum 85. Geburtstag von Otto Volk*, Würzburg 1977.
[3] O Volk, *Gesammelte Abhandlungen*, Würzburg 1990.
[4] O. Folkas, *Matematika ir pritaikomieji mokslai*, Kosmos, No. 4, 1924, 309-313.
[5] O. Folkis, *Matematika ir pasaulėžiūra*, Logos, No. 1, 1924, 64-67.
[6] O. Folkis, *Apie matematišką pažinimą*, Logos, No. 2, 1925, 85-114.
[7] Central State Archive of Lithuania.

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE
1-NP/2014

KALINA BARTNICKA*

ACCESS TO HIGHER EDUCATION AND STUDY FOR POLES IN THE SECOND HALF OF 19TH CENTURY

DOSTĘP POLAKÓW DO WYKSZTAŁCENIA I NAUKI W DRUGIEJ POŁOWIE XIX W.

A b s t r a c t

In the 19th century, in Poland divided among Russia, Austria and Prussia, the occupants hindered access to education for Poles. Fighting the restrictions, the Poles organized scientific institutions, published texts enabling self-study and founded high and academic private schools. In the Polish Kingdom in 19th century these schools were mostly clandestine, becoming legal at the beginning of 20th century. In Galicia, polonized high schools and universities in Lwów and Kraków educated students, including women, from all occupied territories. Many studied abroad. People educated in the second half of 19th century rebuilt the system of higher education in Poland Reborn.

Keywords: *partitions of Poland, Germanization*, *Russification*, *universities, secondary schools*, *underground education*, *women's education*, *"Flying University"*, *The Manual for a Self-Learner*, *Society of Science Courses*

S t r e s z c z e n i e

W XIX w., w Polsce podzielonej między Rosję, Austrię i Prusy, zaborcy utrudniali Polakom dostęp do wykształcenia. Walcząc z ograniczeniami Polacy organizowali instytucje naukowe, wydawali publikacje ułatwiające samokształcenie, zakładali średnie i wyższe szkoły prywatne. W Królestwie Polskim w XIX w. przeważnie tajne, od początku XX w. legalne. W Galicji spolonizowane szkoły średnie i uniwersytety we Lwowie i Krakowie kształciły studentów, w tym kobiety, ze wszystkich zaborów. Wiele osób studiowało za granicą. Ludzie wykształceni w drugiej połowie XIX w. odbudowali szkolnictwo wyższe w Polsce Odrodzonej

Słowa kluczowe: *rozbiory Polski*, *germanizacja, rusyfikacja, uniwersytety*, *szkoły średnie*, *tajne nauczanie*, *edukacja kobiet*, *„Uniwersytet Latający"*, *Poradnik dla samouków*, *Towarzystwo Kursów Naukowych*

* Kalina Bartnicka, Institute of History of Science, Polish Academy of Sciences, Warsaw.

Throughout the whole 19th century, from the collapse of Commonwealth of Two Nations (the political union of the Grand Duchy of Lithuania and the Kingdom of Poland) in 1795 until 1918, Poles had no longer a state of their own that would support the development of teaching and science. The 19th century saw the Polish schooling system in each partition gradually integrated into the occupying states' systems. The foreign rulers had no interest in developing the Polish scientific life and Polish schools. Such a policy prevailed in all three partitions but was the most hurtful in the Russian partition.

The mid-19th century saw a deterioration of the Polish schooling system and scientific life on the former territory of Poland now partitioned among three occupying countries. This resulted directly from a deliberate political decision: the foreign invaders' repressions following the collapse of the Polish uprising against Russia in 1831. Polish people's access to studies and ability to participate in the European scientific life, also with regard to exchange of ideas in pure sciences, depended on their access to universities and scientific thought abroad. A young, talented and scientifically ambitious young person had to meet the following criteria:

1. Receive education at secondary level confirmed by a secondary school (*gimnazjum*) certificate (*matura*) because *matura* was the only certificate giving access to university studies at home and abroad.
2. Start studies at home, at a school available to secondary school graduates or leave for abroad to start or complete university studies.
3. Possess or obtain financial assets enabling them to pursue their interests and scientific research after finishing university with a Master's or Doctor's degree.
4. Be able to live off scientific work or possess funds that would keep the research going.
5. Establish contact with domestic and foreign communities that stimulate scientific work.

At that time there were only two universities operating in the whole former territory of Poland, in Galicja (the Austrian partition): one in Lwów (Lemberg), developed according to Austrian patterns, and the Jagiellonian University of Krakow, which was practically Germanized in1850. These two establishments did not teach in Polish, nor did they educate the Polish youth from the other partitions. In the Russian partition the universities were dissolved, and in the Prussian partition there were no universities whatsoever. In all three partitions the only schools granting their alumni the right to study at a university were men's gimnazjums. In the Prussian and Austrian partitions they became Germanized, while in the Russian partition their number was dramatically reduced, and their curriculum given a strictly philological character.

For more than thirty years few young men completing secondary school in the so-called Kingdom of Poland were legally eligible to embark on university education. In a country bereft of universities in those days even those few had nowhere to go to study. The foreign rulers forbade teachers or students to travel to a university abroad, even to the University of Krakow! A legal departure required obtaining a permit from the tsar himself. The youth already studying abroad were ordered to return home. Those who declined this order lost a possibility to work in public administration. "An unlawful use of a passport" while abroad was punishable[1].

[1] J. Skowronek, *Nauka i nauczanie w okresie międzypowstaniowym* (*Science and Teaching Between the November and January Uprisings*), [in:] *Dzieje Uniwersytetu Warszawskiego 1807–1915*

Few young people had an opportunity to study at Russian universities that educated the teachers for secondary schools in the Kingdom of Poland[2]. This offered little contact with Europe's scientific thought anyway as it was a period of distrust nursed by the Russian authorities towards the very institution of university, as well as towards any associations of the Russian professorial community with its Western European counterparts that had grown since the 1820s.

The 1840s saw Tsar Nicholas I impose a stricter supervision over universities, professors, students, and the contents of the academic programs. Access to university studies had been limited (quantitative limits, raised tuition fees), a ban on trips abroad was issued, and censorship of imported books (including scientific literature) exacerbated. The rising repressiveness stemmed from distrust towards universities and Europe's scientific thought. Suspicion and the tendency to isolate the Russian youth from a foreign influence reached their climax in the period between the Spring of Nations and the mid-1850s[3]. As it stood, the difficulties to obtain a passport endured by young people in the Russian partition shut the door to university studies in Western Europe, limited the flow of ideas, and made it impossible to access the European scientific thought.

Some sixty years later, in 1918 Poland regained its independence. The newly reborn country had to put in place a new schooling system, universities, and academic and non-academic scientific centres. It needed professorial staff, scientific infrastructure, and young people keen and eligible to take up university studies (the requirement of the *matura* certification). The time was extremely unfavourable due to the social unrest, political instability, economic, social and cultural consequences of the partitions, as well as the enormous material, moral and demographic ravage inflicted by World War One. And yet, in these circumstances, it took just two years for a new and quite efficient system of higher education to be set up and running. In 1920 as many as ten schools of higher education were operating in Poland: five universities (in Lwów, Kraków, Poznań, Warszawa, and Wilno), two technical universities (in Lwów and Warsaw), the Mining Academy in Kraków, the Veterinary Academy in Lwów, and Warsaw University of Life Sciences. One need add to this group also private schools that would acquire their academic status soon: the Catholic University of Lublin and the Free Polish University (*Wolna Wszechnica Polska*) in Warsaw. Also, a network of non-academic schools of higher education was created. The departmental positions were filled with care and usually aptly in didactic and scientific respects. In July

(*The History of the Warsaw University 1807–1915*), ed. by Stefan Kieniewicz, PWN, Warszawa 1981, p. 207.

[2] R. Wroczyński, *Dzieje oświaty polskiej* (*The History of the Polish education*), vol. II, 1795–1945, (ed. II) „Żak", Warszawa 1996, p.100-107; see also Karol Poznański, *Oświata i szkolnictwo w Królestwie Polskim* (*The Education and Schooling System of the Kingdom of Poland*) *1831–1869*, vol. 1–2, Warszawa 2001.

[3] K. Bartnicka, *„Jaki powinien być uniwersytet rosyjski?" – Sprawy uniwersyteckie w świetle ankiety Ministerstwa Oświecenia Narodowego w 1849 r.* ("*What Should a Russian University Be Like?*" *– University business in light of the National Education Ministry's survey n 1849*), [in:] *Rozprawy z dziejów oświaty* (*Essays on History of Education*), ed. by Józef Miąso, vol. XXXVII, Warszawa 1996, p. 91-117.

1920 the *Bill on Academic Schools* was passed in parliament[4]. The positions of lecturers and heads of departments were taken up by people educated in the second half of the 19th and at the turn of the 20th centuries.

Such a swift restoration of the Polish system of higher education was a cause of pride to many people at the time. “Poland commenced its statehood with a step that proved its nation’s high culture: by creating universities” – said in 1920 Juliusz Makarewicz, lawyer, professor, and later also Lwów University’s Chancellor[5]. A dozen or so years later, Adam Wrzosek, a Professor of Medicine at the Jagiellonian University, and Head of Science and Higher Education Department at the Ministry of Religious Affairs and Public Education (MWRiOP), wrote: “In the history of our higher education so far, there hasn’t been a finer time than the first year after Poland regained its independence”[6]. Also today, the pace at which the higher education system was created, the “European” scientific level of the professorial staff, and the scientific achievements of the interwar period rightly deserve admiration.

One may ask where the professorial staff of these new universities came from. What was Polish men and women’s access to higher education in the three partitions in the period prior to Poland regaining its independence? How did the scientific associations come into being on the former Polish territories under partitions that would entertain an academic level enabling them to receive new scientific ideas and conceive them? As far as I know, such questions have not been yet raised in the literature dealing with problems of education of young Polish scholars. Relation rather than analysis has been the subject of study. Aside from universities’ own monographs, usually on the peripheries of some synthetic historical studies of science or education, the object of interest lay in the socio-economic problems concerning access to higher education or the political engagement of students[7]. At the level of secondary education, the object of scientific attention were not the pedagogical or scientific aspects of teaching, but above all the problems of national subjugation, and later the patriotic character of different forms of underground teaching. My remarks are meant to outline a problem in a longer time perspective and on a national scale, which has yet to be researched.

x

Since the time of the Enlightenment a conviction has been prevalent among Polish people that the development of education and culture is the way to rise from a political

---

[4] *Ustawa z dnia 13 lipca 1920 roku o szkołach akademickich* (*Bill on Academic Schools of 13 July 1920*), Official Journal of the Republic of Poland, 1920, N° 72, item 494, publ. by Dorota Zamojska, *Akademicy i urzędnicy. Kształtowanie ustroju państwowych szkół wyższych w Polsce 1915–1920* (*Academics and clerks. The shaping of the system of state higher schools in Poland 1915–1920*), Warszawa 2009, p. 248-267.

[5] J. Makarewicz, *Społeczna rola najwyższych uczelni* (*The social role of higher education institutions*), Lwów 1920, p. 3; cited by: D. Zamojska, *op. cit*., p. 15.

[6] A. Wrzosek, *Szkolnictwo wyższe. Uniwersytety* (*Higher education. Universities*), [in:] *Dziesięciolecie Polski Odrodzonej 1918–1928* (*The decade of a Reborn Poland 1918–1928*), Warszawa 1933, ed. II, p. 554, cited by D. Zamojska, *op. cit*., p. 16.

[7] D. Zamojska, *op. cit*., p. 9.

collapse[8]. This conviction led to the implementation of a comprehensive school system reform in Poland in the 1770s by the National Educational Commission (KEN), which put the Commonwealth ahead of its neighbouring countries. The reform involved the universities (*Akademia*) in Krakow and Wilno, and integrated them into the schooling system. The schools became homogenised and modernised in terms of organisation, management, curricula, methods of teaching, and teacher training. The level and scope of education at secondary school were separated from those at university. Admission into university required a completion of a secondary school, i.e. departmental or sub-departmental (wydziałowe i podwydzialowe)[9] or passing an exam that verified one's eligibility to study at a university. The universities, now called "main schools", had their study offer expanded, received a modern organisational structure and were supplemented by a number of assisting research institutes[10].

The educational needs in society led to a growing interest in schools of all levels, and their growth ensued despite the collapse of the state (1795) until the November Uprising in 1831. Gradually, however, the time for teaching mathematical and natural sciences was reduced and the number of hours for the teaching of languages was rising[11]. However, until the November Uprising 1830–1831, the Polish people had a say in the teaching and schooling policies, particularly in the territory annexed by Russia[12], where they undertook

---

8 Similar views in response to Prussia's debacle in the Napoleonic Wars led at the beginning of the 19th century to Prussia having its schools structure put in order by Wilhelm von Humboldt and creating the Berlin University in 1810.

9 They varied in the number of teachers and classes, and lesson organization: as far as teaching curriculum was concerned, there were no major differences. Separate teachers were put in place to teach mathematics and physics, with the latter also teaching natural history. Mathematics was intended to deliver practical knowledge, useful in everyday life, management and measurements, and also train the student in "thorough and accurate thinking". The object of physics is to explore and understand "(...) the natural causes and effects". See Grzegorz Piramowicz, *Uwagi o nowym instrukcji publicznej układzie /.../* (*Reflections on the National Educational Commission's new public school curriculum /.../)*, Warszawa 1776, publ. by Stanisław Tync, *Komisja Edukacji Narodowej (Pisma Komisji i o Komisji). Wybór źródeł* (*Writings on the National Educational Commission – an Anthology*), Wrocław 1954, p.161-183; Tadeusz Mizia, *Szkoły średnie Komisji Edukacji Narodowej na terenie Korony* (*The National Educational Commission high schools on the Crown's territory*), Warszawa 1975, p. 159-170.

10 The new university structure introduced division into two colleges in place of the traditional faculties: humanities, social studies and theology constituted the so-called Moral College, whereas mathematics, physics, natural history, and medicine constituted the Physics College; see *Ustawy Kommissyi Edukacyi Narodowej dla stanu akademickiego i na szkoły w krajach Rzeczypospolitej przepisane* (*The National Educational Commission's Acts on the Commonwealth's academic class and on schools*) (1783), publ. by S. Tync, *op. cit*., p. 589-593: Chapter 2 "Main Schools", points 14-16.

11 By contrast to KEN curricula, the proportions in subjects taught were distorted with mathematical and natural subjects suffering cuts. See: A. Winiarz, *Szkolnictwo Księstwa Warszawskiego i Królestwa Polskiego (1807–1831)*, Lublin 2002, p. 231: "The total weekly teaching time in classes 1–6 was 192 hours, of which 102 involved (the teaching of) languages". Latin and literature alone took up 44 hours a week.

12 From 1807 the schooling system in Polish central territories including the Duchy of Warsaw, and from 1815 the Kingdom of Poland united by personal union with Russia, was governed by

to retain the encyclopaedic curriculum in secondary schools, which were very well organised[13].

Pure science and natural studies were available in the Russian partition at the universities in Wilno and Warszawa, and in the Austrian partition in Kraków and Lwów. Since the late 18th century the Lwów University had been administered like other Austrian universities and was totally dependent on the Austrian regional (gubernium) and central (Vienna) authorities. The departments' structure was traditional, and the teaching was performed in Latin and German. Neither this university nor the Germanised *gimnazjums* in Galicja underwent KEN's school system and educational reforms, as eastern Galicja had been severed from Poland in the first round of partitions.

The Austrian authorities imposed their methods on the Krakow University, temporarily Germanising it between 1805–1809. Pure and natural sciences were parcelled out to different departments. In the first three decades of the 19th century the university was going through a very hard time[14]. The political and organisational changes were not helpful in stabilising the academic staff and raising its scientific level. The university was not playing an important role at that time, however some outstanding scholars did grace its premises, such as the Austrian astronomer Joseph Johann Littrow or mineralogist and botanist Balthazar Hacquet. 1809 saw the resumption of lectures in Polish. First Hugo Kołłątaj,

---

Polish educational authorities: the Educational Chamber (Izba Edukacyjna), and in the years 1816–1831 the Government's Commission for Religious Faiths and Public Education (Komisja Rządowa Wyznań Religijnych i Oświecenia Publicznego). The lands annexed directly by Russia saw the creation of the Vilnius Scientific District (Okręg Naukowy Wileński) with schools from the 8 western governorates (*gubernia)* answering to the Vilnius University.

[13] In the Kingdom of Poland in 1830 there were 15 provincial secondary education schools (szkoły wojewódzkie) with some 5800 students and a few more were set to be established. Following the 1831 defeat of the Uprising and the successive reorganisations, in 1855 the Kingdom of Poland was home to government-licensed seven-level philological gimnazjums with 1504 students. In the late 1860s the gimnazjums accommodated only 1882 students which was several times fewer than that in 1830. See Adam Winiarz, *Szkolnictwo Księstwa Warszawskiego i Królestwa Polskiego (1807–1831)* (*The Schooling System of the Duchy of Warsaw and the Kingdom of Poland 1807–1831*), Lublin 2002, p. 185-264.

In the Vilnius Scientific District, in the years 1803/4–1832, the Vilnius University administered the total of 102 secondary education schools. Admission into the university was granted on completion of gimnazjums (each *gubernia's* leading schools) as well as county schools (szkoły powiatowe) which made up the majority of schools in the Vilnius Scientific District. About 53 schools were closed down in the years 1831–1835 or soon after. All schools were Russified. 13 gubernia-level gimnazjums were retained while the levels of county-level schools was lowered, therefore hindering their graduates' access to universities. See Leszek Zasztowt, *Kresy 1832–1834. Szkolnictwo na ziemiach litewskich i ruskich dawnej Rzeczypospolitej*, (*The Kresy. 1832–1834. The Schooling System on the Lithuanian and Ruthenian Lands of the Former Commonwealth*), Warszawa 1997, p. 376-380, Anex III.

[14] Between 1795–1809 Kraków was in the Austrian partition. In the years 1809–1815 it was incorporated into the Duchy of Warsaw. The Congress of Vienna created a diminutive state of the Free City of Kraków, also known as the Republic of Kraków. On its liquidation in 1846, Kraków was definitively annexed by Austria.

and later Stanisław Staszic undertook attempts to modernise the university[15]. Several talented Polish professors were appointed heads of departments: the mathematician Karol Hube, the chemist Józef Markowski, the physicist Roman Markiewicz, and the astronomer Józef Łęski. In the latter part of the 1820s, the distinguished mineralogist Ludwik Zejszner was made the head of his department. After the collapse of the November Uprising, this was the only Polish university until the mid-19th century. After 1849 the University of Krakow was Germanised for a dozen or so years.

At the newly opened in 1816 Imperial University of Warsaw consisting of five departments, mathematical studies along with the Philosophy Section and the Natural Sciences Section became part of the Philosophical Sciences Faculty. The departments of advanced algebra, experimental and applied philosophy, chemistry, mineralogy, and botany were developing successfully. The Mathematics Section soon established its position and “acquired a good reputation, whereas its professors enjoyed respect and recognition”[16].

The Imperial University of Wilnius, which had achieved a high scientific level prior to the November Uprising, comprised four faculties/divisions of science: Physical and Mathematical; Doctoral and Medical; Moral and Political; and Literature and Fine Arts. In 1803, the *Act of Confirmation* granted by Tsar Alexander I declared the university would provide “all sciences, superior skills, and free arts”[17]. According to the university’s *Statutes*, the Physical and Mathematical Faculty consisted of ten departments: physics, chemistry, natural history, botany, farming, pure advanced mathematics, applied advanced mathematics, astronomy, civil architecture, as well as the position of astronomer observer. The professors included distinguished scholars, such as the renowned mathematician Jan Śniadecki, and his no less renowned brother, the chemist and physician Jędrzej Śniadecki.

The Warsaw and Wilnius universities were developing quickly in scientific respect and attracted many young people. The number of students at the University prior to the November Uprising reached over one thousand (or including students of the Wilnius gimnazjum closely cooperating with the university-nearly 1800 students)[18]. In the academic year 1828/1829,

---

[15] Irena Danuta Sieniuc, *Szkoła Główna Krakowska w dobie Ks. Warszawskiego* (*The Kraków Main School in the era of the Duchy of Warsaw)*, „Rocznik Krakowski” („Kraków Yearbook”), vol. XXXIII, book 2, Kraków 1954, p. 77-78. The author states that the prospective teachers, lawyers and theologians at the Philosophy Faculty had to study courses in “lower mathematics, philosophy and physics, and medics had to additionally attend their own dedicated programme”.

[16] M. Wawrykowa, *Uniwersytet Warszawski w latach 1816-1831 (Warsaw University 1816–1831)*, [in:] *Dzieje Uniwersytetu Warszawskiego 1897–1915*, *op. cit*., p. 109 and the following.

[17] *Akt potwierdzenia Imperatorskiego Uniwersytetu w Wilnie* (*The Act of Confirmation of the Imperial Vilnius University*), of 4 April 1803, publ. by Michał Baliński, *Dawna Akademia Wileńska* (*The Ancient Vilnius Academy*), Petersburg 1862, p. 552-558, Annex XVIII; See *Ustawy czyli ogólne postanowienia Imperatorskiego Wileńskiego Uniwersytetu i szkół jego wydziału* (*Acts, that is general decisions made by the Imperial Vilnius University and its Schools*), of 18 May 1803 r., publ. by Michał Baliński, *Dawna Akademia*, *op.cit.*, p. 559-596, Annex XIX, in Russian and Polish.

[18] Following considerable fluctuations during the Napoleonic Wars, after 1815 the Wilno students community was quickly rising. See Daniel Beauvois, *Szkolnictwo polskie na ziemiach litewsko-ruskich 1803–1832* (Polish Schools on Lithuanian and Ruthenian lands 1803-1832). Volume I, *Uniwersytet Wileński* (Wilno University), Rome-–Lublin 1991, p. 273), [in:] 1830, the Wilno

the University of Warsaw had 964 students, 55 of whom were studying physics, and 42 mathematics at the Mathematic-Physical Department[19].

From its outset, Warsaw University had attracted young people not only from the Kingdom, but also from the territory of Lithuania, Ukraine, and Belarus. Incidentally, those from the eastern lands usually chose to study at the Law and Administration Department[20]. It may be that because pure science at the Wilno University had achieved a high level, young people from those lands would usually choose to study it in Wilno rather than Warsaw.

Warsaw of the latter half of the 1820s saw preparations for the opening of a technical university, Polytechnic, and professors were being educated abroad. In order to create equal opportunities at university entrance exams, the Preparatory School for the Polytechnic Institute was created. The successive classes of preparatory students it educated were becoming first-year students at the newly created Polytechnic. In 1829 "Almost all the departments were staffed, and it started operating as a centre of technical thought because entrepreneurs were calling on its professors with queries regarding the modernisation of their companies or products"[21].

The creation of new universities, the growing interest of young people in academic education, the creation of repositories and research institutes, the thorough education of the young academic staff (scholarships, foreign study trips), as well as the development of and maintenance of contacts with foreign scholars (inviting them as honorary members, correspondence, etc), was all interrupted and consequently suppressed by repressions inflicted by the Russian authorities following the collapse of the November Uprising. The universities in Wilnius and Warsaw were shut down, as were the newly created Polytechnic Institute and the Warsaw Society of Friends of Learning. The scientific repository was removed and taken away. The Kingdom was deprived of its educational autonomy: in 1839 the Warsaw Educational District was created, to be directly supervised by the Ministry of Education in Petersburg. The number of secondary schools was drastically reduced and a strictly philological curriculum was imposed on them, while the duration of learning gradually shortened. And all this transpired despite the significant increase of the population and the continually rising educational needs of the Polish society[22].

University had 1300 students in 1830, which made it Russia's biggest in terms of numbers, with the Petersburg University coming next with a little more than 800.

[19] D. Beauvois, *op. cit*, p. 327; in the years 1822/1823 and 1825/1826 the Mathematical and Physics Faculty was chosen by 305 and 316 students, respectively.

[20] M. Wawrykowa, *op. cit*., p. 158.

[21] A. Winiarz, *op. cit*., p. 404.

[22] Before the second partition (1793), the Commonwealth was home to 10 million people; in the mid-19th century the territory of a similar size accommodated some 16 million, and in 1910 more than 34 million. On regaining its independence, according to the 1921 census Poland was home to some 27.7 million people; see: Ireneusz Ihnatowicz, Antoni Mączak, Benedykt Zientara, *Społeczeństwo polskie od X do XX wieku* (*Polish Society from the 10th to the 20th century*), Warszawa 1979, p. 457-478; Andrzej Chwalba, *Historia Polski 1795-1918* (History of Poland), Kraków 2000, p. 24.

Although other schools and educational institutions provided a bigger scope and a higher level of education, they still did not offer sufficient preparation for university education[23]. However, the seven-grade *real* gimnazjum provided a higher scientific level and a broader scope of study, to a certain extent filling the void left by the lack of a technical university, training students for "industrial purposes". It provided its students with quite a large knowledge of scientific subjects, while the number of languages and hours dedicated to studying them was reduced; it taught students arithmetic, geometry, mineralogy, physics, chemistry, natural history, technology, machine construction, geography, and world history. It was extremely popular and the number of its students grew threefold from 349 to 1070 between 1840 and 1850.

Impeding, and in fact suppressing, Polish people's learning and scientific aspirations by the Russian authorities was characteristic of the whole period 1831–1915. Such a policy prevailed in all the three partitions but was the most hurtful in the Russian partition[24], which came to encompass more than 66% of the former Commonwealth's territory and 45% of the population. The degradation of Polish education was extremely afflicting and upsetting for the Polish society which understood the gravity of education and the cultural, social, economic consequences of educational repressions. Attempts to overcome them were undertaken since the mid-19th century: efforts were made in all partitions to restore the Polish educational system, especially in the Russian and Austrian partitions. The fight against constraints proceeded differently in each partition, and the beginning of the changes took place in the 1860s.

x

Following the ascent to the throne by Tsar Alexander II (1855), the anti-Polish course of Russian policy was mitigated, among other things, with regard to the educational policy in the Kingdom of Poland. In 1857, the Medical and Surgical Academy was opened in Warsaw. In 1861, the Kingdom's educational autonomy was restored (KRWRiOP). Count Aleksander Wielopolski was appointed as its head and soon undertook to draw up an educational reform. In May 1862, the tsar approved *The Bill on Public Education in the Kingdom of Poland*[25]. It restoring the Polish school's national character, and introducing a progressive school structure. Secondary schools were organised on two levels: 5-grade county schools (szkoły powiatowe) and 7-grade gimnazjums[26].

---

[23] For example, the Agronomic Institute restored in 1836 (later renamed as the Agricultural and Forestry Institute) in Warsaw's Marymont, the pharmaceutical, and veterinary and lower veterinary schools, and the so-called "additional courses" at Warsaw gimnazjum.

[24] The Russian partition encompassed the so-called Kingdom of Poland formally united with Russia by personal union and the so-called "annexed lands" incorporated directly into the Russian Empire.

[25] Karol Poznański, *Reforma szkolna w Królestwie Polskim w 1862 r.* (*School Reform in the Kingdom of Poland in 1862*), „Monografie z dziejów oświaty" ("Monographs on the History of Education"), ed. by Łukasz Kurdybacha, vol. X, Ossolineum, Wrocław 1968.

[26] The biggest influence on the concept of high school was exerted by Tytus Chałubiński (1820–1889), doctor of medicine and naturalist, professor at the Medical and Surgical Academy in Warsaw, and

The objective of the gimnazjums was set out in detail: it was to prepare young people "both for various walks of life and to [enable them] listen to refined lectures at a Main School"[27]. The gimnazjum's curriculum had balance restored in the teaching of various groups of subjects, with the mathematics-natural science block being strengthened, as 34% of the effective learning hours were dedicated to such subjects as: logic, arithmetic, geometry, measurements and algebra, plane and advanced trigonometry, descriptive geometry, general mechanics, analytical geometry, physics, mathematical-physical geography, chemistry, and natural history. The new curriculum afforded the students better preparation for university studies.

The Kingdom maintained two university-level institutions: the Main School in Warsaw with the departments: philology-historical, law and administration, medical, and mathematics--physical. Puławy was home to the multi-disciplinary Polytechnic and Agricultural--Forestry Institute. The didactic and scientific staff was swiftly completed for both schools and recruitment went on. The graduates of philological gimnazjums were admitted without examinations to all departments, as were the students of the Medical-Surgical Academy that had been converted into the Medical Department of Warsaw's Main School. Other candidates had to sit an entrance exam which was identical for all departments and addressed mainly the knowledge of humanities and social issues. The examinations showed poor results as the secondary education in the Kingdom was narrowly focused and its level was low[28].

The objective of the Historic-philological and Mathematic-Physical departments was to educate teachers but the intellectual quality of the academic staff surpassed the needs of the gimnazjum teachers and largely reflected the scientific ambitions of the School's creators[29]. In the autumn 1862, 721 students began their study, 120 of them at the Mathematical--Physical department. The needs of the industry were taken into account when this department was being created. Aside from mathematics, the curriculum included also mechanics, geodesy and technology, astronomy, mineralogy, and human anatomy and physiology[30].

---

later at the Warsaw Main School; and Józef Korzeniowski (1797–1863), a well-known writer with an extensive educational experience. They both advocated the importance of thorough general education based on combination of humanities and mathematical and natural sciences.

[27] *Ustawa o wychowaniu publicznym w Królestwie Polskim* (*The Act on Public Education in the Kingdom of Poland*), 1862, Section Two: *O zakładach naukowych średnich* (*On Secondary Scientific Facilities*), art. 56; publ. by Stefan Wołoszyn, *Źródła do dziejów wychowania i myśli pedagogicznej* (*Sources for History of Education and Pedagogical Thought*), vol. II, *Pedagogika i szkolnictwo w XIX stuleciu* (*Pedagogy and Schooling System in the 19th century*), ed. II, altered, „Strzelec", Kielce 1997, p. 482.

[28] Students who failed the exam and graduates of *real* gimnazjums' 6th grade (which did not grant them admission into university) could attend a preparatory class that would enable them to complement their education in Polish, Greek,, Latin, algebra, and geometry.

[29] K. Poznański, *op. cit*., p. 207-236: Chapter VII. *Realizacja Ustawy o wychowaniu publicznym w Królestwie Polskim na polu szkolnictwa wyższego* (*Implementation of the Act on public education in the Kingdom of Poland in the field of higher education*).

[30] S. Kieniewicz, *Akademia Medyko-Chirurgiczna i Szkoła Główna (1857–1869*) (*The Medical--Surgical Academy and Warsaw Main School 1857-1869*), [in:] *Dzieje Uniwersytetu Warszawskiego*, *op. cit*., p. 306.

After the exam, like at the Main School, the inauguration of lessons followed for 168 students of the Institute in Puławy.

Between 1862 and 1868, the Mathematical-Physical department of the Main School educated 826 students (28% of the total). The Mathematical section proved more attractive than the Natural Science section. 105 people graduated with a Master's degree, none with a Doctor's degree, eight habilitations were pursued: Władysław Kwietniewski, Władysław Zajączkowski and Aleksander Czajewicz in mathematics; Jan Kowalczyk in astronomy; Erazm Langer in chemistry; Edward A. Strasburger in botany; August Wrześniowski in zoology; and Jan Trejdosiewicz in geology. Some students obtained doctoral degrees abroad. Many mathematics students continued their engineering studies in Russia and made a career there[31].

The Mathematical-Physical department of the Main School was divided into the mathematical and natural sciences parts. The Department's professors included distinguished professors and graduates of the Dorpat University: biologist, one of the first university scholars of evolution, Benedykt Dybowski (1833–1930) and botanist Konstanty Górski (1802–1864), as well as the world-renowned discoverer in the field of artificial dyes, chemist Jakub Natanson (1832–1884). However, none of the seven mathematics professors belonged to such outstanding personalities.

The opening of the Main School resounded throughout the whole country as it was taking place in a tense political situation. The outbreak of an armed uprising with students joining in the fighting ranks threatened shutdown of the School by the tsarist authorities. The inauguration speeches and political commentaries emphasised the responsibility of the youth for the School's well-being[32].

Wielopolski's school reform revealed the weaknesses of secondary schools created between 1831 and 1863 and activated Polish people in the Russian partition to challenge the Russian authorities' repressive policies following the collapse of the January Uprising. Efforts to rebuild the educational system and pursue scientific work began to be associated with the concept of serving the nation. The conviction was set forth: "the guarantee for the nation's survival may only be achieved by sustaining its cultural identity and individuality, as well as by its presence in the European intellectual community"[33]. The Main School was developing successfully and its creators had vast plans for its further development. It operated briefly, however (1862–1869), as there was too little time, scientific resources and funding to implement these plans. The didactic staff never managed to stabilise. In mathematics and science there was not enough time for "research schools" to form and there were no conditions for independent research or exchange of scientific ideas[34]. However, after 30 years of no university in the Kingdom of Poland and the isolation from

---

[31] *Historia Nauki Polskiej* (*History of Polish Science*), ed. by Bogdan Suchodolski, vol. IV. *1853–1918*, edited by Zofia Skubała-Tokarska, Wrocław 1987, p. 381-385; this important but little known problem is discussed by Jerzy Róziewicz, *Polsko-rosyjskie powiązania naukowe (1725–1918)* (*Polish-Russian scientific links*), Wrocław 1984.

[32] In discussions questions were asked: "what can a country gain on losing its only scientific institution which was supposed to restore Poland's bygone intellectual glamour and finesse to its future generations", cit. by K. Poznański, *op. cit.*, p. 225.

[33] D. Zamojska, *op. cit.*, p. 16.

[34] S. Kieniewicz, *op. cit.*, p. 318-319.

European science, the Major School had opened the door to young men in the Kingdom of Poland to university studies.

In January 1863, an armed uprising broke out in the Kingdom of Poland, which was suppressed in the spring 1864. Again, severe political and economic repressions came to torment the people of the Kingdom of Poland. In 1872 schools organised according to laws that came into effect in Russia in 1871 and 1872, underwent complete Russification. There were 6 and later 7-grade "real gimnazjums". While in 1870/71 there were 21 gimnazjums (including two "*real*" gimnazjums) with 6836 students, in 1904/05 there were 23 classical gimnazjums with 10,301 students.[35] The number of governmental schools was insufficient therefore private schools were allowed to exist but they gave no *matura* certificates to their graduates. One way to evade the restrictions constraining the secondary schools system were commercial schools which included 7-grade schools with their curriculum similar to that of classical gimnazjums. They were not supervised by the educational authorities but by the Ministry of Finance, which was less restrictive towards the Polish population.

x

Girls' schools, private boarding schools and convent-run schools, while different in terms of the number of grades and the level of teaching, were all strictly supervised by the authorities. In 1827, the first government-licensed secondary school for girls was founded in Warsaw the so-called Institute for Governesses (Instytut Guwernantek), which was intended to educate girls to become home teachers or teachers for private girls' schools. After the November Uprising the Institute was Russified and converted into a 6-grade boarding school so called Instytut Aleksandryjski for Girls' Education in Puławy, with a curriculum similar to that in a girls' *gimnazjum*. The principle was to educate 200 female students including 100 on a scholarship. The Institute was run under the auspices of the Empress, so it was sometimes referred to as Instytut Maryjski, taking the name after her. In the 1860s it was transferred to Warsaw, where, as it seems, it was integrated in the network of governmental girls' gimnazjums.

In the 1860s, the Russian authorities began to develop government-licensed schools for girls in the belief that this would accelerate the Russification of the Polish society. As a result, many 6-, and later 7-grade general education girls' gimnazjums were created (with one optional eighth pedagogical grade), with Russian as the official language. The graduates obtained the licence to work as a private teacher. While in 1870/71 there were 9 *gimnazjums* in the Kingdom, in 1904/5 there were 14 gimnazjums with 6,200 students, of whom only 36.4% were Polish![36]. Their learning curricula were similar to those of boys' classical gimnazjums, except that classical languages were replaced by "female" works and

---

[35] M. Szymański, *Higiena i wychowanie fizyczne w szkolnictwie ogólnokształcącym Królestwie Polskim 1815–1915* (*Hygiene and physical education in the Kingdom of Poland's general education schools 1815–1915*), Wrocław 1979, p. 122. The curriculum was characterised by, according to K. Poznański, *Reforma szkolna*, *op. cit*., p. 317: "excess of classical subjects". For 9 years including the preliminary grade, "Latin and Greek took up 85 hours weekly, whereas mathematical and naturalist subjects only took 37".

[36] M. Szymański, *op. cit*., p. 123.

the so-called “talents”: dancing, singing, musical instruments, drawing. Given that the length of the learning time at girls’ schools was shorter than that at boys’ schools, female students completed a narrower curriculum.

Generally, private boarding schools were not allowed to provide education at the level of higher government-licensed gimnazjums. The lessons of the Polish language, literature and history, and the higher gimnazjums curricula were therefore taught illegally, often as secret courses.

x

In 1869, the Polytechnic and Agricultural Forestry Institute in Puławy was made the Russian Institute of Farming and Forestry without the technical profile, whereas the Main School in Warsaw was in 1869 converted into the 4-departmental Imperial Warsaw University, with Russian scientific and educational staff. The Polish professors were removed if they did not possess Russian scientific titles of Master or Doctor (degrees or titles from other universities were not recognised) or if they were unable to lecture in Russian[37].

Warsaw University was different from its Russian counterparts in the selection of departments of the particular faculties, especially the Historical-Philological and Legal Departments. The Mathematical-Physical Department was almost identical to those of Russian universities, comprising11 divisions: pure mathematics, analytical and practical mechanics, astronomy and geodesy, physics, experimental and analytical chemistry, mineralogy, geology and palaeontology, physical geography, botany, zoology, technical chemistry, and agronomical chemistry. The staffing was unstable (especially in the Historical-Philological department)[38] and many positions of heads of departments remained vacant.

Warsaw University was open to young people from the Kingdom of Poland who had completed a secondary school in the Warsaw Educational District. School graduates from other regions had to obtain permits from the district educational supervisor. The objective was to isolate the youth living in the Kingdom from the young people living on the “annexed” lands. Furthermore, in order to isolate young people living in Russia’s western governorates from the Polish influence in the Kingdom, the authorities recommended admitting the former into universities in the Russian interior[39].

---

[37] I. Ihnatowicz, *Uniwersytet Warszawski w latach 1869–1899* (*Warsaw University 1869–1899*), [in:] *Dzieje Uniwersytetu Warszawskiego*, *op. cit*., p. 419, 421.

[38] I. Ihnatowicz, *op. cit*., p. 424-438. While the Mathematical-Physics Faculty employed 10 professors and 5 *docents,* over the course of 30 years 54 people filled these positions. Poles constituted some 21% of this staff; with some even taking the positions of Dean (physicist Stanisław Przystański, mathematician Tytus Babczyński, geologist Karol Jurkiewicz). Apart from the Russians, (67%), the Faculty employed also Germans (11%).

[39] I. Ihnatowicz, *op. cit*., p. 440-441; a Ministry of Education circular letter of 11/23 July 1899 banned providing accommodation to high school graduates from the Wilno Educational District. These students were allowed to study in Moscow, Petersburg and Dorpat. In 1886 and 1887 decrees were issued regulating the limits for numbers of students of Jewish origin. The Warsaw educational authority set the limit at 10% but it was not strictly obeyed. Jews constituted 15%–20% of Warsaw’s students population.

The graduates of classical gimnazjums were theoretically supposed to be admitted without examinations. Others, including graduates of "real" gimnazjums, had to take exams in the "missing subjects", usually Greek and Latin. It is estimated that in the years 1870–1900 Warsaw University educated about 10.000 students including 6.500 Polish and several hundred Jewish and German ones[40].

The Imperial Warsaw University was not a hub of innovative scientific thought. Its contacts with scholars from foreign universities were sporadic, and its educational work with scientific objectives in mind was negligible. For thirty years, until 1900, only 21 master's degrees and 9 doctoral degrees were granted (doctors of medicine, exclusive). The Polish had no chance to pursue a scientific career in Warsaw so they sought scientific degrees elsewhere. But for many the studies in Warsaw were the beginning of their scientific work. If they returned to Warsaw, they had to organise their scientific work outside the government-licensed universities. The state patronage began to be overtaken by Polish social institutions.

At the end of the 19th century Warsaw saw at last the creation of a Russian school, namely the Polytechnic Institute. In 1898, after many years of efforts and calls by the Polish society for the establishment of a technical university in Łódź, Nicholas II approved of the opening of 4-department Russian Polytechnic Institute. Most of the didactic staff were Russian, but 90% of the students were Polish. In 1905, following the school strike and the resulting boycott of Russian schools in the Kingdom of Poland, the Institute emptied, it was partly closed by the authorities in the years 1905–1908, and in 1915 evacuated to Russia[41]. However, the Institute did at times employ Polish professors, such as Wiktor Biernacki, Tadeusz Tołwiński, Aleksander Wasiutyński, Józef Jerzy Boguski, Mieczysław Pożaryski, and Tadeusz Miłobędzki. These scholars, as well as the few Poles employed in some departments of the Imperial University, did not evacuate, but, as the temporary staff, joined the new Polish universities that were being created then in Warsaw. They also continued teaching in the framework of the Warsaw Society of Friends of Learning.

x

As early as the 1880s, the Kingdom of Poland saw the development of a vigorous movement of clandestine teaching and studying on its territory, in various forms and at various levels. For example, institutions at university level were created embracing larger

[40] The proportion of Polish students, calculated on the basis of their confession, was decreasing from 86% to 60.7%, the Russian (Orthodox) students' proportion was rising from 2% to 19%. Following the school strike of 1905 and the Polish boycott of Warsaw University, the Catholic youth constituted 5% of the total 1556 students, with a growing tendency to 14% of the total 2062 students in the year 1914/15; the Orthodox students' proportion was 17% and 74%, respectively; see I. Ihnatowicz, *op.cit*., p. 442; Halina Kiepurska, *Uniwersytet Warszawski w latach 1899–1915* (*Warsaw University 1899–1915*), [in:] *Dzieje Uniwersytetu*, *op. cit*., p. 552.

[41] See: J. Miąso, *Szkoła Przygotowawcza do Instytutu Politechnicznego i późniejsze starania o kształcenie inżynierów w Królestwie Polskim* (*Polytechnic Institute Preparatory School and subsequent efforts aimed at education of engineers in the Kingdom of Poland*), [in:] *150 lat wyższego szkolnictwa technicznego w Warszawie 1826–1976* (*150 years of higher technical education in Warsaw 1826–1976*), Warszawa 1979, p. 31-44.

groups of young people, other forms and institutions provided opportunities to compensate for the deficiencies of official secondary schools; yet other forms provided self-study opportunities, while research workshops supported financially individual researchers. An important role among the institutions operating independently of the government-licensed schools, and therefore in a separate educational system, was played by an underground Polish university created by Jadwiga Szczawińska-Dawidowa, operating between 1885–1905 in Warsaw, educating mainly women (but not exclusively), called "The Flying", or "Petticoat University" ("Latający" or "Babski Uniwersytet")[42].

"The Flying University" was maintained by its students' fees. It comprised three faculties: Social Sciences, Historical-Philological, and Mathematical and Natural Sciences. The whole course was intended to last five or six years. The students were taking tests and final exams, received student books and certificates of the subjects they had completed. The University" engaged Warsaw scholars who had no opportunity to be employed at a government-licensed university. The lectures and classes for a dozen or so strong groups were held at private flats of the students or professors. For reasons of secrecy, the flats were often changed. The numbers of students kept growing: from about 200 a year at the outset, to about one thousand at the end of the century, when the lectures were also attended by male--students of the Imperial University and the Polytechnic Institute, who were complementing their knowledge in the field of humanities and social sciences. The local police were bribed. When the Petersburg authorities found out about the university a few years later, they did not seem particularly disturbed. It may be that they considered it as a non-menacing institution that provided an outlet for young women's political activeness.

The Mathematics and Natural Sciences department held courses in the following subjects: mathematics (algebra, geometry, basics of trigonometry), physics, organic and inorganic chemistry, cosmography, mineralogy with geology, anatomy and physiology of plants, plant systematic, human anatomy and physiology, zoology, and hygiene. The lectures were held in a one-year or two-year cycle, usually for two hours a week. Among the lecturers one may find such names as: mathematicians Samuel Dickstein or a former professor of the Main School Władysław Kwietniewski, the physicist Jerzy Józef Boguski, the geographer Wacław Nałkowski, zoologist Józef Nusbaum-Hilarowicz, or bacteriologist Odo Bujwid. The chemistry courses were held at the Laboratory of the Industry and Agriculture Museum and at a workshop of the Leppert and Karpiński Paint Production Plant, whereas clandestine mineralogy classes at ... Warsaw University's Mineralogy Lab, whose curator was the distinguished petrographer and mineralogist Józef Morozewicz. The Mathematics and Natural Sciences Faculty seems to have been the least academic in nature. Mathematics and natural science subjects at girls' gimnazjums were taught in a very limited scope, therefore, at the Floating University they were addressed probably at the boys' gimnazjums educational level[43]. The school had its code

[42] J. Miąso, *Tajne nauczanie w Królestwie Polskim w XIX i XX wieku* (*Underground teaching in the Kingdom of Poland in the 19th and 20th centuries*), [in:] *Wybrane prace z historii wychowania XIX–XX w.* (*Selected works on history of education 19th–20th centuries*), „Żak", Warszawa 1998, p. 101-116.

[43] J. Miąso, *Tajne nauczanie*, *op. cit*., p. 106-107.

of conduct and curriculum, as well as its own library created by Eugenia Kierbedziowa, a private Scientific Papers Reading Room, to which some lecturers contributed their own books[44]. In 1906, the Flying University was converted into the openly operating Society of Educational Courses (Towarzystwo Kursów Naukowych – TKN)[45].

The Flying University's principle was to "provide quite solid education" to many women who had no opportunity of studying abroad. It provided educational preparation to those interested in leaving. "Nearly all women from the Kingdom of Poland, who had obtained completion certificates from foreign universities by 1905 (...) had earlier studied at the Floating University. (...) Its lecturers were almost all the Warsaw scholars deprived of the possibility of normal work at the Russified Warsaw University"[46].

Since 1898, an invaluable source for keen students was the world's unique publication series *The Manual for a Self-Learner* (*Poradnik dla samouków*), initiated and published by Stanislaw Michalski and Aleksander Heflich, and financially supported by the Mianowski Foundation for the Promotion of Science (Kasa Mianowskiego). It was an intermediary between an encyclopaedic publication and an academic book. In *The Manual*, the self-learner received a general discussion of a selected area of knowledge supported by recommended literature with tips for the sequence of reading, as well as a set of questions which allowed evaluating their level of the acquired knowledge. *The Manual* was, to a certain extent, a substitute of the formal school and university education, and made possible studying in Polish to those who were deprived of this opportunity[47].

Political prosecutions in the Kingdom of Poland, the school strike of 1905 and the boycott of the Russian universities in the Kingdom of Poland drove many young women and men to study abroad from the turn of the 20th century onward. In 1909/10, in Europe, except Austria and Russia, about there were 1400 students from the Kingdom and Lithuania, while 2500 in Russia; in total 11.831 people studied outside the Kingdom of Poland. In 1915, 821 Kingdom citizens studied at both universities in Galicja. In total, 18,500 Polish students studied outside the Kingdom of Poland[48]. But many people were unable to leave, for different reasons, financial or family-related.

The Russian authorities, pressured by revolutionary sentiment and domestic unrest, made a series of concessions in the field of education. Among other things, they allowed creating private elementary and secondary schools, for boys and girls, with Polish as the official language, but without the entitlements of a government-licensed school. A problem arose, however, as to the further education of the graduates of these schools, because the certificates they received from these schools had no administrative power, and consequently, they did not

---

[44] This Reading Room was the origin of the current Public Library of Warsaw.

[45] H. Kiepurska, *Wykładowcy Towarzystwa Kursów Naukowych (1906–1915)* (*Speakers of the Educational Courses Society 1906–1915*), [in:] *Inteligencja polska pod zaborami. Studia* (*Studies on Polish intelligentsia under partitions*), ed. by R. Czepulis-Rastenis, Warszawa 1978, p. 261--309; S. Brzozowski, *Zabór rosyjski – Królestwo Polskie* (*The Russian partition – the Kingdom of Poland*), [in:] *Historia Nauki Polskiej*, *op. cit*., vol. IV, p. 445-489.

[46] J. Miąso, *op. cit*., p. 102.

[47] Jan Piskurewicz, *Warszawskie instytucje społecznego mecenatu nauki w latach 1869–1906* (*Social patronage of education in Warsaw 1869–1906*), Wrocław 1990, p. 125-126.

[48] H. Kiepurska, *Uniwersytet Warszawski*, *op. cit*., p. 556.

grant the students the right to enter universities. Hopes were growing among Polish people with regard to achieving some degree of autonomy for the Kingdom and some concessions on the part of the authorities concerning higher education which required educated didactic staff. All these factors led social activists along with scholars and people with political and economic influence to undertake efforts to legalise the operations of the Floating University, and consequently received permission from the Russian authorities to create the Society of Science Courses, TKN. TKN founders included Henryk Sienkiewicz, Tadeusz Korzon, Leopold Kronenberg, Stanisław Leszczyński, Antoni Osuchowski, Karol Benni, Antoni Kryński, Ignacy Chrzanowski, Piotr Drzewiecki, J.A. Święcicki and Stanisław Kalinowski[49]. The so-called "protector members" and "lifelong members", which titles were given to people supporting the TKN with substantial sums of money, included the Natansons, Michał Bergson, Stanisław Rotwand, Ignacy Paderewski, Stefan Dziewulski, industrialists, bankers, social institutions, an so on. Also the lecturers of TKN courses became members. The Society was a social institution maintained entirely from members' fees, donations, and fees collected from students.

The TKN operated legally since 1906. Its statute was registered as late as 1907 and stated that the Society's objective was "to provide higher education to and facilitate the pursuit of scientific work of people with appropriate preparation", and "to promulgate science news among society at large"[50]. TKN was led by the principle of the freedom of studying and teaching, it had its chancellor, and consisted of faculties headed by deans and managed by faculty boards. The number of faculties over the next few years including wartime years was rising. In 1906 the following faculties were created: Natural Science, Humanities, Technical, and Agricultural (from 1913, Horticultural); in 1915, Physics & Mathematics Faculty and the interdisciplinary Pedagogical Institute; in 1916 Forestry Faculty; and 1918, Political & Social Sciences Faculty.

Salaries were not high, so TKN lectures were largely an additional occupation for teachers, and the departments' staffing was changeable. The lecturers were required to possess habilitation licences and individual research achievements, or at least present good potential for individual research work. The candidates' scientific and moral qualifications were examined carefully and in general, as the future would show, without mistakes. The candidates had to be approved by the Russian authorities. Very many lecturers would later fill professorial positions at universities in the Second Republic of Poland. Their work for TKN was considered, to a certain extent, to contribute to their didactic and scientific tenure.

Originally, pure science subjects were allocated to the Natural Sciences Faculty, which was sizeable, with the number of natural subjects rising. In 1915, the Physics & Mathematics Faculty was singled out and created. Yearly, it had about 30 students. Mathematics was lectured by Samuel Dickstein, Zygmunt Janiszewski, Stefan Kwietniewski, Stefan Mazurkiewicz, Jan Krassowski, Wacław Sierpiński, Lucjan Zarzecki; astronomy by Tadeusz Banachiewicz and Jan Krassowski; physics by Marian Grotowski, Stanisław Kalinowski,

---

[49] H. Kiepurska, *Wykładowcy* (*Speakers*), *op. cit*., p. 263; the author considers S. Kalinowski as one TKN founders as he had given lectures on mathematical and naturalist subjects yet before the establishment of the TKN, which were subsequently included into TKN curriculum.

[50] Cited by H. Kiepurska, *Wykładowcy* (*Speakers*), *op. cit*., p. 263.

Józef Wierusz Kowalski, Ludwik Silberstein, and Bruno Winawer; chemistry by Edward Bekier, Józef Boguski, Tadeusz Miłobędzki, Stanisław Glixelli, Hilary Lachs, Jan Bielecki, Kazimierz Sławiński, Ludwik Szperl.

This institution was co-educational and admitted people even without the *matura* certificate if they had completed the 7th grade of the gimnazjum, in the belief that they would catch up on the one grade without difficulty. Some classes were run in the evenings to enable working people to attend them. The curriculum involved both lectures and classes, students took tests and exams, and the level of teaching was high. The number of students attending TKN courses in the years 1906–1920 is estimated to have been about 25.000, of whom 70% were women. The passing of exams was honoured by universities in the Galicja, France and Switzerland. The TKN created a secular form of a "free university" which in 1920 was converted into Wolna Wszechnica Polska (Free Polish University), soon received partial academic rights, and after World War II gave rise to Łódź University.

x

In 1860, Galicja received autonomy within the Austro-Hungarian monarchy, which proved to be a very favourable situation for Polish educational and academic aspirations. From 1867 the Galicja's schooling system was administered by the National School Board (Rada Szkolna Krajowa), which was autonomous of the Austro-Hungarian authorities. Polish was again the official language at most rural schools, in secondary schools, at the Lwów and Kraków universities and at the Polytechnic Institute in Lwów. In 1872 the school attending duty was introduced. A quick growth followed of 8-grade classical gimnazjums for boys, as well as that of real, government-run and private gimnazjums.

In 1860, there were 16 gimnazjums with 4850 students in the Galicja; in 1883/1884 – 25 gimnazjums and "real gimnazjums" and 5 "real schools" with about 11 000 students; in 1908 – 45 gimnazjums and "real gimnazjums" and 11 "real schools" with about 35 000 students. Until the end of the 19th century girls' general education secondary schools (departmental or *liceums*) did not prepare their students to take *matura* final exams. Since 1896 women could take *matura* exams as external students at boys' gimnazjums. Since 1897, the women citizens of Austria received the right to enter universities' philosophical departments, and soon also medicine departments, as regular students. In 1896 Kraków saw the opening of the "first girls' private gimnazjum". In 1900 the first 20 students received their *matura* certificates. By 1914 three other girls' gimnazjums had been created in Kraków. At the same time, first girls' gimnazjums were created in Lwów (first *matura* in 1904) and in provincial towns: by 1912, 5 classical and 9 "real gimnazjums" had been created, providing *matura* certificates with the right to enter universities. Ukrainian girls were admitted into 5 Ukrainian private boys' gimnazjums from 1907. Girls' liceums, which prepared students for the *matura* examination, posed certain competition for the *gimnazjums*[51].

---

[51] R. Dutkowa, *Żeńskie gimnazja Krakowa w procesie emancypacji kobiet (1896–1918)* (*Girls'* gimnazjums *of Kraków in the process of women's emancipation*), Kraków 1995, p. 12-44, 88; of the 944 female high school graduates of Kraków's oldest gimnazjums 709 took up studies at the Jagiellonian University.

The educational level of gimnazjums and its graduates caused criticism of the Jagiellonian University's Senate and the Higher Schools Teachers Union. In 1913–1914 the criticism was directed at the philological-historical-aesthetic bias of the gimnazjum curriculum, obsolete teaching methods, and the erudite, detached from social needs, curriculum of "real gimnazjums", where pure science and natural sciences were taught in place of Greek. The critics pointed out that the candidates had insufficient education, insufficient knowledge of the Polish language, were incapable of thinking and learning, and all this was caused by too quick development of secondary schools and the excessive scope of its social expansion.

At the time of the autonomy, Galicja's Polonised universities blossomed, with new departments, and new research institutes being created. The professorial staff kept growing and scientific contacts with foreign universities were expanding. The education of the young academic staff was developing successfully and the number of students grew considerably. Lwów University in 1914 accommodated more than 5500 students, and its staff amounted to 147 employees including 67 professors. The Jagiellonian University at the time had about 3000 students with 264 academic staff including 79 professors. After 1905, boys and girls from the Russian partition began flowing in great numbers to Galicja's secondary schools and universities. The composition of the students and academic staff had acquired a national and co-educational character[52].

Pure science subjects were allocated to the philosophical departments. In the 19th century they had attained an equal status with other subjects. Various forms of attracting students and young academics allowed for picking the most talented individuals and consequently supporting their scientific development (seminars, positions of university assistant, scholarships, study trips and foreign studies, habilitations, positions of university reader)[53]. For example, at the Jagiellonian University, where the full-time positions of university assistants had been reserved until 1920 only for the Philosophy and Medical Departments, the number of assistants grew from 11, including 6 at the Philosophy Department, in 1860/1961, to 113 in 1917/1918, including 51 in the Philosophy Department. Professors were helped in conducting classes by exhibitioners and the so-called "teaching fellows", who could be students in their senior years.

Two-year scholarships had existed at the Chemical Laboratory since 1856, at the Physics Section since 1894, and as part of the Mathematics seminar as late as 1916. Some future professors had taken advantage of scholarships, for example chemistry scholarships were granted to Karol Olszewski, Tadeusz Estreicher, Michał Siedlecki; physics scholarships to Zdzisław Krygowski, Tadeusz Godlewski, Stanisław Loria; mathematics scholarship to Franciszek Leja. From the 1880s professors were also assisted free of charge by cadets (Polish: *elewowie*, *elewi*). Full time academic positions were held also by adjunct

---

52 R. Wroczyński, *Dzieje oświaty polskiej* (*The History of the Polish education*), *op. cit*., p. 184-208; data on teaching staff, see p. 200; in the years 1891–1929, the Jagiellonian University's 585 assistants included 164 coming from the Kingdom of Poland, 32 from Russia, 8 from the Prussian partition; the 206 *docents* included 53 from the Kingdom of Poland, 7 from Russia, 8 from the Prussian partition.

53 U. Perkowska, *Kształtowanie się zespołu naukowego w Uniwersytecie Jagiellońskim (1860–1920)* (*The shaping of the Jagiellonian University's scientific staff*), Wrocław 1975.

professors, but these had to possess senior scientific qualifications. The position of adjunct professor was held by such scholars as astronomers Daniel Wierzbicki, Lucjan Grabowski and Władysław Dziewulski (the latter two were later professors in the reborn Poland).

Research and scientific specialisation were developing which led to the creation of new departments. In pure science and natural sciences, the originally single chairs were gradually expanded into a series of new chairs of physics, mathematics, chemistry, earth sciences, astronomy, and natural sciences. At the Philosophy Faculty in Kraków the number of chairs grew from 14 to 52 (including 3 chairs of each of physics, chemistry and mathematics), it included 26 sections and 25 seminars including 2 mathematical. In Lwów, the number of chairs grew from 13 with 12 professors in 1863, to 36 chairs with 44 professors, 27 university readers, 23 assistants, and 11 junior lecturers; including 2 chairs of each of mathematics, physics, and chemistry. Seminars were launched, also in pure science, which spurred scientific development, creation of research workshops, and introduced methodological research concepts.

In Kraków, during the autonomy of Galicja, at the Philosophy Department 144 habilitations were completed and approved. Although the biggest number involved habilitations in the history of Polish literature (10) and 7 in each of classical philology, philosophy, modern world history, but also 5 habilitations in each of mathematics, chemistry, and zoology. The universities of the Galicja were home to the elite personnel of the future academic community of the 2nd Republic, or even the first years following World War Two.

Among the professors working at both universities there were many distinguished scholars. Sometimes it is hard to assign one name to one university only because one could observe frequent transfers from one university to another, and an exchange of professors between the universities of Galicja. Their disciples or visitors from other universities (including the Russian partition) pursued habilitation here and consequently joined universities in Galicja or abroad. In pure science, Kraków and Lwów are associated with some great names of distinguished mathematicians, physicists, and chemists[54]. They sustained lively contacts with academic communities in Germany, Austria, France, Switzerland, Russia, or Britain. During the second half of the 19th century, and especially at the turn of the 20th century, both universities had climbed from the provincial level to become vibrant academic centres keeping up with the advances of world science.

x

---

[54] S. Brzozowski, *Warunki organizacyjne życia naukowego w trzech zaborach* (*Organizational conditions for scientific life in the three Partitions*), [in:] *Historia nauki polskiej*, *op. cit.*, vol. IV, part I–II, especially p.74-142, 245-268. For example, the following Kraków academics: physicists Marian Smoluchowski and Zygmunt Wróblewski, Wacław Dziewulski, Władysław Natanson, historian of pure science, physicist and mathematician Ludwik Antoni Birkenmajer, mathematicians Franciszek Mertens, Kazimierz Żorawski, Stanisław Zaremba, Franciszek Leja, chemists Karol Olszewski, Jan Zawidzki, Ludwik Bruner, Stanisław Glixelli. In Lwów: mathematicians Józef Puzyna Zygmunt Janiszewski, Wacław Sierpiński, Stanisław Ruziewicz, Hugon Steinhaus; physicists Feliks Kreutz, Oskar Fabian, Marian Smoluchowski (before he moved on to Kraków in 1913), Jan Stock, Tomasz Stanecki, chemist Bronisław Radziszewski and others.

Unable to study domestically, ambitious women set out to study abroad. These departures were sporadic in the 1860s and 1870s; in the 1880s more than 70 departed, and in the later years they travelled in great numbers. The entrance into university depended on whether one possessed the *matura* certificate. In the Russian partition, Polish women could take the *matura* at a boys' gimnazjum as external students. On many occasions, however, they had to complement their general education and language knowledge deficiencies. They usually suffered from poverty and their studies lingered on. The foundations which financed Polish students' scholarships favoured men. All the same, women succeeded in completing their studies, getting doctoral degrees, and even achieving scientific recognition. They were enlarging the community of Polish intelligentsia that was up to date with the latest achievements of European science.

Usually Polish women chose to study medicine and philosophy, more rarely natural and pure science. They travelled to Switzerland (Zurich, Geneva, Basel, Lausanne, Freiburg, Neuchatel) and Belgium (Brussels, Gent, Leuven). To study pure science, they usually set off to Paris (7 students in the 1880s, 20 in the 1890s including Maria Skłodowska)[55]. The biggest group represented women from the Kingdom of Poland, especially Warsaw. Jan Hulewicz, an expert of these issues, brings up an opinion from 1895 stating that: "On average, the women students from Warsaw are better prepared in scientific respects than others as they have had an opportunity to take part in collective classes, that are common in Warsaw, with the best professors"[56]. This comment most probably referred to the courses which evolved into the Flying University. After women were allowed admission into Galicja's universities, most Polish women set their sight on the universities in Kraków and Lwów. In 1906, the opportunity opened up for women to attend legal courses organised in Warsaw by the Society of Science Courses.

x

Young people in the Prussian partition usually went to study at German universities. Despite multiple (II!) attempts the Polish people did not manage to get their own university in Poznań[57]. The Prussian authorities were interested in having Polish young people studying in Germany. It was only at German universities that scholarship students of the Society for Scientific Assistance[58] were allowed to study. Besides, students going to a 9-grade German

---

[55] J. Hulewicz, *Sprawa wyższego wykształcenia kobiet w Polsce w wieku XIX* (*The question of higher education of women in the 19th century*), Kraków 1939, especially p. 192-225, chapter IX „Studia Polek za granicą w latach 1870–1900" (Polish women students abroad 1870–1900).

[56] J. Hulewicz, *op. cit*., p. 202.

[57] D. Mycielska, *Drogi życiowe profesorów przed objęciem katedr akademickich w niepodległej Polsce* (*Life stories of Polish professors before they took charge of university faculties in the independet Poland*), [in:] *Inteligencja polska XIX i XX wieku. Studia*, *op. cit.*, vol. II, Warszawa 1981, p. 247.

[58] Witold Molik, *Z dziejów kształcenia polskiej inteligencji na obczyźnie. Polscy studenci w uniwersytetach niemieckich 1871–1914* (*History of education of Polish intelligentsia in exile. Polish students at German universities 1871–1914*), [in:] *Inteligencja polska XIX i XX wieku* (*Studies on Polish intelligentsia in the 19th and 20th centuries*), ed.by Ryszarda Czepulis-Rastenis, Warszawa 1987, p. 248-249.

gimnazjum were unable to command modern languages well enough to study in French or English.

In the years 1849–1870 the biggest number of students at German universities arrived from the Grand Duchy of Poznań and Prussia, and relatively fewer from the other partitions. Most of them headed to Prussian universities in Wrocław (Breslau), Berlin, Greifswald, Halle, Münster, as well as Heidelberg, Münich, Leipzig, Freiburg, Würzburg, and Goettingen. After 1871 the number of trips picked up. In the years 1871–1914, at 21 German universities (including Koenigsberg and Strasburg) 243 Polish students studied in a single year (1871) which number grew to 676 in 1914[59]. They usually studied medicine (in 1914 as many as 36.7%). The philosophy department encompassing the subjects of study for future teachers attracted 10% of Polish students in 1875. In the subsequent years, the interest in philosophy gradually decreased to bottom 3.1% of Polish students in 1914. Natural sciences attracted 4–6% of Polish students, only temporarily in the years 1895–1906 this number rose to about 10%. Interest in agricultural studies was growing systematically, in 1916 it reached 16%. Women were allowed to study at German universities in 1900.

Few students from the Russian partition went to study at German universities. For example, in the late 1880s only some 50 people, comparing to the 1200 studying at Russian universities. Statistics prove that graduates from German universities clearly marked their presence among university professors, renowned politicians or activists. This confirms the high level of German science at that time. Few students left Galicja for Germany, and if they did, it was usually to complete their studies, and not at Prussian, but at southern German universities[60]. Even though the young didactic personnel at Galicja universities did have a lively contact with German science, the biggest attraction for the young academics were university centres in Vienna, Berlin, Leipzig and Munich, and Paris. Medicine academics usually studied in Breslau, representatives of pure science were attracted first of all by Goettingen, and then Berlin and Leipzig; they also studied in England and France. France attracted especially mathematicians who also travelled to Switzerland, Netherlands, and sporadically to Austria. Thanks to them, Galicja universities kept abreast with European science, and new ideas were flowing into university education of Polish youth.

x

In 1914, the three partitions were home to seven schools of higher education: in Galicja there were two Polish universities (Lwów and Kraków) and the Polytechnic in Lwów; in the Russian partition there were Russian schools: Warsaw's Imperial University, Polytechnic Institute, and Veterinary Institute; in Puławy the Polytechnic and Agricultural Forestry Institute, called Nowoaleksandryjski, because Puławy received the new name of New Alexandria[61]. There was no university in the Prussian partition. The Warsaw and Puławy Russian higher education institution and their the professorial staff left Poland in 1915: the University to Rostov-on- Don, and the Polytechnic to Nizhny Novgorod.

---

59 W. Molik, *op. cit*., p. 254-267.
60 W. Molik, *op. cit*., p. 267.
61 D. Mycielska, *Drogi życiowe* (*Life stories*), *op. cit.*, vol. II, p. 244-245.

After 1918, the academic system in Poland, with the exceptions of Kraków and Lwów, had to build up their staff from scratch. This especially referred to the new universities in Poznań and Wilno, as well as Warsaw. However, some Polish professors, lecturers and students remained in Warsaw's high schools or the Polytechnic Institute despite the boycott of these institutions by Polish youth following the 1905 strike. The first Polish staffing of both Warsaw University and and Politechnic Institute was completed in 1915, when following the evacuation of Russian institution the German occupying authorities decided to open university and polytechnic with Polish as the official language. It should be noted that preparations had been launched as early as 1906: the Society for Science Courses – TKN – had been working on launching a Polish university in Warsaw. Benon Tadeusz Miłobędzki, a chemist and assistant at the Polytechnic Institute's Non-Organic Chemistry Section between 1899–1915, and later professor at Warsaw's SGGW and Poznań University, explains this in 1949 in his letter to Wojciech Świętosławski, reminiscing about the years of the strike and the boycott: "wise people who ran the strike decided that students should boycott the Russian school. But the professors, teachers and assistants were ordered not to vacate their positions and continue fighting in legal ways. The point was that in case the strike was defeated, which was not in any way improbable, the school should not be left completely without Polish teachers. The point at stake at the University and the Polytechnic was to defend the resources, libraries so they were not removed to Russia. Incidentally, we managed to keep hold of them"[62].

x

"In times of war the muses fall silent", goes the classic dictum derived from Cicero. In the background for Poland's collapse in 1795 and its restoration in 1918 wars were waged on the European and world scale: at the turn of the 18th and 19th century the Napoleonic Wars, and in the years 1914–1918 the World War One. The national restoration coincided with clamour of arms. In the case of Poland, this dictum did not find much confirmation. Among the nine muses, daughters of Zeus and the titaness Mnemosyne (the patron goddess of poetry, art, dance, and science), Urania was the muse of astronomy and geometry, and one of her sisters, Clio, was the muse of history. The muses suffered hardships on our land but they never fell silent. Both Clio and Urania can be considered the patrons of conference, dedicated to the period from the mid-19th century to 1918, the end of the World War One. In our case, the domain of Urania, the mathematical and natural sciences, was crucial in the second part of the nineteenth and early twentieth centuries. The answer to the question of how it was possible is given by history, the domain of Clio.

---

[62] Cited by H. Kiepurska, *Wykładowcy Towarzystwa Kursów Naukowych (1906–1915)* (*Speakers of the Educational Courses Society 1906–1915*), cited by *Inteligencja polska pod zaborami. Studia*, *op. cit*., vol. 2, Warszawa 1981, p. 293-294.

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE

1-NP/2014

MARTINA BEČVÁŘOVÁ*

THE ROLE OF CZECH MATHEMATICIANS IN THE BALKANS (1850–1900)[1]

ZNACZENIE CZESKICH MATEMATYKÓW NA BAŁKANACH (1850–1900)

A b s t r a c t

From 1860s, the number of mathematicians, teachers and authors of monographs, textbooks and papers in Bohemia increased noticeably. This was due to the improvement of education and the emergence of societies. During 1870s and 1880s many candidates for teaching mathematics and physics were without regular position and income. Some of them went abroad (especially to the Balkans) where they obtained better posts and started to play important roles in the development of "national" mathematics and mathematical education. The article describes this remarkable phenomenon from the history of the Czech mathematical community and analyzes its influence on other national communities.

Keywords: *mathematics and mathematical education in Bohemia and Balkans*, *history of mathematical societies*, *history of mathematics*, *the 19th century*

S t r e s z c z e n i e

Od lat 60. XIX w. w Czechach wyraźnie wzrosła liczba matematyków, nauczycieli matematyki, autorów monografii, podręczników i artykułów. Było to spowodowane rozwojem matematycznej edukacji i powstaniem towarzystw. W latach 70. i 80. XIX w. wielu czeskich kandydatów na nauczycieli matematyki i fizyki w szkołach średnich nie miało stałej posady i dochodów. Część z nich udawała się za granicę (szczególnie na Bałkany), gdzie zajmowali lepsze stanowiska i znacząco przyczynili się do rozwoju „narodowej" matematyki i edukacji matematycznej. Artykuł prezentuje ten niezwykły fenomen w historii czeskiego środowiska matematycznego i poddaje analizie jego wpływ na inne społeczności narodowe.

Słowa kluczowe: *matematyka i edukacja matematyczna w Czechach i na Bałkanach*, *historia towarzystw matematycznych*, *historia matematyki*, *wiek XIX*

* Martina Bečvářová, Department of Applied Mathematics, Faculty of Transportation Sciences, Czech Technical University in Prague and Department of Mathematics Education, Faculty of Mathematics and Physics, Charles University in Prague, becvamar@fd.cvut.cz.

[1] The text summarizes the long time author's research, complements and broads her monographs and studies [2–5, 7–11].

1. Introduction

1.1. The development of Czech secondary schools and universities in the 19th century

In the first half of the 19th century, Bohemia became the industrial backbone of Austria. The growing tendencies to centralize and improve an industrial production required a rapid development of technical schools where an important role was played by teaching of mathematics and in particular geometry which became an integral part of education. The expansion of the technical universities required a development of secondary education in view of increasing demands on the professional preparedness of the teachers and students. This pressure led naturally to creation of a new type of secondary schools (technical secondary schools, the upper forms of grammar schools, schools of commerce) and a reform of classical grammar schools. At the same time, it led to an increasing number of vacancies for teachers and tightening up the demands on their preparation. Therefore teaching methods at "classical" universities were reformed and focused on the education of future teachers, doctors and lawyers. In the second half of the 19th century in Bohemia, due to the rise of nationalistic movements, the Czech and German communities (living together for a long time) separated. This separation was also reflected in science and education. An important feature of that period was the process in which Czech science was "becoming independent". It was accompanied, on the one hand, by protracted national conflicts and, on the other hand, by expensive constructions of new schools, the establishment of new associations and the development of the Czech scientific terminology, journals and monographs. As a consequence, finances were drained and the development of the Czech science delayed[2].

Up to the end of the 1850s, the education system of secondary schools and universities was solely in German. Only since 1861, the first Czech secondary schools were built. In the period between 1861 and 1865 some subjects at the state secondary schools were taught in Czech, while the teaching of others remained in German. In the second half of the 1860s, the German and Czech secondary schools were coexisting with same standard. Thus, the graduates of the Czech stream of education who entered universities started to require lectures in their mother tongue. In the 1860s, the efforts of Czech political representatives and intellectuals as well as the movement of university students to have their studies in Czech language required an establishment of Czech mathematical lectures at the Prague Technical University (1864). At first, they existed in parallel with German ones that had better teachers and more funding. The arrival of better qualified Czech teachers and students who have been educated at Czech secondary schools led to the strengthening of positions of Czech mathematical "departments" at the Prague Technical University (later the Czech Technical University in Prague) and the establishment of the similar lectures at the Prague University (1871). The professional standard of the Czech mathematical lectures were comparable to the German ones and even began to surpass them in student enrolment. At the end of the 19th century, the importance of the Czech mathematical departments was increasing, because of the growth in the number of their teachers and students. On the other hand, the number

---

[2] For more about the reasons leading to the establishment of the independent Czech educational system see [5].

of German students was decreasing, because most of the German professors considered Prague to be merely a temporary place on the way to Vienna or Germany.

1.2. Czech mathematical textbooks and translation of "classical" and modern books

The tendencies to write Czech textbooks for elementary subjects of the higher classes of middle schools and the lower ones of the secondary schools were very popular between 1850s and 1860s. At first, the textbooks had a character of temporary texts. The first high-quality mathematical textbooks for secondary schools were written in the beginning of the 1860s by J. Fleischer, V. Jandečka, D. Ryšavý, J. Smolík, F. Šanda and V. Šimerka. In the 1870s, the efforts for improving teaching and the replacement of old textbooks by new ones that would comply with the new curricula grew stronger. These textbooks were written together by the above professors of secondary schools and by some university professors (for example F.J. Studnička and K. Zahradník). The first textbooks of mathematics for the students of the Prague Technical University were published in the mid 1860s thanks to G. Skřivan, F.J. Studnička, Em. Weyr, Ed. Weyr. Most of them (but for the university students) were written after the year 1871, i.e. after introducing Czech mathematical lectures at the Prague University, and especially after 1882, i.e. after establishing the Czech University in Prague.

Firstly, Czech authors wrote textbooks according to foreign models and in respect to their professional interests; they published them either at their own expense in various publishing houses or at the expense of richer booksellers. They faced not only a lot of professional problems (the absence of domestic models, imperfect terminology and methodology) but also financial ones – there were few readers. The activities in the 1860s and 1870s cannot be considered a systematic creation of textbooks, because in most cases they were mere revisions or "free copies" of older ones. It should be noted that this trend did not manifest itself only in mathematics but also in other disciplines. The situation improved in the 1880s when textbooks were published by *Jednota českých mathematiků* (The Union of Czech Mathematicians). It was the first systematic and profitable effort of publishing Czech textbooks for secondary schools as well as for universities. At the end of the 19th century, Czech textbooks for secondary schools complied with European standards.

The first attempts to translate classical works of mathematicians and some modern monographs to Czech[3] occurred in the 1860s. The first translations of mathematical works were published in the 1870s[4]. Their authors were active members of *Jednota českých mathematiků*

---

[3] It should be noted that in those times the Czech scientists tried to translate one of Aristotle's work on logic. A.J. Vrťátko translated in 1860 his book *Categories* and issued it under the title *Aristotle's Categories.* More details on Czech translations of mathematical works of classics and modern monographs see [5].

[4] At the beginning of 1870s, Emil Weyr translated two monographs written by the Italian geometer Luigi Cremona *Sulle trasformazioni geometriche delle figure piane* and *Introduzione ad una teoria geometrica delle curve piane,* Martin Pokorný then translated the famous textbook written by the German mathematician Richard Baltzer *Die Elemente der Mathematik* and Karel Zahradník added the translation of the important work of the Italian mathematician Giusto Bellavitis *Saggio di applicazioni di un nuovo metodo di geometria analitica* (*Calcolo delle equipollenze*).

(The Union of Czech Mathematicians) who graduated at universities and started to work with youthful enthusiasm. Further translations were made in the 1880s[5]. However, most of the mathematicians focused on the compilation of original works, monographs and Czech textbooks. Further translations appeared only at the beginning of the 20th century[6]. Czech mathematicians paid particular attention to the translation of one of the most outstanding mathematical work of all time – *The Euclid's Elements* – i.e. the book that influenced development and teaching of mathematics since the third century before Christ[7]. In addition, sections of the work by René Descartes (1596–1650), Blaise Pascal (1623–1662) and Bernard Bolzano (1781–1848) were translated[8].

Translation activities were moulded by the professional interests of individual translators and therefore could not be systematic. The translations of modern mathematical works were inspired above all by an attempt to make the newest results of world mathematical research accessible to readers and enrich the domestic professional literature. On the other hand, the translations of classical works were motivated by an attempt to gain some personal prestige and prove that Czech mathematical terminology could compete with that of Greek and Latin.

1.3. Czech professional associations and their activities

An interesting feature of the 19th century was a gradual formation of scientific institutions which – in spite of the initial lack of finances and a small number of experts – organised lectures and scientific discussions, published professional publications (journals, monographs and textbooks) and issued reports on various activities. Scientific associations that originated at the end of the 18th and in the first half of the 19th centuries combined Czech and German speaking specialists of various branches and were usually bilingual. Their activities were not considerably influenced by nationalistic conflicts. After the fall of Bach absolutism (1859), the Czech society formed enough space for various activities and for the formation of various

---

5 At the beginning of 1880s, F.J. Studnička translated the famous article written by Bernard Bolzano *Rein analytischer Beweis des Lehrsatzes, dass zwischen je zwey Werthen, die ein entgegengesetztes Resultat gewähren, wenigstens eine reelle Wurzel der Gleichunge liege*.

6 It was the translation of three Archimedes' works (M. Valouch, *Archimedovo měření kruhu* (On the Measurement of a Circle), Výroční zpráva c. k. státního vyššího gymnasia v Litomyšli, 1903, 25 pages; M. Valouch, *Archimeda Syrakusského Počet pískový* (The Sand Reckoner), Výroční zpráva c. k. státního vyššího gymnasia v Litomyšli, 1905–1906, 13 pages (reprint 1993); F. Vrána, *Archimédův výklad Eratosthenovi o mechanických způsobech zkoumání* (*Z řečtiny přeložil Fr. Vrána*) (The Method of Mechanical Theorems), 3. výroční zpráva c. k. státního gymnasia v Prostějově za školní rok 1908/09, Prostějov, 1909, p. 2-18). For more details on the Czech translations of Archimedes' works see [6].

7 There was an unsuccessful attempt of the Union of Czech Mathematicians in 1870–1871; the attempt of Josef Smolík (1832–1915), who translated the whole *Elements* at the end of 1880s and whose translation remained in the form of a manuscript; the translation made by František Fabinger (1863–1938), who translated and published the first book of *Elements* in 1903, and the successful complete translation made by František Servít (1848–1923), which was published by the Union of Czech Mathematicians in 1907. For more information see [12].

8 For more details on the Czech translations see [5].

associations. In the mid of the 1860s, they started to show their professional and language particularities.

The development of Czech mathematics was influenced considerably by the foundation of *Spolek pro volné přednášky z mathematiky a fysiky* (The Association for Free Lectures of Mathematics and Physics). At the beginning, it brought together Czech and German university students of mathematics and physics and later, students of the Technical University became its members. In 1869, this association changed into *Jednota českých mathematiků* (The Union of Czech Mathematicians) and influenced Czech mathematics and physics for decades to come[9]. The association became a convenient centre of mathematical activities that were connected closely with those of the universities and professors, and bounded together the university with the high school teachers and students, the teachers of the elementary schools and recruited new people who were interested in mathematics and physics.

It should be noted that mathematics in Bohemia was also pursued in the mathematics and natural sciences section of the *Královská česká společnost nauk* (The Royal Czech Scientific Society, founded in 1770) and in the similar section of the *Česká akademie císaře Františka Josefa pro vědy, slovesnost a umění* (The Czech Academy of František Josef for Science, Literature and Arts, founded in 1890)[10].

The work of associations included also publication, educational and popularisation activities. The newly created journals on mathematics influenced the development of Czech mathematical terminology and teaching for many years. They described the main trends in mathematics and opened the room for publications, reviews and educational activities of members of the Union, amateur mathematicians, teachers and students.

In 1871 and 1872, the Union published its summaries under the title *Zpráva o činnosti Jednoty českých mathematiků v Praze …* (The Report on the Activity of the Union of Czech Mathematicians in Prague …)[11], in which it informed about its professional and cultural activities. In the period between 1873 and 1875, the Union published its bulletin *Věstník Jednoty českých mathematiků* (The Bulletin of the Union of Czech Mathematicians) that provided information about all its activities and contained summaries of the most important recent Czech and foreign scientific literature. In 1878, the bulletin was replaced by *Annual Reports*[12]. In 1870 and 1871, the Union published three reports that included professional articles[13]. They received a wide acceptance among the Czech

---

[9] For more about the foundation of the *Association for Free Lectures of Mathematics and Physics* see [13].

[10] For more about the Czech scientific associations see [5].

[11] *Zpráva o činnosti Jednoty českých mathematiků v Praze za první a druhý ročník, totiž od 14. října 1869 do 15. října 1871* (The Report on the Activity of the Union of Czech Mathematicians in Prague during the First and Second Year …), nákladem Jednoty českých mathematiků, Praha, 1871, 23 pages, *Zpráva o činnosti Jednoty českých mathematiků v Praze za třetí ročník, totiž od 15. října 1871 do 7. července 1872* (The Report on the Activity of the Union of Czech Mathematicians in Prague during the Third Year …), nákladem Jednoty českých mathematiků, Praha, 1872, 38 pages.

[12] They have been published every year in the extent of 10-15 pages.

[13] *První zpráva Jednoty českých mathematiků* (The First Report of the Union of Czech Mathematicians), Jednota českých mathematiků, Praha, 1870, 86 pages. It was edited by Mírumil Neumann and Karel

professors of mathematics and physics at the secondary schools, as well as among the students at the universities and secondary schools, and became a model for future Czech mathematical journals. It must be noted that the authors of individual articles were Czech beginners in physics and mathematics who were engaged significantly in the development of the Czech science. In 1872, the Union decided to publish its own journal titled *Časopis pro pěstování mathematiky a fysiky* (The Journal for Cultivation of Mathematics and Physics) which faced a lot of problems for several years, especially financial ones resulting from the lack of regular subscribers. Nevertheless, it has always been a wide field for the activity of Czech authors, students and teachers of the secondary schools because of its policy to address a broad audience of readers. In addition, the contributions of the best mathematicians were published there.

In 1875, the role of the "professional" journal was taken over by a new international journal called *Archiv mathematiky a fysiky* (The Archive of Mathematics and Physics). However, it became clear very soon that the editors of the new journal overestimated both their possibilities and the interest of the Czech society in mathematics and physics. This is why the journal ceased to exist in 1878 after the publication of only two volumes. After that, the journal *Časopis* … returned to its original objective, i.e. publishing professional, educational, didactic and informative articles. It kept this function up to the end of the 19th century[14]. In addition, the journal was a link between the Czech intellectuals scattered all over Austria and Hungary and the Prague centre of the Union.

1.4. Czech professional mathematical works

For more than three decades in the second half of the 19th century, Czech mathematicians tried to show that they could compete with the German mathematicians and even surpass them in many respects. These efforts required a lot of time and energy. Not until the last quarter of the 19th century the works of Czech professional mathematicians reached a standard where they were able to keep up with the individual trends of science and also understand them and contribute to the global scientific research. Since the 1880s Czech mathematical works have specialised in individual branches and approached more critically the subjects of research. Our mathematicians paid attention to the newest results in descriptive and projective geometry, the theory of matrices and determinants, quadratic forms and analysis and from the beginning of the 20th century they started to contribute to the development of individual mathematical disciplines (for example M. Lerch, K. Pelz, J. Sobotka, F.J. Studnička, Em. Weyr, Ed. Weyr, K. Zahradník). Some of them published also their important results in foreign languages (German, French and even Italian) to make

Zahradník. *Druhá zpráva Jednoty českých mathematiků* (The Second Report of the Union of Czech Mathematicians), Jednota českých mathematiků, Praha, 1870, 96 pages + 1 tablet; M. Neumann and A. Pánek were its editors. *Třetí zpráva Jednoty českých mathematiků* (The Third Report of the Union of Czech Mathematicians), Jednota českých mathematiků, Praha, 1871, 96 pages + 1 tablet which was edited by M. Neumann and A. Pánek.

[14] It must be noted that this journal is still published. In 1991, it changed its name to *Mathematica Bohemica* with the subtitle *The Journal for Cultivation of Mathematics*. It is published in English and has exclusively professional character.

them accessible to the European mathematical community. On the other hand, they published Czech versions of their works that appeared in a foreign language, as well as informative, popularising or methodological articles in the local journals[15].

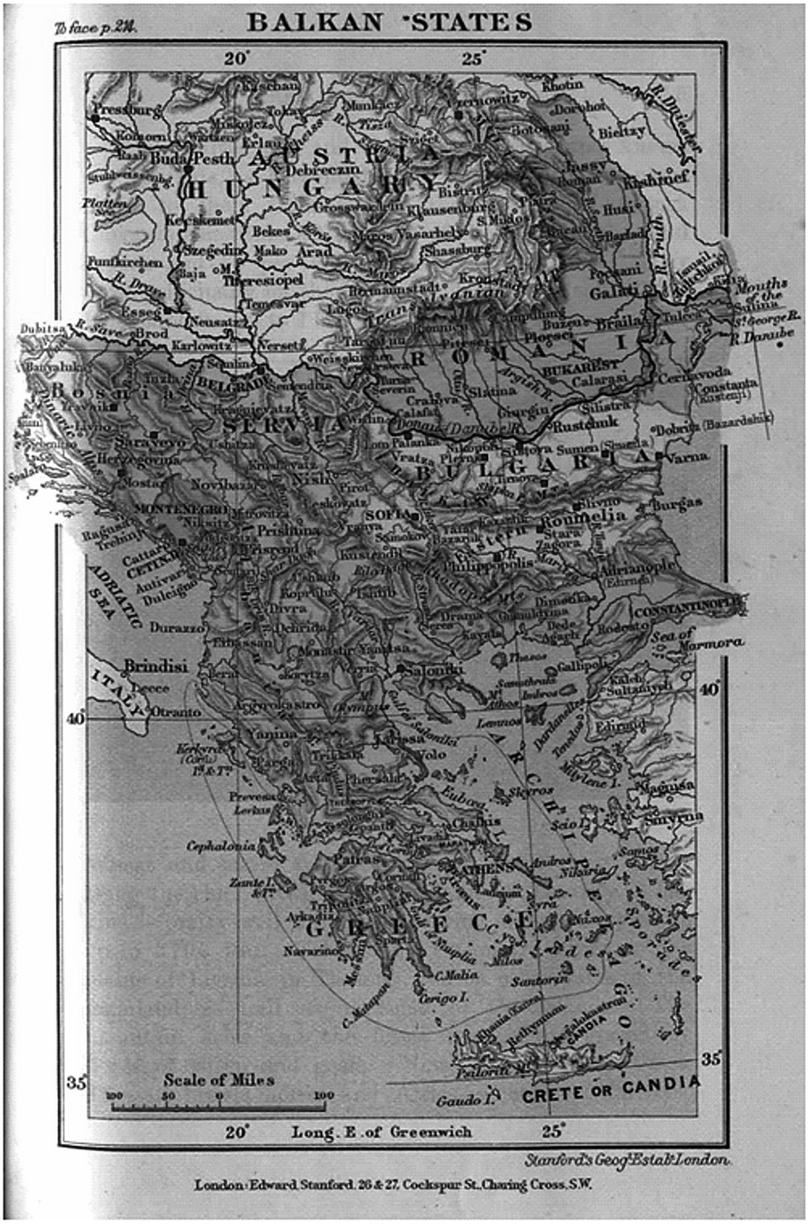
2. Czech mathematicians abroad

One of the main aims of teaching mathematics at the Prague University was the preparation of future teachers of secondary schools. The rigid state control of their education and the well worked-out and thought-out educational system enabling their professional development and career contributed in two first decades of the second half of the 19th century to the improvement of the teaching of mathematics and natural sciences as well as to the development of the secondary schools and the education of our population. Nevertheless, the rules that satisfied the needs of the third quarter of the 19th century when there was a shortage of teachers lost their validity at its end and in fact brought the development in this field to a standstill. Since the end of the 1870s the number of members of the Czech mathematical community increased in contrast with the shortage of jobs at the Czech universities and secondary schools. In that period many good teachers could not find work as professors at the secondary school level and they often worked as supply-teachers for five to ten years. As a result, many teachers changed jobs or went abroad[16]. Many first-class Czech teachers went to South-East Europe to other countries that were part of the Austrian--Hungarian Empire – such as modern day Croatia and Slovenia – and other Balkan countries – later Serbia, Bosnia, Herzegovina, Bulgaria etc. – where they contributed to the development of national science and education that – in comparison with that in Bohemia – were lagging.

After their arrival, they learned the respective foreign language and began to create curricula for the teaching of mathematics and descriptive geometry at the secondary schools and universities. For their colleagues-teachers, they wrote the first methodological manuals

[15] For more information see [5].

[16] Some Czech mathematicians and physicsts went also to other Western Europe and countries of the Austrian-Hungarian Empire. They were searching for better career, broadening the horizon of their knowledge and contacts with the best mathematical centres of Western Europe, as well as possibility to publish their scientific and popular works there. They usually came back after some time and worked as professors at prestigious secondary schools or universities. Czech mathematicians, who were employed at German schools in Germany, Switzerland or other countries of the Austrian-Hungarian Empire, taught, researched and published their professional works, because they were in a much more developed and cultural environment than their colleagues who stayed in the Balkans. Nevertheless, they kept contacts with the Czech mathematical community, monitored vacancies and in many cases returned to Bohemia and tried to find good jobs there, relying on their contacts abroad and their wide experience. For example, Čeněk Hausmann (1826–1896) and Václav Láska (1862–1943) lectured at the Technical University in Lvov (Galicia), Johann Josef Partl (1802–1869) taught at the real school in Budapest (Hungary) and Čeněk Hausmann lectured at the Technical University in Budapest, Emanuel Czuber (1851–1925), Josef Finger (1841–1911) and Jan Sobotka (1860–1931) spent many time at the Technical University in Vienna (Austria), Emil Weyr (1848–1894) lectured at the University in Vienna and Karel Pelz (1845–1908) at the Technical University in Graz (Austria), Matyáš Lerch (1860–1922) lectured at the University in Freiburg (Switzerland). For more information see [3].



Balkan States (1899)[17]

[17] Map from *Stanford's Compendium of Geography and Travel: Europe* (volume 1, 1899, p. 214). Places where mathematicians and teachers from Bohemia worked for a longer time are marked with square.

about the teaching of mathematical subjects in their mother tongues. For their pupils they created the first brief teaching manuals and collections of mathematical exercises (at first published in the lithographical form or within the annual reports of the secondary schools – see for example J. Pexider, A.V. Šourek, K. Zahradník). During the few first years, they translated Czech textbooks of mathematics and descriptive geometry to other languages (for example A.V. Šourek and V. Šak). They set a form for the first generations of students educated in their mother tongues. In the second phase of their "mission" – usually at the end of the first decade of their stay – they were inspired by Czech models and wrote new textbooks for the secondary schools and universities (for example V. Láska, F.V. Splítek, V. Šak, A.V. Šourek, K. Zahradník). These textbooks were widespread and used until the end of the World War I. Thanks to their quality education, high professional standard and all around activities they contributed to the creation of the mathematical terminology that has been used – except for a few modifications – until today (for example A. Studnička, A.V. Šourek, K. Zahradník). On the basis of their good experience from Bohemia they led local mathematical communities to the unification of professional associations (for example J. Finger, A.V. Šourek, K. Zahradník) and initiated publishing professional, educational and popularisation periodicals (for example F.V. Splítek, A.V. Šourek, K. Zahradník). In addition, they participated in the international promotion of the results of professional and pedagogical research (A.V. Šourek, K. Zahradník). All their activities were inspired by those developed in our country in the 1860s and 1870s. On one hand, the Czech society lost some quality experts, but on the other hand, the Czech teachers at the secondary schools and universities contributed to the birth of the "national" mathematics in the Slavonic countries in Southern Europe.

During their active life they kept in contact with their Czech colleagues. They were founders or correspondents of *Jednota českých mathematiků* (The Union of Czech Mathematicians, for example J. Finger, J. Laun, T. Monin, J. Pexider, C. Plch, A.V. Šourek, J.S. Vaněček, K. Zahradník), followed an eye about the development in Bohemia and in professional periodicals of their new homeland informed regularly about the activities of the Union, Czech textbooks, monographs and journals. In addition, they wrote reviews and contributed to the *Časopis pro pěstování mathematiky a fysiky* (The Journal for Cultivation of Mathematics and Physics, for example C. Plch, T. Monin, K. Zahradník), *Zprávy Královské české společnosti nauk* (The Reports of the Royal Czech Scientific Society, for example K. Zahradník) or *Rozpravy České akademie věd* (The Transactions of the Czech Academy of Sciences, for example K. Zahradník).

3. The most prominent Czech personalities in the Balkans

In what follows we shall mention only the Czech mathematicians who translated Czech or German textbooks to other languages or, being influenced by the Czech literature, wrote textbooks in them, created mathematical terminology and gained recognition for the development of the regional secondary schools and universities[18].

[18] For more about the development of the Czech mathematical community in the second half of the 19th century see [5].

Croatia

Teacher	Place	School	Period
Jan Pexider[19] (1831–1873)	Zagreb	Secondary school (gymnasium)	1864–1873
Josef Laun[20] (1837–1915)	Rijeka Zagreb	Secondary school (gymnasium) Secondary school (gymnasium)	1864–1865 1865–1868
Karel Seeberg (1835–?)	Vinkovci, Sinj	Secondary school (gymnasium)	1865–1867
Josef Silvestr Vaněček[21] (1848–1922)	Osijek	Real school	1873–1875
Karel Zahradník (1848–1916)	Zagreb	University	1875–1899

Slovenia

Teacher	Place	School	Period
Rudolf Schnedar (1828–1862)	Ljubljana	Real school	1860–1862
Josef Baudiš (1825–1898)	Gorizia (Italy, today)	Secondary school (gymnasium)	1860–1864
Josef Finger[22] (1841–1925)	Ljubljana	Real school	1870–1874

[19] Jan Pexider devoted his life to work in Croatia. As one of the first Czechs he began to translate from German to Croatian language the textbooks of mathematics and physics for secondary schools. Because of his premature death he did not influence the development of the teaching of these subjects in Croatian language in a significant way. For more information see Věstník Jednoty českých mathematiků (The Bulletin of the Union of Czech Mathematicians), 1, 1873, no. 1, p. 5, no. 4, p. 35, 41, 50, 51, and 2(1874), no. 1, p. 13-14; Program gimnazije u Zagrebu 1864–1873 (The Report of Gymnasium in Zagreb during years 1864–1873), Zagreb, 1873, and [1].

[20] In 1864, Josef Laun became a teacher at the grammar school in Rijeka. From 1865 till 1868, he taught at the grammar school in Zagreb, and then he left the teaching profession. He studied at the Faculty of Law at Prague University and afterwards started to run a farm in Kněževes. For more information see [13].

[21] In 1873, Josef Silvestr Vaněček obtained a professorship at the grammar school in Osijek. In 1875, he was named a teacher at the secondary school in Jičín (Bohemia). From 1878 to 1879, he studied mathematics and descriptive geometry in France. After his return to Bohemia, he taught again in Jičín (until 1906). In 1884, he unsuccessfully tried to become an associate professor of mathematics at the Czech University in Prague. In 1895, he unsuccessfully ran for the post of professor of descriptive geometry at the Czech Technical University in Prague. From 1880 to 1890, alone or jointly with his brother M.N. Vaněček prepared more than thirty works related to geometric problems. For more information see M. Bečvářová, *J.S. Vaněček a L. Cremona (nově objevená korespondence)*, [in] J. Bečvář, M. Bečvářová (eds.), *34. mezinárodní konference Historie matematiky*, Poděbrady, 23. až 27. 8. 2013, Matfzypress, Praha, 2013, p. 63-80.

[22] In 1870, Josef Finger became a professor of mathematics and physics at the technical secondary school in Ljubljana. After 4 years he left Slovenia and went to the grammar school in Hernals in Vienna, where he started to teach in 1874. In 1876–1878, he taught at the secondary school

Bosnia and Herzegovina

Teacher	Place	School	Period
Cornelius Plch (1838–1889)	Travnik (today Tornik in Serbia)	Secondary school (gymnasium)	from 1870s up 1889
Alois Studnička (1842–1927)	Sarajevo	Technical school	1893–1907

Bulgaria

Teacher	Place	School	Period
Antonín Václav Šourek (1858–1926)	Sliven Plovdiv Sofia	Secondary school Secondary school University	1880–1881 1881–1890 1890–1926
František Vítězslav Splítek (1855–1943)	Svistov Salonica (today Thessalonica in Greece) Sofia Gabrovo Plovdiv	Secondary school Secondary school Secondary school Secondary school Secondary school	1880–1883 1883–1888 1888–1889 1889–1891 1891–1915
Theodor Monin (1858–1893)	Sliven Sofia	Secondary school University	1881–1886 1889–1891
Vladislav Šak (1860–1941)	Sliven Sofia	Secondary school Secondary school University	1882–1886 1886–1907 1891–1894 1907–1908

3.1. Croatia

In 1875, the Czech mathematician **Karel Zahradník**[23] went to Zagreb to the newly created University of František Josef. Until 1890, he was the only professor of mathematics there. He taught algebra, calculus, higher analysis, geometry, numbers theory and probability[24].

---

in Leopoldstadt near Vienna. In 1876, he became a private docent at the University in Vienna where he gave lectures until 1890. In 1878, he was promoted to an extraordinary and in the year 1884 an ordinary professorship of mechanics and graphic static at the Technical University in Vienna, where he has taught until his retirement in 1911. In 1905, he became a protector of the first mathematical associations of students founded at the Technical University in Vienna. Its aim was to support students' publications and lectures in mathematics and natural sciences. It is probable that J. Finger influenced significantly the activities of this association. And it should be noted that Czech mathematician Gabriel Blažek, one of the founders of the *Association for Free Lectures on Mathematics and Physics,* tried to establish a similar association in the school-year 1863–1864 at the University in Vienna. More about Finger's life see [5, 13].

[23] As for his life, see [2, 3, 5, 9, 10].

[24] Not until 1890 was the teaching of mathematics conducted by another mathematician. That year D. Segen (Zahradník's first student about to take a doctor's degree) began to give lectures on geometry. V. Varićak, a student of Zahradník, started to give lectures on mathematical analysis four years later.

Since 1886 he was the head of a “mathematical seminar” for talented students. It was here that the first professional works of Croatian mathematicians originated. In 1896––1899, he worked as a director of the mathematical institute at the university in Zagreb. After his arrival to Zagreb he formulated the first mathematical curricula and rules for individual examinations including the final one. He supervised examinations of teachers of all mathematical subjects at Croatian schools where the Croatian language was used. During his more than twenty-year stay in Zagreb he educated the first Croatian teachers and mathematicians of the secondary schools and prepared some to take a doctor’s degree. In the course of all these activities he was inspired by the work of his teacher and friend František Josef Studnička (1836–1903), whom he considered to be his mentor. He tried to follow Studnička’s Prague activities.

In the 1870s, he translated his papers (written in German or Czech languages) and published them in the Croatian language; later he also published in this language his original results and wrote textbooks for the secondary schools and universities. In 1878, he published in Zagreb his book *O determinantih drugoga i trècega stupnja. Za porabu viših srednjih učilišta* (On Determinants of Second and Third Order. For Higher Classes of the Secondary Schools)[25] which he translated to Czech the next year and published in Prague under the new title *Prvé počátky nauky o determinantech. Pro vyšší střední školy* (The First Start of the Theory of Determinants. For Higher Classes of the Secondary Schools)[26]. The booklet was based on his lectures in 1876/1877 for the Croatian university freshmen. At the end of the 19th century, his lectures *O determinantima. Predavanja u zimskom semestru godine 1897/8* (On Determinants. Winter Semester 1897/8)[27] and *O plohama i o krivuljama u prostoru. Predavanje u ljetnom semestru godine 1898* (On Planes and Curves in the Space. Summer Semester 1898)[28] were published in the Croatian language. These were the first Croatian textbooks of mathematics. Thanks to him *Kapesní logarithmické tabulky F.J. Studničky* (Studnička’s Pocket Logarithmic Tables) were published in Croatian. At the end of the 1870s, he started to translate Studnička’s textbook *Algebra pro vyšší třídy středních škol* (Algebra for Higher Classes of the Secondary Schools)[29] for the secondary schools, but the Croatian government did not allowed its publication[30].

Zahradník laid the foundations of Croatian mathematics and contributed significantly to the development of the Croatian mathematical community. He participated in the mathematics and natural sciences section of the Croatian Academy of Sciences, where he gave professional and popularisation lectures and published his works. He influenced also the development of the mathematical section of the journal *Rad Jugoslavenske akademije znanosti i umjetnosti u Zagrebu* (The Transactions of the Yugoslavian Academy

[25] Zagreb, 1878, 39 pages.

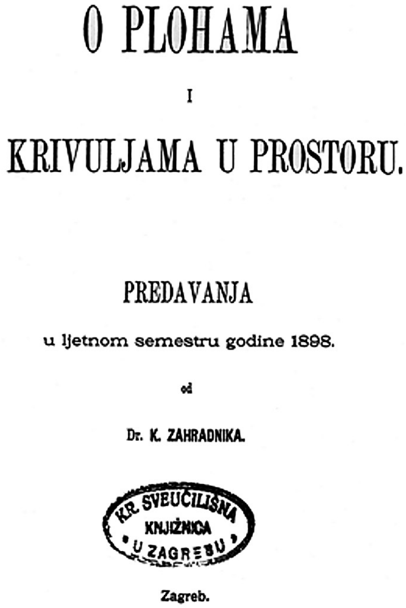
[26] Praha, 1879, 48 pages.

[27] Zagreb, 1898, 112 pages.

[28] Zagreb, 1898, 152 pages.

[29] Studnička’s textbook was firstly published in 1877, secondary in 1879. In 1878 and 1879, F.J. Studnička published German version of his textbook.

[30] For more about this affair can be found in Zahradník’s letters deposited in the F.J. Studnička’s estate in Literary Archives of the Treasure of National Literature in Prague. For more information see [14].


O PLOHAMA

I

KRIVULJAMA U PROSTORU.

PREDAVANJA

u ljetnom semestru godine 1898.

od

Dr. K. ZAHRADNIKA.

KR. SVEUČILIŠNA KNJIŽNICA U ZAGREBU

Zagreb.

Title page of Zahradník's lectures *On Planes and Curves in the Space. Summer Semester 1898*

of Science and Arts in Zagreb). While his work is still recognized and his name still well-known in Croatia[31], he is almost forgotten in Bohemia, though he cooperated with the Union of Czech Mathematicians until the end of his life.

3.2. Bosnia and Herzegovina

In 1893, **Alois Studnička**, a secondary school teacher of drawing and geometry and the brother of university professor of mathematics F.J. Studnička, went to Sarajevo. He was invited by the government of Bosnia and Herzegovina to help create an educational system for cabinet-makers, kettle-smiths, locksmiths and other trades. He became the director of the Crafts School which he headed until his retirement in 1908. In Sarajevo, where he worked until the end of his life, he elaborated the curricula for similar schools in Sarajevo, Mostar, Celovac (Klagenfurt) and Linz. He influenced significantly the development of the Serbian educational system and helped the birth of technical terminology in cabinet-maker trade, draughtsmanship and black-smith trade. His activity in this field contributed to the creation of the large collection of technical teaching aids for various crafts. This collection was deposited in the Vienna Technical Museum[32].

[31] His portrait was on diplomas granted by the Croatian Ministry of Culture and Sports to the best participants of the Mathematical Olympiad in 2000.

[32] As for his life see [14].

3.3. Bulgaria

In 1880s, Bulgaria got rid of Turkish hegemony and began to build its own educational system. Czech mathematician **Theodor Monin** spent a few years of his life there; he taught at the grammar school in Sliven in 1881–1886. He came back to the Czech Technical University in 1886 and became the assistant of František Tilšer (1825–1913), professor of descriptive geometry. However, in the next year the Bulgarian government called him to the new university in Sofia and he became the first Bulgarian university professor of mathematics. He started to develop "Bulgarian" mathematics with a great fervour, but unfortunately he fell seriously ill in 1891 and had to return to Bohemia. That is why he could not accomplish his plans to write several Bulgarian mathematical textbooks[33].

Professors of the grammar school in Sliven (1882) (T. Monin is in the second row, second from the left)

After completing his studies at the secondary technical school in Písek and at the Technical Universities in Vienna and Prague **Antonín Václav Šourek**, another Czech mathematician, became a professor of mathematics at the grammar school in Sliven in 1880. He spent only one school year there. Then he went over to the grammar school in Plovdiv, where he remained for 9 years. In 1890, he was promoted to the professorship of mathematics at the grammar school in Sofia and at the same time to the external professorship of mathematics at the Sofia University. In 1893, after the death of Theodor Monin, he was relieved from his duties at the above-mentioned secondary school and devoted all his time to the university, where he

[33] As for his life see [3–5, 7, 11, 16].

was appointed to the ordinary professorship in 1898 and where he stayed until 1914. In this period, namely in 1893, he also became a professor of descriptive geometry at the Military Academy in Sofia (he taught there for 9 years). In 1895, he started to give lectures on the same subject in the courses for the headquarters. In the years between 1895 and 1912, he lectured on perspective at the Academy of Painting in Sofia. His bad health forced him to leave Sofia and to move to Rome in 1914. There he became an unsalaried secretary of the military attaché. At the beginning of 1916, he went to Bern where he took care of Bulgarian war prisoners. He returned to the Sofia University in 1921 and continued to teach there until his death[34]. Since his arrival to Bulgaria he had contributed to the development of Bulgarian mathematics and its teaching at secondary schools and universities. He remained in close contact with Czech mathematicians and their Union and during his whole life tried to apply the Czech experience and connections to the development of Bulgarian mathematics and to the educational process at secondary schools as well as at universities.

Šourek's literary activity was very extensive. He published his first Bulgarian textbooks in 1880 and covered several branches of mathematics, namely plane trigonometry (1883)[35] and solid geometry (1883)[36], analytic geometry (1885)[37], spherical trigonometry (1889)[38] and descriptive geometry (1888, 1889)[39]. The textbooks were complemented by methodological annuals, collections of algebra exercises (1885, 1886)[40] and some smaller works. In the course of their writing, he was inspired by Czech textbooks written by F.J. Studnička, J. Smolík, E. Taftl, A. Strnad, F. Hromádko etc. Šourek also translated Studnička's logarithmic tables from Czech to Bulgarian and furnished them with a detailed explanation of the rudiments of algebra; they were published in 1882[41]. At the end of the 1890s, he also translated to Bulgarian Strnad's textbook *Geometrie pro vyšší třídy reálných gymnázií* (Geometry for Upper Classes of Grammar Schools) and Taftl's textbook *Algebra pro vyšší třídy středních škol* (Algebra for Upper Classes of Secondary Schools)[42].

---

[34] For more information see [3–5, 11, 15–17].

[35] Ch.G. Danov, Plovdiv, 1883, 128 pages, 54 pictures.

[36] Ch.G. Danov, Plovdiv, 1883, 123 pages, 116 pictures.

[37] Litographie, Plovdiv, 1885, IV + 154 pages, 250 pictures.

[38] Plovdiv, 1889, 97 pages, 49 pictures.

[39] First part, Plovdiv, 1888, IV + 237 pages, 367 pictures, 6 tablets; second part, Plovdiv, 1889, IV + 197 pages, 342 pictures, 11 tablets.

[40] Plovdiv, 1885, IV + 120 pages; Plovdiv, 1886, IV + 86 pages.

[41] The second edition of these tables is from 1888 and the third one from 1895. Studnička's tables (either in Czech or Bulgarian version) were used at Bulgarian secondary schools even in the first half of the 20th century.

[42] Alois Strnad (1852–1911) was a brilliant Czech secondary school teacher, an active member of the *Union of Czech Mathematicians* and an author of successful Czech textbooks for secondary schools. See A.B. Стърнад: *Геометрия за висшите класове на реалните гимназии* (A.V. Strnad: Geometry for Upper Classes of Grammar Schools), I.–IV. volume, Ch.G. Danov, Plovdiv, 1896, 161 pages, 122 pictures, 74 pages, 22 pictures, 96 pages, 32 pages.
Emanuel Taftl (1842–1920) was a secondary school professor of mathematics and physics. He taught at secondary schools in Hradec Králové and Klatovy. He became famous by the above textbook that had six editions. See E. Тафтл: *Алгебра за горните класове на гимназиалните училища* (E. Taftl: Algebra for Upper Classes of Secondary Schools), Ch.G. Danov, Plovdiv, 1899, 412 pages.

His teaching texts for his university students were written and published in the 1890s; they covered the field of analysis (1890/1891), analytic geometry (1891, 1892, and 1894), algebra (1891/1892), synthetic geometry (1891/1892) and descriptive geometry (1893/1894)[43]. Czech textbooks written by F.J. Studnička, Eduard and Emil Weyr certainly served as his inspiration. The Military Academy in Sofia published in 1895 Šourek's work about projection methods in geometry named *Учебник по начертателна геометрия. Част I. Ортогонална и котирана проекция* (Textbook on Descriptive Geometry. The First Part. Orthogonal and Orthogonal One-Plane Projection)[44]. At the beginning of the 20th century, Šourek decided to revise and extend his Bulgarian lectures and they were subsequently published in the lithographic form (projective geometry (1909), differential geometry (1911) and analytical geometry (1912, 1914))[45]. He also published the monograph *Учебник по дескриптивна геометрия* (Textbook on Descriptive Geometry)[46] (1914) that was an extended and complementary version of his university lectures. Unfortunately, he did not live sufficiently long to see his last monograph *Основи на проективната геометрия. Част перва: Проективност, колинеарност и реципроцитет на геометр. форми от трите разреда* (Elements of Projective Geometry. First Part. Projection, Colinearity and Reciprocity of Geometrical Figures of the Third Orders)[47] published in 1926 which summarised and extended his university lectures.

Šourek was one of the most renowned "Bulgarian" mathematicians between 1850 and 1930. He contributed significantly to the establishment of the *Физико-Математическото Дружество в София* (Physical and Mathematical Society in Sofia, founded 1898) and together with a few colleagues played an important role in its birth and in the development of its activities. He also helped in the foundation of the *Списание на Физико-Математическото Дружество в София* (The Journal of Physical and Mathematical Society in Sofia) in 1904. This journal stimulated the scientific activity of the younger Bulgarian generation and allowed its members to present their professional works. Šourek is also considered to be the founder of the Bulgarian terminology in descriptive

[43] *Лекции по алгебраичен анализ* (Lectures on Analysis), Plovdiv, 1891, IV + 288 pages, 21 pictures; *Аналитична геометрия на равнината заедно с криви линии* ... (Analytical Geometry in the Plane ...), Sofia, 1891, IV + 321 pages; *Аналитична геометрия на пространството* ... (Analytical Geometry in the Space ...), Sofia, 1892, 187 pages (second print, Sofie, 1894, VI + 334 pages); *Лекции по висша алгебра* ... (Lectures on Higher Algebra ...), Sofia, 1892, IV + 180 pages; *Лекции по синтетична геометрия* ... (Lectures on Synthetic Geometry ...), Sofia, 1892, IV + 238 pages; *Лекции по дескриптивна геометрия* ... (Lectures on Descriptive Geometry ...), Sofia, 1894, IV + 334 pages.

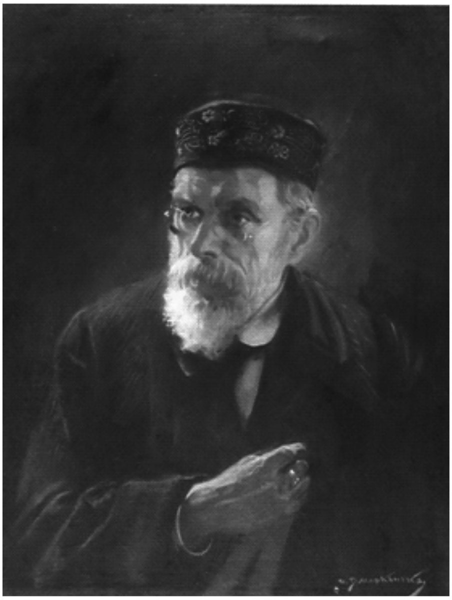
[44] Sofia, 1895, IX + 271 pages, 349 pictures and 69 pictures on the 12 tablets.

[45] *Проективна геометрия* ... (Projective Geometry ...), Sofia, 1909, 512 pages, 581 pictures; *Лекции по диференциална геометрия* ... (Lectures on Differential Geometry ...), Sofia, 1911, 317 pages; *Аналитична геометрия* ... (Analytic Geometry ...), Printed House I. Georgiev and K. Minkov, Sofia, 1912, IV + 93 pages, 49 pictures; *Лекции по диференциална геометрия* ... (Lectures on Differential Geometry ...), Sofia, 1914, 320 pages.

[46] Edition Univerzitní biblioteka, vol. 3, Printed House Sofia University, lexikographie, Sofia, 1914, XXIV + 616 pages, 846 pictures.

[47] Edition Univerzitní biblioteka, vol. 56, Printed House I.K. Božinov, Sofia, 1926, XVIII + 313 pages, 338 pictures.

geometry. Thanks to his good knowledge of Bulgarian and other languages (Czech, German, French, and Italian), his deep sense of syntax, close cooperation with philologists and above all to his perfect knowledge of descriptive geometry itself, he developed a very successful system of the essential terms with wide possibilities of a more detailed evolution. Thanks to his method and prestige among the members of the Bulgarian mathematical community, most of his terms are still used without any change or at most with only small modifications.



Antonín Václav Šourek

Q 1949

УЧЕБНИКЪ

по

НАЧЪРТАТЕЛНА ГЕОМЕТРИЯ

Съставилъ

АНТ. В. ШОУРЕКЪ,

I.228

I.

ОРТОГОНАЛНА И КОТИРАНА ПРОЕКЦИЯ.

СОФИЯ

1895.

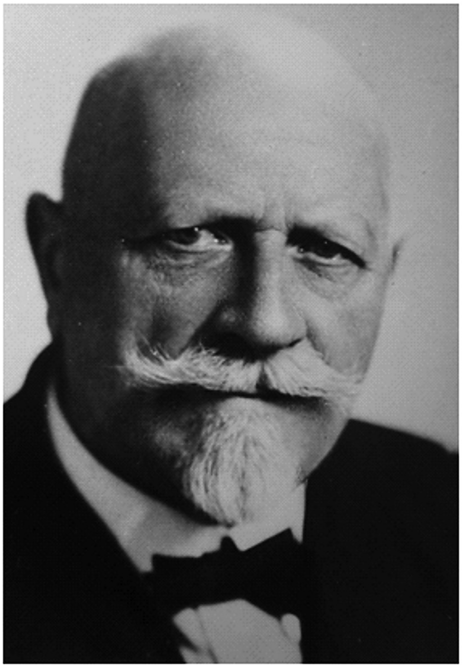
Title page of Šourek's *Textbook on Descriptive Geometry* (1895)

Bulgaria was a place of work also for Czech mathematician **František Vítězslav Splítek**. After his graduation at the Czech Technical University in Prague in 1880, he accepted an offer from the Bulgarian Ministry of Education to help in the development of Bulgarian secondary schools. Firstly, he taught in Svistov. In 1883, he became a teacher in Salonica (today Thessalonica in Greece), but he had to leave his position for political reasons. For Bulgarian students in Greece, he wrote two mathematical textbooks named *Аритметика* (Arithmetic) (Plovdiv, 1885) and *Геометрия с чртание в четире степени* ... (Geometry with Drawing at Four Levels. The First Level. Geometric Figures in the Plane and Their Ornamental Drawing) (Plovdiv, 1886)[48]. In 1888, he returned to Bulgaria and became a professor at the grammar school in Sofia. He also taught at the grammar school in Gabrovo (between 1888 and 1889) and at the state secondary school in Plovdiv (between 1891 and 1915). He rejected the proposed professorship at the Sofia University because he thought that he was not sufficiently qualified for it.

[48] E. Dionne, Plovdiv, 1885, 1 tablet; E. Dionne, Plovdiv, 1886, 106 pages + 163 pictures.

Splítek wrote very successful and popular textbooks on technical drawing for the students of the lower classes of Bulgarian secondary schools (*Руководство по геометрическо чертание* (Instruction for Geometric Drawing) (Plovdiv, 1895), *Геометрия с геометрическо чертание за основните училища* (Geometry with Drawing for Primary Schools) (Plovdiv, 1895), *Учебник по геометрия и геометрическо чертание. I. степен* (Textbook on Geometry and Drawing. The First Level) (Plovdiv, 1896) and *Учебник по геометрия и геометрическо чертание. II. степен* (Textbook on Geometry and Geometric Drawing. The Second Level) (Plovdiv, 1897)).

Splítek's pedagogical and cultural activities outside the school in Svistov and Plovdiv were known and popular. He founded two special associations, which joined teachers from primary and secondary schools as well as people from different cultural and political spheres. Thanks to his activities, a new Bulgarian journal for pedagogy, education and school problems and laws was founded[49].



František Vítězslav Splítek

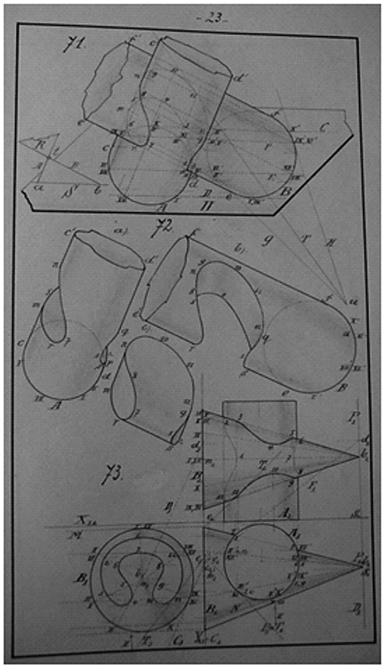


Table 23 from Splítek's manuscript *Descriptive Geometry* (The private family archive of the Splítek – Lukáš, Sofia)

The educational system at Bulgarian secondary schools was influenced significantly also by **Vladislav Šak**, Czech mathematician and geometer. He obtained an ordinary professorship at the grammar school in Sliven in 1882. In 1886, he moved to the grammar

[49] For more information see [3–5, 8, 11, 18].

school in Sofia and taught there until 1907. He was also a private docent at the Sofia University between 1891 and 1894. He lectured on spherical and analytic geometry, analysis and algebra. Finally, he was professor of mathematics at the Sofia University for the 1907/1908 school year. Then he came back to Prague and started to teach mathematics and Bulgarian language at the Czechoslovak School of Commerce. During the first Balkan War, he was a war reporter in Bulgaria. The Austrian police held him in prison between 1916 and 1917 because of his cooperation with Tomáš Garrigue Masaryk (1850–1937) and Edvard Beneš (1884–1948), later Czech presidents. After the war he held important functions in Bulgarian diplomacy. He was an honorary consul (between 1920 and 1922) and a general consul of the Bulgarian Kingdom (between 1922 and 1932)[50].

БЪЛГАРСКО-ЧЕХСКА
КОНВЕРЗАЦИЯ

СЪСТАВИЛЪ

ВЛАДИСЛАВЪ ШАКЪ,

бившъ професоръ при университета въ София, учитель по български езикъ при чехско-славянската търговска академия въ Прага.

ПРАГА
ИЗДАТЕЛЬ ЙОС. Р. ВИЛИМЕКЪ
Клонъ: Виена, Шотенрингъ 1.

Title page of Šak's *Bulgarian-Czech Conversation* (1914)

V. Šak (second row, third from the left) and the first graduates of the grammar school in Sliven (1884)

Šak translated two Czech textbooks to the Bulgarian – *Algebra pro I., II. a III. třídu reálných gymnázií a trojtřídní měšťanské školy* (Algebra. Textbook for 1st, 2nd and 3rd Classes of Grammar Schools) (Plovdiv, 1886)[51] written by Václav Starý and *Deskriptivní geometrie pro vyšší třídy reálných gymnázií* (Descriptive Geometry for Upper Classes of Secondary Schools) (Plovdiv, 1895)[52] written by Čeněk Jarolímek. They were used at Bulgarian secondary schools until the World War I. In addition, he wrote one of the first Bulgarian articles named *Няколко думи верху изучаването по дескритивната геометия* (Some Thoughts of Teaching Descriptive Geometry) (1897/1898) dealing with

[50] For more information see [3–5, 11, 16].
[51] Translators: V. Šak and T.P. Šiškov.
[52] Translators: V. Šak and T.P. Šiškov.

the methodology of teaching descriptive geometry[53]. He had a wide range of interests – he wrote poems, libretti, short stories, feuilletons and critical articles about the state of Bulgarian politics and economy. He also issued Bulgarian-Czech and Czech-Bulgarian Dictionaries and Bulgarian Grammar in Czech language for Czech students[54]. For Czech readers, he translated the works of Bulgarian writers and poets[55].

It should be noted that at the end of the 1870s and the beginning of the 1880s a lot of Czech engineers, doctors, teachers, natural scientists, lawyers and even artists went to Bulgaria. They participated there in the building of the new Bulgaria that did its best to free itself from Turkish influence and approached European traditions[56].

4. Conclusion

The Czech mathematical community that was formed and kept developing since the middle of the 19th century was able to export its successful and versatile activities out of the Czech territory, particularly to the Balkans where the nationalistic movements began with a delay of about twenty years. As we have described, the Czech mathematicians played an important role in the development of the "national" mathematical communities, scientific societies, and educational systems.

References

[1] J. Bečvář (ed.), *Jan Vilém Pexider*, edition Dějiny matematiky, volume 5, Prometheus, Praha 1997.

[2] M. Bečvářová, J. Čižmár, *Karel Zahradník (1848–1916). Praha – Záhřeb – Brno*, edition Dějiny matematiky, volume 46, Matfyzpress, Praha 2011.

---

[53] Отчет на Софийската държавна мъжка гимназия за учебната 1897–1898 година. Втора година (Second Report on the Sofia State Grammar School for Boys in the School Year 1897/1898), Sofia, 1898, pp. 40–56.

[54] *Българско-чехски речник* (Bulgarian-Czech Dictionary), J. Otto, Praha, 1911–1914, 647 pages; *Чехско-български речник* (Czech-Bulgarian Dictionary), J. Otto, Praha, 1911–1914, 1071 pages; *Kniha bulharsko-české konverzace – Българско-чехска конверзация* (Bulgarian-Czech Conversation), J.R. Vilímek, Praha, 1914, 325 pages.

[55] For more information see [4].

[56] For more information on the Czech-Bulgarian relations see for example *Česko-bulharské kulturní vztahy v době obrození*, Práce z dějin slavistiky, volume 14, Praha, 1990; D. Grigorov, M. Černý (eds.), *Úloha české inteligence ve společenském životě Bulharska po jeho osvobození*, Velvyslanectví Bulharské republiky v České republice, Praha, 2008; D. Hronková, *Kapitoly z minulosti česko-bulharských vztahů I., II.*, Praha, 2005, 2007; J. Rychlík, *Dějiny Bulharska*, Lidové noviny, Praha, 2000; V. Todorov, *Úvod do bulharistiky. Průvodce po dějinách česko-bulharských vztahů*, Praha, 1991; J. Tomeš, *Co daly naše země Evropě a lidstvu*, volume 2, second edition, Evropský literární klub, Praha, 1999. For those interested in a deeper study of the development of Czech-Bulgarian relations, Bulgarian literature can be recommended, especially that which originated in the last twenty years and which analyzed the share of Czech intelligence to restore the Bulgarian country after its liberation from Turkish rule.

[3] M. Bečvářová, *Czech Mathematicians and Their Role in the Development of National Mathematics in the Balkans*, [in:] M. Bečvářová, Ch. Binder (eds.), *Mathematics in the Austrian--Hungarian Empire. Proceedings of a Symposium held in Budapest on August 1, 2009 during the XXIII ICHST*, edition History of Mathematics, volume 41, Matfyzpress, Prague 2010 (second edition, Matfyzpress, Prague 2011), pp. 9-31.
[4] M. Bečvářová, *České kořeny bulharské matematiky*, edition Dějiny matematiky, volume 40, Matfyzpress, Praha 2009.
[5] M. Bečvářová, *Česká matematická komunita v letech 1848–1918*, edition Dějiny matematiky, volume 34, Matfyzpress, Praha 2008.
[6] M. Bečvářová, *Archimédovy práce česky*, [in] J. Bečvář, M. Bečvářová (eds.), *29. mezinárodní konference Historie matematiky*, Velké Meziříčí 22.08–26.08.2008, Matfyzpress, Praha 2008, pp. 92-102.
[7] M. Bečvářová, *Teodor Monin (1855–1893) – první vysokoškolský profesor matematiky v Bulharsku*, Homo Bohemicus, Izdanie na Bochemija klub, volume 1, 2008, pp. 5-14.
[8] M. Bečvářová, *František Vítězslav Splítek (1855–1943) – zapomenutý učitel, matematik, etnograf a spisovatel*, [in:] D. Grigorov, M. Černý (eds.), *Úloha české inteligence ve společenském životě Bulharska po jeho osvobození*, Velvyslanectví Bulharské republiky v České republice, Praha 2008, pp. 185-201.
[9] M. Bečvářová, *Život i djelo Karela Zahradníka*, [in:] S. Mardešić (ed.), *Karel Zahradník 1848–1916*, Hrvatska akademija znanosti i umjetnosti. Spomenica preminulim akademicima, svezak 134, Zagreb 2007, pp. 9-36.
[10] M. Bečvářová, *Life and Work of Karel Zahradník (1848–1916)*, [in:] T. Motlíček, Rechcígl M. (eds.), *Moravia from World Perspective*, 22th World Congress of Czechoslovak Society of Arts and Sciences, volume 2, Ostrava, Repronis, 2006, pp. 276-283.
[11] M. Bečvářová, *Kořeny bulharské matematiky*, [in:] M. Bečvářová (ed.), *27. mezinárodní konference Historie matematiky*, Velké Meziříčí, 25.08.–29.08.2006, sborník sylabů, Praha 2006, pp. 14-16.
[12] M. Bečvářová, *Eukleidovy Základy, jejich vydání a překlady*, edition Dějiny matematiky, volume 20, Prometheus, Praha 2002.
[13] M. Bečvářová, *Z historie Jednoty (1862–1869)*, edition Dějiny matematiky, volume 13, Prometheus, Praha 1999.
[14] M. Bečvářová-Němcová, *František Josef Studnička (1836–1903)*, edition Dějiny matematiky, volume 10, Prometheus, Praha 1998.
[15] B. Dolapčiev, *A.V. Šourek (1857–1926)*, Pokroky matematiky, fyziky a astronomie, volume 8, 1963, pp. 168-170.
[16] S. Hineva, I. Tzenova, *Čeští geometři na sofijské univerzitě*, Pokroky matematiky, fyziky a astronomie, volume 36, 1991, pp. 237-242.
[17] J. Sobotka, *Vzpomínka na Antonína V. Šourka*, Časopis pro pěstování mathematiky a fysiky, volume 56, 1927, pp. 1-6.
[18] J. Šedivý, *Učitel F.V. Splítek*, Matematika a fyzika ve škole, volume 13, 1982/1983, pp. 69-70.

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE

1-NP/2014

DANUTA CIESIELSKA*, STANISŁAW DOMORADZKI**

ON MATHEMATICAL LECTURES AT THE JAGIELLONIAN UNIVERSITY IN THE YEARS 1860–1918. ESSAY BASED ON MANUSCRIPTS

O MATEMATYCZNYCH WYKŁADACH W UNIWERSYTECIE JAGIELLOŃSKIM W LATACH 1860–1918. OPRACOWANIE NA PODSTAWIE RĘKOPISÓW

A b s t r a c t

This article is a partial report of research on mathematical education at the Jagiellonian University in the period 1860–1945. We give a description of the selected lectures: *Calculus of Probability* by Michał Karliński, *Analytic Geometry* by Franciszek Mertens, Marian Baraniecki's lectures, *Higher seminar* (*Weierstrass preparation theorem*) by Kazimierz Żorawski, *Principles of Set theory* by Zaremba and *Analytic function* and *Number theory* by Jan Sleszyński. Moreover, short biographical notes of professors of mathematics of the Jagiellonian University Michał Karliński, Franciszek Mertens, Marian Baraniecki, Stanisław Kępiński, Kazimierz Żorawski, Stanisław Zaremba and Jan Sleszyński – are given.

Keywords: *mathematics in Cracow in XIX century and the beginning XX century*

S t r e s z c z e n i e

Niniejszy artykuł jest fragmentem badań autorów w zakresie historii matematycznej edukacji w Uniwersytecie Jagiellońskim w latach 1860–1945. Prezentujemy opis wybranych wykładów: Michała Karlińskiego *Rachunek prawdopodobieństwa*, Franciszka Mertensa *Geometrię analityczną*, informacje o wykładach Mariana Baranieckiego, Kazimierza Żorawskiego zajęcia w wyższym seminarium na temat twierdzenia przygotowawczego Weierstrassa, Stanisława Zaremby *Wstęp do teorii mnogości* oraz Jana Sleszyńskiego *Teorię funkcji* (analitycznych) i *Teorię liczb*. Prezentujemy również krótkie biografie profesorów matematyki UJ: Michała Karlińskiego, Franciszka Mertensa, Mariana Baranieckiego, Stanisława Kępińskiego, Kazimierza Żorawskiego, Stanisława Zaremby oraz Jana Sleszyńskiego.

Słowa kluczowe: *matematyka w Krakowie w XIX i na początku XX w.*

* Danuta Ciesielska, Institute of Mathematics, Faculty of Mathematics, Physics and Technical Science, Pedagogical University of Cracow.

** Stanisław Domoradzki, Faculty of Mathematics and Natural Sciences, University of Rzeszów.

1. Historical background

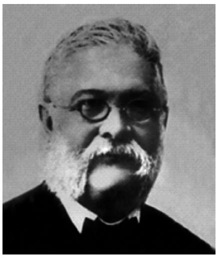
From 1846 to 1918 Kraków was a city in the Austro-Hungarian Empire, situated in the Kingdom of Galicia and Lodomeria. In 1861 Galicia got autonomy; the National Parliament and local government were formed in the capital city Lvov (Lemberg). The December Constitution of 1867 guaranteed the right to instruction in Polish at the universities (see: [5]). In the Kingdom of Galicia and Lodomeria Polish cultural, artistic and scientific life developed. There were four universities: the Jagiellonian University in Kraków, Lvov University, Lvov Polytechnic and for a time the University in Czernowitz (later Bukovina Duchy). For further studies see [14] and [19].

2. From 1860 to 1895

2.1. Mathematical staff in the period 1860–1895

In that period the staff of the Philosophical Faculty at the Jagiellonian University was small and there were only a few professors who published original mathematical papers: Jan Kanty Steczkowski[1], Franciszek Mertens and Marian Baraniecki. As a result, the number and scope of mathematical lectures were very limited. Michał Karliński lectured on classical calculus, calculus of variation, probability and mathematical geography, Franciszek Mertens on analytic geometry, trigonometry and algebraic equations), Marian Baraniecki on theory of determinants, algebraic equations, geometry and Number Theory), Ludwik Birkenmajer[2] gave lectures on the history of mathematics. For detailed studies see: [19] and [20].

2.2. Calculus of Probability by Michał Karliński



Franciszek Michał Karliński (1830–1906) was appointed to the Chair of astronomy and mathematics at the Jagiellonian University in 1862. Karliński worked primarily on observational astronomy, and – when the eye disease made celestial observations impossible – on meteorology. To Karliński's lecture duties belonged both astronomy lectures, as well as higher mathematics. After that lectures were separated between specialists. In addition, in 1874–1877 (untill the foundation of the Chair of Geography at the University). Karliński taught mathematical geography and theory of geographical maps. Karliński's surviving notes show, among others, that in the nineteenth century Poland was familiar with world literature in probability. In his notes Karliński mentioned the works of such masters as Jacob Bernoulli (*Ars conjectandi* 1713), A. de Moivre, Nicholas, Daniel and Johann Bernoulli, J. d'Alembert, J. Lagrange, P.S. Laplace (*Théorie*

---

1 Jan Kanty Steczkowski (1807–1872), a mathematician, professor of the Jagiellonian University, the author of a series of mathematical books for students published by Scientific Society of Kraków (Towarzystwo Naukowe Krakowskie: 1815–1872).

2 Ludwik Antoni Birkenmajer (1855–1929), a historian of sciences, mathematician, physicist, astronomer and professor of the Jagiellonian University, a member of Polish Academy of Arts and Sciences.

analitique des probabilities 1812). In the preliminary remarks to the lecture Karliński wrote: "When, in studying the truth, gaps or breaks occur as a result of our ignorance so that it is impossible to find out what is in contradiction to the acknowledged truth, whereas we cannot prove the total compatibility of what we already know with what we want to know or investigate, then, instead of essential truth we have probability".

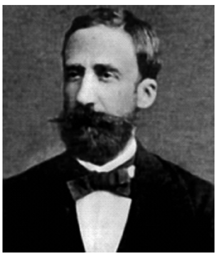
Karliński classified probability as:

- *Philosophical* – "it occurs when we conclude the uniformity of rule from a multitude of cases",
- *Aesthetic* – "in the fine arts, it consists in our ability to consider as true and real the thing which is presented by the artist, either as complete, or as happening in accordance to the assumptions made by him and to the fundamental conditions of art, or more succinctly, in comparison of what artist tells us with what experience has taught us".
- *Mathematical* – "it is the relationship that exists between the number of cases favorable to some event, or, as it is commonly said, of chances, and the number of all probable cases, however under the assumption that all cases are equally possible".

In Karliński's lecture one can find thoughts about axiomatic definition of probability terms: "probability of the favorable effect is $v = \frac{a}{a+b}$, while the probability of unfavorable effect is $v_1 = \frac{b}{a+b}$. Both these probabilities are fractions whose sum $v + v_1 = \frac{a+b}{a+b} = 1$. Unity is the symbol of certainty ..." (a is the number of favorable effects, b – unfavorable).

Karliński gave a lot of historical information, solved problems about playing cards and dice, and about drawing lottery balls from an urn. His manuscript deserves attention because of the mathematical terminology. For each term Karliński gave a Polish, Latin, French and German name, and noted some historical facts, such as the name of an inventor, the time of introduction and the origin of the introduced term. The second part of the lecture was devoted to the applications of probability theory. "According to already presented rules for calculating the direct probability – writes Karliński – we are able to calculate in advance the benefits and losses connected with uncertain accidents of any kind". He discussed issues concerning lottery, and devoted much attention to the mathematical concept of expectation.

2.3. *Analytic Geometry* by Franciszek Mertens



Franciszek Mertens (1840–1927) was born in Środa (Prussian Emporium, now Poland) and died in Vienna. Mertens studied at the University of Berlin where he attended lectures by Weierstrass, Kronecker and Kummer. In 1865 he obtained his doctorate with a dissertation *De functione potentiali duarum ellipsoidium homogenearum*; his advisors were Kummer and Kronecker. He worked first at the Jagiellonian University in Kraków from 1865 to 1884. He moved to the Polytechnic in Graz, and in 1894 he became an ordinary professor of mathematics at the University of Vienna. Mertens worked on a number of different topics including potential theory, geometrical applications of determinants, algebra and analytic number theory. He published 126 papers but is probably best known for the conjecture on the Möbius function (so-called Mertens' conjecture). Since a proof of it would have implied the truth of the Riemann hypothesis, many mathematicians attempted to prove Mertens' conjecture, but in 1985 Andrew Odlyzko and Herman te'Rielee disproved it.

In 1880's Franciszek Mertens lectured on the theory of determinants and projective geometry at the University, though the lecture was entitled *Analytic geometry*. Some copies of the lithographed notes from this lecture survived. One copy is a property of the Library of the Faculty of Mathematics and Computer Sciences of the Jagiellonian University (Bibl. Inst. Matem. U.J. sygn. 216, L. inw. 234). In 1880's this copy was a property of the Mathematical Seminar in Kraków. Another copy of the book is available at the Jagiellonian Library (see: [16]). The notes were taken by Leon Wątorski, a member of the Mathematical and Physic Society of Studensts' of the Jagiellonian University and the lithographed copies were produced by J. Pacanowski. The book has 462 pages

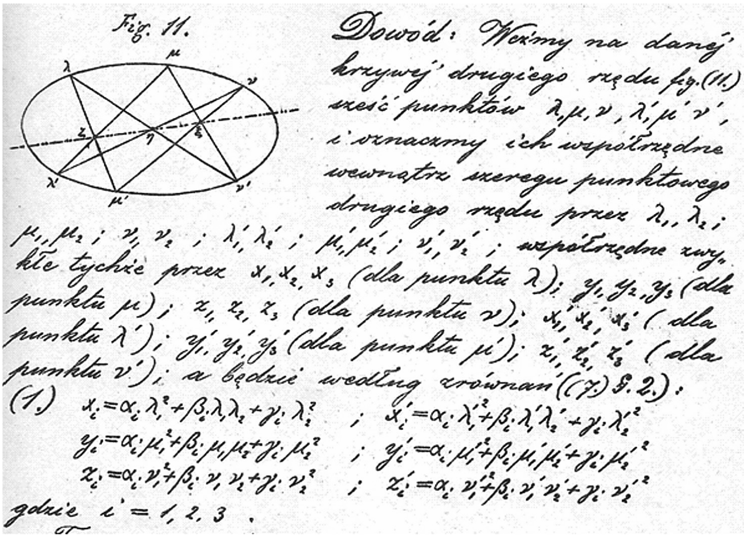


Fig. 11.

Dowód: Weźmy na danej krzywej drugiego rzędu fig.(11.) sześć punktów $\lambda, \mu, \nu, \lambda', \mu', \nu'$, i oznaczmy ich współrzędne wewnątrz szeregu punktowego drugiego rzędu przez λ_1, λ_2; μ_1, μ_2; ν_1, ν_2; λ_1', λ_2'; μ_1', μ_2'; ν_1', ν_2'; współrzędne zwykłe tychże przez x_1, x_2, x_3 (dla punktu λ); y_1, y_2, y_3 (dla punktu μ); z_1, z_2, z_3 (dla punktu ν); x_1', x_2', x_3' (dla punktu λ'); y_1', y_2', y_3' (dla punktu μ'); z_1', z_2', z_3' (dla punktu ν'); a będzie według zrównań ((7) §. 2.):

$$(1)\quad \begin{aligned} x_i &= \alpha_i \lambda_1^2 + \beta_i \lambda_1 \lambda_2 + \gamma_i \lambda_2^2 \;; & x_i' &= \alpha_i \lambda_1'^2 + \beta_i \lambda_1' \lambda_2' + \gamma_i \lambda_2'^2 \\ y_i &= \alpha_i \mu_1^2 + \beta_i \mu_1 \mu_2 + \gamma_i \mu_2^2 \;; & y_i' &= \alpha_i \mu_1'^2 + \beta_i \mu_1' \mu_2' + \gamma_i \mu_2'^2 \\ z_i &= \alpha_i \nu_1^2 + \beta_i \nu_1 \nu_2 + \gamma_i \nu_2^2 \;; & z_i' &= \alpha_i \nu_1'^2 + \beta_i \nu_1' \nu_2' + \gamma_i \nu_2'^2 \end{aligned}$$

gdzie $i = 1, 2, 3$.

Fig. 1. Proof of Pascal Theorem by Mertens

and consists of 12 chapters. The notes start with foundations of the theory of determinants and its application to solving algebraic equations in many variables. Mertens gave a definition of determinant and its analytic (Laplace) form, basic applications (e.g. Cramer's Rule and the Kronecker-Capelli-(Rouché-Fontené-Frobenius) Rule). In the second part of the notes the theory of algebraic curves is discussed. Mertens introduced projective coordinates in both forms: Plücker's and Möbius's homogeneous coordinates[3]. He also

[3] Homogeneous coordinates were introduced by August Ferdinand Möbius in his 1827 work *Der barycentrische Calcül*. Julius Plücker in 1832 suggested to assign six homogeneous coordinates, for the homogeneous coordinates of two points in projective 3-space, as determinants. A generalization of this methods is called Plücker embedding. Homogeneous coordinates can be used to locate the point of intersection of two algebraic curves. Władysław Zajączkowski in early

gave fundamentals of the theory of quadratic forms and classification of curves of degree 2. Mertens presented the fundamental theorem of projective geometry and the theory ended with: Desargues theorem, Pascal hexagon theorem and Brianchon's theorem. All theorems were completely discussed and proved.

In 1880's Mertens introduced in Kraków the basics of Felix Klein's Erlangen Program. In 1872 Klein proposed to focus on projective geometry and group theory to produce completely new characterization of geometry. Klein in his consideration, emphasized projective geometry as a unified way of looking at various geometries and Mertens was following his idea, presenting the projective space as fundamental for any geometry. For the detailed studies on the perception of analytic geometry[4] in Poland see [13] and for the presentation of the most comprehensive Polish lecture on projective space in XIX century see [8].

2.4. Marian Baraniecki and his lectures

Marian Alexander Baraniecki (1848–1895) attended a gymnasium in Warsaw and studied mathematics at the University in Warsaw (Szkoła Główna Warszawska) since 1865. He graduated from the Imperial University of Warsaw[5] in 1870 and he obtained a master's degree in mathematical and physical sciences. Then he studied at the Jagiellonian University in Kraków and in the Leipzig University, where in 1871 he obtained a PhD on the basis of the dissertation *Über die gegeneinander permutable Susbstitutionen*. Three years later, after further studies in St. Petersburg and Moscow, he obtained a master's degree in pure mathematics on the basis of a dissertation about hypergeometric functions (for a career, this degree meant more than PhD but less than habilitation). Baraniecki was a teacher in high schools in Warsaw and between 1876 and 1885 he was a reader at the Imperial University of Warsaw. In 1885, he was offered a job at the Jagiellonian University and at the Polytechnic School in Lvov. He chose Jagiellonian University, where he headed the Mertens' Chair. Baraniecki published several papers on algebra and the theory of functions in *Memoir of the Society of Sciences in Paris* (*Pamiętniki Towarzystwa Nauk Ścisłych w Paryżu*) and publications of Academy of Arts and Sciences. His book on theory of determinants, published in 1879 in Paris by the Kórnik Library, and the lithographed *Course of algebraic analysis (1879–1880) and The initial synthetic lecture of properties of conic*, were a basis for major Polish university-level courses on the theory of determinants. Baraniecki distinguished himself as one of the founders of the Mathematical and Physical Library – a publishing series founded in Warsaw in 1884 with support of Mianowski Fund. The series was directed at readers at various levels of education, up to university level.

1880's presented at the Lvov University in his lecture on analytic geometry a proof of Bézout's theorem on algebraic curves involving Plucker coordinates.

4 Unfortunately J. Dianni, the author of the paper on perception on analytic geometry in Poland, did not distinguish between Cartesian and projective methods in geometry. She discussed them jointly, to some disadvantage for the latter.

5 The Warsaw University (Szkoła Główna Warszawska) was closed in 1869 by the tsarist authorities as an act of repression after January Uprising. In 1870 the Imperial Warsaw University was founded.

In Kraków Baraniecki taught algebra and geometry and gave courses in mathematics for naturalists. He also had classes in the lower seminar for mathematician. The scope of his lectures and classes was related with the theory of determinants, set theory, algebraic equations and numerical theory of functions of a complex variable and periodic functions analytic geometry and synthetic theory of surfaces and lines of double curvature, the study of invariants and coefficients of double and triple forms. He died at the age of 46 years. His scientific and teaching activity and efforts to elevate the mathematical culture are not adequately appreciated.

The successor of Baraniecki at the Chair was Stanisław Kępiński (1867–1908), who studied mathematics and physics at the Faculty of Philosophy of the Jagiellonian University in the years 1885–1889. In 1896 he was appointed as an extraordinary professor of mathematics at the Jagiellonian University, and in 1899 he became the Dean of the Faculty of Mathematics in Lvov Polytechnic School.

3. From 1895 to 1918

3.1. Staff and regular lectures

In 1895 a new era of mathematics at the Jagiellonian University began. Prominent Polish mathematicians started their academic career and brought to the university modern mathematics. Michał Karliński continued lecturing on *Calculus* and *Spherical trigonometry*. Kazimierz Żorawski, who started his career at the Jagiellonian University in 1895, lectured on: *Higher mathematics*, *Differential geometry*, *Analytic geometry*, *Elementary geometry*, *Curves and surfaces*. He had also a special class for students, called a higher seminar. Stanisław Zaremba, who arrived to Kraków in 1900, lectured on: *Higher algebra*, *Projective geometry*, *Calculus*, *Analytic function*, *Differential equations* and *Analytic geometry,* and held some seminars for students. Maurycy Pius Rudzki[6] lectured on *Mechanics*, Cezary Russjan[7] on *Ordinary differential equations*, *Partial differential equations*, *Calculus of variations* and *Projective geometry*. At the university there were some special lectures founded by a private foundation: Ludwik Birkenmajer's *History of mathematics,* Antoni Hoborski's[8] *Calculus* and Alfred Rosenblatt's[9] *Curves and Surfaces of Second Degree*. Also Jan Sleszyński, who had

---

[6] Maurycy Pius Rudzki (1862 Uhryńkowce, now Ukraine – 1916 Kraków), an astronomer and geophysicist. He studied in Lvov, Vienna and Odessa. He obtained PhD at the Vienna University and Odessa University. A professor of the Jagiellonian University, member of Polish Arts and Sciences in Kraków.

[7] Cezary Russjan (1867 Makieievo, now Ukraine – 1937 Charkov), a mathematician. He studied in Kiev and Odessa, where he graduated in mathematics. He also studied in Berlin, Paris and Leipzig. He was a docent of the Odessa University, professor of the Jagiellonian University, Lvov Polytechnic and Charkov University.

[8] Antoni Hoborski (1879 Tarnów – 1941 Sachsenhausen), a professor of mathematics at the Mining Academy in Kraków and Jagiellonian University. Rector of the Mining Academy in Kraków.

[9] Alfred Rosenblatt (1882 Kraków – 1946 Lima), a professor of mathematics at the Jagiellonian University and St. Marc University in Lima, a member of *Academia Nacional de Ciencias Exactas, Físicas y Naturales del Perú.*

many lectures: *Number theory*, *Analytic function*, *Probability*, *High algebra*, *Methodology of mathematics*, *Determinants*, was a beneficiary of the foundation (see [9]). For detailed study see: [19] and [20].

3.2. Special lectures

In 1900 Stanisław Zaremba joined the university and started his academic career presenting the modern methods of mathematics in a special lecture: *On the Dirichlet Boundary Condition and Related Problems*. The idea of the presentation of modern mathematics was continued and Kazimierz Żorawski, Jan Sleszyński, Alfred Rosenblatt and Antoni Hoborski contributed to this project. Many of this lectures were sponsored by a private foundation. Each professor presented two lectures: Kazimierz Żorawski – *Curves and Surfaces* (1909, 1910); *Kinematics*[10] (1911), Stanisław Zaremba – a very important lecture *Principles of the Sets Theory* (1912) and *Theoretical Physics* (1915) and Jan Sleszyński – *Mathematical Logic* (1913) and *Probability* (1912). Alfred Rosenblatt, a private docent with no chair, presented a lecture entitled *Algebraic curves*[11] (1913).

In the Jagiellonian Library there is the legacy of Aleksander Birkenmajer[12]. In this collection there are many handwritten notes of mathematical, physical and other lectures, and among them there are notebooks entitled: *Principles of Set Theory* by Stanisław Zaremba (see: [1]), *Number theory* by Jan Sleszyński (see: [2]), *Analytic Function* by Jan Sleszyński (see: [3]) and *Higher Seminar in mathematics* by Kazimierz Żorawski (see: [4]).

3.3. *Weierstrass preparation theorem* by Kazimierz Żorawski

Kazimierz Żorawski (1866−1953) was born in Szczurzyn near Ciechanów. After graduation from a classical gymnasium in Warsaw and four years of study at the Imperial University of Warsaw, Żorawski graduated in 1888 with a first degree in mathematics. He continued his study in Leipzig, where he was a student of Lie, and in Götingen. Żorawski obtained a PhD in mathematics from the Leipzig University. In 1892 he moved to Galicia and was appointed as a lecturer at the Lvov Polytechnic. A year later he got habilitation at the Jagiellonian University. In 1895 he obtained the I Chair of mathematics at the Jagiellonian University. In 1917 Żorawski was a Rector of the Jagiellonian Univeristy. Two years later he moved to Warsaw, to work in the Ministry of Education, Warsaw Polytechnic and Warsaw University. He was a member of the Polish Academy of Arts and Sciences. Żorawski wrote more than 70 papers, mostly

[10] This was a special lecture meant for broad audience.

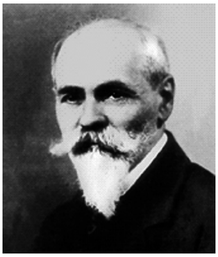
[11] In that period, Rosenblatt was interested in algebraic geometry. He published about 200-page monograph on algebraic surfaces in 1912 and 28 papers on algebraic geometry before 1929.

[12] Aleksander Ludwik Birkenmajer (1890–1976), a historian of exact sciences and philosophy, biblilogist, professor of the Jagiellonian University and Warsaw University, an expert in the field of research of Copernicus. A son of Ludwik Birkenmajer and a grandson of Franciszek Michał Karliński. He was a member of Polish Academy of Arts and Sciences in Kraków and of Royal Historical Society in London.

in real and complex analytic and differential geometry, differential equations, kinematics and the theory of continuous groups of transformations.

The Weierstrass preparation theorem states that an analytic function of several complex variables in an open neighborhood of a point factors into a monic polynomial in one (fixed) variable whose coefficients are analytic function of the remaining variables, and a function not vanishing in the neighborhood. This theorem, until mid-1930s, was a fundamental tool for investigating a function at singular points. About 1912, Żorawski presented the Weierstrass preparation theorem[13] to the students of the Faculty in the classes of higher mathematical seminar. He also sketched a proof of the theorem and discussed the type of singularity of selected points and functions using the theorem.

3.4. *Principles of Set theory* by Zaremba



Stanisław Zaremba (1863−1942) was born in Romanówka (Austro-Hungarian Empire, now Ukraine) and died in Kraków. He attended a German gymnasium in St. Petersburg. In 1886 he graduated from the Institute of Technology in St. Petersburg. In 1887 he went to Paris, where he studied mathematics. He obtained a doctorate from Sorbonne in 1889 on the basis of the thesis *Sur un problème concernant l'état calorifique d'un corps solide homogène indéfini*, advisored by Darboux and Picard. Zaremba stayed in France until 1900, making many contacts with mathematicians of the French school at that time; publishing his results in French mathematical journals made him well known and highly respected by leading French mathematicians such as Poincarè and Hadamard. In 1900 Zaremba returned to Poland, where he was appointed to the Chair in the Jagiellonian University. Much of Zaremba's research was in partial differential equations and potential theory. He also made major contributions to mathematical physics and crystallography. He studied elliptic equations and he contributed mostly to the Dirichlet principle. Zaremba wrote more than 100 papers and 7 books[14].

Principles of Set Theory (see: [1]). The lecture took place in the summer semester of the academic year 1910/1911, on Saturdays from 8 to 9 am (see: [18]). Let us recall that

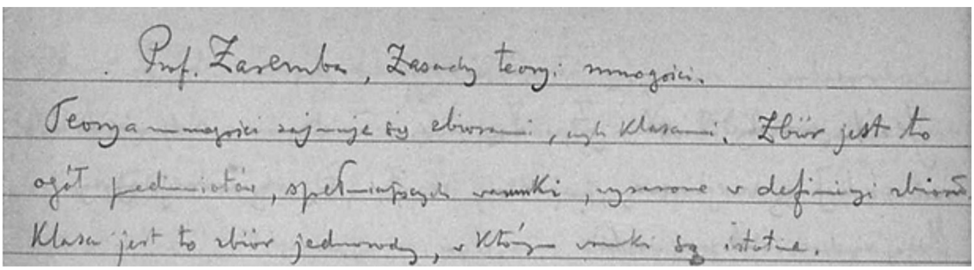
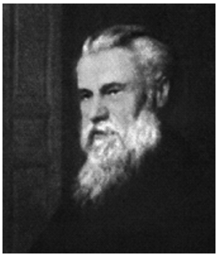

Prof. Zaremba, Zasady teoryi mnogości.
Teorya mnogości zajmuje się zbiorami, czyli klasami. Zbiór jest to
ogół jednostek, spełniających warunki, wyrażone w definicyi zbioru.
Klasa jest to zbiór jednorodny, w którym warunki są istotne.

Fig. 2. An extract from the *Principles of the Sets Theory* by Zaremba

[13] The notes of the student A. Birkenmajer entitled *Higher seminar with prof. Żorawski*.
[14] The is no complete bibliography of Zaremba.

in the academic year 1909/1910 in Lvov Wacław Sierpiński lectured, first time in Polish lands, on set theory (see: [14]). Unfortunately, no copy of notes from this lecture survived. However, fortunately, in the Jagiellonian Library there is the notebook entitled *Principles of Set Theory*. The notes were taken during Zaremba's lecture. Zaremba presented 'naive set theory', and in fact he gave a study of the structure of the real number line. Let us list some problems selected from the book: On the matter of the set theory, decimal representation of a real number and unique decimal representation, on the continuum nature of the interval (0,1), on the continuum nature of any interval. Zaremba's original argument on equicardinality of the square and the interval is finally given.

3.5. *Analytic function* and *Number theory* by Jan Sleszyński



Jan Sleszyński (1854−1931) was born in Łysianka (Lisianka), now Ukraine. In 1871 he graduated form a secondary school in Odessa and studied mathematics at the Odessa University, from which he graduated in pure mathematics in 1875. He studied at the University of Berlin under Karl Weierstrass and obtained his doctorate in 1882. From 1883 to 1909 he was a professor of mathematics at the Odessa University. In 1911 Sleszyński was appointed to the III Chair of mathematics at the Jagiellonian University. Sleszyński worked at the Jagiellonian University until his retirement in 1924. In 1921 he became a member of Polish Academy of Arts and Sciences. Sleszyński was probably the very first to state: "The point of civilization is the exchange of ideas. And where is this exchange, if everybody writes and nobody reads?".

Sleszyński, recognized in Poland as a logician, was in fact a prolific mathematician. His main work was on continued fractions, least squares and axiomatic proof theory based on mathematical logic. Sleszyński is an author of the first rigorous proof of a restricted form of the Central Limit Theorem and the Sleszyński–Pringsheim theorem on convergence of certain continued fractions. In 1910 Kazimierz Żorawski, the president of the Foundation of Dr. Kretkowski[15], was looking for a candidate for the III Chair of Mathematics. The candidate should be a well-known Polish-speaking professor of pure or applied mathematics. Sleszyński, who just retired from the Odessa University, accepted the offer. From 1911 to 1918 Sleszyński lectured at the University on *Number theory* (1911, 1915), *Analytic functions* (1911, 1913, 1915–1918), *Theory of determinants* (1913, 1916–1918), *Differential calculus* (1918), *Calculus of Probability*[16] (1912, 1916) and *Mathematical logic* or *Methodology of mathematics* (1913, 1915, 1916, 1918).

The lecture *Analytic functions* (see: [3]) by Sleszyński was arranged in a modern style. Sleszyński suggested to students not only classical monograph by Jordan and Goursat but

[15] Władysław Kretkowski (1840–1910), a mathematician and engineer, graduated from Sorbonne and Imperiale École des Ponts et Chauseés in Paris. Kretkowski obtained a PhD in mathematics from the Jagiellonian University in 1882. In his last will he bequested all his property for the development of mathematics in Kraków, especially suggesting endowment of the III Chair of Mathematics at the University.

[16] It was classical probability.

also quite new Stolz's *Grundzüge der Differential und Integralrechnung*. The notes consist of 151 pages and of 25 separate pages for the table of contents. Pages from 2 to 25 are devoted to metrical (topological) properties of real line and complex plane, pages from 25 to 58 to the theory of real function, and 59 to 151 to analytic function of a one complex variable. Probably Sleszyński was the very first person who lectured on topology (sic!) at the Polish university. Starting with metrical properties of the real line he defined: an accumulation point of a set and a closed set Next, he defined a set dense in itself, a perfect set and proved a theorem on such sets. He also discussed Weierstrass theorem on boundary classes, fundamental definitions and theorems on real line; supremum and infimum and the Bolzano theorem, convex set. etc. His treatment of the theory of function of real variable was classical, so we will move here to a discussion of analytic functions of one complex variable. Sleszyński first gave a foundation of a 'class theory' in complex plane. By a set or class of complex numbers he meant all complex numbers which satisfy a set of assumptions. In the theory of classes Sleszyński gave only the 'definition of inclusion' and did not discussed any problem of the (naïve) sets theory. He gave many definitions and theorems of classical topology, in particular he discussed limit point and accumulation point, the class (set) of all accumulation points of any given class. He introduced a (topological) classification[17] of classes: closed (zamknięta), open (otwarta), everywhere dense (wszędzie gęsta), perfect

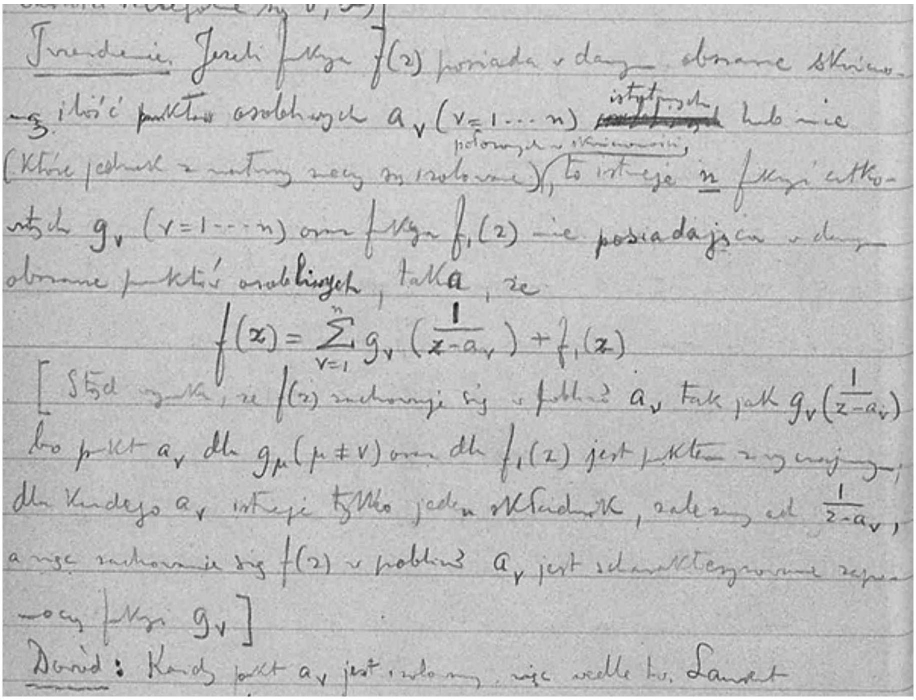
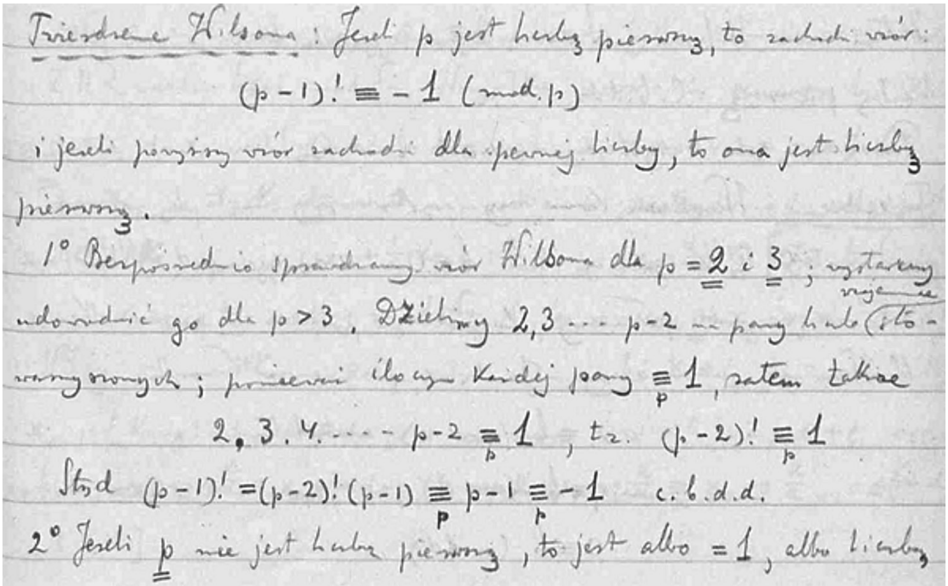


Fig. 3. An extract from the *Analytic functions* by Sleszyński

[17] In parentheses there are Polish name proposed by Sleszyński.

(doskonała), connected (ciągła[18]), simply connected (zwięzła[19]), convex (wypukła) and some fundamental properties of these classes, for example the fact that a convex set is simply connected.

After these metrical (topological) preparations Sleszyński introduced the theory of analytic function with a definition of derivative of a function at a point and Cauchy--Riemann equations (a necessary condition for differentiability). Next he defined the derivative of a function of complex parameter, a curvilinear integral, and concluded with Weierstrass mean value theorem for curvilinear integral. Next Sleszyński switched his consideration to series. He started with the classical definition of convergent and absolutely convergent series and Cauchy-Mertens theorem. Then he discussed differentiability, uniqueness of power series and Weierstrass theorem, power series, Cauchy-Hadamard theorem, differentiability and uniqueness and integrability of power series. He finished this part with Euler Formulae. The Cauchy integral formula and Taylor series are given. Next he introduced Liouville[20] series and defined isolated singularities, gave their classification and finally presented classical theorems on singularities and their application – Riemann principle, Casorati--Werierstrass[21], Cauchy residue and the definition of the winding number. Fundamental facts on entire functions – Louville theorem and Mittag-Leffer theorem – and uniqueness theorem


Twierdzenie Wilsona: Jeśli p jest liczbą pierwszą, to zachodzi wzór:

$$(p-1)! \equiv -1 \ (\mathrm{mod.}\ p)$$

i jeśli powyższy wzór zachodzi dla pewnej liczby, to ona jest liczbą pierwszą.

1° Bezpośrednio sprawdzamy wzór Wilsona dla $p = 2$ i 3; wystarczy udowodnić go dla $p > 3$. Dzielimy $2, 3 \ldots p-2$ na pary liczb wzajemnie stowarzyszonych; ponieważ iloczyn każdej pary $\underset{p}{\equiv} 1$, zatem także

$$2 \cdot 3 \cdot 4 \ldots p-2 \underset{p}{\equiv} 1, \ \text{tz.} \ (p-2)! \underset{p}{\equiv} 1$$

Stąd $(p-1)! = (p-2)!\,(p-1) \underset{p}{\equiv} p-1 \underset{p}{\equiv} -1$ c.b.d.d.

2° Jeśli p nie jest liczbą pierwszą, to jest albo $= 1$, albo liczbą

Fig. 4. An extract from the *Number theory* by Sleszyński

[18] Now ‘spójna’.
[19] Now ‘jednospójna’.
[20] Sleszyński meant Laurent series.
[21] Sleszyński did not mention the name of Sochocki, who was first to prove this theorem. For a detailed study see e.g.: St. Domoradzki, *Julian Karol Sochocki (1842–1927)*, Opuscula Mathematica (1522), Kraków 1993, pp. 137-142.

completed the theory of analytic function. In the last part of the lecture Sleszyński made some remarks about multi-valued functions and Riemann surface.

Number theory (see: [2]) by Sleszyński. The notebook has 83 pages and there is no date of the lecture, but it was probably in the summer semester of the academic year 1912/13. There is no table of contents. Probably the notebook is not complete.

Sleszyński presented basic facts on the numbers theory: divisibility and Euclid's algorithm, definition of prime and composite numbers and the theorem on the infinite number of prime numbers. He gave some facts about divisibility of the integers: the divisibility rules, definitions of greatest common divisor (GCD) and least common multiplite (LCM) for two or a finite number of integers and some fundamental theorems, Gauss algorithms for GCD and LCM of two integers. Next Sleszyński switched to the theory of congruences starting with definition of congruence relation for two numbers and moving to the Gaussian function[22], Euler theorem for prime numbers and Fermat little theorem. He gave the explicit form of the Euler totient function. Then he considered polynomials with integer coefficients and problem of solving linear congruencies, and presented an algorithm for solving a reduced congruence. Sleszyński also disscused Diofantine equations and gave some classical facts: Wilson theorem, Chinese remainder theorem and general Euclid algorithm. He introduced Gauss' function and symbol, Legendre's symbol and Jacobi's symbol. Continued fractions, one of his main areas of research, are also broadly disscused. Finally, the theory of quadratic residues and Eisenstein theorem are presented.

This work was partially supported by the Centre for Innovation and Transfer of Natural Sciences and Engineering Knowledge, University of Rzeszów.

References

Archive records

[1] *Principles of Set Theory by Stanisław Zaremba*, Archive records of Aleksander Birkenmajer, Jagiellonian Library, Manuscripts, Przyb. 570/75.

[2] *Number Theory by Jan Sleszyński*, Archive records of Aleksander Birkenmajer, Jagiellonian Library, Manuscripts, Przyb. 570/75.

[3] *Analytic Function by Jan Sleszyński*, Archive records of Aleksander Birkenmajer, Jagiellonian Library, Manuscripts, Przyb. 570/75.

[4] *High seminar in mathematics by Kazimierz Żorawski*, Archive records of Aleksander Birkenmajer, Jagiellonian Library, Manuscripts, Przyb. 570/75.

[5] *Nauki matematyczno-fizyczne w Krakowie w w. XIX by A. Birkenmajer*, Archive records of Aleksander Birkenmajer, Jagiellonian Library, Manuscripts, Przyb. 400/75.

[22] Sleszyński meant Euler totient function, for which Gauss gave an explicit presentation.

Books and articles

[6] H. Barycz, *Uniwersytet Jagielloński w życiu narodu polskiego* (*Jagiellonian University in the life of the Polish nation*), Wrocław 1948.

[7] Ch. Binder, *The Appointment Policy in the Austrian-Hungarian Empire*, [in:] (eds.), M Bečvářová, Ch. Binder, Mathematics in the Austrian-Hungarian Empire (proceedings of the Symposium held in Budapest on August 1, 2009 during the XXIII ICHST History of Mathematics, volume 41, Prague 2010, 43-53.

[8] D. Ciesielska, *Geometria analityczna według W. Zajączkowskiego*, 33. Mezinarodni Konference Historie Matematiky, Univerzita Karlova v Praze, Prague 2012, 187-202.

[9] D. Ciesielska, *Sprawa doktoratu Władysława Kretkowskiego*, Dzieje matematyki polskiej, Wydawnictwo Uniwersytetu Wrocławskiego, Wrocław 2013, 7-37.

[10] K. Ciesielski, A. Pelczar, Z. Pogoda, *Franciszek Mertens (1840–1927)*, [in:] (ed.), B. Szafirski, Złota Księga, Wydział Matematyki i Fizyki UJ, Kraków 2000, 301-311.

[11] K. Ciesielski, Z. Pogoda, *On Mathematics in Kraków Through Centuries*, Roczniki PTM Ser. II: Wiadomości Matematyczne 48, no. 2, 2012, 313-324.

[12] K. Ciesielski, *100th Anniversary of the Jagiellonian University Students' Maths Society*, Math. Intelligencer 17, no. 1, 1995, 42-46.

[13] J. Dianni, *Recepcja geometrii analitycznej w Polsce w wiekach XVII–XIX* (*Perception of analytic geometry in Poland in the centuries XVII–XIX*), Kwartalnik Historii Nauki i Techniki XIX, 1974, 677-700.

[14] S. Domoradzki*, The Growth of Mathematical Culture in the Lvov Area in the Autonomy Period (1870–1920)*, Matfyzpress, Prague 2011.

[15] S. Domoradzki, *Franciszek Mertens (1840–1927)*, Opuscula Mathematica, 1522, 1993, 109-117.

[16] F. Mertens, *Wykład geometryi analitycznej* (*Lecture on Analytic geometry*), Towarzystwo matematyczno-przyrodnicze uczniów Uniwersytetu Jagiellońskiego, Kraków 1882.

[17] J. Hachaj, P. Jakóbczak, *Wykłady Profesorów Stanisława Zaremby i Kazimierza Żorawskiego w świetle notatek Aleksandra Birkenmajera* (*Lectures of Professors Stanisław Zaremba and Kazimierz Żorawski through notes of Aleksander Birkenmajer*), Roczniki PTM Ser. VI: Antiquitates Mathematicae 1, 2007, 7-14.

[18] R. Murawski, *Twierdzenia limitacyjne*, [in:] *Nauka – możliwości i ograniczenia*, Warszawa 2013, 33-41.

[19] Z. Opial, *Zarys dziejów matematyki w Uniwersytecie Jagiellońskim w drugiej połowie XIX wieku* (*An Outline of the History of Mathematics at the Jagiellonian University in the second half of the nineteenth century*), [in:] *Studia z dziejów Katedr Wydziału Matematyki, Fizyki, Chemii Uniwersytetu Jagiellońskiego* (*Studies on the History of Departments of the Faculty of Mathematics, Physics, Chemistry of the Jagiellonian University*), (ed.), S. Gołąb, Jagiellonian University, Kraków 1964.

[20] *Spis wykładów* (*Index of lectures*), c.k. Uniwersytet Jagielloński, Kraków, academic years: 1870/1871–1917/1918.

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE
1-NP/2014

SERGEY S. DEMIDOV*

THE ORIGINS OF THE MOSCOW SCHOOL OF THE THEORY OF FUNCTIONS

GENEZA MOSKIEWSKIEJ SZKOŁY TEORII FUNKCJI

A b s t r a c t

The school known as the Moscow school of the theory of functions or the school of D.F. Egorov – N.N. Luzin, originated in 1910s within the framework of the Moscow philosophical-mathematical school. As a matter of fact, its birth was transplanting into the Moscow soil of the French studies on set theory and the theory of functions (E. Borel, H. Lebesgue, R. Baire). This school appeared as an attempt of Muscovites to reach the front line of modern mathematical studies in an area alien to interests of mathematicians from St.- Petersbourg. The attempt has turned successful: its result was creation (in a very short period) of one of the most effective European schools with its own subjects of studies (analytic sets etc.). As a result of the activity of this school Moscow became one of the leading world mathematical centers. Already in the late 1920s, the research done in this school (through the works of P.S. Aleksandrov, A.O. Gelfond, M.V. Keldysh, A.Ya. Khinchin, A.N. Kolmogorov, M.A. Lavrent'ev, L.A. Lyusternik, P.S. Novikov, L.S. Pontryagin, A.N. Tikhonov, P.S. Urysohn etc.) went out very far from the problems which marked the beginning of the Moscow school of the functions theory.

Keywords: *Moscow mathematical school of the theory of functions*, *set theory*, *theory of functions of a real variable*, *analytic sets*, *D.F. Egorov*, *N.N. Luzin, W. Sierpiński*

S t r e s z c z e n i e

Szkoła, znana jako Moskiewska Szkoła teorii funkcji lub Szkoła D. F. Jegorowa – N.N. Łuzina, powstała w drugiej dekadzie XX w. w ramach Moskiewskiej Szkoły Filozoficzno-Matematycznej. W rzeczywistości jej powstanie było przeniesieniem na grunt moskiewski francuskiej szkoły teorii mnogości i teorii funkcji (wyniki E. Borela, H. Lebesgue'a, R. Baire'a). Szkoła w Moskwie była próbą wejścia matematyków moskiewskich do światowej czołówki matematycznej w dziedzinie, która nie była przedmiotem badań matematyków z Sankt Petersburga. Tak pomyślana szkoła okazała się sukcesem: w bardzo krótkim czasie powstał tam jeden ze światowych ośrodków matematycznych z własną tematyką badań. Badania prowadzone w tej szkole już w końcu lat 20. XX w. (dzięki pracom P.S. Aleksandrowa, A.O. Gelfonda, M.W. Kiełdysza, A. J. Chinczyna, A.N. Kołmogorowa, M.A. Ławrentiewa, L.A. Lusternika, P.S. Nowikowa, L.S. Pontriagina, A.N. Tichonowa, P.S. Urysohna i innych), odeszły bardzo daleko od problemów, które zapoczątkowała Moskiewska Szkoła teorii funkcji.

Słowa luczowe: *Moskiewska szkoła teorii funkcji*, *teoria mnogości*, *teoria funkcji zmiennej rzeczywistej*, *zbiory analityczne*, *D.F. Jegorow*, *N.N. Łuzin*, *W. Sierpiński*

* Sergey S. Demidov, S.I. Vavilov, Institute for the History of Science and Technology, Faculty of Mathematics and Mechanics, M.V. Lomonosov Moscow State University.

1. Mathematics in Moscow in the early twentieth centary

By the early twentieth century, in the mathematical Moscow there was the following situation. A kind of school was formed around the Moscow Imperial University and the Moscow Mathematical Society, known in history as the Moscow philosophical and mathematical school [1]. One of the main characteristic features of this school were deep philosophical interests of its members, who wished to understand the mathematics – its subject and the methods used in it – in the broad philosophical context. Muscovites were in opposition to positivism, then highly fashionable in academic circles. Their propensity to the idealistic philosophy (including religious one), in particular, to the ideas of Leibniz, extremely popular at that time in Moscow philosophical circles, is well-known. The most influential mathematician in Moscow at that time, a professor of the Moscow University and a corresponding member of the Russian Academy of Sciences N.V. Bugaev (1837––1903), was an original philosopher, the author of a philosophical system of "evolutionary monadology", which had an evident impact on the very subject of Muscovites' mathematical research. One of the consequences of Bugaev's philosophical views was distinguishing the phenomenon of "discontinuous" in his teachings about the nature and the society. Contrasting his worldview with the analytical world outlook which dominated hitherto, whose core was Laplace's determinism and whose mathematical expression was the analysis of extremely smooth (analytic!) functions completely determined by being prescribed in an arbitrarily small neighborhood of any point of the area of their definition (it is a mathematical expression of the idea of total determinism!), Bugaev preached the idea of building a new mathematics, whose center should be the theory of discontinuous functions [2]. He began to build such a theory, which he called arithmology, together with his disciples. The starting point in this direction for Bugaev was the theory of functions of the number theory for the study of which considerable analytical apparatus has been created in mathematics. Thus the class of functions studied by Bugaev consisted of the piecewise smooth functions and the limits of sequences of such functions. Despite all the efforts made by his school, such theory turned out quite poor. One of the latest and most gifted of his students, D.F. Egorov (1869–1931), started his scientific career with arithmology (he devoted to arithmology his first paper [3] published in 1892), immediately dropped the subject, considering it hopeless (as a gifted mathematician, he had a remarkable intuition).

He chose differential geometry as his future field of studies – one of the main lines of research of the Moscow philosophical and mathematical school, which has grown from the work of K.M. Peterson (1828–1881). These studies (by Peterson, B.K. Mlodzeevskii (1858–1923), D.F. Egorov) became widely known and turned Moscow into an important European geometrical center [4, 5].

Another important area of research of the Moscow School was applied mathematics.

This direction, which began in Moscow with N.D. Brashman (1796–1866), received in this period a remarkable development in the works of N.E. Zhukovskii (1847–1921) and his students (S.A. Chaplygin, etc.). Zhukovskii's results (in particular, his work on the water hammer, which allows one to solve the persistent problem of failures in pipelines) made his name well-known in Europe and brought his school a prominent place among the contemporary schools of applied mathematics [5].

Studies in other branches of geometry (K.A. Andreev, A.A. Vlasov), results in number theory (Bugaev), in complex analysis (P.A. Nekrasov), in probability theory (Nekrasov), and in other areas of mathematics turned Moscow into an important mathematical center. However, this was not sufficient for the young ambitious Muscovites: they were not satisfied with the position of mathematicians who, although recognized abroad, were neglected by academic Petersburg. In the capital of the Empire P.L. Chebyshev school reigned, which tried to spread its decisive influence on the entire Russian mathematical community.

2. Mathematics in St. Petersburg and the conflict of mathematical communities of the two capitals

The mathematicians from Petersburg became famous in the world by their research in the areas which were developed in the outstanding studies of their common teacher P.L. Chebyshev (1821–1894). These were: number theory (E.I. Zolotariov, A.N. Korkin), probability theory (A.A. Markov, A.M. Lyapunov), constructive theory of functions (A.A. Markov, V.A. Markov), applied mathematics (Lyapunov), and mathematical physics (Lyapunov, V.A. Steklov). These studies were (and still are) highly appreciated in the world mathematical community and (what is especially important for us!) served as the basis for an even higher self-esteem of the St. Petersburg mathematicians. From their point of view (of course we are talking about prevailing opinions among them) it was necessary to develop only those parts of mathematics that have applications. This is evidenced by the list of the main areas of their research, in which number theory looks like odd man out. This section made it to the list, in a certain sense, by accident. Just arrived from Moscow to St. Petersburg, a young ambitious Chebyshev could not reject an offer of an influential academician V.Ya. Bunyakovskii to assist him with the preparation of a volume of Euler’s works on the number theory for publication. Having plunged into the world of Euler’s ideas, Chebyshev immersed himself into it and took so much interest in it that he grew into one of the classics of the theory of numbers. For St. Petersburg mathematicians who did not recognize the subjects without an applied orientation (hence their cold attitude toward the geometry of Lobachevskii, even when his ideas gained worldwide recognition, their opposition to Riemann’s “decadent constructions”, to S. Lie’s ideas etc.), it became necessary to search for “excuses” in order to engage in research in the field of number theory. One of these “excuses” was the fact that the methods originating in number theory turned out to be applicable in other parts of mathematics, in particular, in mathematical analysis (number theory as “a forge” for the new methods of mathematical analysis!).

Positivism, the rejection of any idealistic philosophy and the militant atheism, were dominant in the worldview in the mathematical community of St. Petersburg. They became the basis of their negative attitude to the religion and the religious philosophy and caused their strongly negative view of Moscow mathematicians. Such attitudes also determined their rejection of Cantor’s works on the set theory, which were often provided with theological introductions.

The studies of Muscovites on differential geometry were not supported by the mathematicians of the northern capital either, because these studies did not lead to applications, which were then considered rather important. As a result, the two mathematical communities were in confrontation (which should be considered in the context of a cultural confrontation between the two capitals [6]).

Since these communities determined the climate in the country (almost all professors in each Russian university graduated either from Moscow or from St. Petersburg University), this conflict gave rise to tensions in Russian mathematics as a whole. The acuteness of the conflict was tempered largely by Chebyshev himself – a native of Moscow, he maintained good relations with many Moscow mathematicians, supporting them in various undertakings (such as in the creation and activities of the Moscow Mathematical Society), and in the election to the Academy. But after his death, when A.A. Markov became the leader of the St. Petersburg mathematical community, the conflict escalated rapidly. We have already talked about the fact that mathematicians from Petersburg did not have any special respect for the results of Muscovites on differential geometry. Their attitude to Zhukovskii (a "Moscow celebrity", as V.A. Steklov sarcastically called him in his letters) was contemptuous. This tension was constantly manifested in the mathematical public life of the country and led to open clashes, which often ended with the scandals at the meetings of the Moscow Mathematical Society.

So criticism from St. Petersburg mathematicians against the works of academician V.G. Imshenetskii on the theory of integration of linear differential equations, supported by Muscovites, turned into a very hot battle at a meeting of the society, after which Imshenetsky returned to the hotel and died. Another well-known case of conflict, which also happened at a meeting of the Moscow Mathematical Society, were the attacks of St. Petersburg mathematicians on S.V. Kovalevskaya due to gaps in the demonstrations in her famous studies on the motion of a rigid body around a fixed point. Muscovites rose to defense of Kovalevskaya against A.A. Markov's aggressive attacks. And although, as we have said, their results were quite highly appreciated in the West, the Muscovites were not satisfied with the position to which they were actively pushed by the Petersburg academicians. They wanted to see themselves also at the forefront of the modern mathematical research. They did not want to compete with the colleagues from St. Petersburg in their favorite subjects, since this necessarily put them in the position of walking in the footsteps of St. Petersburg school.

For them it was necessary to find their own way, even further distanced from St. Petersburg ways. And they did find the way.

3. Birth of the Moscow school of function theory

In 1903 Bugaev died and his disciples, the young professors B.K. Mlodzeevskii and D.F. Egorov, became leaders in the Department of Mathematics at Moscow University. They made a lot of efforts to modernize the teaching of mathematics at Moscow University. They tried, firstly, to teach at the most modern level, and secondly, to acquaint students with the latest achievements of mathematics and the latest trends in their special courses. So Mlodzeevskii already in 1900/1901 lectured on the theory of functions of a real variable, and in the next academic year he repeated the lectures. The synopsis of these lectures compiled by a student P.A. Florenskii (1882–1937) (later the famous philosopher and theologian) has been preserved – see [7]. In this synopsis we find the exposition of the principles of set theory, and an introduction to the theory of functions of a real variable – a new discipline developed in 1890s on the basis of Cantor's set theory by the French mathematicians E. Borel (1871–1956), H. Lebesgue (1875–1941) and R. Baire (1874–1932). It is important to note that Mlodzeevskii not only introduced his students to the latest variant of the function theory, but connected it with Bugaev's ideas, with his arithmology. Cultivated in the atmosphere of Bugaev's preaching of the importance of building of the theory of discontinuous functions, the young Moscow mathematicians saw such a theory in the constructions of French mathematicians. And nothing prevented Muscovites from starting their own research on the theory of sets and functions of a real variable. If the Petersburg mathematicians were repelled by set theory because of its theological framing proposed by Cantor, the Muscovites found it rather attractive. The possibility to study efficiently the world of discontinuous functions made the new topics particularly attractive for them. Various aspects of this theory were discussed at the meetings of the students' circle organized by Florenskii at the Moscow Mathematical Society [8]. In 1908 I.I. Zhegalkin defended his thesis on transfinite numbers [9]. In 1903 Florenskii published in the literary magazine "Put" (Path) the first Russian exposition of the set theory [10]. The problems of set theory and of discontinuity were the topic of his Candidate's thesis "The idea of discontinuity as an element of the world outlook" [11], defended in 1904. The student N.N. Luzin was one year younger than his friend Florenskii and was under his influence [12, 13]. After graduating from university Florenskii was recommended by Zhukovskii for further studies at the University for the preparation of the master's thesis, but he did not use this recommendation and went to the Moscow Theological Academy – it was his conscious choice to devote himself to philosophy and theology. He delegated his function of the Secretary of the students' circle to Luzin [13]. And although Luzin, when entering the mathematics department of Moscow University, did not intend to devote himself to mathematics (his goal was to get an engineering degree and the training at the University was to be only a step in achieving this goal), in the course of training he became extremely interested in it. Under the guidance of Egorov his mathematical talent was revealed (its presence was a surprise even to himself). In 1906 he defended his graduation essay "On a method of the integration of differential equations" and passed state exams. In this way Luzin completed his studies at the university and was recommended by Egorov to continue the training for the preparation of master's thesis. By the end of 1909, he passed his master's examinations, which did not take him long as he studied these topics in his student years. Reflecting on the direction of his further studies, he took classes at the Faculty of history and philology, where he attended lectures on theoretical philosophy and on various areas of modern philosophy (in particular, L.M. Lopatin's lectures). In the autumn of 1910, when he (already in the rank of private-docent) was preparing to start his teaching at the University, an order came from the Ministry of Education to send him on a mission to Göttingen and Paris "for improvement in the mathematical sciences". Of course, he received such a gift as a result of Egorov's efforts, who exercised for this all his influence. In Göttingen he read a lot, worked (mostly in the theory of trigonometric series), and talked with the local mathematicians. In December he moved to Paris; his stay there turned out to be truly momentous. There he began to work in Hadamard's seminar, coming into personal contact with E. Picard, E. Borel, Lebesgue, Denjoy, etc. We can judge the creative atmosphere of Luzin's Parisian life in this period by his correspondence with D.F. Egorov [14]: his amazing creative enthusiasm, his contacts with the leaders of the French school of function theory – with Borel and Lebesgue, the beginning of his friendship with Denjoy. This correspondence allows one to feel the atmosphere in which the Moscow school of function theory was born. During this period, D.F. Egorov pondered a question that led him to the proof of the theorem which is known now as Egorov theorem and was published [15] in 1911 in the Comptes Rendus of the French Academy of Sciences (the correspondence [14] enables one to reconstruct the creative process of the demonstration of this theorem – see [16]). In that period Luzin was working on the problems which formed the content of his article [17], published the following year in the same journal and containing the theorem on the C-property (more extensive articles containing this result appeared in the same year in Russian [18]), known in mathematics as Luzin theorem (a similar result was published in 1905 by G. Vitali [19], which however, passed then unnoticed – see [20]).

These two articles became the foundation of the Moscow school of function theory, one of the most glorious in the first third of the twentieth century.

4. The first steps of the Moscow school of function theory

At the end of 1911 Luzin settled in Paris. His work progressed well and with Egorov's help his study tour, which ended in 1913, was extended. In the spring semester of 1914, he attended Picard's lectures on selected chapters of the function theory, the lectures of M. Bocher, a visiting professor from the United States, on the recent results in the theory of linear differential equations of the second order, Borel's lectures on the generalization of the notion of an analytic function. He participated in the sessions of Hadamard's seminar in Collège de France.

The most important, of course, was his work on problems of the theory of functions of a real variable and of set theory (he spent a lot of time reflecting on the problem of continuum) [21]. Returning to Moscow, he began in the fall his teaching at the University: a course of analytical geometry and a special course on the theory of functions of a real variable. The ground for the reception of the latter course was prepared by Egorov, who ran a spring semester seminar on the subject. As his disciple recalled later [22, c. 475]: "It is this special course and the accompanying seminar (...) that became a center from which the Moscow school of function theory grew (...)" The first generation of his disciples was raised at this seminar.

In 1915 Lusin published his thesis *The integral and the trigonometric series* [23], the defence of which was held on 27 April of the following 1916. The opponents were D.F. Egorov and L.K. Lakhtin. The historical and mathematical analysis of its content can be found in the book of A.P. Yushkevich [5, p. 572], who, in particular, wrote: "«The integral and the trigonometric series» was Luzin's invaluable contribution to the metric function theory. On the basis of the concept of measure the author studies properties of measurable functions, of the integral, of the derivative and of other central concepts of analysis". The result of the defence was a triumph. The Council decided to "(...) approve N.N. Luzin for the degree of the Doctor of pure mathematics (i.e. bypassing the Master's degree – S.D.)

because this thesis has special scientific merit (...)" [24, p. 18]. In the same year he was approved for a position of an extraordinary professor. The rise was quick and extraordinary. This was the heyday of Luzin's talent. A truly charismatic personality, he rallied around him the talented youth, who literally adored a young professor. All of them felt like the true creators of the new science and like members of a knight order, which they called Luzitania. These are the names of the first "knights": M.Ya. Suslin, D.E. Menshov, A.Ya. Khinchin, P.S. Aleksandrov, P.S. Uryson, V.N. Veniaminov, V.S. Fedorov.

The creative atmosphere of Luzitania promoted the early appearance of the first results of its members. In 1916 the notes of A.Ya. Khinchin (*Sur une extension de l'intégrale de M. Denjoy*. T. 162) appeared in Comptes Rendus of the French Academy of Sciences, in which he applied his notion of "asymptotic derivative" to the generalization of the concept of the Denjoy integral. P.S. Aleksandrov (*Sur la puissance des ensembles mesurables.* B. T. 162) demonstrated that every uncountable Borel set has the cardinality of the continuum, and D.E. Menshov (*Sur l'unicité du développement trigonométrique*. T. 163) constructed an example of a trigonometric series which has coefficients different from zero and converges almost everywhere to zero. Finally, in 1917 in the same journal a brilliant article of M.Ya. Suslin (*Sur une definition des ensembles mesurables B sans nombres transfinis*. T. 164) was published, which marked a turning point in the history of the Moscow school of function theory. A history of this note is the following. In the summer of 1916 Luzin assigned to his student the task to analyze critically the work of Lebesgue *Sur la représentation des fonctions analytiques* (1905). Trying to prove Lebesgue's assertion that the projection onto a straight line of a two-dimensional Borel set is a Borel set (Lebesgue considered this statement obvious), the student found that it was not true: using a construction introduced by Aleksandrov, he constructed an example in which such a projection is not a Borel set. W. Sierpiński, who resided in those years in Moscow and worked together with Luzin (how the fate has thrown a young Polish mathematician in Moscow – see below), described this event [25, c. 33]: "I witnessed how Suslin informed Luzin about an error of Lebesgue and handed him the manuscript of his first paper. Luzin very took seriously to the report of a young student and confirmed that he had indeed found a mistake in the work of the famous scientist".

The theory of new sets, which received the name of Suslin sets or analytical sets, became the last word in set theory and its development started immediately by Luzin himself. His first work, "where the set theory got its notable further development" [26, c. 130], was published in the same volume of Comptes Rendus as the work of Suslin. The new subject – theory of analytical or Suslin sets – became later central for the Luzin school. The milestone in its development was Luzin's book *Leçons sur les ensembles analytiques et leurs applications*, published in Paris in 1930 with a preface by Lebesgue (a Russian edition [27] appeared only in 1959).

Moscow school of function theory became one of the most striking phenomena in the European mathematical life of the first quarter of the twentieth century. Its development was rapid, despite the gravity of the situation in which Russia found itself in that period: the First World War, the Revolution and the subsequent Civil War. The attractive force of Luzin's personality (in those days it was exceptional – see the memories of L.A. Lyusternik [28]), the beauty of topics which opened before the Muscovites, the possibility for them to feel themselves at the epicenter of the nascent new mathematics, finally (and we should not forget about it!) Egorov's activity, who played the role of the unquestionable moral authority and the guardian of principles, created in Moscow the extremely favorable conditions for the scientific work, which continued even in the most unfavorable period of the years 1918–1921, when Luzin and his disciples, in search of sustenance, left Moscow. When from time to time Luzin came to the capital, all those who happened to be at that time in Moscow gathered for a seminar. Despite these harsh conditions the studies were going on very intensively. When in 1922 Luzin finally returned to the university and his seminar started to work in regular mode, the circle of his pupils was joined by L.A. Lyusternik, N.K. Bari, M.A. Lavrentiev, L.G. Shnirel'man, P.S. Novikov, L.V. Keldysh, A.N. Kolmogorov, and V.I. Glivenko. Luzin's older students became masters themselves and established their own seminars, in which they studied questions different from Luzin's topics.

The first to separate were the members of the topological circle of P.S. Aleksandrov and P.S. Uryson, including their own students A.N. Tikhonov, V.V. Nemytskii, N.B. Vedinisov, L.A. Tumarkin, and L.S. Pontryagin. A.Ya. Khinchin began to apply the methods of the measurable function theory to number theory and obtained important results in the metric number theory. Under his guidance L.G. Shnirelman and A.O. Gelfond began their research in number theory. M.A. Lavrentiev created his own school in the theory of functions of a complex variable (M.V. Keldysh, etc.). Finally A.Ya. Khinchin and A.N. Kolmogorov started their research in probability theory. The research of the school in set theory and theory of functions of a real variable. became an excellent common ground for all these studies, the results of which gained worldwide recognition, But in their research Luzin's students went in different directions, sometimes quite distant from each other (and, most importantly, from their Master). The school broke up and in the process of this disintegration a new formation began to develop, which in turn, became (together with the Leningrad school) the basis for the Soviet school of mathematics, one of the most influential ones in the second half of the twentieth century.

It is interesting to note that the arrogant attitude of mathematicians from Petersburg (in the 1920s already named Leningrad) was kept long enough. There is an anecdote, popular in Russian mathematical community. According to that anecdote, V.A. Steklov, displaying Luzin's thesis and leafing through its pages in which there were not as many formulas as there were in the works of the mathematicians from St. Petersburg, summed it up: "Is this mathematics? No, this is philosophy". In 1926, when the significance of the work of Luzin's school apparently should have been clear to mathematicians, another representative of the same school, academician Ya.V. Uspenskii in his letter to A.N. Krylov, discussing the candidates for the elections to the Academy wrote [29, p. 193-194] the following: "I feel deep disgust for this direction and firmly believe that this fashion will soon pass, especially if we take into account the criticism of Brouwer and Weyl, who raised strong objections not only against the entire colossus erected by Cantor and Lebesgue, but also against the facts which since the days of Weierstrass were considered as firmly established". The conflict lasted until the mid-30s and was put to the end by (...) I.V. Stalin. In the course of his reform of the Soviet science the Presidium of the Academy and a number of leading academic institutions (including the V.A. Steklov Mathematical institute) were transferred in 1934 to Moscow – "the headquarters of the Soviet science" had to be located close to the overlord.

The conflicting sides were forced to live and work together by the "will of the monarch". As a result, there was a fruitful synthesis of the Moscow and the St. Petersburg traditions, which laid the foundations for the Soviet mathematical school.

5. Concluding remarks on W. Sierpiński and on the parallels in the development of the Moscow and Warsaw schools of set theory and of the function theory

In our story we mentioned the name of the Polish mathematician W. Sierpiński, who witnessed the events of the Moscow mathematical life of 1915–1918 years and participated in them [25]. The events of the World War I brought him to Moscow (The entry of Russia into the war in 1914 took him on its territory. Because he was at that time a citizen of Austro-Hungary, he was interned in Vyatka. By efforts of B.K. Mlodzeevskii and D.F. Egorov he received the right of residence in Moscow, where he spent several years, closely associating with Egorov and Luzin). There he became friends with Luzin, with whom he kept creative relationship for many years [30, 31]. It was in Moscow that Sierpiński obtained, by his own admission (see a fragment of his letter to I.G. Melnikov from May 9, 1966 [30, c. 362]), his first significant results in set theory, published in 1916 in the Paris Comptes Rendus. Between 1915 and 1918 he published 36 papers, 4 of them in collaboration with Luzin. Their cooperation, despite some theoretical differences, for example, on the question of the axiom of choice, continued in subsequent years.

(On the Moscow period of Sierpinski's life and on the philosophical spirit reigning in Moscow mathematics in that time see E. Medushevski's article [32].) The "Russian component" of Sierpiński's biography cannot be reduced to the contacts with Luzin and his entourage. Born in Warsaw, after finishing the gymnasium he studied at the Warsaw University, where an outstanding representative of the St. Petersburg school G.F. Voronoi (1866–1908) was his teacher. Under his supervision Sierpiński did (1904) his first scientific study: he improved Gauss' result about the number $A(x)$ of the integer points in the circle $u^2 + v^2 \leq x$. The communications of the Polish and Soviet mathematicians in research on set theory (for example, in the theory of projective sets) and the theory of functions of a real variable are a special story, still waiting for its researcher. In conclusion I would like to draw attention to some parallels in the history of the Moscow and Warsaw schools of the theory of sets and functions.

Moscow school of function theory, as we have said, arose from Muscovites' search for topics to enable them to go out into the forefront of modern research, moreover, topics that would be independent of the interests of the Petersburg school, with which Muscovites were in the confrontational relationship. The theory of sets and functions of a real variable turned out to be such topics.

For Polish mathematicians (Sierpiński, etc.) the urgent task was to find areas of research which would allow them in the shortest possible time to create a mathematical school in Poland and, moreover, a school whose research would be at the forefront of modern mathematics.

To them, the same theory of sets and functions turned out to be such areas. The school was created in the confrontation with the old Polish mathematical center – with the mathematicians, grouped around Krakow University. The roots of the confrontation of the Moscow mathematicians and Petersburg mathematicians were ideological, and all other factors (including personal ones) were only secondary (although they would from time to time come to the forefront). In the Polish case the personal factor played a much more significant role. S. Zaremba's dominance in Krakow, his personal preferences, particularly his mathematical habits (he was "a pure classicist" and the set-theoretic direction evoked his strong antipathy) caused the departure from Krakow of many young mathematicians (including S. Banach, S. Kaczmarz, O. Nikodym).

Luzin wrote about this in his letter to Denjoy September 30, 1926 [33, c. 318-319]: "On returning from Paris to Moscow, I spent some days in Warsaw since Mr. Sierpiński invited me to meet him and to familiarize myself with his school (…) I would like to inform you about my mathematical impressions that I got in Warsaw (…).

Polish mathematicians, with whom I met, live in different cities – in Warsaw , Krakow, Lvov, Kovno, Vilno. From conversations with them I got a pretty clear view of mathematical life in Poland.

It seems to me that the mathematical life in Poland follows two completely different ways: one of them is inclined to the classical parts of mathematics, and the other to the theory of sets (functions). These ways exclude one another in Poland, being the irreconcilable enemies, and now a fierce struggle is going on between them".

The "classical side" forms a group, wrote Luzin [33, c. 319-320], around the Krakow University and the Krakow Academy and its leader is S. Zaremba. This group stands in opposition to the school of Sierpiński, the studies of which focused mainly on the theory of functions of a real variable and set theory. The representatives of this school took the leading positions in Warsaw and Lvov. These schools were in a state of war, the success of which, apparently, is predetermined: Warsaw and Lvov must win. That perspective was considered by Luzin as dangerous for the development of the Polish mathematics – this development gained unilateral character, and as a result, mathematics detached from its roots [33. c. 320]. In my opinion – wrote Luzin [33, c. 319] – such situation is dangerous because the exclusive attention to set theory and the neglect of the branches of classical mathematics seems to me to be too narrow, too one-sided".

(The situation was similar in many respects to the Russian one – there, the adherents of the traditional mathematics grouped around Leningrad mathematicians, and a new trend that was growing out of Cantor's set theory and the theory of functions of a real variable grouped around Muscovites: of Luzin and his school. And here and there the relationships were confrontational. But Russian scales made the situation not so acute: the rapid growth of research on new topics in Moscow did not threaten the development of the traditional mathematics in St. Petersburg, especially since one of the most important European schools of the time operated there – the school of Chebyshev).

Luzin told Sierpiński about his concerns, and the latter replied as follows [33, c. 320]: "Yes, this is really a serious danger, but greater than the dominance of one way is the danger of the lack of any way.

Before the advent of the Warsaw way mathematics in Poland didn't exist as there were separate scientists each of whom was interested in different things and did not have disciples.

This is why their works often had only a personal interest and were devoid of any scientific significance. Undoubtedly, this lack of personal creative initiative was caused by the lack of the public control, of the general mathematical opinions and of recognition of their works.

It was necessary, therefore, to create a broad mathematical environment, and it was created by the Warsaw school. As for our narrowness, I hope that it will decrease and disappear afterwards. The choice of the function theory as a basis for a common mathematical movement is the consequence of its simplicity".

Sierpiński proved to be right: Polish mathematics rather quickly went beyond the theory of sets and functions of a real variable and already in the 1930s established itself as one of the Europe's leading. Its potential was so powerful that even the tragedy that Polish science experienced during the Second World War (the extermination of a number of outstanding Polish mathematicians, the departure of talented young people to the West) has not stopped the process of its active development.

Luzin, discussing the situation that evolved in the Polish mathematics by 1926, of course, meant also a situation which was similar in many respects, that of mathematical Moscow at that time: the expanding of research topics by his students led to the disintegration of Luzitania. As we said before, hitherto a united community, rallied around him, their recognized master, was then divided into a number of new schools headed by his former students, who chose the direction of their research sometimes very far from his own interests. Luzin felt very painfully this decay and the loss of close ties with his disciples, trying to understand what was happening and to find the correct line of conduct. As we know, he was not so successful. The conflict that occurred with some of his disciples led to the notorious "affaire of academician N.N. Luzin" and could have ended tragically for him [34].

Many Soviet and western mathematicians stood up for Luzin; a special role in that campaign belonged to his old friend W. Sierpiński [35]. Fortunately, all ended well for him, though the wound inflicted by the circumstances of this "affaire" on the corps of the Soviet mathematical community did not heal for a long time.

References

[1] Nekrasov P.A., *Moskovskaja filosofsko-matematicheskaja shkola i ejo osnovateli*, Matematicheskij sbornik, T. 25, vyp. I., C. X–XIV, 1904, 3-249.

[2] Demidov S.S., *N.V. Bugaev i vozniknovenie Moskovskoj shkoly teorii funkcij dejstvitel'nogo peremennogo*, IMI, vyp. 29, 1985, 113-124.

[3] Egorov D.F., *Nekotorye sootnoshenija iz teorii integralov po deliteljam*, Matematicheskij sbornik, T. 16, 1892.

[4] Egorov D.F., Mlodzieiowski B.K., *Notice sur Karl Michailovitch Peterson*, Annales de la Faculté des Sciences de Toulouse, 2, V. 5, 1903, 459-479.

[5] Jushkevich A.P., *Istorija matematiki v Rossii do 1917 goda*, Nauka, Moskva 1968.

[6] Demidov S.S., *Saint-Pétersbourg et Moscou, deux capitales*, [in:] *La mathématique*, T. 1: *Les lieux et les temps*, Bartocci C., Odifreddi P. (Eds.), CNRS Editions, Paris 2009, 683–703.

[7] Medvedev F.A., *O kurse lekcij B.K. Mlodzeevskogo po teorii funkcij dejstvitel'nogo peremennogo, prochitannyh osen'ju 1902 g. v Moskovskom universitete*, IMI, vyp. 30, 1986, 130–147.

[8] Polovinkin S.M., *O studencheskom matematicheskom kruzhke pri Moskovskom matematicheskom obshhestve v 1902–1903 gg*, IMI, vyp. 30, 1986, 148-158.
[9] Zhegalkin I.I., *Transfinitnye chisla*, Izd-vo Moskovskogo Universiteta, Moskva 1907.
[10] Florenskij P.A., *O simvolah beskonechnogo*, T. 2, Novyj put', 1904, 173-235.
[11] Florenskij P.A., *Vvedenie k dissertacii „Ideja preryvnosti kak jelement mirosozercanija"*, IMI, vyp. 30, 1986, 159–176.
[12] Demidov S.S., Parshin A.N., Polovinkin S.M., *O Perepiske N.N. Luzina s P.A. Florenskim*, IMI, vyp. 31, 1989, 116-125.
[13] Demidov S.S., Parshin A.N., Polovinkin S.M., *O Perepiske N.N. Luzina s P.A. Florenskim*, IMI, vyp. 31, 1989, 125-191.
[14] *Pis'ma D.F. Egorova k N.N. Luzinu*, Aleksandrov P.S., Medvedev F.A., Jushkevich A.P. (Eds.), IMI, vyp. 25, 1980, 335-361.
[15] Egoroff D.Th., *Sur les suites de fonctions mesurables*, Comptes Rendus Acad. Sci., T. 152, Paris 1911, 244-246.
[16] Bogachjov V.I., *K istorii otkrytija teorem Egorova i Luzina*, IMI, V. 13 (48), 2009, 54-67.
[17] Lousin N.N., *Sur les propriétés des fonctions mesurables*, Comptes Rendus Acad. Sci., T. 154, Paris 1912, 1688-1690.
[18] Luzin N.N., *K osnovnoj teoreme integral'nogo ischislenija*, Matematicheskij sbornik, T. 28, 1912, 266-294.
[19] Vitali G., *Opere sull'alalisi reale e complessa. Carteggio*, Pepe L. (Ed.), Cremonese, Bologna 1984, XI-524.
[20] Borgato M.T., *Giuseppe Vitali: research on real analysis and relationship with Polish and Russian mathematicians*, Proceedings of the 8th Congress of the International Society for Analysis, its Applications, and Computation: *Progress in Analysis*, vol. 3, Peoples' Friendship University of Russia, Moskva 2012, 253-260.
[21] *Otchjot o zagranichnoj komandirovke dlja nauchnyh zanjatij privat-docenta Moskovskogo universiteta Nikolaja Luzina*, IMI, vyp. 8, 1955, 57-70.
[22] Bari N.K., Golubev V.V., *Biografija N.N. Luzina*, [in:] Luzin N.N., *Sobranie Sochinenij*, T. 3, Izd-vo AN SSSR, Moskva 1959, 468-483.
[23] Luzin N.N., *Integral i trigonometricheskij rjad*, Moskva 1951.
[24] *Nikolaj Nikolaevich Luzin (k 100-letiju so dnja rozhdenija)*, Kuznecov P.I. (Ed.), Znanie, Moskva 1983.
[25] Serpinskij V., *O teorii mnozhestv*, Prosveshhenie, Moskva 1966.
[26] Tihomirov V.M., *Otkrytie A-mnozhestv*, IMI, vyp. 8, 1955, 129-139.
[27] Luzin N.N., *Lekcii ob analiticheskih mnozhestvah i ih prilozhenijah*, Keldysh L.V., Novikova P.S. (Eds.), Fizmatgiz, Moskva 1959.
[28] Ljusternik L.A., *Molodost' Moskovskoj matematicheskoj shkoly*, T. 22, Vyp. 1(133), 137-161; Vyp. 2(134), 199-239; Vyp. 4(136), 146-188, UMN, 1967.
[29] Ermolaeva N.S., *Novye materialy k biografii N.N. Luzina*, IMI, vyp. 31, 1989, 191-202.
[30] Mel'nikov I., *Vaclav Serpinskij*, IMI, vyp. 24, 1979, 361-365.
[31] *Pis'ma V. Serpinskogo k N.N.Luzinu*, Volkov V.A., Medvedev F.A. (Eds.), IMI, vyp. 24, 1979, 366-373.
[32] Medushevskij E., *Matematiki i filosofy*, Russkij mir, 7, 2012, 176-198.
[33] *Pis'ma N.N. Luzina k A. Danzhua*, Djugak P. (Ed.), IMI, vyp. 23, 1978, 314-348.
[34] Demidov S.S., Ljovshin B.V. (Eds.), *Delo akademika Nikolaja Nikolaevicha Luzina*, SPb: RHGI, 1999.
[35] Dugac P.N., *Luzin. Lettres à Arnaud Denjoy avec introduction et notes*, Archives Internationales d'Histoire des Sciences, Vol. 27, 1977, 179-206.

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE
1-NP/2014

STANISŁAW DOMORADZKI*, MYKHAILO ZARICHNYI**

ON SOME ASPECTS OF THE SET THEORY AND TOPOLOGY IN J. PUZYNA'S MONUMENTAL WORK

O NIEKTÓRYCH APSEKTACH TEORII MNOGOŚCI I TOPOLOGII W MONUMENTALNYM DZIELE PUZYNY

A b s t r a c t

The article highlights certain aspects of the set theory and topology in Puzyna's work *Theory of analytic functions* (1899, 1900). In particular, the following notions are considered: derivative of a set, cardinality, connectedness, accumulation point, surface, genus of surface.

Keywords: *set theory*, *point-set topology*, *surface topology*, *mathematics at the edge of XIX and XX centuries, history of complex analysis*, *University of Lvov, Józef Puzyna*

S t r e s z c z e n i e

W artykule uwypuklono wybrane aspekty dotyczące teorii mnogości i topologii w dziele Puzyny *Teorya funkcyj analitycznych* (1899, 1900). Odniesiono się m.in. do następujących pojęć: pochodna zbioru, moc zbioru, spójność, punkt skupienia zbioru, powierzchnia, rodzaj powierzchni.

Słowa kluczowe: *teoria mnogości*, *topologia teoretyko-mnogościowa*, *topologia powierzchni*, *matematyka na przełomie XIX i XX w., historia analizy zespolonej*, *Uniwersytet we Lwowie, Józef Puzyna*

* Stanisław Domoradzki, Faculty of Mathematics and Natural Sciences, University of Rzeszów.
** Mykhailo Zarichnyi, Professor of the Faculty of Mechanics and Mathematics, Ivan Franko National University of Lviv.

1. Introduction

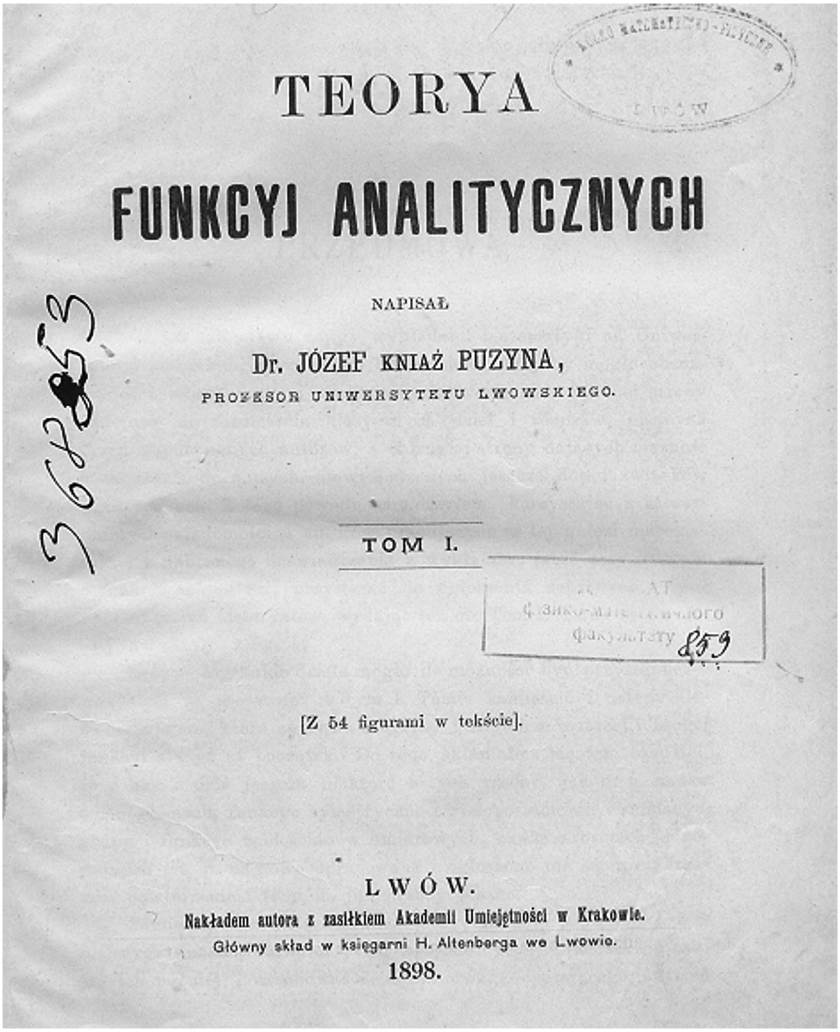
Józef Puzyna was a Polish mathematician. He is recognized as a precursor of the Lvov School of Mathematics. He was born on the 18th of April, 1856 in Nowy Martynów, a place near Rohatyń (now Ukraine). Let us recall the most important events from his biography (see, e.g. [4, 6, 15] for details). After studying at the Lvov University in 1875–1882 as W. Żmurko's student, and at the University of Berlin as K. Weierstrass' student, and, after finishing the doctorate degree in 1883 on the basis of the dissertation *O pozornie dwuwartościowych określonych całkach podwójnych* (*On seemingly bivalent definite double integrals*) at the Lvov University, he associated his scientific and teaching activities only with the Lvov University. In 1885 he got habilitation and taught mathematics as an assistant professor. He headed the Department of Mathematics as an associate professor in the period of 1889–1892 and since 1892 as a professor until his death. He was a very good lecturer and lectured on many branches of mathematics. He also held positions of responsibility at the university: he was the rector in 1904/5 academic year, and vice-rector in 1905/6, Dean of the Faculty of Philosophy in 1894–1895. In 1907 Puzyna participated in the work related to a survey conducted among all university professors of mathematics of the Monarchy. The purpose of this survey was to develop a memorandum which was submitted to the Minister of Religious Affairs and Education in Vienna. The memorandum showed the necessity of increasing the number of chairs of mathematics at universities of the Monarchy. Since 1917, Puzyna served as the President of the Mathematical Society in Lvov. Among his scientific descendants there were Franciszek Leja, Hugo Steinhaus, Antoni Łomnicki, Wacław Sierpiński, Stanisław Ruziewicz.

Puzyna died in 1919 in Stryj.

Józef Puzyna

Józef Puzyna was extremely devoted to the issues of teaching mathematics, both in schools and at the universities. From his numerous reviews one can learn that he paid a lot of attention to the contents of textbooks emphasizing the role of general ideas in exposition of the material. As Puzyna wrote, "a student of mathematics should know about those who for centuries made it possible for us to get that knowledge in a general and comfortable form that we can enjoy today".

One of Puzyna's main achievements was his monograph *The theory of analytic functions.* When J. Puzyna asked the Ministry of Religion and Enlightenment in Vienna for a grant to publish *The theory of analytic functions*, he received a negative response as there was a shortage of resources for research (as well as positions). The book was published in two volumes [16, 17]. Both volumes were self-published by the author, with some support by Academy of Sciences and Arts in Kraków.



TEORYA

FUNKCYJ ANALITYCZNYCH

NAPISAŁ

Dr. JÓZEF KNIAŻ PUZYNA,

PROFESOR UNIWERSYTETU LWOWSKIEGO.

TOM I.

[Z 54 figurami w tekście].

LWÓW.

Nakładem autora z zasiłkiem Akademii Umiejętności w Krakowie.

Główny skład w księgarni H. Altenberga we Lwowie.

1898.

Fig. 1. The title page of Puzyna's monograph, Vol. 1

In the present paper we touch on questions concerning expositions of elements of the set theory as well as topology in Puzyna's monograph. We use some materials already published by the first author.

2. Elements of the set theory

The set theory was created by Georg Cantor in 1874. Its foundations were expounded in Cantor's paper [3]. Immediately after its inception, the new theory actually divided the mathematical world. Some mathematicians (Frege, Dedekind, Hilbert) fully accepted it while others, including Poincaré and Hermann Schwarz, categorically rejected it.

A new wave of interest in the set theory emerged in the early 20th century, when the famous paradoxes of the set theory were discovered. In particular, it became clear that the concept of the set of all sets led to contradiction.

The history of the set theory, or rather part of it related to Georg Cantor, as well as penetration of the ideas of set theory in Polish mathematics is described in detail in the book [19]. However, we have to remark that not much is said about Puzyna, although his significant achievements in this area are emphasized.

The history of the development of the set theory in Poland usually begins with the name of W. Sierpiński, who became interested in this theory in 1908 and gave the first lecture on the set theory at the Lvov University in 1909. Sierpiński drew attention of his students to this subject. Several of his works on the general set theory and theory of functions of a real variable were published in the "Wektor" magazine in 1912–1913. He wrote a book *Zarys teoryi mnogości* (*The outline of the set theory*) in 1919. But it was Puzyna who was the pioneer in introducing the language of set theory, and used the language of intuitive topology in teaching mathematics. Note that *Studya topologiczne* (*Topological studies*) appear in the list of courses taught at the Jan Kazimierz University in Lvov already in 1898.

The book was immediately noticed in Poland as well as abroad. In particular, Placyd Dziwiński wrote in "Kosmos" (XXIV, 1900): "Already the first volume drew attention of the world by the richness of its content and independent treatment of the subject". Here Dziwiński also cited a report from "Naturae Novitates", where its author criticizes that the book was written in an "incomprehensible" language. Nevertheless, the reviewer in "Naturae Novitates" emphasizes that the monograph is "an original work from the beginning to the end".

Decades later, Puzyna's book was characterized by Saks and Zygmund in the monograph [18] as follows: "This work is a veritable encyclopedia of Analysis: in addition to the "Theory of Analytical Functions – partially in beautiful Weierstrass presentation – includes knowledge of Set Theory and Topology (Analysis Situs), Group Theory, Algebra, Differential Equations, Harmonic Functions. If it appeared in any of the more prevalent foreign languages, it would have a further, increasingly sophisticated editions, with all the makings for becoming a classic textbook. Today, after 40 years since the year of the original, a new development of the comprehensive work by Puzyna and adapting it to modern forms of treatment of the subject is beyond capabilities the authors of this book. (...)".

We can assume that the exposition of the material, based on set theory, seemed quite revolutionary. Puzyna's book was published before the invention of well-known paradoxes of the set theory.

The third part of the monograph by Puzyna is called "the theory of sets". The material begins with a definition of finite and infinite field of real and imaginary (complex) variable.

Literatura podręcznikowa w obcych językach z zakresu Teorii Funkcyj Analitycznych jest bardzo obfita i podawanie jej na tym miejscu byłoby nawet kłopotliwe. Natomiast w języku polskim obok jednego przekładu (E. Goursat, *Kurs Analizy Matematycznej*, t. II, z francuskiego przełożył T. J. Łazowski, Warszawa 1919) mamy jedno tylko dzieło oryginalne, mianowicie: J. Puzyna, *Teoria Funkcyj Analitycznych*, t. I (str. XVIII+549), Lwów 1898, t. II (str. XIV+693) Lwów 1900. Dzieło to jest prawdziwą encyklopedią Analizy: obok właściwej Teorii Funkcyj Analitycznych — wyłożonej częściowo w pięknym Weierstrassowskim ujęciu — zawiera wiadomości z zakresu Teorii Mnogości i Topologii (Analysis Situs), Teorii Grup, Algebry, Równań Różniczkowych, Funkcyj Harmonicznych. Gdyby ukazało się w którymkolwiek z bardziej rozpowszechnionych języków obcych, doczekałoby się dalszych, coraz doskonalszych wydań, mając wszelkie dane, by stać się podręcznikiem klasycznym. Dziś, po 40 latach od chwili ukazania się oryginału, nowe opracowanie wszechstronnego dzieła Puzyny i przystosowanie go do współczesnych form ujmowania przedmiotu przerasta możliwości autorów tej książki. Wybrali przeto drogę opracowania książki nowej, jakkolwiek o niewspółmiernie węższym zakresie i skromniejszych ambicjach.

Fig. 2. A fragment from the Introduction to the Saks' and Zygmund's monograph

The boundary of a domain is defined rather informally. The author remarks without precise definition that the boundary can be formed by (parts of) curves and points. One of the most important notions here is that of neighborhood. Neighborhoods at infinity are also considered in the book. It is proved that any infinite countable set of points contains an accumulation point (which may be infinity).

The notion of a derived set was introduced by Cantor in 1872. The (first) derivative of an infinite set P is denoted by P'. If P' is infinite, then one similarly defines the second derivative P'' etc. If the set $P^{(\nu)}$ is finite, then Puzyna writes that there is no derivative of the $(\nu + 1)$-st order i.e., $P^{(\nu+1)} = 0$ (this means that this derivative is the empty set).

By the definition, the first order sets are those whose some finite derivative is empty. Otherwise, they are called the second order sets. Puzyna provides an example from Mittag-Lefler's paper [10] of a set of reals P such that $P^{(\nu-1)}$ is countable and $P^{(\nu)}$ is empty (i.e. the degree of P is ν). The set of rationals on the segment (0,1) is an example of the second order set.

If the points of the derivative of a set P do not belong to P (i.e. if $PP' = 0$), then P is called an isolated set (the set of isolated points).

According to Cantor, the sets P such that $PP' = P'$ are said to be closed. The everywhere dense sets are also defined.

The intersection of all finite derivatives of a set P is denoted by $P^{(\omega)}$. The equation $P^{(\omega)} = 0$ characterizes the first order sets.

The derivatives of transfinite order are also defined. Puzyna neither provides the definition of a transfinite (ordinal) number nor cites Cantor's paper [2] in which the transfinite numbers are introduced. Puzyna does not strive to be precise in these considerations and proceeds by using rather informal description. He first defines the derivatives

$$P^{(\omega+1)}, P^{(\omega+2)}, P^{(\omega+3)}, \ldots \qquad (*)$$

Similarly as in the case of $P^{(\omega)}$ he defines the derivative $P^{(2\omega)}$ as the common part of the derivatives (*). Without further explanations (and without exposition of the theory of well-ordered sets), the author provides the following table for all the countable ordinal numbers as the degrees of the derivatives:

I.		1	2	...	ν...
II.	ω	$\omega+1$	$\omega+2$	...	$\omega+\nu$...
	2ω	$2\omega+1$	$2\omega+2$	...	$2\omega+\nu$...
	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
	$\nu_1\omega$	$\nu_1\omega+1$	$\nu_1\omega+2$	...	$\nu_1\omega+\nu$...
	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
	ω^2	ω^2+1	ω^2+2	...	$\omega^2+\nu$...
	$2\omega^2$	$2\omega^2+1$	$2\omega^2+2$	...	$2\omega^2+\nu$...
	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
	$\nu_2\omega^2$	$\nu_2\omega^2+1$	$\nu_2\omega^2+2$	...	$\nu_2\omega^2+\nu$...
	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
	$\nu_2\omega^2+\nu_1\omega$	$\nu_2\omega^2+\nu_1\omega+1$	$\nu_2\omega^2+\nu_1\omega+2$	...	$\nu_2\omega^2+\nu_1\omega+\nu$...
	$\vdots$		$\vdots$	...	$\vdots$
	$\nu_k\omega^k+\ldots+\nu_1\omega$	...	...	...	$\nu_k\omega^k+\ldots+\nu_1\omega+\nu$...
	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$
	ω^ω	$\omega^\omega+1$	...	...	...

Fig. 3. A fragment from Puzyna's monograph: ordinal numbers

The numbers in this table are called transfinite numbers. Without formally defining the notion of well-ordering Puzyna however notices its fundamental property, namely that for every element of such a (well ordered) set there exists a well defined immediate subsequent element of this set. The finite (resp. infinite countable) ordinal numbers are called the numbers of class I (resp. of class II).

Note that the first uncountable ordinal is usually denoted by ω_1 not ω^ω and it will be seen later that the latter notation leads to a confusion. Note also that it was hardly possible to provide in the monograph a strict exposition of the theory of well-ordered sets.

Then Puzyna provides Mittag-Leffler's examples of set of reals P such that:

a) $P^{(\omega)}$ = point zero, $P^{(\omega+1)} = 0$,
b) $P^{(\omega+\nu)}$ = point zero, $P^{(\omega+\nu+1)} = 0$,
c) $P^{(2\omega)}$ = point zero, $P^{(2\omega+1)} = 0$.

Puzyna does not define the notion of cardinality. The countable sets are defined as the sets that can be exhausted by means of a process of successive elimination of their elements. He uses the term the sets of the first cardinality for the finite and countable infinite sets.

Some fundamental properties of these sets are established, in particular:

a) any subset of a set of the first cardinality is of the first cardinality,
b) the union of any countable family of sets of the first cardinality is also of the first cardinality.

Without formal proofs it is explained later that the well-ordered sets of the second class are of the first cardinality.

Next, Puzyna considers the cardinality of the segment (0,1). He denotes this cardinality by ω^ω. The explanation uses the expansion of real numbers into continued fractions. Finite fractions are in one-to-one correspondence with the set of all maps of n into itself, i.e., n^n. Similarly, the set of all irrational numbers in (0,1) is in one-to-one correspondence with the set of all maps of ω into itself, i.e., the set ω^ω. Note that the latter is the upper bound of n^n, where n is natural. This is in a sense similar to some of Euler's arguments or to the proofs in the style of non-standard analysis. The notation ω^ω appears already on page 97.

It denotes the ordinal number which is the least upper bound of all countable cardinals. One can hardly find an explanation of this notation, but a few pages later it leads to an erroneous conclusion concerning the continuum hypothesis.

Returning to the set (0,1), the author shows its cardinality is that of cardinality of the mentioned set of all irrational numbers in (0,1). Therefore, the cardinality of the set of all real numbers is ω^ω. Then, using the completeness of the set of reals, he proves that the cardinality of the set (0,1) is uncountable.

Page 108 contains the (clearly wrong) conclusion that the cardinality ω^ω immediately follows the countable cardinality. Puzyna calls such sets to be of class II.

Then the following question is considered: what is the cardinality of a subset in the n-dimensional real domain? At the very beginning, the author considers the (closed) n-dimensional cube. It is interesting to note that the notation for this set rather differs from the modern style and is the following:

$$(x_1, \ldots, x_n) = (0 \ldots 1, 0 \ldots 1, \ldots, 0 \ldots 1)$$

(Here we see that Puzyna does not use the symbol $\in$ (or the script epsilon) for the set membership, despite the fact that Peano used this notation already in 1889.)

In order to prove that the n-dimensional cube has the same cardinality as the unit segment, Puzyna first passes to the set of points with all irrational coordinates (earlier, it is established that the setter set is of the same cardinality). Then he uses the trick of forming one number out of n using the decimal expansions.

Note that this question was later asked by W. Sierpiński.

In the footnotes, Puzyna mentions G. Peano's article *Sur une courbe, qui remplit toute un aire plaine*, "Mathematische Annalen", T. 36, (1890), p. 157. In this article Peano discusses the considerably more complicated problem of existence of continuous maps from the unit segment onto the square.

It is proved that, for any countable set in the unit cube, there exists a point in the cube such that every its neighborhood contains a point of the set (the so-called accumulation point). In modern terminology, this is precisely the proof of compactness of the cube in the Euclidean space. The method used in the proof is that of dividing of the square into four equal parts. The required accumulation point is that of intersection of the family of descending squares containing the infinite set of points of the countable set under consideration. It is remarked that similar arguments work also in the n-dimensional domain.

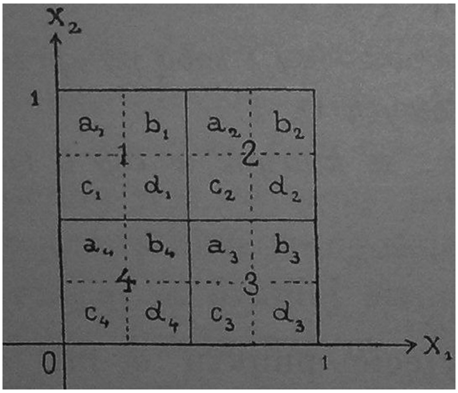


Fig. 4. A figure from Puzyna's monograph: division of the square

Since no precise definition of set is given, the author explains that "the *entire*, bounded or unbounded, domain should be regarded as a set". These sets are called continua or continuous domains. The definition of a continuum is, however, that of an open set. Also, Puzyna introduces the notion of the boundary of an open set (continuous domain).

The author does not define the notion of compactness explicitly. In the subsequent sections, the property of compactness is needed in the proof of the fundamental theorem of algebra, therefore the proof enclosed in the monograph seems to be incomplete.

Later the sets of the first and the second cardinality are considered in the *n*-dimensional domains. Puzyna proves that these domains are of the same cardinality as the set of reals.

The notion of continuum is defined as the set of points satisfying the following property: all the points in a neighborhood of any of its points belong to the set as well. The notion of connected domain („obszar zwarty" in Polish; note that in the modern Polish mathematical terminology „zwarty" means "compact") is rather informal, it sounds as follows: a "continuum" is a set such that from any of its point one can pass to any other its point through points only belonging to this „continuum". It is proved that the complement to any countable set in a continuum is also a continuum. Note that the modern form of the notion of connectedness was hardly known to Puzyna.

It is remarked that the notions of upper bound and lower bound are derived from the set theory.

Section III is concluded with the notion of stereographic projection. This map is a homeomorphism between the plane and the punctured unit sphere. Puzyna uses the term "pokrewieństwo" ("kinship") for this map and speaks of a "circumference kinship" (circumference-preserving homeomorphism) or "isogonal kinship" (conformal map).

3. Topology of surfaces

Let us turn our attention to Part V of the monograph that deals with Riemann surfaces. We already remarked that not all mathematicians preferred using the language of the set theory in their research.

The exposition here starts with the definition of a closed surface. However, this definition is necessarily not rigorous as the author avoids using charts, i.e. homeomorphisms onto domains of Euclidean spaces. Actually, we find here an informal description of surfaces.

The simply connected surfaces are introduced by means of intuitive definition. These are the surfaces that satisfy the following properties:

1) Every curve connecting two points of the surface can be transformed into another one so that it does not leave the surface in the process of transformation. The endpoints of the curve either are the same or can change.
2) Every connected curve contained in the surface can be shrunk to an arbitrary point, while remaining on the surface in the process of shrinking.
3) If the surface possesses the boundary, then every simple (non-self-intersecting) curve that connects two distinct points of the boundary divides the surface into two separate parts.

In modern terms, the author implicitly uses the notion of homotopy (isotopy) of continuous maps in this definition.

It is remarked that the boundary of any simply connected surface cannot contain two closed separate pieces and that any cut of a simply connected surface yields two simply connected surfaces.

Then n-connected surfaces are also introduced. These are the surfaces in which one can make n–1 cuts such that the result of cutting is a simply connected surface.

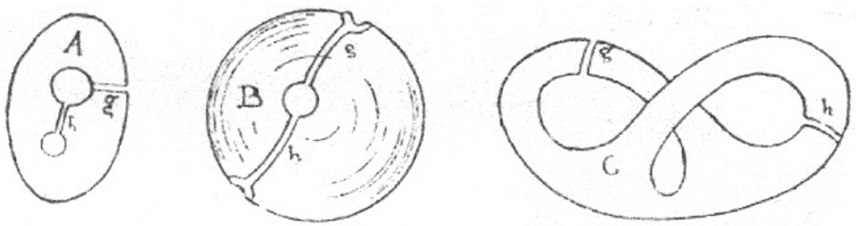


Fig. 5. Examples of 2-connected surfaces are: a planar annulus, sphere with two holes etc. The third figure in the above picture is an example of a 3-connected surface. The proofs of statements on the surfaces are based on intuitive approach

A closed curve on a surface (either having self-intersections or not) is called a *circumference* on this surface. A circumference is reducible (contractible, in modern terminology) if it can be deformed within the surface to a point, otherwise it is called irreducible. The notion of a complete system of irreducible circumferences is introduced and it is shown that every irreducible circumference can be uniquely represented as an equivalent one to a combination of circumferences from a chosen complete system. Actually, the homotopy classes of circumferences form the fundamental group of a surface and the mentioned complete system of irreducible circumferences is precisely a set of generators of this group. The notion of fundamental group was introduced by H. Poincaré in [14]; this article is cited by Puzyna despite the fact that he avoids using here the language of the group theory.

Next, the notion of genus of a surface is defined. The genus is the half of the number of cuts needed to obtain a simply connected surface.

All the above considerations work for oriented surfaces and it was implicitly assumed earlier that the surfaces under consideration possess this property. Perhaps the simplest example of a non-oriented surface is the Möbius band. Puzyna cites V. 2 of *Werke* by Möbius [11].

When describing the notion of deformation of surfaces Puzyna uses the informal terms:
1) points that are infinitely close remain so in the mapped surface,
2) finitely distanced points are also finitely distanced in the mapped surface.

He formulates the following statement: Given two surfaces, one can deform one of them into the other whenever they are of the same connectedness and have the same number of the circles on the boundary. The arguments given in the monograph are informal and cannot be considered as a proof of this fact.

In the case of closed surfaces, the following statement is formulated: every closed (oriented) surface can be deformed into a sphere with finite number of handles. This is the classification theorem for oriented surfaces, which is known to be a powerful and complicated result. The theorem was stated in various forms by different authors.

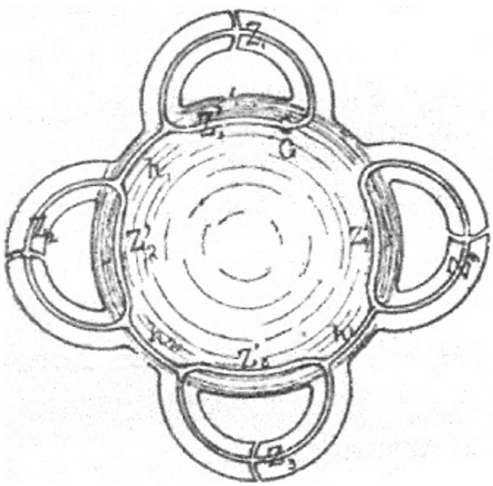


Fig. 6. A figure from Puzyna's monograph: Sphere with handles

The exposition of this proof is again based on an intuitive approach.

A generalization of Euler's theorem to (triangulable) surfaces is also given. This allows the author to consider the Euler characteristic of a surface.

The following section contains a description of the construction of the Riemann surfaces, first, at a neighborhood of a branching point. This construction is illustrated by the following pictures.

It is proved that the algebraic notion of genus of any Riemann surface can be also described in topological terms. Actually, the genus is a topological invariant of a surface.

The material also contains various information on algebraic curves. In particular, an analysis of singularities of the algebraic curves by means of the quadratic maps is given.

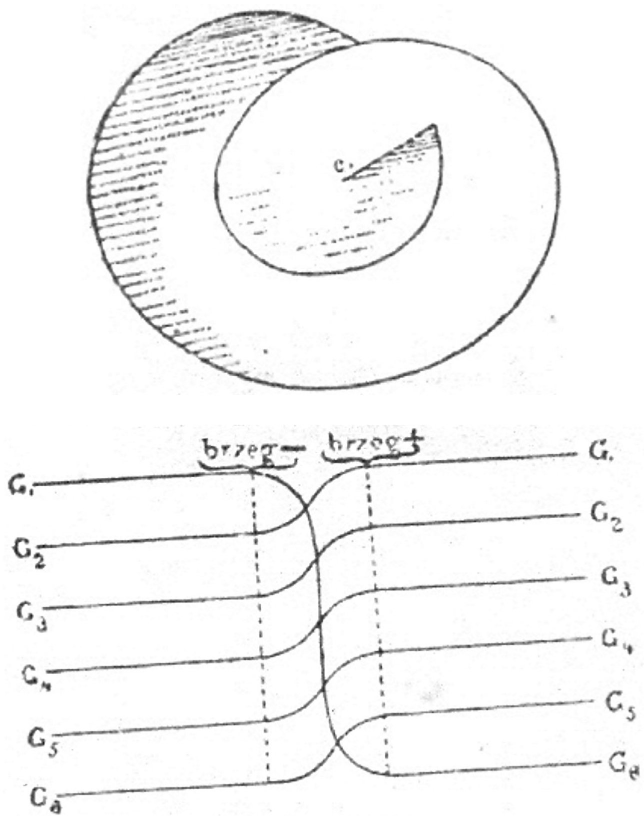


Fig. 7. Neighborhood of a branching point

It is interesting to look on Puzyna's book from the point of view of the unity of mathematics. The introductory parts contain material from set theory and geometry, as well as algebra, in particular group theory.

The exposition of the material is rigorous throughout the book. However, in some places the style becomes rather narrative when the author deals, e.g., with topology of the plane.

Note that even simply formulated and intuitively evident statement of the planar topology can have complicated proofs, and the famous Jordan curve theorem is a good example supporting this statement.

4. Conclusions

The material of Puzyna's book demonstrates that the author belonged to the part of mathematical community that accepted the most fundamental ideas of set theory. One can hardly overestimate the importance of the monograph for the further development of the set theory in Poland.

At the same time, in the monograph one can find an approach to exposition of topological notions which is not based on set-theoretical language. Describing the topological properties of (Riemann) surfaces Puzyna prefers the intuitive and visual arguments, rather in the spirit of Poincaré. This combination of styles is somewhat eclectic, but can be justified from a didactic point of view.

This work was partially supported by the Centre for Innovation and Transfer of Natural Sciences and Engineering Knowledge, University of Rzeszów.

References

[1] M. Bečvářová, *Czech Mathematicians and Their Role in the Development of National Mathematics in the Balkans*, in M. Bečvářová, Ch. Binder (eds.), Mathematics in the Austrian-Hungarian Empire. Proceedings of a Symposium held in Budapest on August 1, 2009 during the XXIII ICHST, edition History of Mathematics, volume 41, Matfyzpress, Prague 2010 (second edition, Matfyzpress, Prague 2011), 9-31.

[2] G. Cantor, *Beitrage zur Begrundung der transfiniten Mengenlehre II* (tr.: *Contributions to the Founding of the Theory of Transfinite Numbers II*), Mathematische Annalen, vol. 49, 1897, 207-246.

[3] G. Cantor, *Ueber eine Eigenschaft des Inbegriffes aller reellen algebraischen Zahlen*, J. Reine Angew. Math., vol. 77, 1874, 258-262.

[4] S. Domoradzki, *The Growth of Mathematical Culture in the Lvov area in the Autonomy period (1870–1920)*, Matfyzpress, Prague 2011.

[5] S. Domoradzki, *Teoria mnogości w dziele Józefa Puzyny Teorya funkcyj analitycznych*, [in:] W. Więsław (ed.), *Dzieje matematyki polskiej*, Instytut Matematyczny Uniwersytetu Wrocławskiego, Wrocław 2012, 45-58.

[6] R. Duda, *Lwowska szkoła matematyczna*, Wydawnictwo Uniwersytetu Wrocławskiego, Wrocław 2007.

[7] R. Duda, *Selected Mathematical Achievements of Poles: a Historicial Perspective*, The Contribution of Polish Science and Technology to World Heritage, Kraków–Warszawa 2010, 57-76.

[8] R. Duda, *Die Lemberger Mathematikerschule*, JB 112, Heft 1, 2010, 3-24.

[9] C. Goldstein, J. Gray, J. Ritter (eds.), *L'Europe mathématique: histoires, mythes, identités*, Paris 1996.

[10] G. Mittag-Leffler, *Sur la représentation analytique des fonctions monogènes uniformes*, Acta Mathematica, vol. IV, 1884, 1-79.

[11] A.F. Möbius, *Werke*, Vol. 2, 1858.

[12] G. Peano, *Sur une courbe, qui remplit toute un aire plaine*, Mathematische Annalen, vol. 36, 1890, 157-160.

[13] A. Płoski, *O dziele Józefa Puzyny „Teorya funkcyj analitycznych"*, [in:] S. Fudali (ed.), Materiały z II Ogólnopolskiej Szkoły Historii Matematyki, Szczecin 1988, 237-243.

[14] H Poincaré, *Analysis Situs*, 1895.

[15] Y.G. Prytula, *Remarks on the history of mathematics in Lviv up to the middle of the XXth centuary, Lviv mathematical School in the period 1915–45 as seen today*, Banach Center Publications, Institute of Mathematics Polish Academy of Sciences, vol. 87, Warszawa 2009, 17-26.

[16] J. Puzyna, *Teorya funkcyj analitycznych*, vol. I, Lwów 1898.

[17] J. Puzyna, *Teorya funkcyj analitycznych*, vol. II, Lwów, 1900.
[18] S. Saks, A. Zygmund, *Funkcje analityczne*, Monografie Matematyczne X, Warszawa–Lwów––Wilno 1937.
[19] G.I. Sinkievich, *tr. Georg Cantor and Polish school of the set theory*, Saint-Petersburg 2012.

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE
1-NP/2014

ROMAN DUDA*

SOME REMARKS CONCERNING RECEPTION OF MATHEMATICS IN CENTRAL-EASTERN EUROPE IN THE YEARS 1850–1920

KILKA UWAG ODNOŚNIE DO RECEPCJI MATEMATYKI W EUROPIE ŚRODKOWO-WSCHODNIEJ W LATACH 1850–1920

Abstract

In the flow of mathematical ideas from the West to Central-Eastern Europe one can distinguish several typical forms: 1) foreign mathematicians, invited to cultivate mathematics upon new ground (e.g. Euler in Russia); 2) domestic mathematicians who completed their studies abroad and continued research after returning home (e.g. W. Buniakowski or M. Ostrogradski in Russia); 3) domestic mathematicians who dared developing new directions, thus initiating original schools of mathematics (e.g. N. N. Lusin in Russia). A separate phenomenon was a startling discovery of non-euclidean geometry (N. N. Lobatchevsky in Russia, J. Bolyai in Hungary).

Keywords: *cultivation of mathematics*, *continuation of research*, *mathematical journal*, *founding a school in mathematics*

Streszczenie

W przepływie idei z Zachodu do Europy Środkowo-Wschodniej można wyróżnić kilka typowych form: 1) matematycy obcy, zapraszani do wdrażania matematyki na nowej glebie (np. L. Euler w Rosji); 2) matematycy rodzimi, którzy po studiach za granicą kontynuowali badania w zakresie tamtejszej problematyki (np. W. Buniakowski I M. Ostrogradski w Rosji); 3) matematycy rodzimi, którzy odważyli się na rozwijanie nowych kierunków, kładąc w ten sposób podwaliny pod oryginalne szkoły (np. N. Łuzin w Rosji). Osobnym zaskakującym wydarzeniem było odkrycie geometrii nieeuklidesowej (N. N. Łobaczewski w Rosji, J. Bolyai na Węgrzech).

Słowa kluczowe: *uprawianie matematyki*, *kontynuacja badań*, *czasopismo matematyczne*, *założenie szkoły matematycznej*

* Roman Duda, Professor emeritus, Institute of Mathematics, University of Wrocław.

In the period 1850–1920 there were considerable differences in the development of mathematics (and of other exact sciences as well) between Western and Central-Eastern Europe. The West was then further ahead and so the dominant flow of ideas was from the West to the East. At that time Central-Eastern Europe had been still on the acquiring side.

Domination of the direction from the West to the East does not mean, however, that there was no flow in the opposite direction. In our part of Europe some new ideas of great scientific value also occurred, just to mention Mendeleyev's periodic table, Lobachevski's or Bolyai's geometry, Smoluchowski's statistical physics. Phenomena of such a kind, however, were rare and rather exceptional.

Being on the acquiring side means a transfer of scientific ideas from the outside, and their reception (assimilation) on the so far alien soil. Although the aim of this note is to describe this process, its course and its effects in the specific area of mathematics, it seems, however, that analogous processes can be also observed with respect to other exact sciences.

Ideas live in people and move with them. For this note particularly important are people who possessed some mathematical ideas and worked actively upon their development and transmission. Quite often such a role was played by foreigners like Leonhard Euler (1707–1783) or Jakob Bernoulli (1759–1789) in Petersburg, Christian Martin Bartels (1769–1826) in Kasan, Simone Antoine Lhuillier (1750–1840) in Poland, Czech mathematicians in Bulgaria [2], Otto Volk (1892-1989) in Lithuania [1]. All of them were people from abroad who came to their new destination in our part of Europe as men with already established reputation and who successfully ingrained their ideas into minds of local people. Their role was to initiate development and their influence has been of great value.

Emerging scientific institutions in Central-Eastern Europe required new personnel and so it seemed reasonable to send talented people abroad with the task of gaining education there and to share their knowledge with local students after return. In the period considered it was a common practice and some of those who returned became quite influential at home. Such was the case of Jan Śniadecki (1756–1830) in Poland, who after return from Paris had reformed university in Cracow, wrote some manuals and taught extensively. Later he became the rector of Vilnius university and continued his reforms and teaching there. He deserves much credit for the raising of (then-low) mathematical culture in Poland.

Such local people who first studied in leading mathematical centres in Western Europe and then returned filled with knowledge, ideas and enthusiasm, and who as a rule kept in contact with their teachers, were essential for further development. They were important not only because of their number (in that period there were many of them) but mainly because of their close contact with the local territory. After their return home, they could more easily (than their foreign predecessors) recognize local needs and evaluate local chances. In consequence they were usually more effective in training new adepts and creating new trends – sometimes even original schools in themselves.

Such native bearers of knowledge (acquired elsewhere) were taking two distinct courses of action at home: continuation or starting something new. These two kinds will be now described and exemplified.

More often there was a continuation. This was a case of a man who mastered a skill in a specific area of mathematics abroad and distinguished himself there by a Ph.D. thesis and/or original publications, and who successfully continued research in that area after return. Acquaintances he struck up made it for him relatively easy to publish results obtained at home and to gain in that way some appreciation abroad. In such a way the man could become known in the community and his mathematics could achieve wider circulation.

Some early examples are from Russia:

Wiktor Buniakowski (1804–1889). He studied in Paris, defending his Ph.D. thesis in 1825. After return to Russia, he had spent his life in Petersburg where he worked both at the university and in the Academy. He was a professor of the university in the years 1826–1864 and became an adjunct to the Academy in 1826, since 1830 its ordinary member and from 1864 until the end – its deputy president. His interests embraced number theory, mathematical analysis and calculus of probability, and his best known result was the Buniakowski-Schwarz inequality, proved independently by both named mathematicians (Buniakowski in 1859, Schwarz in 1884).

Michail Ostrogradski (1801–1861). A similar course of life. He studied 1816–1820 in Kharkov and then 1822–1828 in Paris. After return to Russia, he also had spent his life in Petersburg where he was teaching in several institutes of higher education and at the same he became an adjunct to the Academy in 1828 and ordinary member since 1830. His interests in mathematics were diverse, including mathematical analysis, mechanics and mathematical physics, but those most important were concerned with the propagation of heat. He discovered the well-known Ostrogradski formula relating triple integral with respect to a volume and double integral with respect to the surface of that volume.

The two men, Buniakowski and Ostrogradski, were the first Russian-born mathematicians who gained an international status. There was, however, a delay in their recognition due to the fact that Russia was then at the periphery of scientific world and many of their results were published in Russian, a language then hardly known outside the Russian empire.

Two other examples are from Poland:

Stanisław Zaremba (1863–1942). Born in Ukraine, he studied in Petersburg and Paris, receiving Ph.D. degree in 1889 in Paris. In 1900 he accepted an invitation from the Jagiellonian university and spent the rest of his life in Cracow. Zaremba was interested in problems of mathematical analysis related to physics, particularly in partial differential equations. He was highly valued by H. Poincaré [14] and by H. Lebesgue, among others [7].

Kazimierz Żorawski (1866–1953). After studies at the Russian university in Warsaw he got a scholarship to continue abroad. He went to Göttingen and Leipzig. Sophus Lie became his master in Leipzig and there Żorawski got his Ph.D. After return to the home country he stayed briefly in Lvov and from 1893 in Cracow, where some years later he was joined by Zaremba. Żorawski worked in the theory of Lie groups, publishing in Polish and German. Highly evaluated by S.Lie and E. Cartan, he was the only Polish mathematician mentioned by F. Klein in his account of mathematics in the XIX century [9].

The two men, Zaremba and Żorawski, were the first Polish mathematicians who gained an international status in modern times [16] (earlier there were some Polish mathematicians enjoying international recognition, but it happened only in the XV–XVI centuries).

Because of studies abroad and the "continuation" process after return, as in the just--described cases of Petersburg and Cracow, communities appeared which understood modern mathematics and maintained contacts with leading mathematicians abroad. Some of these communities were supported by newly founded journals (publishing, however, mostly in native languages). The general level of mathematics in Central-Eastern Europe was thus rising and in such conditions a totally new phenomenon could appear: some new domestic leaders could have nourished and then developed new ideas, bringing about their own students, and thus raising "schools" of mathematics which also influenced the development abroad. Particularly influential were two such "schools": in Moscow and in Warsaw.

Bolesław Młodziejewski (1858–1923) was a Russian mathematician of Polish origin. Born in Moscow, he studied there but completed his studies in Zürich, Paris and Göttingen. After return, he became professor of the Moscow university in 1892. His lectures on the theory of real functions and his seminar begun to disseminate ideas of the French school of that theory in Moscow [11, 12]. Młodziejewski has been joined by D.F. Egorow (1969–1931), I.M. Żegałkin (1869–1941) and N.A. Bugajew (1837–1903), and those in turn by their students including P.A. Florenski (1882–1937), N.N. Lusin (1883–1950) and S.P. Nowikow (1883–1964). Of the latter trio the most eminent mathematician was Lusin. He had an opportunity to spend several longer periods in Paris and it was his "Lusitania", as the group of students surrounding him was called, which began the great history of the Moscow school of mathematics [18]. The school was not a direct continuation of the French school, but it started a new domain of mathematics, so-called descriptive set theory [13], and greatly influenced some others, including topology, functional analysis, and probability theory. It included great talents of P.S. Aleksandrov (1896–1982), P.S. Urysohn (1898–1924), S.L. Sobolew (1908–1989), A.N. Kolmogorov (1903–1987) and many others. Despite difficult times (Soviet terror, "Luzinshchina" [3], isolation from the outside world), the Moscow school was an extraordinary phenomenon, soon to become one of the leading mathematical centres in the XX century.

The Warsaw school of mathematics developed along similar lines. Its origins are going back to Lvov (not to Cracow, as one might suppose). In 1908 Wacław Sierpiński (1882–1969) became a docent at the Lvov university and soon thereafter a professor. He taught theory of sets and its applications to real functions and topology, introducing some new ideas of his own. More important, Sierpiński began to gather around himself a group of ambitious young men like Stefan Mazurkiewicz (1888–1945), Stanisław Ruziewicz (1889–1941) or Zygmunt Janiszewski (1888–1920). Members of the group began to publish original results from the area of their interest but this promising seedbed of a new mathematical centre was soon dispersed. After the outbreak of the World War I in 1914 Sierpiński was interned in Russia (where he happened to be on holidays), Janiszewski volunteered to enlist in Polish troops, Mazurkiewicz returned to his native Warsaw, and Ruziewicz was drafted into the Austrian army. The group ceased to exist.

At the end of the war Janiszewski and Mazurkiewicz found themselves in Warsaw. There was a public inquiry on the needs of Polish science and the three men – Zaremba, Janiszewski, Mazurkiewicz – responded. Zaremba proposed to send young men abroad, to secure them teaching positions in secondary schools after return, to encourage their research, and eventually to offer university posts to the best ones among them [17]. It was nothing new, just an old model of "continuation". In contrast to that, Janiszewski proposed a totally new approach, which can be summarized in a few points:

1) to select one specific area of mathematics, possibly a new one (not with a long tradition behind);
2) to concentrate an attention of all young people upon that area;
3) to create within the group an atmosphere of cooperation, exchange of ideas and mutual aid;
4) to support the group with a newly founded journal devoted specifically to the chosen area and in which articles would be published only in internationally recognized languages [6].

While the first three points could be seen as a summary of experiences of the Lvov group, the fourth one was an original idea of Janiszewski himself.

All points of the Janiszewski's proposal could be disputed. And so they were, both within the country and outside of it. The choice of one specific area of mathematics and concentration of all efforts upon it brought an evident peril of the loss of whole generation of talented people if something went wrong, e.g., if the choice was ineffective. The atmosphere of openness and mutual aid contrasted sharply with the prevailing one of competition. There was no single mathematical journal in the world with the limited scope, while the ban on Polish language could be offensive to many. For instance, H. Lebesgue argued that a journal with a limited scope was doomed because supply of good papers would soon cease [15]. When Lusin pointed out the peril of the domination of one way, Sierpiński responded that it was better to have one than none [5].

Mazurkiewicz was the third to respond the inquiry [10]. He emphasized the necessity of good libraries, of new journals and good books, of scholarships etc. He followed Janiszewski but preferred different development in distinct academic centers.

Janiszewski not only wrote a proposal but also began to collect articles for the first issue of the journal which he founded and named "Fundamenta Mathematicae". The first issue appeared in 1920. He was helped first by Mazurkiewicz and then also by Sierpiński who then just arrived (in 1918) from Moscow. After inviting two logicians, Jan Łukasiewicz (1878–1956) and Stanisław Leśniewski (1886–1939), the five men formed the first Editorial Board of the "Fundamenta Mathematicae" and the article of Janiszewski became the program of the Warsaw school of mathematics, of which Janiszewski, Mazurkiewicz and Sierpiński were obvious leaders. Although Janiszewski soon died, the school became a great success. The jubilee issue of "Fundamenta Mathematicae" 25 (1935) gathered the best world mathematicians of the time, and in 32 issues of it, published in the years 1920–1939, 946 papers appeared, two thirds of which were from Poland and one third from abroad.

Cracow, under the leadership of Zaremba and influenced by him, kept itself apart from the Warsaw movement, but the model of Janiszewski has been duplicated in Lvov. The natural leaders of Lvov mathematics were then Hugo Steinhaus (1887-1972) and Stefan Banach (1892–1945), and the two men founded another journal "Studia Mathematica", limited to functional analysis and supporting the Lvov group. In 9 issues of "Studia Mathematica", published in the years 1929–1940, 161 papers appeared, of which 110 came from the Lvov group.

The Lvov group grew into the Lvov school of mathematics [4] and the Warsaw and Lvov branches were a great success and formed together the so-called Polish School of Mathematics. Its characteristic feature was free use of non-effective methods of proof (based on the Axiom of Choice, measure theory, Baire category etc.), and it had immense influence upon the development of set theory, point-set topology, functional analysis and mathematical logic.

Each of the two schools, Moscow and Warsaw, was like a springboard to elevate Russian or Polish mathematics, respectively, to the world level and common recognition. The Moscow school was more numerous and lasted more than twice as long as the Polish one did, and its influence was stronger and deeper. In both cases, however, it was politics which had the last word. In Moscow politics forced an isolation of the Russian science, including mathematics, from the outside world in the thirties, and put an abrupt end to the Moscow school of mathematics in the seventies [18]. In Poland it was World War II which nearly annihilated the whole intellectual life in the country.

Putting politics aside, there remains the general picture of emerging science in peripheral countries: reception of modern ideas by people returning from studies abroad, their extended continuation in the home country, and eventual creation of original schools. Some totally original ideas which also arose (N.N. Lobatchevsky in Russia, J. Bolyai in Hungary) did not serve as a real stimulus for the elevation; the history shows that they were fought against and recognized only much later. E.g., Buniakowski and Ostrogradski alike actively fought against geometrical ideas of Lobatchevsky.

References

[1] J. Banionis, *The famous mathematician of Lithuanian University Otto Theodor Volk (1892–1989)*, this issue. See also: *Matematyka na Litwie 1920–1940* (*Mathematics in Lithuania 1920–1940*), Wiadom. Mat. 42, 2006, 63-72.

[2] M. Bečvařová, *Czech mathematicians and their role in the development of national mathematics in the Balkans 1850–1951*, History of Mathematics, 41, Prague 2011, 33-42.

[3] S.S. Demidov, B.V. Lefshin (eds.), *Deło akademika Nikołaja Nikołajewicza Łusina* (*The case of academician Nikolai Nikolaevich Lusin*), Sankt-Petersburg 1999.

[4] R. Duda, *Lwowska Szkoła Matematyczna* (*The Lvov School of Mathematics*), Wrocław 2007 (*English translation in print*).

[5] P. Dugac, *N. Lusin. Lettres à Arnaud Denjoy avec introduction et notes*, Arch. Intern. Hist. Sci. 27, 1977, 179-206; translated fragments (into Polish): Wiadom. Mat. 25.1, 1983, 65-68.

[6] Z. Janiszewski, *Stan i potrzeby matematyki w Polsce* (*State and needs of mathematics in Poland*), Nauka Polska 1, 1918, 11-18; reprint in English in the book: S.M.G. Kuzawa, *Modern Mathematics. The Genesis of a School in Poland*, New Haven 1968, 112-118; reprint in Polish: Wiadom. Mat. 7.1, 1963, 3-8.

[7] *Jubilé scientifique de M. Stanislas Zaremba*, Kraków 1930.

[8] J.-P. Kahane, *Aperçu sur l'influence de l'école mathématique polonaise 1918–1939*, Centre Scientifique de l'Académie Polonaise des Sciences à Paris, 1992; Polish translation: *Próba oceny wpływu polskiej szkoły matematycznej lat 1918–1939*, Wiadom. Mat. 31, 1995, 163-175.

[9] F. Klein, *Die Entwicklung der Mathematik im XIX. Jahrhundert*, 2 volumes, Berlin 1927 (volume 2, 202).

[10] S. Mazurkiewicz, *O potrzebach matematyki w Polsce* (*On the needs of mathematics in Poland*), Nauka Polska 2, 1919, 1-5.

[11] F.A. Medvedev, *Francuzskaja szkoła teorii funkcyj i mnożestw na rubieże XIX–XX ww.* (*The French school of the theory of functions and of sets at the turn of XIX–XX c.*), Moscow 1976.

[12] F.A. Medvedev, *O kurse lekcyj B.K. Młodziejewskogo po tieorii funkcyj dejstwitelnogo peremennego, proczytannych osenju 1902 g. w moskowskom uniwersitete* (*On the course of lectures on theory of functions of a real variable read by B.K. Młodziejewski in the autumn 1902 at the Moscow university*), Istoriko-matem. Issledow. 30, 1986, 223-246.
[13] C. Phili, *La génese et le développment de la théorie descriptive des ensembles*, Istoriko-mat. Issledow., II series, Moscow 2010.
[14] H. Poincaré, *Sur l'équation* $\Delta u + \xi u = 0$*. Analyse d'un mémoire de M. Zaremba*, Bull. Sci. Math. 26(2), 1902, 337-350.
[15] W. Sierpiński, *O polskiej szkole matematycznej* (*On the Polish school of mathematics*), [in:] *Wkład Polaków do nauk. Nauki ścisłe. Wybór artykułów* (*Contribution of Poles to Science. Exact Sciences. A. Selection of Articles*), Biblioteka Problemów 101, Warszawa 1967, 413-434.
[16] W. Ślebodziński, *Wspomnienia matematyka z lat 1903–1968* (*Recollections of a mathematican from the years 1903–1968*), Wiadom. Mat. 12.1, 1969, 17-31.
[17] S. Zaremba, *O najpilniejszych potrzebach nauki w Polsce ze szczególnym uwzględnieniem matematyki* (*On the most pressing needs of science in Poland with a particular attetion to mathematics*), Nauka Polska 1, 1918, 1-10.
[18] S. Zdravkovska, P.L. Duren, *Golden Years of Moscow Mathematics*, American Mathematical Society, 2007.

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE
1-NP/2014

JAN KOROŃSKI*

WŁADYSŁAW ZAJĄCZKOWSKI AND DIFFERENTIAL EQUATIONS IN POLAND IN THE SECOND HALF ON THE NINETEENTH CENTURY

WŁADYSŁAW ZAJĄCZKOWSKI I RÓWNANIA RÓŻNICZKOWE W POLSCE W DRUGIEJ POŁOWIE DZIEWIĘTNASTEGO WIEKU

A b s t r a c t

The paper contains some scientific information on Władysław Zajączkowski (1837–1898) and on his first Polish monograph about ordinary and partial differential equations. Moreover, the aim of this paper is a presentation of selected scientific results of Polish mathematicians publishing in the nineteenth century in the field of ordinary and partial differential equations. Some more details about the publications on differential equations in the 19th century written by Polish mathematicians can be found in [3–9].

Keywords: *ordinary differential equations*, *partial differential equations*, *differential equations in Poland in the nineteenth century*

S t r e s z c z e n i e

W artykule zawarto pewne informacje naukowe o Władysławie Zajączkowskim i jego pierwszej polskiej monografii z równań różniczkowych zwyczajnych i cząstkowych. Ponadto przedmiotem pracy jest prezentacja wybranych rezultatów naukowych matematyków polskich publikujących w drugiej połowie dziewiętnastego wieku w dziedzinie równań różniczkowych zwyczajnych i cząstkowych. Pewne szczegółowe informacje o publikacjach z równań różniczkowych w dziewiętnastym wieku napisanych przez matematyków polskich można znaleźć w [3–9].

Słowa kluczowe: *równania różniczkowe zwyczajne*, *równania różniczkowe cząstkowe*, *równania różniczkowe w Polsce w dziewiętnastym wieku*

* Jan Koroński, Faculty of Physics, Mathematics and Informatics, Institute of Mathematics, Cracow University of Technology.

1. Introduction

We present some information (in the context of differential equations) about Zajączkowski, Zaremba and Żorawski up to 1900. Their papers had the greatest importance in the second half of the 19th century. We devote a lot of attention especially to Władysław Zajączkowski (1837–1898), giving information on important original scientific results, biography and bibliography of Władysław Zajączkowski.

Zaremba and Żorawski are better-known mathematicians, while Zajączkowski seems to be a bit forgotten (wrongly so) [6]. The vast majority of Zaremba's and Żorawski's scientific activities took place only in the twentieth century.

2. Władysław Wojciech Zajączkowski (1837–1898)

Władysław Wojciech Zajączkowski was born on April 12, 1837, in Strzyżów near Rzeszów [6] in the family of the pharmacist Jan and Alojza neé Tokarska. He graduated from a gymnasium in 1855. In the same year he began studying mathematical, physical and natural sciences at the Jagiellonian University, where in 1858 he was appointed as an assistant at the Department of Physics. In 1861 he received the degree of Doctor of Philosophy on the basis of the paper about Cracow's barometric relations: *Stosunki barometryczne Krakowa jako przyczynek do klimatologii tegoż*, Rocznik Towarzystwa Naukowego Krakowskiego XXXI (1864), 183–246. In the years 1861–1862 Zajączkowski completed his studies at the universities of Göttingen, Berlin and Vienna. In 1862 he habilitated at the Jagiellonian University in mathematics [6] on the basis of his work about the Euler and Fourier integrals. Next he worked at the Jagiellonian University (years 1862–1864) as an assistant professor in the Department of Elementary Mathematics. Since 1864 he was a lecturer at the Main School of Warsaw where he was delivering lectures in analytical mechanics, analytical geometry, integral calculus, and differential equations. In 1867 he was awarded the doctoral mathematical degree having written a thesis on theory of differential equations with partial derivatives of the first order. When the Russian authorities closed the Main School in 1869 he continued his lectures in the Russian Imperial University of Warsaw up to the 1872. After his appointment as a professor of mathematics at the Technical Academy in Lviv (later the Polytechnic School) in 1872 he moved from Warsaw to Lviv, where he stayed for the rest of his life. In the academic years 1878–1879 and 1885–1886 he held the office of the Rector of the Polytechnic School. At the same time he was lecturing on analytic geometry and the theory of differential and integral equations in the Imperial University of Lviv as an assistant professor (since 1881) of this university.

In the years 1886–1891 he held the honor of being a member of the National School's Council. He was a member of the Scientific Society of Cracow, the Academy of Sciences in Cracow, the Pedagogical Society in Lviv and the Society of Sciences in Paris.

Zajączkowski was one of the greatest Polish mathematicians of the 19th century. His research papers dealt mainly with differential equations, analytic geometry and mathematical methods of physics. He was an author of almost 60 publications [Z1-Z58], including 10 scientific books and academic textbooks (lecture notes); several textbooks were reprinted two or three times. He also published 30 scientific papers in mathematical journals. Several previously published papers were repeated in an expanded version in other languages. Moreover, he published several works on history, popular science, or teaching.

Fig. 1. Władysław Wojciech Zajączkowski (1837–1898)

In 1867 Zajączkowski published the monograph *Theory of partial differential equations on the first order derivatives* [Z1], on the basis of which he obtained the degree of Doctor of Philosophy at the Warsaw School of Economics. It was printed as a separate book with 82 pages. It was the first book in Polish on the theory of partial differential equations of first order.

Zajączkowski was also the author of the first extensive Polish monograph in the field of differential equations, which is noteworthy. This monograph was enitled *Lecture on the science of differential equations* [Z17] and published in Paris in 1877 by Jan Działyński, who was the owner of The Kórnik Library and a chairman of the Societies of Scientific Aid and Sciences in Paris. This book consisted more than 900 pages.

It was the world`s first such detailed report concerning both ordinary and partial differential equations, reflecting the current state of knowledge in the theory of differential equations up to the 1870s.

In those days the works of Lacroix were translated into Polish by Niemczewski, but they contained only the basics of differential equations. Textbooks containing differential equations published by Duhamel, Sturm, Schlömilch, Boole, Natani and Mingo unfortunately did not match the content and timeliness of Zajączkowski's text.

Therefore, from the historical point of view, it was the most appropriate and the unique global source. A similar account did not exist in the world mathematical literature of 19th century.

A list of Zajączkowski's publications and full information about his life can be found in the paper [6].

Zajączkowski published his papers mainly in Archiv der Mathematik, Communications of the Russian University of Warsaw, in the publications of the Scientific Society of Cracow, Academy of Sciences in Cracow and publications of the Society of Exact Sciences in Paris.

Zajączkowski published twenty papers in the field of differential equations (see the list below).

WYDAWNICTWO TOWARZYSTWA NAUK ŚCISŁYCH W PARYŻU.

WYKŁAD NAUKI

O RÓWNANIACH RÓŻNICZKOWYCH

PRZEZ

WŁ. ZAJĄCZKOWSKIEGO

Doktora Filozofii
Profesora Matematyki w Akademii Technicznéj Lwowskiéj
Korespondenta Akademii Umiejętności w Krakowie
i Towarzystwa Nauk Ścisłych w Paryżu

CENA 25 FR.

PARYŻ
NAKŁADEM WŁAŚCICIELA BIBLIOTEKI KÓRNICKIEJ
PRZEWODNICZĄCEGO
W TOWARZYSTWACH NAUKOWÉJ POMOCY I NAUK ŚCISŁYCH W PARYŻU

1877

Fig. 2. The cover page of the first Polish monograph on ordinary and partial differential equations *Lecture on the science of differential equations* published in Paris in 1877

3. List of Zajączkowski's papers in the field of differential equations

[Z1] *Teoryja równań różniczkowych o cząstkowych pochodnych rzędu Igo. Rozprawa na stopień doktora filozofii Szkoły Głównej Warszawskiej*, Warszawa 1867, IV + 82.

[Z2] *Beweis eines die Pfaff'sche Integrationsmethode betreffenden Lehrsatzes*, Archiv der Mathematik und Physik 47, 1867, 106-109.

[Z3] *Объ особенныхъ интегралахъ линейныхъ дифференцiальныхъ уравненiй 1-го порядка, интегрируемыхъ въ видъ одново первоначаьнаго уравненiя,* Варш. Унив. Извьстiи, 1870, 14.

[Z4] *Замьетка касающаяся теорiи системы линейныхъ уравненiй въ частныхъ производныхъ пєрваго порядка,* Варш. Унив. Извьстiи, н. 2, 1870, 191-205.

[Z5] *Przyczynek do teoryi układu równań liniowych o pochodnych cząstkowych rzędu pierwszego*, Rocznik Towarzystwa Naukowego Krakowskiego, XLII, 1871, 220-236.

[Z6] *O całkach osobliwych zwyczajnych równań różniczkowych rzędu piérwszego*, Rocznik Towarzystwa Naukowego Krakowskiego, XLII, 1871, 366-380.

[Z7] *Замьетка по теорiи особыхъ обыкновенныхъ дифференцiальныхъ уравненiй 1-го порядка,* Варш. Унив. Извьстiи, 1872, 7.

[Z8] *Объ интегрированiи системы совокупныхъ линейныхъ уравненiй въ частныхъ производныхъ пєрваго порядка и тойже функцiи с некоторыми приложенiями,* Варш. Унив. Извьстiи, 1872, 74.

[Z9] *Teoryja równań liniowych o pochodnych cząstkowych rzędu 1go jednéj funkcyi*, Rocznik Towarzystwa Naukowego Krakowskiego XLIV, 1872, 300-332.

[Z10] *Zur Integration eines Systems linearer partieller Differentialgleichungen erster Ordnung*, Archiv der Mathematik und Physik 56, 1874, 163-174 (the same in Russian [Z8], and in Polish [Z9]).

[Z11] *Beitrag zur Theorie der singulären Lösungen gewöhnlicher Differentialgleichungen erster Ordnung*, ibidem 56, 1874, 175-179.

[Z12] *O całkach osobliwych zwyczajnych równań różniczkowych rzędu jakiegokolwiek*, Pamiętnik Akademii Umiejętności w Krakowie, Wydział Matematyczno-Przyrodniczy 1, 1874, 45-56.

[Z13] *Teoryja ogólna rozwiązań osobliwych równań różniczkowych zwyczajnych*, Kosmos 1, 1876, 350-354.

[Z14] *O równaniu różniczkowém* $Xdx+X_1dx_1+...+X_ndx_n=0$*, całkowalném przez jedno równanie pierwotne*, Pamiętnik Towarzystwa Nauk Ścisłych w Paryżu VI, 1875, 1-14.

[Z15] *Całkowanie równań różniczkowych*, 1877 (The item is mentioned in [10] without the data about the publisher and the place of publication; the author of the present article could not locate the original).

[Z16] *Teoryja ogólna rozwiązań osobliwych równań różniczkowych zwyczajnych*, Pamiętnik Akademii Umiejętności w Krakowie, Wydział Matematyczno-Przyrodniczy 3, 1877, 1-23.

[Z17] *Wykład nauki o równaniach różniczkowych*, Wydawnictwo Towarzystwa Nauk Ścisłych w Paryżu, Nakładem właściciela Biblioteki Kórnickiej, Paryż 1877, 904 + 5 nlb.

[Z18] *O pewnéj własności pfafianu*, Rozprawy Wydziału Matematyczno-Przyrodniczego Akademii Umiejętności w Krakowie, VII, 1880, 67-74.

[Z19] *A theorem relating to Pfaffians*, The Messenger of Mathematics, X, 1881, 36-37.

[Z20] *Teoryja wyznaczników o p wymiarach*, Pamiętnik Akademii Umiejętności w Krakowie VI, 1881, 1-31.

[Z21] *Teoryja Fuchsa równań różniczkowych linijowych i jednorodnych z jedną zmienną niezależną*, Pamiętnik Akademii Umiejętności w Krakowie XIII, 1887, 1-47.

[Z22] *Równania różniczkowe*, Lwów 1892, 169.

4. List of Zajączkowski's publications in other fields

[Z23] *Stosunki barometryczne Krakowa jako przyczynek do klimatologii tegoż*, Rocznik Towarzystwa Naukowego Krakowskiego XXXI, 1864, 183-246.

[Z24] *Teorya funkcyi potencyalnéj. Przyczynek do fizyki matematycznej*, Druk C. K. Uniwersytetu Jagiellońskiego, Kraków 1864, 4 nlb + 42 + 1 tabl.

[Z25] *Odczyty z geometryi analitycznej*, Warszawa 1865–1866, 332 (Lithography).
[Z26] *Geometria analityczna: wykład w Szkole Głównej Warszawskiej*, Warszawa 1865, 198 (Lithography).
[Z27] *O obrocie ciała stałego*, 1866 (The item is mentioned in [11] without the data about the publisher and the place of publication; the author of the present article could not locate the original).
[Z28] *Über das Problem der Rotation eines festen Körpers*, Archiv der Mathematik und Physik 46 (1866), 19–25.
[Z29] *Przyczynek do teoryi największości i najmniejszości funkcyi zależnych od ilekolwiek ilości zmiennych*, Rocznik Towarzystwa Naukowego Krakowskiego XXXV, 1867, 223-230.
[Z30] *O matematyce i jej wpływie na wszystkie gałęzie wiedzy ludzkiej*, Mowy na uroczystą inauguracyją Rektora c. k. Akademii Technicznej Dra Feliksa Strzeleckiego profesora Fizyki, Lwów 1872, 12-21.
[Z31] *Szkoła Politechniczna Lwowska*, Przegląd Techniczny, Rocz. 4, t. 7, z. 1, 1-6.
[Z32] *Mowa dr Władysława Zajączkowskiego rektora Szkoły Politechnicznej podczas uroczystego otwarcia roku akademickiego 1878/9*, Lwów 1878, 14.
[Z33] *Zasady algebry i rachunku infinitezymalnego podług wykładu Profesora Doktora W. Zajączkowskiego ułożyli Słuchacze matematyki kursu 1-go 1878/79*, Nakładem Towarzystwa Bratniej Pomocy słuchaczów cesarsko-królewskiej Szkoły Politechnicznej we Lwowie, Lwów 1879, 4 nlb + 347 + 1nlb (Lithographic printing).
[Z34] *Zasady geometryi analitycznej podług wykładu Profesora Doktora W. Zajączkowskiego ułożyli Słuchacze matematyki kursu 1-go 1878/79*, Nakładem Towarzystwa Bratniej Pomocy słuchaczów politechniki we Lwowie, Lwów 1879, 4 nlb + 295 + 1 nlb (Lithographic printing).
[Z35] *Wykład analizy algebraicznej* (lithographic edition – lack of space issue – probably Lviv), 1880/1, 307.
[Z36] *Wykład trygonometryi płaskiej i kulistey* (lithographic edition – lack of space issue – probably Lviv), 1880/1.
[Z37] *Teoryja wyznaczników o p wymiarach*, Pamiętnik Akademii Umiejętności w Krakowie VI, 1881, 1-31.
[Z38] *Zasady rachunku przemienności*, Lwów 1882, 24.
[Z39] *Rachunek różniczkowy i całkowy* (manuscript lithographic copied), Lwów 1881–2, 303 + 1 nlb.
[Z40] *Wykład geometryi analitycznej* (lack of space and the year of issue).
[Z41] *Matematyka. Teoryja linij i powierzchni krzywych*, Lwów 1883, 206 (Litografia).
[Z42] *Geometryja analityczna*, Bibljoteka Matematyczno-Fizyczna wyd. przez M.A. Baranieckiego i A. Czajewicza, Seria 4, tom 4, Warszawa 1884, 511 + 1 nlb.
[Z43] *O zamianie funkcyi całkowitéj i jednorodnéj stopnia 2-go na sumę kwadratów*, Pamiętnik Akademii Umiejętności w Krakowie IX, 1884, 1-44 (Praca ta w Wiadomościach Matematycznych 2, 1898, s. 260, przytoczona jest jako: 18) *Teoryja form kwadratowych*).
[Z44] *Zasady algebry wyższéj*, Nakł. Księgarni Gubrynowicza i Schmidta, Lwów 1884, XII + 286.

[Z45] *Przemówienie jako Rektora Szkoły Politechnicznej we Lwowie przy rozpoczęciu roku szkolnego 1885/6*, Czasopismo Techniczne, 1885, Rocz. 3, Nr 10, 122-125, Idem. – Lwów 1885, 13.
[Z46] *Przemówienie ustępującego Rektora Szkoły Politechnicznej we Lwowie przy otwarciu roku szkolnego 1886/7*, Czasopismo Techniczne, 1886, Rocz. 4, Nr 12, 201-203.
[Z47] *Lekcyja arytmetyki w klasie I-szej*, Muzeum, Lwów 1886, 598-605.
[Z48] *Początki arytmetyki do użytku szkół średnich zastosowane. Na I–ą i II–ą klasę*, Lwów 1887, pp. 135.
[Z49] *Początki arytmetyki i algebry, zastosował do użytku szkół średnich. Cz. II na III i IV klasę*, Nakładem Towarzystwa Pedagogicznego, Lwów 1888, III + 159.
[Z50] *Początki arytmetyki do użytku szkół średnich. Cz. I na I i II klasę*, Wyd. II, Nakładem Towarzystwa Pedagogicznego, Lwów 1889, 163.
[Z51] *Początki arytmetyki i algebry, zastosował do użytku szkół średnich. Cz. II na III i IV klasę*, Nakładem Towarzystwa Pedagogicznego, Lwów 1889, III + 159.
[Z52] Book review: *Kanony logarytmów Wrońskiego*, ed. S. Dickstein, Warszawa 1890, [w:] Muzeum, 1890, 595-599.
[Z53] *Początki arytmetyki. Do potrzeb szkół średnich zastosował dr Władysław Zajączkowski Cz. I na I i II klasę*, Wyd. III, Lwów 1891, 162.
[Z54] *Początki arytmetyki. Do potrzeb szkół średnich zastosował dr Władysław Zajączkowski. Cz. II na III i IV klasę*, Towarzystwo Pedagogiczne, Wyd. III, Lwów 1891, 130 + 2 nlb.
[Z55] *C. K. Szkoła politechniczna we Lwowie. Rys historyczny jej założenia i rozwoju, tudzież stan jej obecny skreślił ...*, Nakładem Szkoły Politechnicznej we Lwowie, Lwów 1894, 170 + 4 tabl.
[Z56] *O inwolucyi punktów na liniach tworzących powierzchni prostokreślnej skośnej*, Rozprawy Wydziału Matematyczno-Przyrodniczego Akademii Umiejętności w XXXII, 1896, 279-301.
[Z57] *Analiza. Kurs 2*, Lwów 1986, 318.
[Z58] Zastosowania analizy do geometrii: podług wykładów Wł. Zajączkowskiego; zebrał L. Böttcher, Lwów, 193 (Litografia).

Remark. In above paragraphs 3 and 4 is the complete bibliography of scientific publications of Władysław Zajączkowski.

5. Scientific information on selected Polish mathematicians who published papers in differential equations up to 1900

5.1. Papers of Stanisław Zaremba up to 1900 in the field of differential equations

Stanisław Zaremba [8–9] studied engineering at the Institute of Technology in St. Petersburg (getting an engineering diploma in 1886). Then he went to Paris, where he studied mathematics for his doctorate at the Sorbonne.

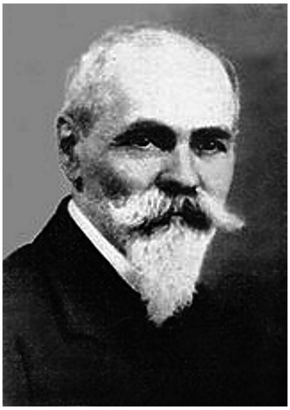


Fig. 3. Stanisław Zaremba (1863–1942)

As a topic for his dissertation Zaremba chose the ideas introduced by Riemann in 1861. His doctoral thesis *Sur un problème concernant l'état calorifique d'un corp homogène indéfini* was presented in 1889. At that time Zaremba got in touch with many mathematicians of the French school. He maintained these ties, engaging in a wide international cooperation after returning to Poland. In particular he collaborated with Painlevé and Goursat. Before 1900 Zaremba taught in secondary schools in France. At that time he concentrated hard on his research. The fact that he published his results in French mathematical journals meant that his work became well known and highly respected by leading French mathematicians such as Poincaré and Hadamard. Zaremba's publications concerned mainly partial differential equations. These publications played a very important role in the development of world mathematical sciences.

In the nineteenth century Zaremba published the following papers in the field of partial differential equations: *Note concernat l'integration d'une équation aux dérivées partialles* (Annales de l'Ecole Normale, 3^{ieme} Série 7, fasc. 3, mai 1890, 135–142), *Sur le problème de Dirichlet* (Annales de l'Ecole Normale, 3^{ieme} Série 14, juillet 1890, 251–258), *Sur la méthode des approximations successives de M. Picard* (Journal the Math*ématiques pures et appliquées* 5^{ieme} Série 3, fasc. 3, 1897, 311–329), *Sur l'équation aux dérivées partialles* $\Delta u + \xi u + f = 0$ s*ur les founctions harmoniques* (Annales de l'Ecole Normale, 3^{ieme} Série 16, octobre 1899, 4270-464).

Most of Zaremba's scientific results were obtained in the twentieth century. They are therefore not the subject of these considerations.

5.2. Papers of Kazimierz Stefan Paulin Żorawski up to 1900 in the field of differential equations

In 1888, after four years of studies at the University of Warsaw, Żorawski graduated with a first degree in mathematics. His work was of such high quality that he was given the opportunity to continue his mathematical studies abroad.

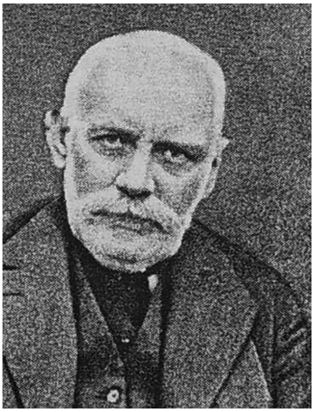


Fig. 4. Kazimierz Żorawski (1866–1953), [2]

He spent some time in Leipzig, where he studied continuous groups of transformations now called Lie groups, and in Göttingen, where he studied differential equations. He was awarded his doctorate in 1891 at the University of Leipzig for his thesis on applications of Lie groups to the differential geometry. After returning to Cracow in 1895, Żorawski continued to teach courses on the analytical and synthetic geometry, the differential geometry, the formal theory of differential equations, the theory of forms, and the theory of Lie groups. The main topics of his research were invariants of differential forms, integral invariants of Lie groups, the differential geometry, and the fluid mechanics.

The most important of Żorawski's work concerning on differential equations written in the nineteenth century was the following paper: *O całkowaniu pewnej kategorii równań różniczkowych zwyczajnych rzędu trzeciego,* Rozpr. Wydz. Mat.-Przyr. Akad. UM. w Krakowie 34, 1898, 141-205. In this paper Żorawski studied the solvability of a class of ordinary differential equations of third order in the form

$$\frac{d^3y}{dx^3} = f\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}\right).$$

A similar problems for ordinary differential equations of the second order was considered by S. Lie and A. Tresse. The Żorawski's paper is a nontrivial generalization of Lie's and Tresse's results.

Żorawski obtained most of his scientific results in the twentieth century. Therefore, they are not the subject of this discussion.

6. General remark on the publications of Polish mathematicians in the nineteenth century in terms of ordinary and partial differential equations

In the 19th century, among the Polish mathematicians who published papers in the field of differential equations there were: Franciszek Karol Mertens (1840–1927), Stanisław Kępiński (1867–1908), Kazimierz Stefan Paulin Żorawski (1866–1953), Józef Puzyna

(1856–1919), Władysław Folkierski (1841–1904), Alojzy Jan Stodółkiewicz (1856–1934), Jan Rajewski (1857–1906), Wawrzyniec Żmurko (1824–1889), Edward Władysław Skiba (1843–1911), Jan Ptaszycki (1854–1912), Władysław Wojciech Zajączkowski (1837–1898) and, of course, Stanisław Zaremba (1863–1942).

Publications of Polish mathematicians in the field of differential equations in the 19th century are diverse and have different scientific value. They are often interesting and important contributions, for example the only work by Mertens on differential equations (*Obliczanie Potencyału dla wielościanów jednorodnych* p. Prof. FR. MERTENSA, Annals of the Scientific Society of Cracow ,Vol. XXXV, 1867 (t. 12 Poczet Trzeci), 343-351), related to determining the volume potential for second-order elliptic partial differential equations in the polyhedral areas. It is interesting and significant because it allowed for the efficient solution of certain boundary problems for the elliptic equations.

Interesting papers in differential equations were written by e.g. Stanisław Kępiński (1867–1908) and Alojzy Jan Stodółkiewicz (1856–1934) [2]. Kępiński was the author of several works on analytic theory of differential equations, for example: *O całkowaniu równań różniczkowych cząstkowych rzędu drugiego* (dissertation, Jagiellonian Uniwersity, 1890); *O całkach rozwiązań równań różniczkowych zwyczajnych liniowych jednorodnych rzędu drugiego*, „Rozprawy AU", seria A, Vol. 26, 1893, 264-328.

Stodółkiewicz was the author of many works on special types of differential equations. For example: *Zastosowanie sposobu Bertranda do całkowania równania różniczkowego o różniczkach zupełnych z wielu zmiennymi.* Memoirs of the Academy of Sciences in Cracow, Vol. VIII, 1883, 137-142; *Całkowanie układów równań różniczkowych o różniczkach zupełnych*, Memoirs of the Academy of Sciences in Cracow, Vol. VIII, 1883, 143-152; *O całkowaniu równań różniczkowych linijnych rzędu drugiego, mających współczynniki linijne, przy pomocy kwadratur*, Memoirs of the Academy of Sciences in Cracow, Vol. IX, 1884, 113-119; *O dwóch szczególnych układach równań różniczkowych o różniczkach zupełnych*, Memoirs of the Academy of Sciences in Cracow, Vol. XII, 1886, 93-95.

Interesting papers in the field of differential equations also were written by other Polish mathematicians mentioned above.

In view of these publications, one can say that many Polish mathematicians were involved in the development of the theory of differential equations in the second half of the nineteenth century. Their research was performed in accordance with the activity of leading scientific centers [1] of the world.

References

[1] R. Bujakiewicz-Korońska, J. Koroński, *Równania różniczkowe do końca XIX wieku*, Matematyka czasów Weierstrassa, Materiały XV Ogólnopolskiej Szkoły Historii Matematyki, Wydawnictwo Uniwersytetu Szczecińskiego, Szczecin 2002, 225-140.

[2] R. Duda, *Matematycy XIX i XX wieku związani z Polską*, Wydawnictwo Uniwersytetu Wrocławskiego, Wrocław 2012.

[3] J. Koroński, *Prace z równań różniczkowych w Pamiętnikach Towarzystwa Nauk Ścisłych w Paryżu*, Zagadnienia Filozoficzne w Nauce (ed. W. Wójcik), LIII, 2013, Copernicus Center Press, Kraków 2013, 199-230.

[4] J. Koroński, *Prace z równań różniczkowych w Pamiętniku Akademii Umiejętności w Krakowie*, Zagadnienia Filozoficzne w Nauce (ed. W. Wójcik), LIII, 2013, Copernicus Center Press, Kraków 2013, 231-262.

[5] J. Koroński, *Publikacje matematyczne w Roczniku Towarzystwa Naukowego z Uniwersytetem Krakowskim Połączonego i w Roczniku c. k. Towarzystwa Naukowego w Krakowie (1817–1872)* (in preparation to print).

[6] J. Koroński, *Władysław Zajączkowski (1837–1898) i jego monografia z równań różniczkowych*, Antiquitates Mathematicae, Vol. 3, 2009, 47-64.

[7] A. Pelczar, *Równania różniczkowe w Polsce. Zarys historii do połowy lat siedemdziesiątych XX wieku*, Wiadomości Matematyczne, t. XXXVII, Warszawa 2001, 63-18.

[8] J. Szarski, T. Ważewski, *Stanisław Zaremba*, Studia z dziejów katedr Wydziału Matematyki, Fizyki, Chemii Uniwersytetu Jagiellońskiego (pod redakcją Stanisława Gołąba), Wydawnictwa Jubileuszowe, Tom XV, Uniwersytet Jagielloński, Kraków 1964, 103-117.

[9] W. Ślebodziński, *Stanisław Zaremba*, Studia z dziejów katedr Wydziału Matematyki, Fizyki, Chemii Uniwersytetu Jagiellońskiego (pod redakcją Stanisława Gołąba), Wydawnictwa Jubileuszowe, Tom XV, Uniwersytet Jagielloński, Kraków 1964, 87-101.

[10] A. Śródka, P. Szczawiński, *Biogramy uczonych polskich. Materiały o życiu i działalności członków AU w Krakowie*, TNK, PAU, PAN, Część III; Nauki Ścisłe, Ossolineum 1986.

[11] A. Śródka, *Uczeni polscy XIX–XX stulecia*, t. IV, S–Ż, Warszawa 1998.

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE

1-NP/2014

JAN KOROŃSKI*

MATHEMATICAL PUBLICATIONS IN THE ANNALS OF THE CRACOW SCIENTIFIC SOCIETY (1817–1872)

PUBLIKACJE MATEMATYCZNE W ROCZNIKU TOWARZYSTWA NAUKOWEGO KRAKOWSKIEGO (1817–1872)

A b s t r a c t

This paper provides a general characteristic of the Cracow Scientific Society (Towarzystwo Naukowe Krakowskie). It existed 1815–1872 and during that time changed its name several times (see below). The Academy of Arts and Sciences (Akademia Umiejętności – AU) was founded in 1872, as a result of the transformation of the Cracow Scientific Society. Moreover, in this paper we present mathematical publications in the Annals of the Cracow Scientific Society.

Keywords: *Cracow Learned Society*, *Mathematical publications*, *Annals of the Scientific Society joined with the Cracow University*, *Imperial–Royal Scientific Society in Cracow*, *nineteenth century*

S t r e s z c z e n i e

W artykule zawarto ogólną charakterystykę Towarzystwa Naukowego Krakowskiego, które istniało 1815–1872 i w tym czasie kilkakrotnie zmieniało nazwę (p. niżej). Akademia Umiejętności – AU powstała w roku 1872. Ponadto zaprezentowano publikacje matematyczne wydrukowane w Roczniku Towarzystwa Naukowego Krakowskiego.

Słowa kluczowe: *Towarzystwo Naukowe Krakowskie*, *publikacje matematyczne*, *Rocznik Towarzystwa Naukowego Krakowskiego z Uniwersytetem Krakowskim Połączonego*, *Cesarsko-Królewskie Towarzystwo Naukowe Krakowskie*, *dziewiętnasty wiek*

* Jan Koroński, Faculty of Physics, Mathematics and Informatics, Institute of Mathematics, Cracow University of Technology.

1. Name of the Society (Towarzystwo Naukowe Krakowskie – Cracow Scientific Society)

Towarzystwo Naukowe z Uniwersytetem Krakowskim Połączone – Its publications bore this name in the period 1817–1833. Full journal title*: Rocznik Towarzystwa Naukowego z Uniwersytetem Krakowskim Połączonego.*

Towarzystwo Naukowe Krakowskie z Uniwersytetem Jagiellońskim Połączone – Its publications bore this name in 1841–1847. Full journal title: *Rocznik Towarzystwa Naukowego Krakowskiego z Uniwersytetem Jagiellońskim Połączonego.*

Towarzystwo Naukowe z Uniwersytetem Jagiellońskim Złączone – Its publications bore this name in 1849–1852. Full journal title: *Rocznik Towarzystwa Naukowego z Uniwersytetem Jagiellońskim Złączonego.*

Cesarsko–Królewskie Towarzystwo Naukowe Krakowskie – Its publications bore this name in 1857–1872. Full journal title: *Rocznik Ces. –Król. Towarzystwa Naukowego Krakowskiego* (*Rocznik Cesarsko–Królewskiego Towarzystwa Naukowego Krakowskiego*).

2. List of mathematical works in the Annals of the Scientific Society of Cracow (in Polish) – (we preserve the original 19th-century spelling)

Volume III (1819)

1. Karol Hube (1769–1845): *O Różnych dowodzeniach twierdzenia, „Że każde zrównanie algebraiczne na czynniki rzetelne pierwszego albo drugiego stopnia, rozłożonem bydź może, a w szczególności, porównanie wiadomego dowodzenia Pana La Place, z dowodem przez Pana Gauss, w roku teraznieyszym w Götindze ogłoszonym".* Rzecz czytana na posiedzeniu Towarzystwa Naukowego dnia 15. Listopada 1816. przez Karola Hube Profesora Matematyki w Uniwersytecie Krakowskim (pp. 91-115).

Volume V (1820)

2. Franciszek Sapalski (1791–1838): *Rozprawa o Teroyi Stereotomii czyli Jeometryi Wykreślnéy*, czytana na posiedzeniu zwyczayném d. 16. Listopada 1817. przez Franciszka Sapalskiego téyże umieiętności w Uniwersytecie Krakowskim Profesora D.F. byłego Officera Artylleryi, ozdobionego orderem Krzyża woyskowego (pp. 229-289).
3. Karol Hube (1769–1845): *O Trygonometryi kulistéy rzecz krótka*, czytana na Posiedzeniu Towarzystwa Naukowego 15. Listop: 1817. przez Karola Hube Prof. Matematyki wyższéy w Uniwersytecie Jagiellońskim (pp. 290-331).

Volume VIII (1823)

4. Karol Hube (1769–1845): *Rosprawa o Wyznaczeniu Bryłowatości klina ostrokręgowego (onglet conique)* przez Karola Hube F. D. Matem: wyższey Profes. w Uniw. Jag. (pp. 115-164).
5. F. Szopowicz (1762–1839): *O znaczeniu ilości,* przez Franciszka Szopowicza F.D. członka Towarzystwa Warszawskiego Przyjaciół Nauk, czytana (pp. 165-190).

Volume IX (1824)

6. Karol Hube (1769–1845): *Rozprawa o początkach Jeometryi Analityczney czyli o linii prostey i płaszczyznie.* Rzecz czytana na posiedzeniu Towarzystwa Naukowego dnia 15. Czerwca r. 1822. przez Karola Hube F. D. Prof. Mat. wyższey w Uni. Jagiel (pp. 76-150).

Volume XI (1826)

7. Karol Hube (1769–1845): *Dalszy ciąg zadań linii prostey i płaszczyzny tyczących się, iako i o tworzeniu się powierzchni krzywych przez linie proste*. Rzecz czytana na posiedzeniu Towarzystwa Naukowego dnia 11. Listopada 1824. roku przez Karola Hube F. D. Pr. Mat. Wyż. w Uniw. Jagiell. (pp. 23-89).

Volume XII (1827)

8. Augustyn Frączkiewicz (1796–1883): *Dowodzenie różnych podań z Trygonometryi płaskiey i Geometryi Elementarney pod Nrem I; daley zagadnienie pod Nrem II; nakoniec wyprowadzenie wzoru P. Abel pod Nrem III* (pp. 151-237).

Volume XIII (1829)

9. Karol Hube (1769–1845): *Rozprawa o Fenomenach niektórych pochodzących z ruchu wirowego ciał, z przydaniem uwag nad przerobieniem współrzędnych i niektórymi twierdzeniami tyczącemi się momentów*. Rzecz czytana na posiedzeniu Towarzystwa Naukowego dnia 15 Maja 1826 roku przez Karola Hube F. D. Matem. Wyż. Profesora. (pp. 91-216).
10. Karol Hube (1769–1845): *Nekrolog ś. p. Józefa Łęskiego F. D. Profesora Astronomii w Uniwersytecie Jagiellońskim i Dyrektora Obserwatorium Krakowskiego, Towarzystwa Król. Przy. Nauk. Członka, przez Karola Hube F. D. Profesora Matem. Wyższey*, czytany na posiedzeniu Towarzystwa Naukowego dnia 14 Grud. 1829 r. (pp. 125-154).

Volume XIV (1831)

11. Karol Hube (1769–1845): *Rozprawa o Twierdzeniach P. Monge, stykania się powierzchni drugiego stopnia tyczących się; uwagi nad dowodzeniem ich przez P. Chasies ogłoszoném, i dowód analityczny twierdzenia: że dwie powierzchnie drugiego stopnia na trzeciey opisane zawsze się dwóch krzywych płaskich przecinaią.* Rzecz czytana na posiedzeniu Towarzystwa Naukowego dnia 12 Maja 1828 roku przez Karola Hube Matematyki Wyższey w Uniwersytecie Jagiellońskim Profesora (pp. 189-216).

Volume XV (1833)

12. Augustyn Frączkiewicz (1796–1883): *DWA WYJĄTKI Z ROZPRAWY O CZWORO-ŚCIANIE*, przez Augusta Frączkiewicza Prof. Mat. w Uniw. Warsz. Członka Tow. Nauk. Krakowskiego (pp. 122-144).

Volume XVI (1841) (t. 1 Poczet Nowy)

13. Karol Hube (1769–1845): *O zasługach MIKOŁAJA KOPERNIKA w Astronomii, na posiedzeniu publiczném Towarzystwa Naukowego Krakowskiego dnia 14 Lutego 1834 r*. czytał KAROL HUBE Rektor Uniwersytetu i Prezes tegoż Towarzystwa (pp. 245-265).

Volume XVII (1843) (t. 2 Poczet Nowy)

14. Karol Hube (1769–1845): *Dwa badania matematyczne powiérzchni skośnych i liczb całych tyczące się*, czytane na posiedzeniu Towarzystwa Nauk. Dnia 24 Kwietnia 1841 r. przez KAROLA HUBE, NN. WW. i Fil. Doktora. Prof. matem. w Uniwersytecie Jagiellońskim (pp. 182-204).

Volume XVIII (1847) (t. 3 Poczet Nowy)

15. Karol Hube (1769–1845): *Wykład treści trzech twierdzeń matematycznych*, Towarzystwu Nauk. Krakow. złożonych d. 24 kwietnia 1841 r. przez K. HUBE, Czł. tegoż Towarz. (pp. 92-107).

Volume XXII (1852) (Zeszyt 1)

16. Jan Kanty Steczkowski (1800–1881): *Rys życia KAROLA HUBEGO Prof. Matematyki w Uniw. Jagiell.* przez Prof. Dra J. K. Steczkowskiego (pp. 239-256).

Volume XXVIII (1861) (t. 5 Poczet Trzeci)

17. Jan Kanty Steczkowski (1800–1881): *Przyczynek do prowadzenia stycznych do krzywych drugiego stopnia czyli przecięć ostrokręgowych,* przez J. K. Steczkowskiego (pp. 321-331).

Volume XXX (1862) (t. 7 Poczet Trzeci)

18. Teofil Żebrawski (1800–1887): *Wiadomość o ADAMIE KOCHAŃSKIM i pismach jego matematycznych*, skreślił TEOFIL ŻEBRAWSKI Czł. Tow. Nauk. Krak. (pp. 1-9).
19. Teofil Żebrawski (1800–1887): *Nowe rozwiązanie podziału kąta na trzy równe części*, przez TEOFILA ŻEBRAWSKIEGO Czł. Tow. Nauk. Krak. (pp. 10-23).

Volume XXXI (1864) (t. 8 Poczet Trzeci)

20. Władysław Zajączkowski (1837–1898): *Stosunki barometryczne Krakowa, jako przyczynek do klimatologii tegoż*, przez W. Zajączkowskiego (pp. 183-246).

Volume XXXV (1867) (t. 12 Poczet Trzeci)

21. Władysław Zajączkowski (1837–1898): *Przyczynek do teoryi największości i najmniejszości funkcyj zależnych od ilukolwiek ilości zmiennych* p. Dra Wł. ZAJĄCZKOWSKIEGO (pp. 223-230).
22. Franciszek Mertens (1840–1927): *Obliczanie Potencyału dla wielościanów jednorodnych* p. Prof. FR. MERTENSA (pp. 343-351).

Volume XXXIX (1870) (t. 16 Poczet Trzeci)

23. Edward Jan Habich (1835–1909): *O szczególnym układzie spółrzędnych i jego zastósowaniu do linij palących* p. E. HABICHA. (pp. 1-23).
24. Wawrzyniec Żmurko (1824–1889): *O styczności kół i kul* napisał W. ŻMURKO (pp. 69-93).

Volume XLII (1871) (t. 19 Poczet Trzeci)

25. Władysław Zajączkowski (1837–1898): *Przyczynek do teoryi układu równań liniowych o pochodnych cząstkowych rzedu pierwszego* p. Prof. ZAJĄCZKOWSKIEGO (pp. 220-236).
26. Władysław Zajączkowski (1837–1898): *O całkach osobliwych zwyczajnych równań różniczkowych* p. Prof. ZAJĄCZKOWSKIEGO (pp. 366-380).

Volume XLIV (1872) (t. 21 Poczet Czwarty)

27. Edward Władysław Skiba (1843–1911): *Przyczynek do teoryi sprężystości* p. prof. Dr. EDW. SKIBĘ (pp. 42-55).
28. Władysław Zajączkowski (1837–1898): *Teoryja równań linijowych o pochodnych cząstkowych rzędu 1go jednéj funkcyi* p. prof. WŁAD. ZAJĄCZKOWSKIEGO (pp. 220-236).

3. General characteristic of the Yearbook of the Scientific Society joined with the University and Imperial-Royal Scientific Society in Cracow

In the years from 1817 to 1872 44 volumes of the Yearbook of the Scientific Society of Cracow were printed. About 300 papers from many fields of science appeared in these volumes. There were papers in mathematics, physics, chemistry, astronomy, geology, mineral springs, biology, medicine, history, literature, law, philosophy and theology. There were more than 80 works of science, among them 28 papers in mathematics (there may be fewer, depending on the criterion according to which the work is classified as mathematical), about 20 works in physics, 20 works of chemistry and 15 works of astronomy. The authors of papers in mathematics were Karol Hube (10 works), Władysław Zajączkowski (5 works), Augustyn Frączkiewicz (2 papers), Franciszek Sapalski, Franciszek Szopowicz, Franciszek Mertens, Edward Jan Habich, and Wawrzyniec Żmurko (one work each); also, Jan Kanty Steczkowski (one paper in mathematics and 2 in astronomy) and Edward Skiba (2 papers in mathematical physics).

References

[1] J. Koroński, *Publikacje matematyczne w Roczniku Towarzystwa Naukowego z Uniwersytetem Krakowskim Połączonego i w Roczniku c. k. Towarzystwa Naukowego w Krakowie (1817–1872)* (in preparation to print).

[2] *Roczniki Towarzystwa Naukowego z Uniwersytetem Krakowskim Połączonego i Roczniki c. k. Towarzystwa Naukowego w Krakowie*, Vol. 1-44, 1817–1872.

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE

1-NP/2014

JAN KOROŃSKI*

MATHEMATICAL PUBLICATIONS IN THE MEMOIRS OF THE ACADEMY OF ARTS AND SCIENCES IN CRACOW (1872–1894)

PUBLIKACJE MATEMATYCZNE W PAMIĘTNIKU AKADEMII UMIEJĘTNOŚCI W KRAKOWIE (1872–1894)

Abstract

This paper concerns a general characteristic of the Academy of Arts and Sciences in Cracow, together with the list of mathematical publications printed in the Memoirs of Academy (1872–1894).

Keywords: *Academy of Arts and Sciences in Cracow*, *mathematical publications*, *Memoirs of the Academy of Arts and Sciences in Cracow, nineteenth century*

Streszczenie

W artykule zawarto ogólną charakterystykę Akademii Umiejętności w Krakowie i listę publikacji matematycznych wydrukowanych w Pamiętniku Akademii (1872–1894).

Słowa kluczowe: *Akademia Umiejętności w Krakowie*, *publikacje matematyczne*, *Pamiętnik Akademii Umiejętności w Krakowie*, *dziewiętnasty wiek*

* Jan Koroński, Faculty of Physics, Mathematics and Informatics, Institute of Mathematics, Cracow University of Technology.

1. General characteristic of the Academy of Arts and Sciences in Cracow and the Memoirs of the Academy of Arts and Sciences in Cracow

The Academy of Arts and Sciences was founded in 1872, as a result of transformation of the Cracow Scientific Society, which had existed since 1815. The Memoirs of the Academy of Sciences in Cracow were a continuation of the Annals of the Scientific Society affiliated with the University of Cracow. This journal was printed in 44 volumes from 1817 to 1872. The first volume of Memoirs of the Academy of Sciences in Cracow was printed in 1874, and the last (eighteenth) volume in 1894. In all Memoirs 123 scientific papers were printed, including 46 mathematical works, and 77 articles in various fields of science. Among them 11 works were related to differential eqations.

2. List of mathematical papers in the Memoirs of the Academy of Sciences in Cracow (in Polish) – (we preserve the original 19th-century spelling):

Volume I (1874)

1. Wł. Zajączkowski: *O całkach osobliwych zwyczajnych równań różniczkowych rzędu jakiegokolwiek* (pp. 45-56).
2. W. Żmurko: *O styczności stożków obrotowych* (pp. 57-64).

Volume II (1876)

3. Dr. Oskar Fabian: *Obliczanie wartości szeregów nieskończonych, zwłaszcza szeregów bardzo słabéj zbieżności* (pp. 37-56).
4. W. Żmurko: *Przyczynek do rachunku przemienności ze szczególném uwzględnieniem znamion największości i najmniejszości całek oznaczonych* (pp. 57-79).
5. Fr. Mertens: *O funkcji oskulacyjnéj Profesora Żmurki* (pp. 113-123).
6. D. Zbrożek: *Teoryja planimetru biegunowego* (pp. 124-133).

Volume III (1877)

7. Dr. Wł. Zajączkowski: *Teoryja ogólna rozwiązań osobliwych równań różniczkowych zwyczajnych* (pp. 1-23).
8. W. Żmurko: *O ważności i zastósowaniu funkcji oskulacyjnéj w rachunku przemienności, oraz odpowiedź na uwagi Dra Mertensa dotyczące tego przedmiotu* (pp. 24--34).
9. W. Żmurko: *O ważności i zastósowaniu funkcji oskulacyjnéj w rachunku przemienności, część druga (dokończenie)* (pp. 94-101).
10. Dr. Ed. Skiba: *Przyczynek do teoryi strun* (pp. 130-154).
11. J. Tetmajer: *Teoryja rozwinięcia funkcyj niewyraźnych* (pp. 155-188).

Volume IV (1878)

12. J. Tetmajer: *Teoryja rozwinięcia funkcyj niewyraźnych. Część trzecia (część I i II, tudzież rozdział I części III są zamieszczone w III tomie Pamiętnika Akademii str. 155-188)* (pp. 1-62).

Volume V (1880)

13. Józef Tetmajer: *Rozwiązanie równań trzech wyrazowych* (pp. 1-27).
14. Józef Tetmajer: *Dodatek do rozwiązania trygonometrycznego równań dwuwyrazowych* (pp. 117-122).

Volume VI (1881)

15. Dr. Władysław Zajączkowski: *Teoryja wyznaczników o* ***p*** *wymiarach* (pp. 1-31).

Volume VII (1882)

16. Jan Nep. Franke: *O inwolucyi sześciu prostych, uważanych jako osi skrętów chwilowych* (pp. 15-26).
17. Władysław Kretkowski: *O przekształceniach pewnych wielomianów jednorodnych drugiego stopnia* (pp. 69-73).
18. Władysław Kretkowski: *O rozwiązywaniu równań algebraicznych ogólnych za pomocą całek oznaczonych* (pp. 158-163).
19. Władysław Kretkowski: *O niektórych wzorach rachunku różniczkowego* (pp. 164-176).

Volume VIII (1883)

20. Dr. Wawrzyniec Żmurko: *O całkowaniu równań różniczkowych linijnych rzędu drugiego o współczynnikach linijnych* (pp. 74-112).
21. A.J. Stodółkiewicz: *Zastosowanie sposobu Bertranda do całkowania równania różniczkowego o różniczkach zupełnych z wielu zmiennymi* (pp. 137-142).
22. A.J. Stodółkiewicz: *Całkowanie układów równań różniczkowych o różniczkach zupełnych* (pp. 143-152).

Volume IX (1884)

23. Dr. Władysław Zajączkowski: *O zamianie funkcyi całkowitéj i jednorodnéj stopnia 2-go na sumę kwadratów* (pp. 158-126).
24. Władysław Kretkowski: *Dowód pewnego twierdzenia tyczącego się dwóch wyznaczników ogólnych* (pp. 1-44).
25. Dr. Józef Puzyna: *O pozornie dwuwartościowych określonych całkach podwójnych* (pp. 45-47).
26. A.J. Stodółkiewicz: *O całkowaniu równań różniczkowych linijnych rzędu drugiego, mających współczynniki linijne, przy pomocy kwadratur* (pp. 113-119).

27. Jan Rajewski: *O całkowaniu równań różniczkowych linijnych rzędu drugiego, w postaci* $(c_2x^2 + b_2x + a_2)y'' + (b_1x + a_1)y' + a_0y = 0$ (pp. 120-160).
28. Dominik Zbrożek: *Zastosowanie wyznaczników w teoryi najmniejszych kwadratów* (pp. 199-218).

Volume X (1885)

29. F. Mertens: *O niezmiennikach jednéj i dwóch form dwulinijowych alternujący* (pp. 26-56).

Volume XII (1886)

30. F. Mertens: *O utworach niezmienniczych form kwadratowych* (pp. 1-93).
31. Dr. Wawrzyniec Żmurko: *Uzasadnienie niektórych ważniejszych uproszczeń algebraicznéj rachuby oparte na bliższém rozważaniu algebraicznego dzielenia* (pp. 1-34).
32. S. Dickstein: *O niektórych własnościach funkcyj alef.* (pp. 35-40).
33. S. Dickstein: *O twierdzeniu Crocchiego* (pp. 41-44).
34. S. Dickstein: *Dowód dwóch wzorów Wrońskiego* (pp. 87-92).
35. A.J. Stodółkiewicz: *O dwóch szczególnych układach równań różniczkowych o różniczkach zupełnych* (pp. 93-95).

Volume XIII (1887)

36. Władysław Kretkowski: *O wyznaczeniu kuli przecinającej pod tym samym kątem ilekolwiek kul danych i o zagadnieniach podobnych* (pp. 81-96).
37. Władysław Kretkowski: *O pewnych zagadnieniach geometrii kulistej* (pp. 97-105).
38. Władysław Zajączkowski: *Teoryja Fuchsa równań różniczkowych linijowych i jednorodnych z jedną zmienną niezależną* (pp. 1-47).

Volume XIV (1888)

39. Dr. Józef Puzyna: *O zastosowaniu uogólnionych form interpolacyjnych Lagrange'a (Tab. I)* (pp. 1-55).
40. Dr. Stanisław Żurakowski: *Dowód twierdzenia H. Wrońskiego* (pp. 56-68).
41. Prof. W. Żmurko: *O powierzchniach sprzężonych z powierzchniami rzędu drugiego* (pp. 208-222).

Volume XVI (1889)

42. S. Dickstein: *Kilka twierdzeń o funkcyjach alef.* (pp. 53-59).
43. Prof. Franciszek Mertens: *O wyznaczniku, którego elementami są wartości n! funkcyj całkowitych* (pp. 60-69).

Volume XVII (1890)

44. Józef Puzyna: *O pewnem twierdzeniu F. Foliego* (pp. 24-45).

45. F. Mertens: *O funkcyjach całkowitych układu mn zmiennych, tworzących m wierszy i n kolumn* (pp. 143-165).
46. Jan Rajewski: *O całkach nieregularnych równań różniczkowych linijowych* (pp. 166-180).

3. Conclusion

Among mathematical papers published in the Memoirs of the Academy of Sciences in Cracow there are eleven papers dealing with differential equations. These papers were written by the following five mathematicians: Alojzy Jan Stodółkiewicz (1856–1934) – four works, Władysław Zajączkowski (1837–1898) – three works, Jan Rajewski (1857–1906) – two works and one work of Wawrzyniec Żmurko (1824–1889) and Edward Władysław Skiba (1843–1911).

These papers can be divided into four thematical groups. The first one, containing five papers, is devoted to linear ordinary differential equations of the second order. The authors of these works are: Wawrzyniec Żmurko – Vol. VIII (1883), Alojzy Jan Stodółkiewicz – Vol. IX (1884), Jan Rajewski – Vol. IX (1884) and Vol. XVII (1990) and Władysław Zajączkowski – Vol. 13 (1887). Stodółkiewicz's and Rajewski's papers from volume IX refer to Żmurko's work from volume VIII. Rajewski's work in the seventeenth volume refers Zajączkowski's work in Vol. XIII. The second group is related to complete differential equations and contains three Stodółkiewicz's works. Two of them are in Vol. VIII (1883), and the third in Vol. XII (1886).

The third group consists of two Zajączkowski's works on singular integrals of differential equations published in Volume I (1874) and Volume 3 (1877).

The last group contains the work, written by Edward Skiba and published in Vol. III (1877), which concerns partial equations of hyperbolic type, specifically the string equation.

Out of the remaining papers, about 20 belong to classical mathematical analysis; 10 to algebra, and 5 to geometry and trigonometry. Including works on differential equations into analysis one can see that about 30 papers out of 46 are devoted to mathematical analysis.

So Polish mathematicians engaged in an active manner in the development of the 19th-century mathematical ideas.

References

[1] J. Koroński, *Prace z równań różniczkowych w Pamiętniku Akademii Umiejętności w Krakowie*, Zagadnienia Filozoficzne w Nauce (ed. W. Wójcik), LIII 2013, Copernicus Center Press, Kraków 2013, 231-262.

[2] *Pamiętnik Akademii Umiejętności w Krakowie*, Vol. 1–18, Kraków (1874–1894).

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE
1-NP/2014

JAN KOROŃSKI*

A NOTE ON MATHEMATICAL PUBLICATIONS OF POLISH MATHEMATICIANS IN THE MEMOIRS OF THE SOCIETY OF EXACT SCIENCES IN PARIS (1870–1882)

NOTATKA O PUBLIKACJACH MATEMATYCZNYCH MATEMATYKÓW POLSKICH W PAMIĘTNIKU TOWARZYSTWA NAUK ŚCISŁYCH W PARYŻU (1870–1882)

Abstract

This paper gives a general characteristic of the Society of Exact Sciences in Paris and its Memoirs, together with the list of mathematical publications printed in the Memoirs (1870–1882).

Keywords: *Society of Science in Paris*, *Mathematical publications*, *Memoirs of the Society of Science in Paris*, *nineteenth century*

Streszczenie

W artykule zawarto ogólną charakterystykę Towarzystwa Nauk Ścisłych w Paryżu. Ponadto zaprezentowano publikacje matematyczne wydrukowane w Pamiętniku Towarzystwa Nauk Ścisłych w Paryżu (1870–1882).

Słowa kluczowe: *Towarzystwo Nauk Ścisłych w Paryżu*, *publikacje matematyczne*, *Pamiętnik Towarzystwa Nauk Ścisłych w Paryżu*, *dziewiętnasty wiek*

* Jan Koroński, Faculty of Physics, Mathematics and Informatics, Institute of Mathematics, Cracow University of Technology.

1. General characteristics of the Society of Exact Sciences in Paris and its "Memoirs"

After the closure of the Warsaw Main School as a result of the January Uprising (1863), Poles in exile in Paris founded the Polish Higher School called the Montparnasse School (since it was located on the Montparnasse Boulevard). This school had initially a charitable character. It often provided accommodation and meals to its students free of charge. The aim of this school was to serve as a substitute of l'Ecole Polytechnique for people who didn't have French nationality, mostly Poles.

Among others, H.G.P. Niewęgłowski (1807–1881), E.J. Habich (1835–1909), K. Szulc (1869–1871), A.E. Sągajło (1806–1877) and W. Folkierski (1842–1904) taught in it. In 1870, the French authorities closed the school. After that, in the same year the Society of Sciences in Paris was founded. The main purpose of the Society of Sciences in Paris was publication of original research in Polish and of didactic papers of Polish authors. The Society existed up to 1882 and issued twelve volumes of the Memoirs of the Society of Science containing papers of about 40 authors. At the initiative of the Society 18 volumes of teaching were also issued. Such prolific publishing activity of the Society was possible thanks to the initiative and financial support of Jan Kanty Działyński (1829–1880), who was a prominent patron of sciences.

The Memoirs of the Exact Society of Sciences in Paris consisted mainly of original papers of Polish mathematicians (working both at home and in exile) in the field of differential and integral calculus, differential equations and partial analytic geometry, higher algebra with the new theory of the determinants and the theory of analytic functions. In "Memoirs" very extensive reviews of teaching were also printed. Occasionally "Memoirs" published valuable works of foreign mathematicians, eg. the habilitation thesis of Riemann. Many of the works printed in the "Memoirs" were related to physics, construction, biology and other natural sciences.

In the Society of Exact Sciences in Paris many creative working mathematicians living in exile were very active, among them Henry G. Niewęgłowski (1807–1881), Adolf E. Sągajło (1806–1877), Władysław Folkierski (1842–1904) and Władysław Gosiewski (1844–1911). Władysław Gosiewski published in "Memoirs" many articles – several treatises in mathematics and the theory of elasticity. Later he became interested in molecular mechanics. In 1872 he returned to Poland and took a teaching job in secondary education and lower-level clerical work, not using his outstanding scientific talent. Besides works of Polish mathematicians from Paris, research results of mathematicians from active scientific centers in Poland were printed in the "Memoirs", including Wawrzyniec Żmurko (1824–1889), Władysław Zajączkowski (1837–1898) and Władysław Kretkowski (1840––1914) (under the pseudonym Trzaska) from Lviv, Marian A. Baraniecki (1848–1895) from Warsaw and St. Petersburg resident Julian K. Sochocki (1842–1927).

2. List of mathematical papers [2] in the Memoirs of the Society of Exact Sciences in Paris (we keep the original nineteenth-spelling):

Below we present a complete list of mathematical articles in volumes 1 up to 12 of the Memoirs of Exact Sciences in Paris.

Volume I (1871)

1. W. Gosiewski: *O funkcyach jednorodnych i jednogatunkowych* (pp. 57-88).
2. W. Żmurko: *Dowód na twierdzenie Hessego, o wyznaczniku funkcyjnym* (pp. 89-92).
3. W. Żmurko: *Przyczynek do teorii największości i najmniejszości funkcyj wielu zmiennych* (pp. 93-100).
4. J.N. Franke: *O względnościach wykreślnych zachodzących między rzutami systemów geometrycznych* (pp. 101-108).
5. W. Kretkowski-Trzaska: *O niektórych własnościach pewnego rodzaju funkcyj jednej zmiennej urojonej.* (pp. 109-111).
6. W. Kretkowski-Trzaska: *O pewnem zastosowaniu wyznaczników funkcyjnych* (pp. 113-121).
7. W. Kretkowski-Trzaska: *O nakreśleniu do trzech kół danych leżących na powierzchni jednej kuli, czwartego koła stycznego leżącego na tejże powierzchni* (pp. 123-132).
8. W. Gosiewski: *Rozbiór krytyczny Dzieł p. G. H. Niewęgłowskiego; Studyum pierwsze Arytmetyka; Studyum drugie Geometrya* (pp. 133-175).
9. Program przedstawionego do konkursu przez Towarzystwo Nauk Ścisłych zadania: Ocenienie prac matematycznych H. Wrońskiego (pp. 176-182).
10. W. Gosiewski: *O funkcyjach jednoczesnych i jednogatunkowych – nota do twier. VI, Roz. II, 17* (pp. 183-186).

Volume II (1872)

11. W. Gosiewski: *Kilka uwag o liczbie różnych wartości, jakie funkcya może przybierać w skutku przestawień zmiennych do niej wchodzących* (pp. 1-26).
12. W. Kretkowski-Trzaska: *Kilka uwag tyczących się funkcyj wielowymiarowych* (pp. 27-38).
13. W. Kretkowski-Trzaska: *Dowód pewnego twierdzenia tyczącego funcyj wielowymiarowych okresowych* (pp. 39-45).
14. A. Sągajło: *Rozbiór krytyczny Dzieła p. Folkierskiego p. t. Zasady Rachunku Różniczkowego i Całkowego, Tom pierwszy, Rachunek różniczkowy* (pp. 47-121).
15. W. Gosiewski: *Przegląd krytyczny Dzieła p. G. H. Niewęgłowskiego pod tytułem: Trygonometrya etc* (pp. 241-245).

Volume III (1873)

16. W. Folkierski: *O równaniach różniczkowych częściowych jednoczesnych* (pp. 1-30;1st paper in Vol. III).

Volume IV (1874)

17. W. Puchewicz: *Teorya funkcyj zmiennej złożonej* (pp. 1-131; 5th paper in Vol. IV).
18. A. Cayley: *O twierdzeniu Wrońskiego* (pp. 1-8; 7th paper in Vol. IV).

19. Sprawozdanie z konkursu naznaczonego przez Towarzystwo Nauk Ścisłych: *Ocenienie prac matematycznych H. Wrońskiego* (pp. 1-3; 8th paper in Vol. IV).
20. Program przedstawionego do konkursu przez Towarzystwo Nauk Ścisłych zadania: *Ułożenie bibliografii piśmiennictwa polskiego z działu Matematyki i Fizyki oraz ich zastosowań, od roku 1831 aż do najnowszych czasów* (pp. 1-2; 9th paper in Vol. IV).

Volume V (1875)

21. K. Maszkowski: *Perspektywa rzutowa jako wynik rzutów prostokątnych na płaszczyzny ukośnie względem siebie położone* (pp. 1-38; 1st paper in Vol. V).

Volume VI (1876)

22. W. Zajączkowski: *O równaniu różniczkowem* $Xdx + X_1dx_1 + ... + X_ndx_n = 0$, *całkowalnem przez jedno równanie pierwotne* (pp. 1-14; 4th paper in Vol. VI).

Volume VII (1877)

23. K. Hertz, S. Dikstein: *Teorya liczb złożonych i ich funkcyj* (pp. 1-60; 3th paper in Vol VII).
24. M.A. Baraniecki: *Rozwinięcie na ułamek ciągły stosunku dwóch całek eliptycznych pierwszego i drugiego gatunku* (pp. 1-8; 5th paper in Vol. VII).
25. M.A. Baraniecki: *O przedstawieniach wymiernych* (pp. 1-30; 6th paper in Vol. VII).
26. A. Sągajło: *Kilka zadań geometryi analitycznej wyłożonych podług najnowszych metod analizy nowoczesnej* (pp. 1-28; 8th paper in Vol. VII).

Volume VIII (1878)

27. M.A. Baraniecki: *Dowód jednego zasadniczego twierdzenia odnoszącego się do hypergeometrycznych funkcyj* (pp. 1-19; 1st paper in Vol. VIII).
28. A. Sągajło: *Krótka wiadomość o przedniejszych poszukiwaniach analizy nowoczesnej nad kołem stycznem do trzech kół danych* (pp. 1-7; 6th paper in Vol. VIII).
29. A. Transon: *Uwagi nad objawem naukowym z powodu wzoru ogłoszonego przez Wrońskiego w roku 1812 i dowiedzionego później przez p. Cayley w roku 1873* (pp. 1-8; 7th paper in Vol. VIII).
30. A. Transon: *Prawo szeregów Wrońskiego (jego foronomia)* (pp. 9-16; 8th paper in Vol. VIII).
31. M.A. Baraniecki: *Zasadnicze wnioski geometryczne z teoryi algebraicznej form kwadratowych podwójnych* (pp. 1-8; 9th paper in Vol. VIII).

Volume IX (1879)

32. B. Riemann: *O hypotezach, które służą za podstawę geometrii, rozprawa p. Riemanna, przetłomaczona i objaśniona przypiskami przez S. Diksteina i W. Gosiewskiego* (pp. 1-3; 6th paper in Vol. IX).

Volume X (1880)

33. J.K. Sochocki: *Wyznaczanie stałych mnożników we wzorach dla linijnej transformacji funkcyi q. – Sumy Gaussa i prawo wzajemności symbolów Legendre'a* (pp. 1-37; 1st paper in Vol. X).
34. M.A. Baraniecki: *O tworzeniu systemu sprzężonego podstawień liniowych* (pp. 1-8; 3th paper in Vol. X).
35. M.A. Baraniecki: *O wyznaczaniu spólnych pierwiastków dwóch równań danych przy pomocy rugownika tych równań* (pp. 1-7; 5th paper in Vol. X).
36. Władysław Kretkowski-Trzaska: *O mnożeniu funkcji kołowych i hiperbolicznych* (pp. 1-7; 9th paper in Vol. X).
37. W. Kretkowski: *Dowód pewnego wzoru Lame'go* (pp. 1-3; 10th paper in Vol. X).
38. M. Szystowski i A. Martynowski: *Rachunek wykreślny na płaszczyźnie* (pp. 1-80; 11th paper in Vol. X).

Volume XI (1881)

39. K. Hertz: *O funkcjach nie mających pochodnych* (pp. 1-24; 1st paper in Vol. XI).
40. W. Żmurko: *Badania w dziedzinie nauki o równaniach oparte na poglądach analityczno-geometrycznych* (pp. 1-150; 4th paper in Vol. XI).

Volume XII (1882)

41. M. Szystowski: *Rachunek wykreślny na płaszczyźnie, Część II* (pp. 1-24; 1st paper in Vol. XII).
42. S. Rychlicki: *O przekształceniu kwadratowem* (pp. 1-19; 2nd paper in Vol. XII p).
43. W. Gosiewski: *O rożniczkowaniu i całkowaniu funkcji rzeczywistej jednej zmiennej niezależnej* (pp. 1-15; 3th paper in Vol. XII).
44. Władysław Kretkowski-Trzaska: *Rozwiązanie pewnego zadania z geometryi wielowymiarowej.* (pp. 1-3; 4th paper in Vol. XII).
45. M.Y. Villarceau: *Zastosowanie teoryi wstaw wyższych rzędów do całkowania równań różniczkowych linijnych* (pp. 1-7; 7th paper in Vol. XII).
46. M.A. Baraniecki: *Ocena książki pod tytułem Algebra przez G. H. Niewęgłowskiego.* (pp. 1-14; 8th paper in Vol. XII).

3. General remarks on the mathematical publications printed in the Memoirs of the Society of Exact Sciences in Paris

In the years from 1870 to 1882 12 volumes of the Memoirs of the Society of Exact Sciences in Paris were published. About 90 publications from many fields of science were printed in these volumes. Among 45 papers on mathematics, five works of foreign mathematicians (Villarceau (1813–1883) – Vol.XII, Riemann (1826–1866) – Vol. IX, Transon (1805–1876) –

two works in Vol. VIII and Cayley (1821–1895) –Vol. IV), 4 competition notes and 4 reviews of didactic works, and 32 original articles written by Polish authors were included.

Most articles published in the Memoirs were written by W. Kretkowski (8), followed by M.A. Baraniecki (7), and next by W. Gosiewski (6), A. Sągajło (3) and W. Żmurko (2). Other Polish authors published – at least one work independently, and sometimes – additional papers as coauthors.

Although differential equations were featured in three works only one can find several works in which differential equations are applied, but these works are dedicated to other scientific fields.

References

[1] J. Koroński, *Prace z równań różniczkowych w Pamiętnikach Towarzystwa Nauk Ścisłych w Paryżu*, Zagadnienia Filozoficzne w Nauce, (ed. W. Wójcik), LIII 2013, Copernicus Center Press, Kraków 2013, 199-230.

[2] *Pamiętnik Towarzystwa Nauk Ścisłych w Paryżu*, Vol. 1–12, Wydawnictwo Towarzystwa Nauk Ścisłych w Paryżu, Paryż (1871–1881).

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE
1-NP/2014

HALINA LICHOCKA*

SWISS EXPERIENCES OF IGNACY MOŚCICKI

SZWAJCARSKIE DOŚWIADCZENIA IGNACEGO MOŚCICKIEGO

A b s t r a c t

In Fribourg, Ignacy Mościcki found favorable conditions for the development of his engineering talents. He was one of the founders of the Swiss nitrogen and electrical industry. He announced the results of his works in Polish, German and French scientific journals. This was followed by rapid adaptation of Mościcki's discoveries and inventions regarding the dielectric properties, the construction of technical high voltage capacitors, the construction of fuses protecting the electrical transmission lines against lightning, production of nitric acid from the air, the construction of devices used for absorption of gaseous substances, etc. His experience Mościcki transferred to Lvov.

Keywords: *nitrogen industry in Switzerland*, *glass capacitors*, *Mościcki's school of engineers*, *chemical research institute*

S t r e s z c z e n i e

We Fryburgu znalazł Ignacy Mościcki sprzyjające warunki dla rozwoju swoich inżynierskich uzdolnień. W Szwajcarii znacznie wzbogacił swoją wiedzę, zdobył doświadczenie i sławę. Był jednym z twórców szwajcarskiego przemysłu azotowego i elektrotechnicznego. Wyniki swoich prac ogłaszał w polskich, niemieckich i francuskich czasopismach naukowych. Tą drogą następowała szybka recepcja odkryć i wynalazków Mościckiego, dotyczących właściwości dielektryków, konstrukcji kondensatorów technicznych wysokiego napięcia, bezpieczników chroniących elektryczne linie przesyłowe przed skutkami wyładowań atmosferycznych, wytwarzania kwasu azotowego z powietrza, urządzeń do absorpcji substancji gazowych itd. Swoje doświadczenie technologiczne oraz własne przemyślenia na temat kształcenia dobrych inżynierów przeniósł Mościcki do Lwowa.

Słowa kluczowe: *przemysł azotowy w Szwajcarii*, *szklane kondensatory*, *szkoła inżynierów Mościckiego*, *chemiczny instytut badawczy*

* Halina Lichocka, Institute of the History of Science of the Polish Academy of Sicences, Warsaw.

1. Electricity and the saltpeter

It happened already during Ignacy Mościcki's [1] first encounters with electrical engineering that he discovered a special interest for it, namely back in the years 1897–1901 when he worked as an assistant at the University of Freiburg. At that time, the depletion of the worldwide deposits of saltpeter, a mineral containing a high amount of sodium and potassium nitrates, was a major concern for scientists, industrialists and politicians. The problem was therefore perceived as severe because saltpeter was the basic material for the production of explosives as well as dyes, artificial silk, fertilizers and many other goods offered by chemical plants. So it was not surprising that efforts were initiated aiming at inventing an industrial method of the synthesis of nitric acid. Another incentive for those who were considering a confrontation with the problem was a quite well-founded belief according to which only water and air, cheap and easily accessible materials, were necessary to obtain nitric acid. The prevailing of such an opinion was due to the fact that the chemical reaction in which nitric oxides were spontaneously produced in the atmospheric air in which electric discharge has just taken place had been known for a long time.

Mościcki found experimenting with electricity and nitrogen very attractive. He devoted much time to them during his Easter holidays 1901. He had already read the latest scientific publications relating to this field. He knew that the synthesis of oxygen and nitrogen was an endothermic reaction and that temperatures reaching 3000°C were needed in order to start it. He also knew that leaving the newly produced oxide in this temperature would lead to its prompt breakdown. Considering all these factors, he came to the conclusion that positive results could be obtained if the synthesis would be conducted in an electric arc powered by alternating current of high voltage and high frequency. He intended to achieve improved thermal conditions by blowing the substrates (the air in this case) through a heated arc.

2. The logistics of the undertaking

The results of these initial experiments of Ignacy Mościcki seemed to herald success. Therefore it happened already in autumn 1901 that a company *Société de l'Acide Nitrique à Fribourg* (Polish: *Towarzystwo Kwasu Azotowego we Fryburgu*, English: Nitric Acid Society in Freiburg) was founded. The company was ready to invest in electrochemical experiments of Mościcki who had given up his assistant position at the University and started working for the Society as a permanent employee [2]. His task was to conduct the necessary research and to prepare patent applications that were to be the property of the shareholders. He started receiving a quite high remuneration which made him totally free from financial worries and allowed to devote all his time to the research work. The cantonal government made three university rooms available to him free of charge, with all the necessary laboratory equipment and an unlimited supply of electricity.

Mościcki felt very responsible for the risk that was inherent in his experiments. He was also very much afraid of failure. That is how he was recalling that time twenty years later: "(…) I was keeping my nose to the grindstone. I made my experiments for days on end while at nights I prepared the theoretical bases for further research. When I encountered an even single difficulty or a setback, a worry pestered me whether I was not trying to bite off more than one could chew" [3].

The major problem emerged when capacitors withstanding voltage of several dozen thousand volt had to be used. It turned out then that such capacitors had not been invented yet. That was the reason for which experiments aiming at producing nitric oxides came to a halt. They had to be stopped altogether for a certain time. Research into dielectrics, necessary to be conducted in that situation, had to be launched instead, in order to determine their electrical breakdown and surface discharge strengths.

3. Necessity is the mother of invention

In order to find solutions to these problems, Mościcki started independent and thorough studies into dielectrics. He took up research into the breakdown strength of glass. He analyzed the same aspect of ebonite, porcelain, natural rubber and other known dielectrics. Judging on the base of the results of his experiments and calculations, he concluded that – compared with all the other materials analyzed by him – glass possessed the best dielectric properties. This conclusion was a decisive factor for the choice of glass as the object of his further experiments.

The research into the properties of dielectrics was the first scientific activity of Ignacy Mościcki. His first reports on them were published in 1904 by the Academy of Science and Art in Cracow (Polish: *Akademia Umiejętności*) [4]. They also appeared in international scientific newspapers [5].

At first, Mościcki constructed plate capacitors, of the type that was in use at the time. He used capacitor plates made of different metals and changed the thickness of the coating. Glass was resistant to high voltage but it became hot too quickly and then it cracked. During these experiments, Mościcki observed that the electrical breakdown of the dielectric layer did not take place in the middle of the plates but on their rims. He came to the conclusion that in such a situation, the thickness of the dielectric layer should be differentiated – it should be thicker at the rims while in the central part of the plate it should be less thick. Capacitors having thicker rims retained the same capacity and resistance but they were much more resistant to an electrical breakdown. Considering all this, he thought that the best shape for such device would be that of a glass tube whose walls would be thicker at the rims. This shape had an additional advantage, namely the whole apparatus did not warm up very much when electric current passed through it. Besides, it was easier to cool it.

This idea was wholly original and deserved to be granted a patent. Its author started elaborating the details and soon after that he launched the laboratory tests. The results were excellent so he prepared a proper application and submitted it to a patent office in Bern. In 1903, he was granted a Swiss patent and a year later – a French one.

As his research into methods of obtaining the nitric acid were – at the time – not so successful as to generate profit, Mościcki felt indebted to the *Socièté de l'Acide Nitrique à Fribourg* and therefore he decided to donate his capacitor patents to the Society. They turned out to be worth over a million francs, which meant that they exceeded all the assets of the society several times.

In order to make use of Mościcki`s patents, a capacitor factory was built in Freiburg. It happened in 1903 and the factory is in operation to this day. Initially, it was called *Fabrique suisse de condensateurs Jean de Modzelewski et Cie*. It was the first factory of high voltage capacitors in Europe [6].

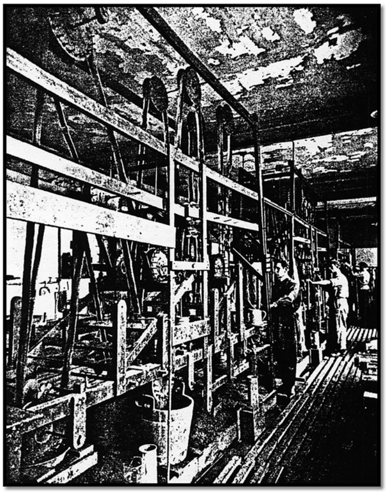


Fig. 1. Factory of capacitors in Freiburg (From the collections of the Museum of Maria Curie Skłodowska in Warsaw)

The capacitors for which Mościcki was granted patents were made of glass and had the shape of a narrow, very elongated bottle with a thickened neck and a rounded thick bottom. Its outer and inner walls were covered with a thin layer of silver that functioned as the capacitor`s plates. A firm galvanic copper coating, fixed on the plates, was there to protect the delicate covers from possible mechanical damage. The whole was later placed in an iron or brass metal sheet, filled with water mixed with glycerin. It was a kind of a cooler. Glycerin prevented the cooling liquid from freezing at low temperatures. Rubber rings, fixed in proper places, guaranteed the tightness of the construction. Wires were put through those rings in order to connect the capacitor plates with current terminals, placed outside.

It was possible to bind these appliances so as to form smaller or bigger batteries that would be resistant to the respectively higher voltage. The batteries of the Mościcki's capacitors were successfully applied in electric circuits with the voltage exceeding 100 kV. They were the best capacitors worldwide at the time.

4. The usefulness of the new capacitors

The demonstration of artificial electrical discharges, organized in a Freiburg laboratory, found a loud resonance in scientific publications devoted to electrical engineering. These electrical discharges were similar to atmospheric phenomena appearing during thunderstorms [7]. The demonstration took place in 1905 during the Congress of

Electrical Engineering. The author of this loud – also in the literal sense of the word – demonstration was Ignacy Mościcki. He wished to demonstrate his latest invention at the Congress, that is the fuses securing the transmission lines and other electric appliances in cases of a sudden increase of voltage. Such temporary increases of voltage, known as overvoltage, were especially dangerous for power lines and power plants as they often caused damage to transformers in the distribution boards.

The fuses invented by Ignacy Mościcki formed an electric system, made of glass capacitors and induction coils. The Capacitor Factory in Freiburg took up their production immediately. As Mościcki used to cede his electrotechnics ideas for the benefit of this factory, the fuses that he constructed were later widely known as the *Giles' valves* (Giles was the name of the factory's director). It was the big hydroelectric power station in Hauterive, the main electricity supplier of Freiburg, that installed the *Giles' valves* in its appliances as the first factory. It was already in 1903 that capacitor fuses were experimentally installed in two main transmission lines there, each of them several dozen kilometers long. The results were very positive. Other power plants – initially in Switzerland, later also in France and in other European countries – soon followed the example of Hauterive.

In 1906, the Mościcki's capacitors were presented for the first time at the world exhibition in Milan and were very much appreciated by specialists. They were honoured with honorary diplomas and a golden medal [8].

The capacitors of the Mościcki's system served reliably in the radio station installed at the Eiffel tower in all the years of the First World War and later. In the interwar period, after it had been found out that other dielectric material could be used instead of the brittle and heavy glass, they lost their significance. In 1920s they were displaced by paper capacitors of Fischer.

5. The design of the industrial synthesis of nitric acid

When the problem of capacitors was already successfully solved and when it was possible to use them for the construction of an electric circuit arc having parameters that seemed to be ideal for conducting the synthesis of nitric oxides, Ignacy Mościcki returned to this subject. After having conducted many experiments and calculations, aiming at making as economical use of energy as possible, he used a system of coils and capacitors of the latest generation, suitable to resist the high voltage. All the experimental results were positive and it did not take long until the method of the production of nitric acid was ready to be applied, after having been examined in a laboratory. Eventually, the time came for this method to be implemented on a larger scale. It was decided that a small experimental factory would be opened in the big factory hall in Vevey. It was Mościcki himself who supervised all the stages of the assembling work.

According to Mościcki, the performance of the experimental factory in Vevey did not turn out to be satisfying. Therefore he found it necessary to undertake efforts aiming at improving the whole technological process. The device that was most energy-intensive and that, at the same time, generated the biggest losses was the stove so this element of the installation captured the most of the constructor`s attention. The result of these efforts was the invention of a stove possessing a system of several electric arcs. A technological test of this stove took place in Freiburg in autumn 1905. The stove prototype with a system of several electric arcs was soon improved by Mościcki in such a way that it could operate continuously and almost wholly automatically. The next invention of Ignacy Mościcki was a vibrant arc. Mościcki constructed the stove prototype with a vibrant flame in 1906.

The construction of electric stoves, however perfect it was, did not solve other problems that were piling up in the process of the production of nitric acid by the use of the Mościcki method. Looking at it from the viewpoint of the improvement of the profitability of the production, one of the most important tasks was to construct absorptive machines for nitric oxides because the absorptive columns, known at the time, did not wholly respond to the requirements. Large amounts of the air-diluted gas mixture of nitric oxides were produced during the process of obtaining the nitric acid. This mixture dissolved in water only slowly and not wholly.

Mościcki solved this problem by constructing absorbing columns which he based on his own ideas. They were later granted patents and used in many countries. The columns found their use not only in the production of the nitric acid but also in other branches of the chemical industry.

At that time, in Norway, a factory operated according to the system of K.O. Birkeland. However, it produced only small amounts of a very diluted nitric acid and nitrates. Mościcki's intention was to design, build and open a big factory that would produce concentrated nitric acid from air and water. He undertook this task, commissioned by the Swiss cantonal authorities. As the factory`s localization, Chippis was chosen.

6. From the project to its realization

When Mościcki was designing and building the factory in Chippis, he was at the same time making its miniature copy at a university laboratory in Freiburg. He did it for his experimental needs because he wished that each element being currently implemented be checked one more time on a laboratory scale [9].

The nitric acid`s factory was built in the valley of Rhone, next to the town of Sierre. Not far from there, large (large also at the time) hydroelectric power plants were situated that satisfied the needs of the stove with a vibrant flame and of other appliances used in the production process. The construction of the factory lasted less than two years. Its equipment included absorptive columns, invented by Mościcki, and condensation appliances than had been granted patents shortly before. It was possible to obtain the 98% concentration of nitric acid by the use of these appliances. In 1910, the first tank containing concentrated nitric acid left the factory in Chippis. It was also the first ever cistern with a highly concentrated nitric acid that had been obtained from the atmosphere using the method of the electrochemical oxidation of nitrogen [10].

The factory's output supplied the chemical industry in Switzerland and the surplus was exported to other European countries. The profitability and receptivity of the market were the reasons for the decision to expand the factory. Mościcki received the next order. This time, the contract foresaw a tenfold increase of the nitric acid that was to be produced there.

It was to be equally pure and should have the same concentration as that produced before. This meant in practice that a new factory had to be built.

The new factory started in 1912 and all the improvements that had been introduced had the patents of Ignacy Mościcki. These were – on a European scale – large factories and they made Switzerland wholly independent from external supplies of nitric acid. This played an important role during the First World War when – due to the blockage of the Central Powers – the delivery of the potassium nitrate from Chile was very difficult. At that time, Switzerland could cover its whole demand for nitrogen compounds from its own production.

7. Profits and loss of the Freiburg period

Mościcki's activities in Switzerland were marked by successes but the work atmosphere did not always allowed freedom and the necessary effectiveness of research. This was due to crossing of scientific ambitions and financial interests of factories, a phenomenon typical for the field of technology. It happened many times that Mościcki, forced by circumstances, sold his ideas – they were not fully elaborated at the time – to different firms, receiving only small sums of money in return. The purchasers implemented a given idea and put appliances on the market after having given them various commercial names. This was the reason for the squandering of many valuable inventions of Mościcki. Today little is known beyond the fact that these inventions should be traced back to him.

Fig. 2. Ignacy Mościcki, Freiburg 1907

However, the industrial technology to obtain nitric acid from air and water, using the energy of electric discharge, has been permanently connected with his name. The capacitors for alternating current circuits of high voltage and frequencies that were immediately used in the biggest radiotelegraphic stations in Europe also brought him fame. Appliances made of a system of coils and capacitors, used to secure power plants and electrical transmission chains from electrical breakdowns that were induced during thunderstorms due to electrostatic discharges in the atmosphere were also associated with his name. Besides, the name “Mościcki” appeared from time to time in the catalogues of firms offering their own electrical products.

Ignacy Mościcki himself positively evaluated the time that started with his arrival in Freiburg. This attitude found resonance in his *Autobiography* in which he wrote: “The summary of my stay in Switzerland is very positive. In the first years of my work as a university assistant, I very much broadened my scientific range. Apart from the chemical knowledge that I had acquired earlier, I got an education in physics and electrical engineering and my knowledge in electrophysics deepened very much. Not only was it my favorite field of science but also this one in which I worked most during the 15 years of my work. All my knowledge comes only from books and possibilities to experiment. The more significant inventive activities that I mentioned before as well as other, having smaller significance, broadened my technological experience. All this resulted in an increase of my intellectual capabilities to solve tasks I had chosen for myself” [11].

8. The reception of the Swiss experiences

In 1912, Ignacy Mościcki received a job offer in Lvov. He was asked to organize from scratch and take over a new chair of electrochemistry and physical chemistry at the Royal Polytechnic School. Mościcki did not spend much time considering the offer. He knew that his salary in Lvov would be over four times smaller than what he used to receive in Switzerland but he also knew that this was a moment in which his dreams about a modern educating path for engineers for the Polish chemical industry were coming true, which he did not expect to happen. Keeping in mind the twenty years of his work abroad, he was aware of the significance of this task. He also felt ready to confront it.

As he intended to continue the research that he had conducted in Switzerland, he purchased – for his own money – the whole equipment of the Freiburg laboratory and dispatched it by rail to Lvov. The devices and appliances occupied a few wagons and weighted over ten tons.

The new professor met with real friendliness from the Royal School of Technology in Lvov. He was given spacious rooms, located on the ground floor of the Main Building, that were made free especially for him where he could organize his laboratory. That should be the first step in creating a research institute. He had been trying to achieve this aim by removing numerous administrative obstacles that piled up at the different levels of the vast university administration. When he came to Lvov, he already had a didactic project ready to be used. This project was based on his own experiences. Mościcki was convinced that even the best theoretical background and laboratory practice were not enough to educate a good engineer. He was of the opinion that such model of education, present at most European universities at the time, did not respond to the needs of the requirements of the methods of production that were advancing very quickly. On many occasions, Mościcki could see the awkwardness of young people who were so accustomed to conducting experiments in laboratories that they felt lost and confused in contact with the real industrial technology [12].

Coming back to his own experiences made him to admit that his individual abilities for innovative work developed under circumstances that he encountered at the University of Freiburg. A good equipment of the university laboratory, readiness to take up current challenges and a real independence in their realization triggered his invention and led him to studying specialist publications and examining solutions that were coming to his mind.

Mościcki intended to introduce such points into the curriculum in his department. He saw many parallels between Freiburg and Lvov. There were no modern industrial plants in Lvov, either, where students could get acquainted with the latest achievements of the factory technology. Therefore he needed a research institute where the best students – those close to the finishing their education as well as young engineers – could participate in the design works, construct prototypes of experimental factories and then implement – on a large scale – projects that had positively undergone the necessary tests. This meant that they could take part in the construction and starting up new industrial plants.

For the time being, all he could do was to organize the laboratory. He wanted it to be as good as possible. The friendliness of people around him was very helpful. The electrical plant in Lvov, under the direction of Józef Tomicki, helped him a lot. The laboratory was supplied with high voltage current and a small distribution board was located at Mościcki's room. The board was covered with mirror panes and everything was done with a great amount of attention. All this – and many other devices that were handy at electrical experiments – was donated by the power station.

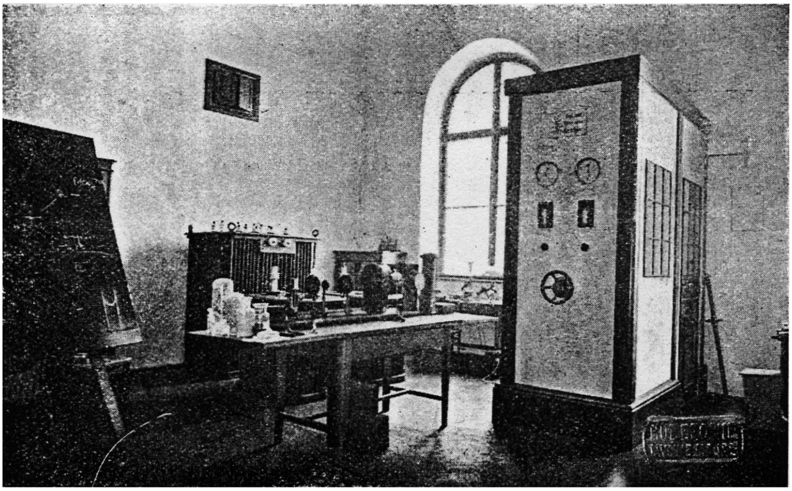


Fig. 3. High-voltage transformer in the cabinet for I. Moscicki in Lvov University of Technology (*Politechnika Lwowska*, Lvov 1932, p. 139)

After a few months during which the necessary equipment was installed, the laboratory was ready. A separate part of it belonged to Mościcki. He conducted his own research there and worked together with more advanced students. The rest of the laboratory was used for experiments that were conducted by students under the supervision of assistants.

Leon Wasilewski, Moscicki's student and later a co-worker, characterized his professor in this way: "(…) Professor Mościcki was a talented man and a wonderful teacher of inventive engineers, architects of the Polish national industry that was emerging at the time. He had an enchanting – and also a little queer – influence on young people which was surprising because he was not a good speaker. In most cases, the professor devoted his lectures to the analysis of the way one had to go in order to reach a given technical solution. It was often a lecture about the development and research inventiveness in technology, usually based on his own experiences" [13].

At the beginning of his stay in Lvov, Mościcki continued his Swiss research. Commissioned by France, he made a project of the factory of nitric acid that was to be built in Mulhouse in Alsace. He also designed a factory of ferrocyanides located in Silesia, in Bory near Jaworzno. The implementation of both projects was interrupted by the outbreak of the First World War. The warfare activities, compulsory enlistment into the armed forces, the moving front line disturbed the normal functioning of the Royal Polytechnic School. In addition, most school space and dormitories were temporarily occupied by a war hospital. In such circumstances, Ignacy Mościcki put a lot of efforts into setting up a research institute that would be independent form the technical university.

He managed to convince two enterprisers – engineers Władysław Szaynok and Marian Wieleżyński who were also pioneers of the Polish natural gas and oil industry – about the necessity to do this. Gas and oil were natural resources of the region surrounding Lvov. On the initiative of Szaynok and Wieleżynski, following companies were set up in the mining region of Borysław: *Natural Gas, Gasoline, Natural Gas Plant in Kałusz* and other. New technological trends for to the gas and oil industry were very much in demand at the time because the war meant positive economic circumstances for them. A research institute, focused on this field, had therefore many chances to succeed.

9. The company *Metan* (English: Methane)

Similarly as earlier in Freiburg, a company was set up in Lvov in 1916 in order to finance research into the technology of the extraction and procession of gas and oil. Because of this advanced scope, it was named *Metan*. Most assets, necessary to launch the *Metan*, were brought in by Władysław Szanyok, an engineer for machines construction (a graduate of the University of Technology in Lviv). He was director of the Oil Bank which – to a significant extent – financed the construction of the gas piping from Borysław to Drohobycz. The initiators of the new company expected that the research would be soon self-financed. However, unlike the Swiss Nitric Acid Society, the company *Metan* was not profit-oriented. Its main aim was to educate engineers and to cooperate in expanding of the chemical industry. Therefore, only recommended persons could become its shareholders because they guaranteed that the founders' intentions would not be distorted in the future.

The initial capital, modest at the beginning, expanded quickly. The first investment of the *Metan* was the setting up of an analytical laboratory in which commissioned work was done, in most cases it was research into fuels, raw materials or half-finished products in order to determine their quality. Next to the laboratory, a mechanical plant was set up that produced devices that were currently necessary for technological works. Research requiring more advanced devices was conducted – wherever it was possible – in the laboratory that Mościcki had organized at the University of Technology. However, this research was often severely disturbed by various war events. In this case, the University of Lvov offered help by putting at the disposal of the company its own laboratory at the Department of Chemistry as well as rooms located in the basement.

Already after the first research topics had been ready and rewarded with patents, it was clear that the company *Metan* had promising future perspectives and that the demand for new technologies, possible to be implemented in a short period of time, was very large. In 1917, a scientific magazine *Metan. A monthly on natural gas industry, published thanks to the efforts of Metan, L. L. Company in Lvov* was launched which served the popularization of the current activities of the company.

10. The transformation of the *Metan* into a research institute

The scope of the research conducted by the *Metan* expanded quickly and the laboratories, small at the beginning, after less than three years after the founding of the company could be transformed into a private research institute known as the Institute for Scientific and Technological Research, having its own office at 3, Leon Sapieha Street. The magazine *Metan* also changed its title, adopting a more appropriate, broader name in 1920, namely: *Chemical Industry.* The numbering of the successive issues was not changed. The *Chemical Industry* is still published.

Despite the disruption caused by the war, many students and assistants from the University of Technology were advancing their engineering talents in the laboratories of the Institute for Scientific and Technological Research. The number of permanent employees was between ten and twenty. The achievements of the institute were impressive. As many as 30 innovative technological solutions were elaborated within five years, which meant over a hundred patents, granted in different countries. All the earnings coming from the sale of patent rights and licenses were assigned to the development of the Institute and to the financing of research.

When the war ended and Poland regained its independence, the Lvov Institute for Scientific and Technological Research was – on the initiative of Mościcki – given as a present to Warsaw. Its new owner was a community association *Chemical Research Institute*. At the shareholders` meeting on March 24, 1922, it was unanimously decided that all the possessions of the *Metan* company would be given to this association. The possessions were considerable in its intellectual and material dimensions.

The major intellectual achievements of *Metan* were named and characterized by Ignacy Mościcki during a lecture [3] delivered at the festive meeting of the Polish Chemical Society in Warsaw on June 1, 1922. Among them there were the following achievements: a new technology of coal, brown coal and peat carbonization; fractional oil distillation; the construction of a gasoline factory in the mining region of Borysław, using the method elaborated in the *Metan*; the method and equipment for separating of brine from the oil emulsion; a very cost-effective technological line for pyrotechnics reactions of oil distillates that had a military meaning; a new way to obtain activated carbon; the technology of electro-winning of alkali hydroxides and chlorine as well as carbon tetrachloride and hydrochloric acid; the method to obtain pure aluminum oxide from clay which allowed to become independent from aluminum deliveries from abroad.

In the material dimension, the company *Metan* possessed devices and appliances that could be used in different branches of the chemical industry. Besides, the profits from the selling licenses increased each year.

The founding meeting of the Chemical Research Institute [14] took place on May 20, 1922. Mościcki was of the opinion that it was necessary to open at least a few such institutes in which new technologies would be developed. These technologies would be appropriate for the country`s needs and for its raw material deposits. Young engineers could extend their qualifications there. They would also be of great importance for further development of national industry. He was deeply convinced that such institutes, directed by outstanding specialists, should be set up as part of universities of technology and also as separate research institutes.

The Chemical Research Institute was intended to function independently from the state budget, as a self-financing association. It was expected that the financial means necessary for its functioning would be coming from the sale of its own ideas concerning technology. This was to be read in the fourth paragraph of its Statutes: “The association is not profit--oriented and its sole aim is to support innovative initiatives of the Polish chemical industry. All its revenues will be assigned for the realization of the Institute`s various aims as well as for its enlargement” [3].

References

[1] Lichocka H., *Ignacy Mościcki*, Wydawnictwo Naukowe Instytutu Technologii Eksploatacji, Radom 2011.

[2] Modzelewski J., *Wspomnienie z pobytu Prezydenta we Fryburgu Szwajcarskim w latach 1900 do 1913*, [w:] *Profesor dr Ignacy Mościcki, życie i działalność na polu nauki i techniki*, Warszawa 1934.

[3] Mościcki I., *O powstaniu Chemicznego Instytutu Badawczego i jego zadaniach z punktu widzenia rozbudowy przemysłu chemicznego w Polsce* (odczyt wygłoszony na posiedzeniu Polskiego Towarzystwa Chemicznego w dniu 1 czerwca 1922 r.), Roczniki Chemii II/1922.

[4] Mościcki I., *Badania nad wytrzymałością dielektryków*, Rozprawy Wydziału Matematyczno--Przyrodniczego Akademii Umiejętności, Seria III, t. 4-A/1904.

[5] Mościcki I., *Über Hochspannungs-Kondensatoren*, Elektrotechnische Zeitschrift, z. 25/1904; *Les condensateurs a haute tension*, L’Eclairage électrique, n. 41, IV/1904; *Elektrische Kondensatoren für die höchsten Spannungen und für Dauerbetrieb*, Fabrique des Condensateurs, Fribourg 1904.

[6] Drewnowski K., *Prace Ignacego Mościckiego w zakresie techniki wysokich napięć w świetle poglądów ówczesnych i obecnych*, [w:] *Profesor dr Ignacy Mościcki, życie i działalność na polu nauki i techniki*, Warszawa 1934.

[7] Mościcki I., *Autobiografia*, Niepodległość, v. 12, Nowy Jork–Londyn 1979.

[8] Drewnowski K., *O zastosowaniu kondensatorów Mościckiego w elektrotechnice*, Lwów 1907.

[9] Drozdowski M. M., *Ignacy Mościcki Prezydent RP – autobiografia*, Warszawa 1993.

[10] Suchowiak L., *Życiorys i działalność Pana Prezydenta Rzeczypospolitej prof. dra h. c. Ignacego Mościckiego*, Roczniki Chemii, z. 23, XIV/1934.

[11] Mościcki I., *Autobiografia* (dalszy ciąg), Niepodległość, v. 13, Nowy Jork–Londyn 1980.

[12] Mościcki I., Kling K., *W sprawie technicznego kształcenia chemików technologów*, Metan, nr 2, III/1919.

[13] Wasilewski L., *Ignacy Mościcki (1867–1946)*, Przemysł Chemiczny 37/1958.
[14] Kling K., Leśniański W., *Powstanie i dotychczasowa działalność Instytutu Badań Naukowych i Technicznych „Metan" oraz jego przekształcenie na „Chemiczny Instytut Badawczy"*, Przemysł Chemiczny VI/1922.

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE

1-NP/2014

ROMAN MIERZECKI*

CHEMICAL IDEAS AND THE DEVELOPMENT OF CHEMICAL AND PETROLEUM INDUSTRY ON THE POLISH TERRITORY SINCE 1850 TO 1920

POJĘCIA CHEMICZNE I ROZWÓJ PRZEMYSŁU CHEMICZNEGO I NAFTOWEGO NA TERENACH POLSKICH W LATACH 1850–1920

Abstract

The information about: 1) the liquefaction of oxygen and nitrogen in 1883 in Kraków, 2) the formulation in Lwow of the hypothesis of vegetal origin of crude oil, 3) the discovery of chromatography in 1903 in Warsaw, is given. The situation of chemical industry in the three parts of Poland partitioned among Russia, Germany and Austria is reported. A special attention is paid to the activity of Ignacy Łukasiewicz, who received for the first time in the world the kerosene from the crude oil, constructed and lighted in Lwow pharmacy in March 1853 the kerosene lamp. In 1854 he excavated petroleum shaft in Bóbrka and in 1856 he built a petroleum refinery in Ulaszowice near Jasło, getting ahead of USA, where the first petroleum refinery at Oil Creek was built five years later, in 1861.

Keywords: *the chemical industry*, *the petroleum industry*, *Poland*, *1850–1920*

Streszczenie

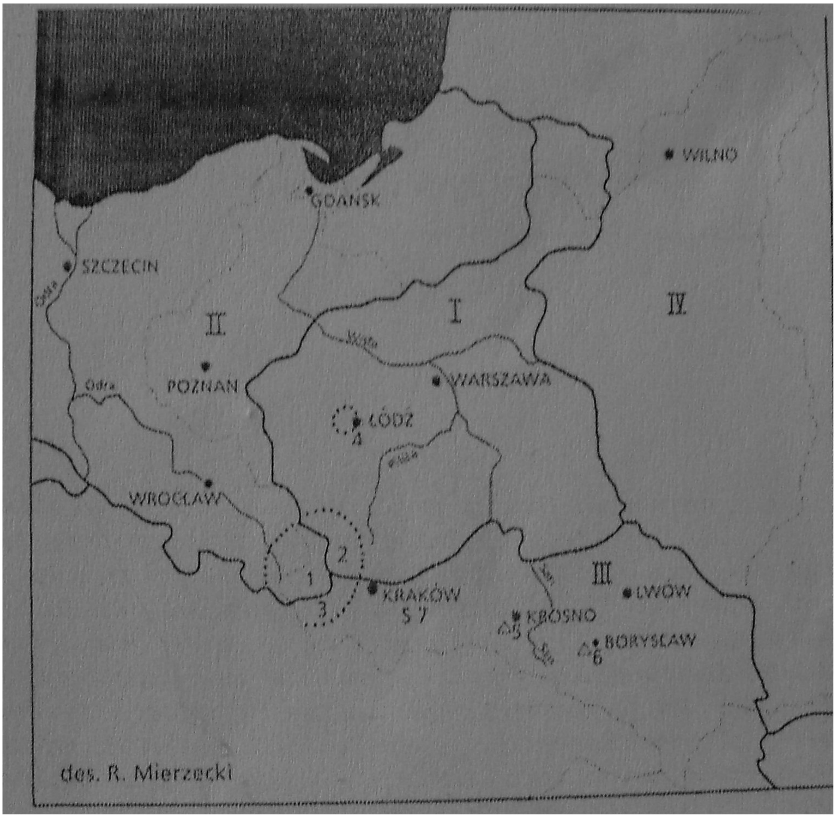
W artykule poinformowano o: 1) otrzymaniu w Krakowie w 1883 r. ciekłego tlenu i azotu, 2) sformułowaniu we Lwowie hipotezy roślinnego pochodzenia ropy naftowej, 3) odkryciu w 1903 r. w Warszawie chromatografii. Omówiono sytuację przemysłu chemicznego w trzech częściach Polski podzielonej między Rosję, Niemcy i Austrię. Szczególną uwagę zwrócono na działalność Ignacego Łukasiewicza, który otrzymał naftę z ropy naftowej, skonstruował i zapalił w marcu 1853 r. w lwowskiej aptece pierwszą w świecie lampę naftową; w 1854 r. wykopał szyb naftowy w Bóbrce, a w 1856 r. zbudował rafinerię ropy naftowej w Ulaszowicach, uprzedzając o pięć lat USA, gdzie pierwsza rafineria w Oil Creek została wybudowana w 1861 r.

Słowa kluczowe: *przemysł chemiczny*, *przemysł naftowy*, *Polska*, *1850–1920*

* Roman Mierzecki, Professor emeritus of the Chemical Department, University of Warsaw.

1. The historical background

In order to understand the situation of Polish science and industry during the second half of the nineteenth century and two first decades of the twentieth century some historical background is necessary. Poland and Lithuania constituted one entity in the eighteen century and this "Commonwealth of Both Nations" was the largest, although not the strongest, country in Europe. From an economic point of view the raw materials found in different part of this vast country were complimentary and formed a good basis for development of chemical industry. However the partitioning of Poland among Russia, Prussia and Austria



Legenda:

_________ Frontiers 1815–1924	1. High Silesia Basin
Frontiers 1923–1939	2. Dąbrowa Basin
	3. Kraków Basin
I Polish Congress Kingdom	4. Łódź Basin
II Duchy of Poznan	5. Krosno Petroleum Basin
III Galicia	6. Borysław Petroleum Basin
IV Russian Empire	7. Salt mines

Fig. 1. The Polish Territory after 1815

at the end of eighteenth century put an end to this unity. The three regions that resulted, fragmented and put apart by the state frontiers, constructed peripheral economic districts of the occupying empires. The three regions inhabited at that time by Poles were:

1) the so-called Polish Congress Kingdom with Warsaw as capital incorporated into the Russian Empire as “Vistula District”,
2) Grand Duchy of Posen (Poznań), East Pomerania and Upper Silesia in 1871 incorporated into the German Empire after an earlier annexation by Prussia,
3) the part annexed by the Austrian Empire, called Galicia, with the capital in Lwów (now Lviv in Ukraine), which after 1872 achieved some measure of autonomy and was governed by Poles (see Fig. 1)[1].

Poland regained independence in November 1918.

2. The Chemical Ideas

Only, in Galicia did Poles have a possibility to develop chemical ideas. In Iagellonian University in Kraków (Cracow) in 1883 the physicist Zygmunt Wróblewski (1845–1888) and the chemist Karol Olszewski (1846–1915) for the first time in the world liquefied oxygen and nitrogen in a stable form. In 1884 Olszewski also liquefied hydrogen, but only in a dynamic state. At the end of the nineteenth century the lowest temperature in the world (–263,9°C) was achieved in the chemical laboratory of the Cracow University. In 1895 Olszewski liquefied and solidified gaseous argon sent to him by William Ramsey, the discoverer of this element[2].

In eighties of the nineteenth century Bronisław Radziszewski (1838–1914), professor of the organic chemistry at the Lwów University developed the idea on the origin of the crude oil, which according to him was to be the result of the fermentation of vegetable remains[3]. He refuted Mendeleiev’s proposition that the crude oil originated from inorganic remains.

In 1903 Mikhail Semenovich Tswiet [Михаил Семенович Цвет] (1872–1919) professor of botany and agronomy at the Russian Imperial University in Warsaw invented chromatography, one of the most important method of chemical analysis.

In 1916 Ignacy Mościcki, at that time professor of the Lwow Polytechnic School, organized in Lwow a society called METAN to promote the chemical industry. It is active to-day in Warsaw as the Ignacy Mościcki Institute of Industrial Chemistry. Ignacy Mościcki was 1926-1939 the President of the Polish Republic.

In 1919 the Polish Chemical Society was organized to unite all people working in Poland with the chemical matters.

---

[1] Roman Mierzecki, *The Polish Chemical Industry; Isolated Developments in a Divided Country*, [in] Ernst Homburg, Anthony S. Travis, Harm G. Schröter, ed., *The Chemical Industry in Europe,1850–1914: Industrial Growth, Pollution, and Professionalization,* Kluwer Academic Publishers, 1998, p. 60.

[2] Zdzisław Wojtaszek, Halina Kuzyk, Alojzy Modrzyniec, Jerzy Dubowy, Krystyna Łopata, *Karol Olszewski*, PWN, Warszawa, Kraków 1990.

[3] Ignacy Z. Siemion, *Bronisław Radziszewski i lwowska szkoła chemii organicznej*, Wrocław 1999, 108-122.

3. The Chemical Industry

The development of chemical industry in each of the three segments of partitioned Poland was different. It was determined by the availability of raw materials in those regions and by political conditions. The invaders were not interested in stimulating the local industry; foreign companies were the owners of most raw materials and plants in Polish territories[4]. The Upper Silesia Basin the most industrialized was rich in coal (the largest deposits in Europe of coking coal) beside the iron ores, large quantities of the zinc ores: calamine and blende[5]. The development of this region was hindered due to proximity to frontiers[6]. Together with the neighboring Basins under Austrian and Russian rules (Cracow and Dąbrowa Basins) the Upper Silesia formed one of the largest industrial regions in Europe. In 1883 the first Thomas converter in Silesia was installed in Friedenshütte (now Huta Pokój). Large amounts of the fertilizer thomasine were produced as byproduct[7]. In nineteenth century the zinc production in Upper Silesia based on calamine and blende exceeded the production in all other regions of Europe[8]. Nevertheless, the chemical industry in Silesia remained underdeveloped, though tar distilling and preparation of raw materials for the dyestuffs industry became an important activity[9]. Not before 1916 did the war situation make it possible to construct there, in Königshütte (now Chorzów), a massive ammonia factory, where nitric acid and nitrogen fertilizers were produced from 1917 onwards. In 1919, after Chorzów had been assigned to Poland, the Germans dismantled much of the equipment and removed the construction plans. In 1922, Polish engineers led by professor Ignacy Mościcki restored the factory to full production[10].

The Duchy of Poznań was considered by the Germans an agricultural region, nevertheless they did not erect there any fertilizers plants. 80 per cent of fertilizers were imported from other parts of the German Empire[11]. By contrast, the sugar industry was substantial for this region. In 1882 a German company constructed a sugar mill in Chełmża (Pomerania), the largest in Europe at that time[12]. However the Poznań and Pomerania sugar factories produced mainly the raw sugar, which was sent for purification to central German countries[13]. In 1882 in Mątwy near Inowrocław the first on Polish territory factory for calcite and crystallized soda was built. In 1890 it produced 30 tons of soda daily[14].

---

4 J. Łukaszewicz, *Przewrót techniczny w przemyśle Królestwa Polskiego 1852–1896*, Warszawa 1963.

5 A. Mączak ed., *Encyklopedia historii gospodarczej Polski do roku 1945*, vol. II, Warszawa 1981, p. 130.

6 *Ibidem*, vol. II, 520.

7 *Ibidem*, vol. I, 297.

8 J. Dębicki, *Przemysł cynkowy*, Warszawa 1927.

9 L.F. Haber, *The Chemical Industry during the Nineteenth Century: A study of the Economic Aspects of applied Chemistry in Europe and North America*, Oxford 1971, 127.

10 E. Kwiatkowski, *W takim żyliśmy świecie*, Kraków 1990, 69-86.

11 A. Mączak ed. *op.cit*, vol. II, 470.

12 *Ibidem*, vol. I, 106.

13 J. Iwaszkiewicz, *Przemysł cukrowniczy w Polsce dawniej i dziś*, Warszawa 1938.

14 Florian Kruszka, Adam Wartalski, *Historia polskiego przemysłu nieorganicznego*, Warszawa 1996, 62.

The Congress Kingdom of Poland was from 1850 a part of Russian economic sphere when the custom frontier between the Kingdom and the rest of Russia was removed. Russian customs duties on raw materials and finished products were high, whereas those for intermediate products were low. Many foreign companies built then in the Kingdom their factories in which their intermediate products were converted into final products, exported without any tax to the whole Russian Empire. It was a reason of rapid progress of production in the Kingdom, greater than in other parts of the Empire[15]. In 1833 a French entrepreneur Philip de Girard constructed a linen factory, the largest in Europe, in a small town (named afterwards Żyrardów) 50 km south-west from Warsaw. The textile industry was developing also in the Łódź region, where in 1889 the first small dye-making factory was set up by Jan Smiechocki. Some years later with the help of Ignacy Hordliczka he increased the dye production, invented a process for fabrication of a sulphur black dye, and built a new much larger dyestuff plant in Zgierz. In the same year1889 in Pabianice near Łódź the firm Schweikert & Fröhlich built a substantial chemical and dye factory. In 1900 the dyestuff production in the Kingdom was as great as 2000 tons and it represented 28 percent of total Russian production[16]. In Warsaw medicines were prepared in several pharmaceutical factories and pharmacies[17]. The sugar industry was active in the Kingdom as well. In 1849–1850 some 33 sugar mills produced 3.500 tons of refined sugar from 50.000 tons of sugar-beets[18]. In 1871 37 sugar mills were active in which 11.654 workers were employed[19]. In 1870 Warsaw and Radom became the centers of the Polish tanning industry[20].

The sulphuric acid needed for dye production was produced from sulphur imported from Sicily. In 1909 some 13.200 tons of this acid were produced that formed 10 per cent of the whole Russian production[21]. In the Dąbrowa Basin the largest ironwork on Polish territory was established as early as 1834[22]. From 1878 local ores and those from other Russian regions were proceeded in open-hearth steel furnaces. At Ząbkowice an electrochemical factory built in 1898 produced chlorine from imported Galician salt. It was converted into chlorinated lime. In 1900 its amount 2.524 tons formed 21 per cent of the production of the whole Russian Empire. In the same electrochemical process, caustic soda was obtained[23].

Galicia was in the worst economical situation. Local raw materials such as sulphur layers in Swoszowice, and phosphate and potassium salts in eastern Galicia were exploited to a very small degree. Only in Cracow Basin the soda plant constructed in 1883

[15] E. Kwiatkowski, *Dzieje chemii i przemysłu chemicznego*, Warszawa 1962, 97.

[16] K. Smoleński, E. Trepka, *Historia przemysłu chemicznego i widoki jego rozwoju w Polsce*, Warszawa 1919; Die chemische Industrie, 1905, **28**, 375-408; *A History of Chemists, Companies, Products and Changes*, Manchester 1987, 79.

[17] I. Kikta, *Przemysł farmaceutyczny w Polsce 1829–1939*, Warszawa 1972.

[18] S.B. Książkiewicz, *Historia polskiego przemysłu chemicznego w latach 1815–1918*, Warszawa 1995, 44.

[19] Z. Przerębel, *Historia cukrownictwa*, Warszawa 1927.

[20] A. Mączak, *op. cit*., vol. I, 283.

[21] F. Kruszka, A. Wartalski, *op. cit*., 9.

[22] A. Mączak, *op. cit*., vol. I, 265.

[23] F. Kruszka, A. Wartalski, *op. cit*., 76.

in Szczakowa did produce 50 tons of calcined soda daily. An important raw material appeared to be the crude oil used for a long time for lubrication purposes.

4. Ignacy Łukasiewicz and the Oil Industry

The crude oil was in the second half of the nineteenth century the greatest raw material resource in the Carpatian region of the Polish territory. 16 liters of petroleum were daily drawn from 30 wells in Borysław[24]. In 1837 in Lwów two dispensing chemists, Joseph Schopf and Gabriel Muling, heated in a retort a mixture of dense and light crude oils and obtained some quantities of gaseous hydrocarbons. These were conveyed by pipes into different parts of their pharmacy and domestic rooms and used for illumination. This cracking process, the first in the world, was rather tedious and found no imitators[25].

Fig. 2. Ignacy Łukasiewicz (1822–1882) painted by Andrzej Grabowski, 1884 (source: chomikuj.pl/Lukasowi/Galeria/Prezentacje+Word.Ignacy+*c5*8lukasiewicz,122899607.docx)

Very important was another attempt at crude oil utilization in 1853. It led to the development of the petroleum industry in south Poland and else where. The individual responsible for this attempt was Ignacy Łukasiewicz (1822–1882) born near Mielec in northern part of Galicia[26]. As a young man he took part in the Polish liberation movement and spent 1847–1849 in an Austrian jail. Such difficult conditions prevented him from undertaking regular studies in science. He began his work in an pharmacy in Rzeszów as an assistant and after for

[24] A. Mączak, *op. cit*., vol. I, 48.

[25] Z. Ruziewicz, *Początki przerobu galicyjskiej ropy naftowej i pierwsze gazowe oświetlenie we Lwowie*, Wiadomości Chemiczne, 1993, **47**, 681-695.

[26] W. Kisielow, *Chemia i technologia chemiczna ropy naftowej w Polsce 1816–1988*, Gliwice 1992.

a brief period of study of mineralogy at the Iagiellonian University in Cracow he qualified in 1852 at Vienna University with a thesis on *Baryta and Anilinum* under the professor of chemistry Joseph Redtenbacher. In the same year he began to work in the large pharmacy "Pod Gwiazdą" (Under the Star) of Piotr Mikolasch in Lwów. There he became interested in petroleum, having some knowledge about this substance from the lectures on mineralogy in Cracow. In 1852 he was asked by the two merchants from Borysław to thicken the crude oil to improve its lubricating properties. Applying the method of fractional distillation, he observed one fraction of a clear liquid. It was kerosene, not known at this time. The merchants wondered if vodka could be extracted from this fraction. Łukasiewicz tried to used kerosene in oil lamps, but it caused an explosion. To overcome this, a tinsmith, Bratkowski, and Łukasiewicz constructed a new lamp with a porous wick, a mica chimney, and air entrance from the bottom (Fig. 3)[27]. This new lamp operated safely when filled with kerosene. The first kerosene lamp lighted in March 1853 in the window of Piotr Mikolasch' pharmacy at the Large Street (now Kopernik Street) in Lwów. Piotr Mikolasch, Ignacy Łukasiewicz and their collaborator Jan Zeh organized a company for production of kerosene.

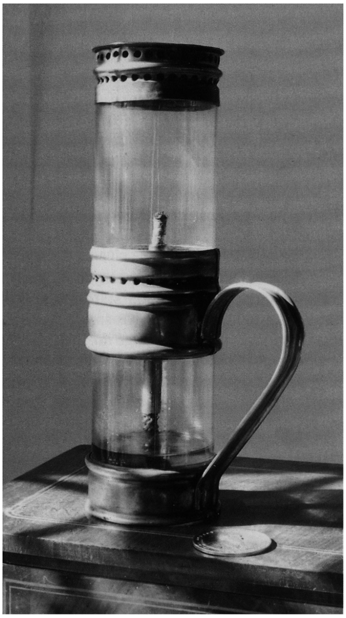


Fig. 3. The reconstruction of the first kerosene lamp (1853) after proposition of I. Łukasiewicz and A. Bratkowski

On the night of 31 July the surgeon Zaorski operated on Władysław Cholecki in a Lwów hospital under light of kerosene lamps. This date is considered as the begin of the world petroleum industry (In USA 29 August 1859 is considered to be such a date, as on this date

[27] *Lampy naftowe ze zbiorów Muzeum Okręgowego w Krośnie i Muzeum-Skansenu Przemysłu Naftowego im. Ignacego Łuksiewicza w Bóbrce*, Krosno 1996, 5.

E.L. Drake sank a 22-meter petroleum well by means of a steam-engine in the farm Willard near Titusville in Pennsylvania). Soon after, the Lwów hospital purchased 500 kg of kerosene for illumination. On the 2-nd December 1853 a patent for "the discovery that the crude oil purified in the chemical way is adapted for the immediate use for technical purposes" was awarded by the Austrian Patent Office to Łukasiewicz and Zeh[28] The Łukasiewicz's kerosene lamps were used on the Austrian Northern Railway. Łukasiewicz's kerosene lamps preceded the American construction of B. Silliman by two years.

Since 1853 Łukasiewicz sought out petroleum reserves in the Carpatian region. In 1854 he drove the first in the world petroleum shaft in Bóbrka near Krosno (now a Museum of Petroleum Industry). In 1856 he built in Ulaszowice near Jasło the first in the world petroleum refinery. The first American petroleum refinery at Oil Creek, Pennsylvania was built five years later, in 1861. In 1862 Łukasiewicz came into contact with American crude oil manufacturers. He adopted their well-sinking method invented in 1859 by E.L. Drake and constructed some new refineries, the largest in 1865 at Chorkówka. The representatives of US-based Standard Oil Company visited Łukasiewicz, studied his constructions, and tried to find in Łukasiewicz's refineries methods to avoid their own difficulties. They offered Łukasiewicz an 20 per cent profit agreement. Łukasiewicz rejected their offer, declaring: 'I have enough of my own money'. Without charge he supplied with kerosene the religious organizations of different creeds and was active in several industry-based social organizations. Łukasiewicz was also active on the social field, he was elected to local self-government organizations and to the Galician Parliament as well.

In 1909, all Carpatians petroleum wells produced 2.700.000 tons of oil; this was equal to 5.22 per cent of the world production of petroleum. After 1909 the production decreased, because of the exhaustion of some oil wells.

[28] Kwaśnicki.prawo.uni.wroc.pl/pliki/Drozen%20o%20Lukasiewicz.pdf

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE
1-NP/2014

VLASTA MORAVCOVÁ*

HISTORY OF DESCRIPTIVE GEOMETRY WITH AN EMPHASIS TO THE BOOM OF DESCRIPTIVE GEOMETRY IN AUSTRO-HUNGARIAN EMPIRE IN THE 19TH CENTURY

HISTORIA GEOMETRII WYKREŚLNEJ ZE SZCZEGÓLNYM UWZGLĘDNIENIEM JEJ ROZKWITU W MONARCHII AUSTRO-WĘGIERSKIEJ

A b s t r a c t

The article shows the development of projection in the Antiquity, the origin of perspective in Renaissance and the development of orthogonal projection from the 16th up to the 18th century before descriptive geometry as a separate discipline of studies was established by Gaspard Monge. Furthermore, the paper presents the expansion of descriptive geometry through Europe in the 19th century with the emphasis on its bloom in the second half of the 19th century in Cisleithania.

Keywords: *descriptive geometry*, *orthogonal projection*, *perspective*, *Cavalier projection*, *stereotomy*, *Gaspard Monge*, *polytechnics in Cisleithania*

S t r e s z c z e n i e

W artykule przedstawione zostały techniki rzutowania w starożytności, początki perspektywy w czasach renesansu oraz rozwój technik rzutowania prostokątnego od XVI do XVIII w., zanim Gaspard Monge uczynił geometrię wykreślną jako osobną naukę. Ponadto w artykule przedstawiono rozprzestrzenianie się geometrii wykreślnej w Europie w XIX w., ze szczególną uwagą na jej rozkwit w drugiej połowie XIX w. w Przedlitawii.

Słowa kluczowe: *geometria wykreślna*, *rzutowanie prostokątne*, *perspektywa*, *rzut kawaleryjski*, *stereometria*, *Gaspard Monge*, *politechniki w Przedlitwaii*

* Vlasta Moravcová, Department of Mathematics Education, Faculty of Mathematics and Physics, Charles university in Prague and Gymnázium Na Pražačce, Praha.

1. Introduction

Descriptive geometry as a science was formed in the end of the 18th century in France by Gaspard Monge, but various methods of projection were used long before. The development of the projection methods was interesting and very important for the formation of descriptive geometry, which is needed for the work of project architects, builders but also for some doctors and other professions.

2. Use of projection in the Antiquity

2.1. Orthogonal projection

We have only a little extant evidence of the use of projection in the ancient times. Orthogonal projection was used mainly in architecture. The top orthogonal views of buildings or temples were carved into stones just like the front orthogonal views of sculptures or columns. In Fig. 1 we can see a papyrus from about 330–390 BC with two orthogonal projections of an Egyptian sphinx. In Fig. 2 there is a column from the Philae island[1]. This column was carved into a stone about 150 BC at full proportion [4]. In these figures there are always two views, but only one view was often used in this period.

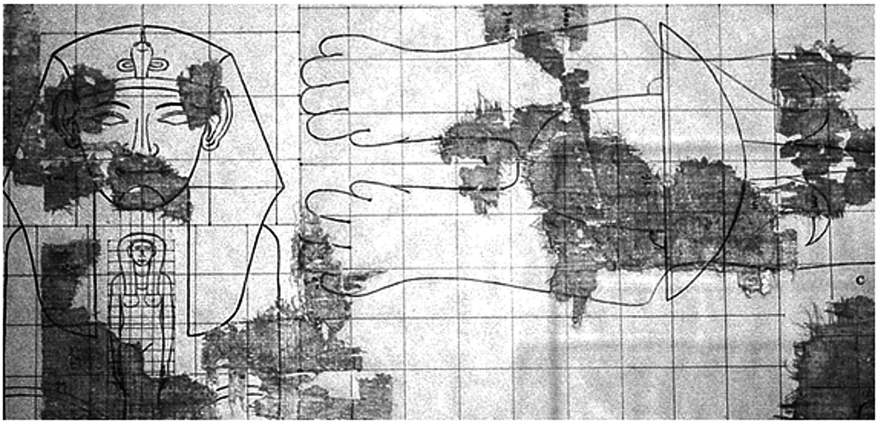


Fig. 1. Papyrus with orthogonal projections of a sphinx [4]

The oldest extant known written comment on the use of projection is in the work *De architectura libri decem* [9] by Marcus Vitruvius Pollio (Fig. 3)[2]. He described the three ways of projections – a top orthogonal view (ichnographia), a front orthogonal view (orthographia) and a view similar to perspective (scenographia). He believed that the views are created on the basis of experience and our vision.

---

[1] The Philae island was an island in the river Nile (Egypt). Now is the island at the bottom of the Aswan dam.

[2] Marcus Vitruvius Pollio was a Roman architect. He lived in the first century BC.

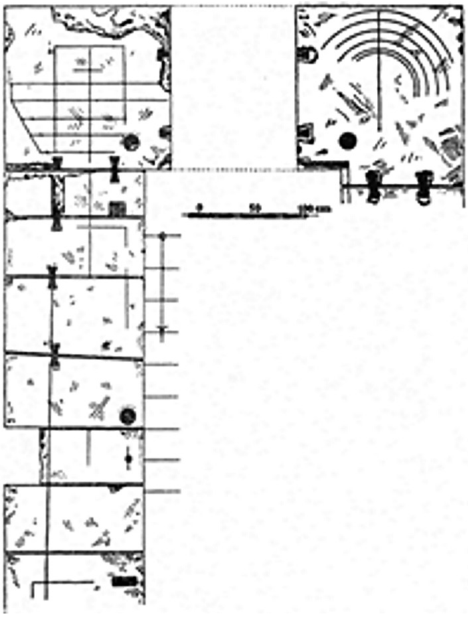


Fig. 2. Column carved into a stone on the Philae island [4]

M. VITRVVII POLLIONIS
DE ARCHITECTVRA
LIBRI DECEM,
CVM COMMENTARIIS
DANIELIS BARBARI,
ELECTI PATRIARCHAE
AQVILEIENSIS:
MVLTIS AEDIFICIORVM, HOROLOGIORVM,
ET MACHINARVM DESCRIPTIONIBVS,
CVM PRIVILEGIIS.
VENETIIS,
Apud Franciscum Franciscium Senensem, & Ioan. Crugher Germanum.
M. D. LXVII.

Fig. 3. Title page of Vitruvius's work [9]

2.2. Attempts at perspective

We cannot talk about the use of perspective in the Antiquity. On the other hand, in some wall paintings (Fig. 4) and frescos from this period we can see some attempts at perspective. No rules of perspective are observed; there are many vanishing points for one direction of lines and other mistakes. The characteristics of perspective were used accidentally with purpose to make the illusion of space more real.

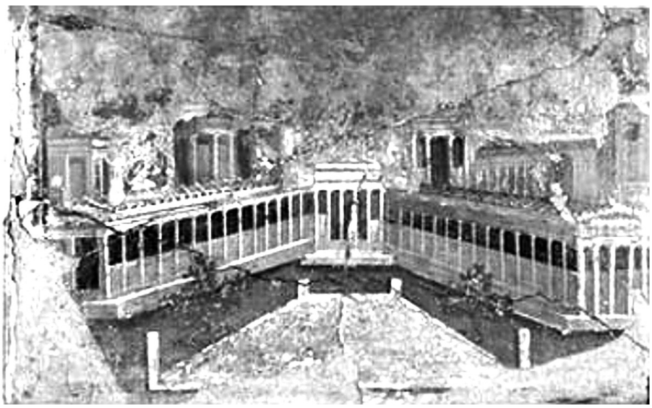


Fig. 4. Wall painting from the ancient city Pompeii created before 79 AD

3. Period of the Late Middle Ages and perspective in Renaissance

The long period of Middle Ages is difficult to investigate. We have few materials from the time before the 13th century to research the development of projection. The most of extant interesting drawings with elements of orthogonal projection were created between 1200 and 1500. The period of Renaissance was the main age of the development and the improvement of perspective because of the works of Renaissance artists.

3.1. Orthogonal projection

Temples or towers were a main topic of drawings in orthogonal projection. Typical feature was the use of arbitrary connection of the top and the front orthogonal view in one picture (Fig. 5)[3], but these two views were not always parts of one object. For example in the plan of the St. Vitus Cathedral in Prague (Fig. 6) by Peter Parler[4] we can see the front orthogonal view of the supporting system and the top orthogonal view of some temple tower.

Fig. 5. Sketch drawn by Villard de Honnecourt in about 1250

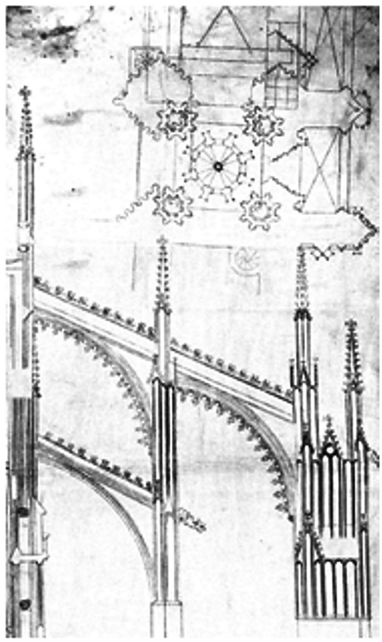


Fig. 6. Plan of the St. Vitus Cathedral from the 2nd half of the 14th centrury

[3] A sketch in Fig. 5 is from the Honnecourt's sketchbook, which is available on <http://classes.bnf.fr/villard/feuillet>.

[4] The plans of the St. Vitus cathedral are deposited in the Library of the Viennese Academy of Fine Arts.

3.2. Witelo‘s work on geometry

The first important Polish geometer was Witelo[5]. He wrote a magnific work *Perspectiva*[6]. This work has ten books. Only the first of them, which contains sixteen definitions, five postulates and 137 propositions, has mathematical character. The book is structured upon the Euclid’s *Element*. In the other books Witelo used the mathematics rules stated in the first one. The main topic of Witelo‘s work was optics, but hand in hand with optics he described many geometric rules connected with central projection[7].

3.3. Improvement of perspective

In the 14th century painters still used perspective in their paintings intuitively, as we can see in a painting by Giotto[8] (Fig. 7). However, many artists made effort to use and describe rules of perspective. Their aim was real illustration of space, as good as possible.

Fig. 7. Intuitive use of perspective in Giotto’s painting from*Scenes from the life of Saint Francis* situated in the Bardi chapel in Florence

During the 15th century Renaissance artists used the correct principles of perspective more frequently[9]. Some famous artists were Filippo Brunelleschi (1377–1446), Piero della Francesca (about 1415–1492), Masaccio (1401–1428), Paolo Ucello (1397–1475) or Leon Battista Alberti (1404–1472). Flawless perspective can be found e.g. in the sketch to the painting *Adoration of the Magi* by Leonardo da Vinci[10] (Fig. 8) or in some paintings by Raffaello Sanzio (1483–1520).

[5] Witelo (about 1230–1280) was born probably in Legnica or Wroclaw (or in some surrounding village). He was a son of Turin’s colonist and Polish women, because he called himself “filius Thuringorum et Polonorum”. About his life we have only little information.

[6] This work was written between 1270–1278. It was firstly published in 1572 in Basel.

[7] For more about Witelo and his work see [12] or [16].

[8] Ambrogio di Bondone (about 1266–1337), known simply as Giotto, was an Italian painter and architect, one of predecessors of the Italian Renaissance.

[9] For more about the use of perspective in art see [3].

[10] Leonardo da Vinci (1452–1519) was an Italian Renaissance painter, architect, inventor, anatomist and writer.

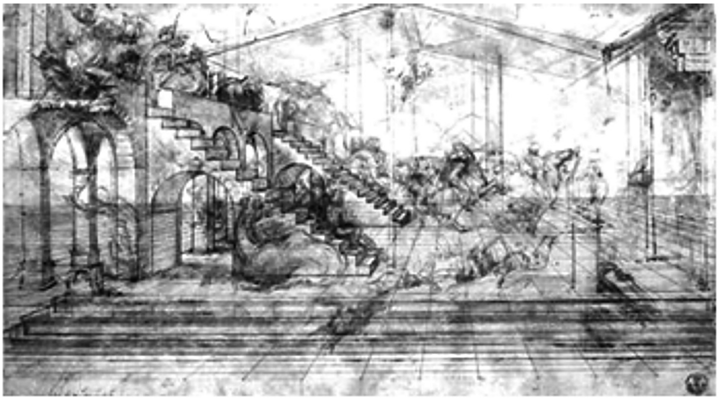


Fig. 8. Sketch to the painting by Leonardo da Vinci

In 1600 Guidobaldo del Monte[11] formulated the basic theorem of perspective, the projections of parallel lines meet at one point (the vanishing point of the parallel lines), in his work *Perspectivae libri sex* [7] (Fig. 9).

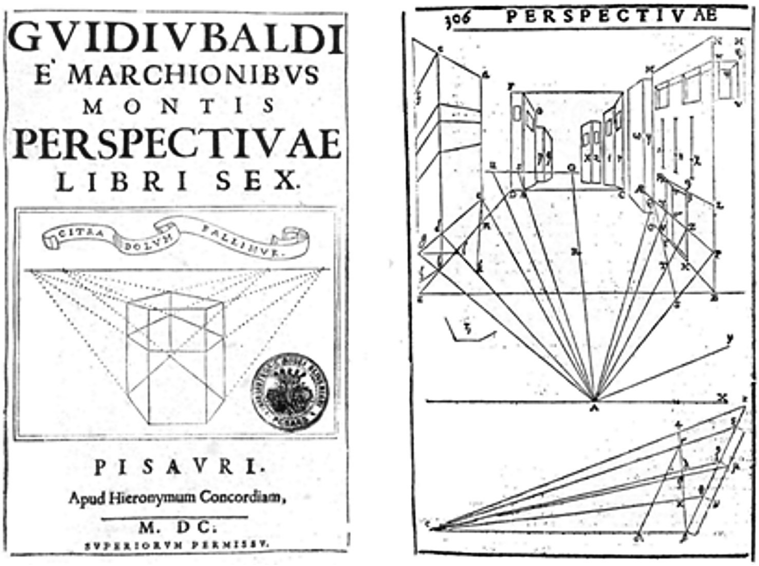


Fig. 9. Title page and one page from the work about perspective by Guidobaldo del Monte

Thanks to Renaissance artists the theory of perspective was concluded at the beginning of the 17th century. Of course, in the subsequent works on perspective constructions were improved and simplified.

[11] Guidobaldo del Monte (1545–1607) was an Italian painter, philosopher and astronomer.

4. Use of parallel projection from the 16th to the 18th century

It is typical for drawings created in the 16th century and later to be drawn according to rules of projections, and they are mainly unequivocal. Moreover, some works created in this period included not only correct illustrations, but also general rules of parallel projection. Therefore, we can call their authors the predecessors of Gaspard Monge.

4.1. Dürer's work on geometry

Some illustrations which look like constructed in Monge‘s projection can be found in *Underweysung der Messung mit dem Zirckel und Richtscheyt in Linien, Ebenen und gantzen Corporen*[12] by Albrecht Dürer[13]. He used orthogonal projection for construction of a cube in five different concrete positions in space (Fig. 10). Dürer started with one simple position (A) and then, using the rotation and symmetry in space, he derived the other positions of the cube (B–E) from the first one.

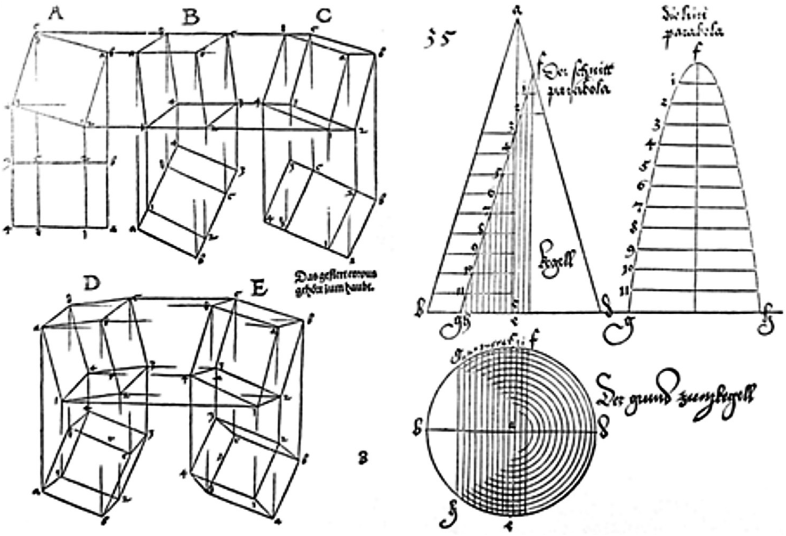


Fig. 10. Dürer's construction of different positions of a cube and construction of parabola

Moreover, Dürer used orthogonal projection and sections of a cone for construction of ellipse, parabola (Fig. 10) and hyperbola. He always chose suitable plane of section and constructed the top and the front view of the section 'point after point'. In this way he obtained

[12] This book was the first big book on geometry in German. It was published in two editions – 1525 and 1538 in Nürnberg. The text is available on <http://digital.slub-dresden.de/werkansicht/dlf/17139/1/cache.off>.

[13] Albrecht Dürer (1471–1528) was a German painter and a graphic artist with interest in mathematics, especially geometry. He researched the use of geometry in art; he was primarily interested in the theory of proportions of human body.

enough information for constructing the conic in real size and he presented the construction on the right side of each page.

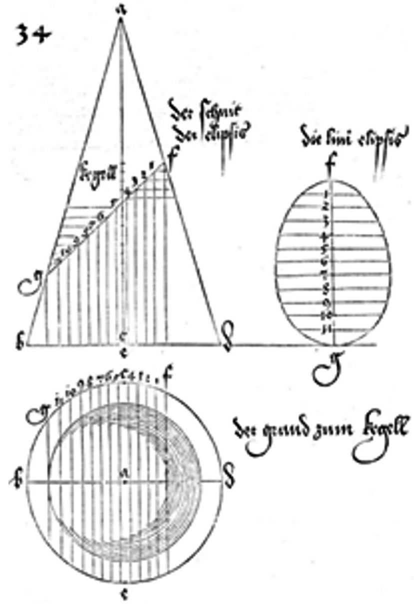


Fig. 11. Mistake in Dürer's construction of ellipse

Dürer's method of the construction of conics is correct, but in his drawing of ellipse he made one mistake. He was probably convinced that the section should be an oval curve with only one axis of symmetry (like an egg) and he constructed it so inaccurately that he really obtained what he expected (Fig. 11).

4.2. Cavalier perspective

For an easy projection of forts (Fig. 13), city plans (Fig. 14), etc. a new method of projection, called Cavalier projection[14], emerged about 1600. It is a special type of oblique projection in which the top view of a building is not distorted in the same way as the height of the building, because the plane of projection is chosen parallel with the ground of the building and the direction of projection is 45° (Fig. 12).

[14] Cavalier projection is sometimes called Cavalier perspective or military perspective. But these names do not refer to perspective as a kind of central projection.

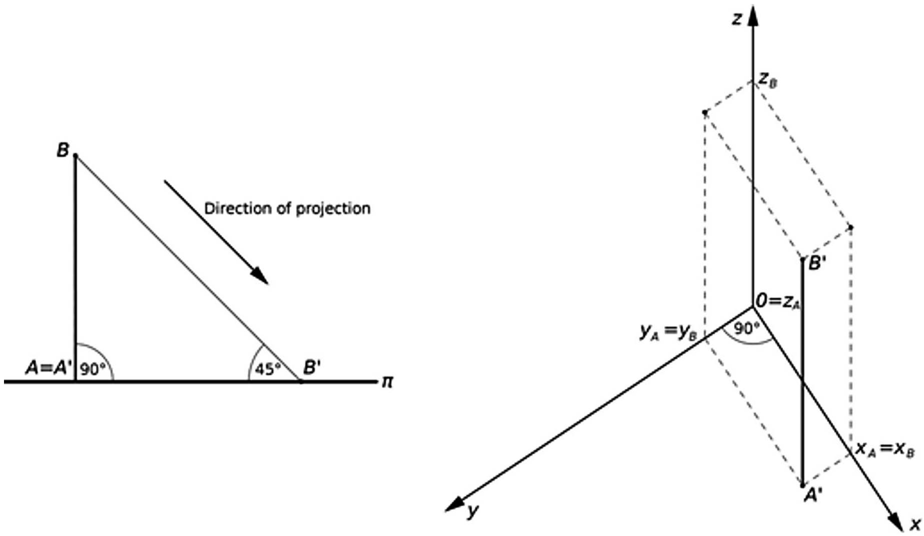


Fig. 12 Principle of Cavalier projection

Fig. 13. Plan of a fort by Jacques Perret from 1602 [4]

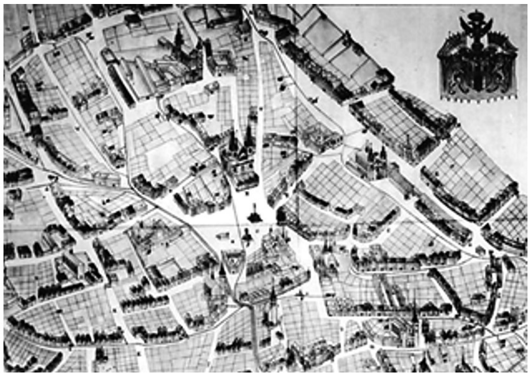


Fig. 14. Plan of a water pipeline by J.V. Veselý from 1720 [14]

4.3. Stereotomy

In essence, Dürer used Monge's projection. Similarly, in many drawings from the period between 16th and 18th century we can find the top and the front view like in Monge's projection (Fig. 15), but these illustrations have common characteristic – they were used only for unequivocal projection of some space object into plane, not for solving of some space problem in plane.

Attempts to generalize the rules of orthogonal projection and use it for solving of space problems originated in works on stereotomy (the theory about cutting stone and wood). The first works on this topic were *Le premier tome de l'architecture* (Paris, 1567) by Philibert de l'Orme[15], *Les secrets de l'architecture* (La Flèche, 1642) by Mathurin Jousse[16] (Fig. 16) or *Brouillon project d'une exemple d'une maniere universelle du S. G. D. L. touchant la pratique du trait à preuves pour la coupe des pierres en l'architecture* (Paris, 1640) by Girard Desargues[17].

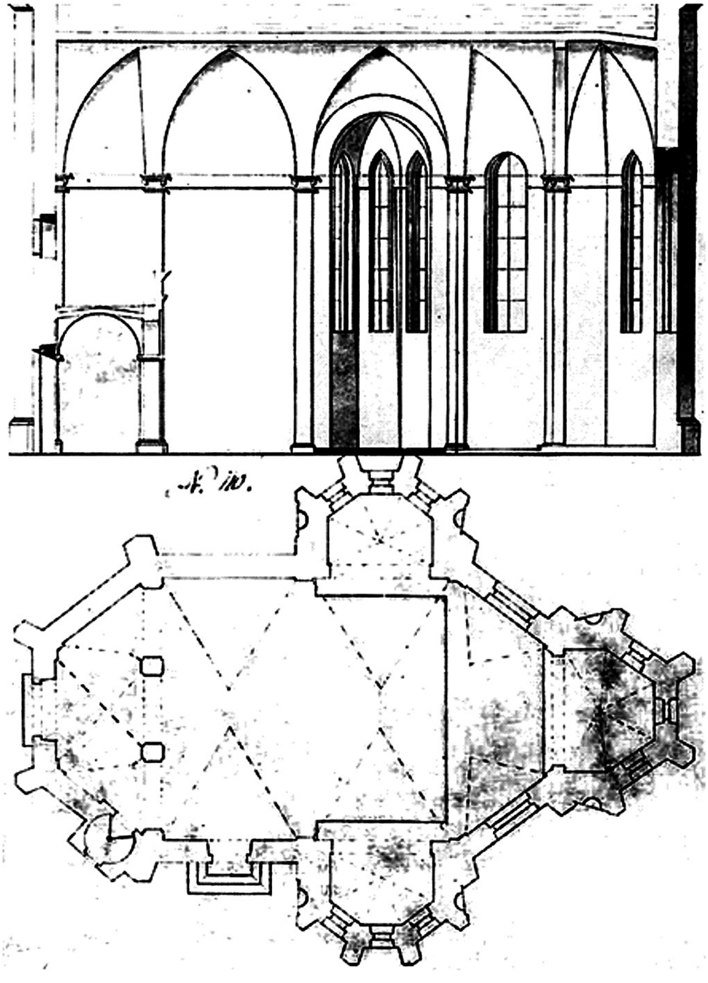


Fig. 15. Drawing of the St. Roch church in Prague from the 1740s [13]

[15] Philibert de l'Orme (about 1514–1570) was a French architect. His first significant work in architecture was the design of the castle *Château de Saint-Maur* in Paris.

[16] Mathurin Jousse (1575–1645) was a French architect. He was the author of many works on building trades (joinery, carpentry, locksmithery, etc).

[17] Girard Desargues Lyonnais (1591–1661) was a French mathematician, architect and engineer. He was interested in perspective, stereotomy and conics. He introduced improper elements and polarity into geometry. He was one of founders of projective geometry.

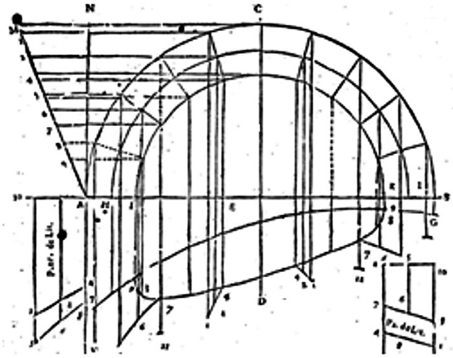


Fig. 16. Orthogonal projection of correct laying of stones under an oval entrance from Jousse's work

LA THEORIE ET LA PRATIQUE
DE LA
COUPE DES PIERRES
ET DES BOIS
POUR LA CONSTRUCTION DES VOUTES
OU
TRAITÉ DE STEREOTOMIE
A L'USAGE DE L'ARCHITECTURE.
TOME SECOND
A STRASBOURG
A PARIS,
M DCC XXXVIII.

Fig. 17. Title page of Frézier's work

A significant work on stereotomy was *La théorie et la pratique de la coupe des pierres et des bois, pour la construction des voutes et autre parties dés bâtimens civils at militaires, ou Traité de stéréotomie à l'usage de l'architecture* (Strasbourg, 1737) (Fig. 17) by Amédée François Frézier[18]. In the first part Frézier described the general principles of orthogonal projection. In the other parts he introduced the applications of projection in stereotomy. This work was probably the most important work on projection before the publication of the Monge's work on descriptive geometry.

[18] Amédée François Frézier (1682–1773) was a French officer, engineer and mathematician. In 1752 he became a member of the French Academy of Sciences.

5. Gaspard Monge and his *Géométrie descriptive*

Gaspard Monge[19] taught a new method of geometry in which he used construction instead of plaster models in Mézières from 1766. However, this teaching was forbidden because of the secrecy of the new method of geometry for military purposes.

After 1794 (during the French Revolution) political situation in France was changed and Monge could teach his new method of geometry, which he called descriptive geometry, at École Normale and École Polytechnique in Paris.

In the school year 1794/1795 Monge published the first edition of his work on descriptive geometry with title *Textes des leçons de géométrie descriptive données à l'École Normale* in the school journal *Séances des Écoles Normales*. In this work descriptive geometry was firstly conceived as a science, therefore Monge is usually called the founder of descriptive geometry.

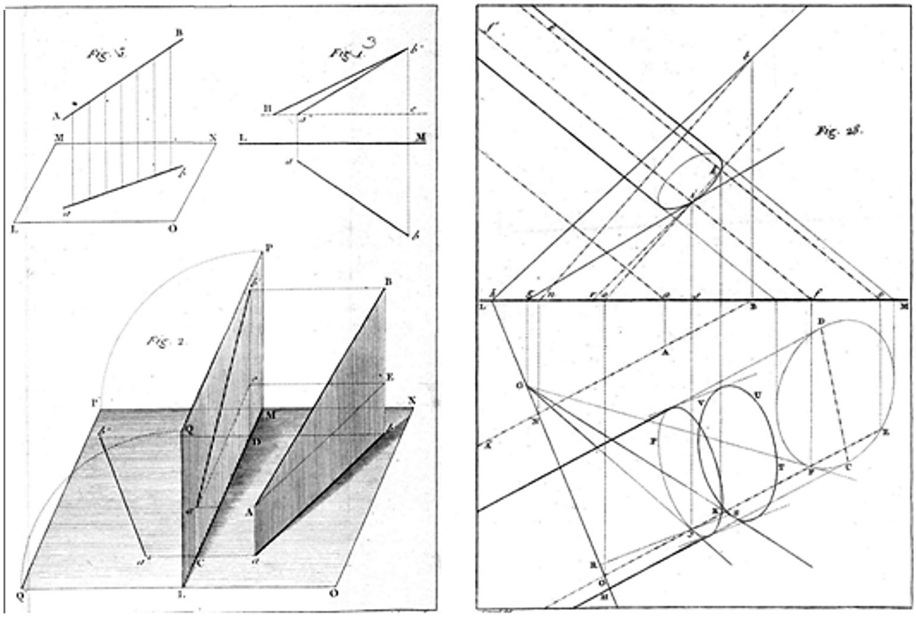


Fig. 18. Illustrations from Monge's work *Géométrie descriptive*

In the 1799 the second edition of Monge's work was published as a book with the title *Géométrie descriptive. Leçons donées aux Écoles Normales, l'an 3 de la Republique* (Fig. 18). The book has five parts. Three of them are about theory and general methods of descriptive

[19] Gaspard Monge (1746–1818) was a French mathematician and physicist. When he was twenty two years old, he became a professor of mathematics at school in Mézières. During the French Revolution he was called at École Normale. After establishment of École Polytechnique Monge was suggested as the president of this school, he refused this position, but taught stereotomy, descriptive geometry and physics here till 1809. For more information about Monge's work and life see [6, 8, 11].

geometry; in the fourth part there are applications of these methods for sections of curved surfaces and in the last part there is theory of curves and surfaces with use of differential geometry.

Monge's work on descriptive geometry was published repeatedly in French. Starting from the fourth edition (1820), it was supplemented with parts *Théorie des ombres* and *Théorie de la perspective* by Barnabé Brisson[20].

5.1. Translations of Géométrie descriptive

Monge's *Géométrie descriptive* was translated into many languages. The book was published in Spanish (1803), English (1809 and 1851), Italian (1838), German (1900) and in Russian (1947). At present, reprints of these editions are generally available[21].

5.2. Monge's students and successors

Other important works on descriptive geometry were published in the first half of the 19th century by Monge's students S.F. Lacroix[22] and J.N.P. Hachette[23]. Another important professor of descriptive geometry at École Polytechnique was Charles François Antoinne Leroy (1780–1854). He wrote *Traité de géométrie descriptive* (Paris, 1834). This work was published fifteen times before 1910 and was translated into German.

All these works (together with Monge's work) and some others influenced the development of descriptive geometry in other countries in Europe.

6. Boom of descriptive geometry in Cisleithania

The development of industry in Europe in the first half of the 19th century caused the development of technical sciences and education during the 19th century. Descriptive geometry gradually began one of obligatory subjects of technical studies. The biggest boom of descriptive geometry came (except of France) in Germany[24], Italy[25], Great Britain[26] and in Austro-Hungarian Empire, primarily in Cisleithania.

[20] Barnabé Brisson (1777–1828) was one of Monge's students at École Polytechnique. After completing his studies he became a building ingeneer, in practice he applied descriptive geometry to building of navigation channels. In 1808 he married Monge's niece Anne-Constance Huart de l'Enclose.

[21] Many reprints are available e.g. on <www.amazon.com>.

[22] Sylvestre François Lacroix (1765–1843) was a French mathematician. From 1794 he helped his teacher Monge with preparing materials for lessons on descriptive geometry. He wrote a work *Essai d'géométrie sur les plans et les surfaces courbes* (Paris, 1795), which was published repeatedly with the title *Complément des élémens de géométrie.*

[23] Jean Nicolas Pierre Hachette (1769–1834) was a French mathematician. He became a successor to Monge at École Normale. He extended the Monge's work on descriptive geometry with two addenda *Suppléments à la Géométrie descriptive de Monge* (1811, 1818).

[24] Among known German geometers we can mention Karl-Wilhelm Pohlke (1810–1876), Guido Schreiber (1799–1871), Bernhard Gugler (1812–1880) or Christian Wiener (1826–1896).

[25] Among known Italian geometers we can mention Vincenzo Flauti (1782–1863), Giusto Bellavitis (1803–1880) or Gino Benedetto Loria (1862–1954).

[26] The axonometric projection has an origin in Great Britain. Regarding this we can mention William Farish (1759–1837) or Peter Nicholson (1765–1844); for more see [5].

In the second half of the 19th century and at the beginning of the 20th century new works and textbooks on descriptive geometry were written and methods of projections (mainly axonometric projections, theory of shadows or photogrammetry) were improved.

6.1. Descriptive geometry teaching at secondary schools

In 1849 (Exner-Bonitz reform of secondary schools in Cisleithania) a new modern type of secondary schools with emphasis on natural sciences and modern languages was established and was called ‘Realschule’[27]. At these schools descriptive geometry has been taught since the 1850s, because students of Realschules often continued their studies at polytechnics and therefore they needed to have knowledge of descriptive geometry.

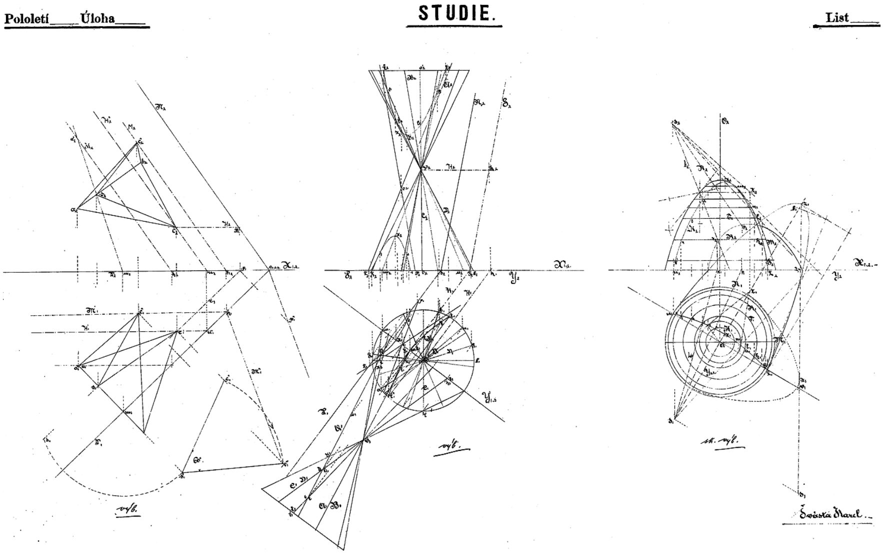


Fig. 19. School-leaving exam work from Czech Realschule in Hradec Králové worked out by Karel Švásta in 1877

The school-leaving exam in descriptive geometry was obligatory at Realschules since the 1870s. Students wrote a five-hour test with three or four exercises. They had to construct them in ink (Fig. 19)[28] and describe the process of the solution. In comparison with the present time the exams were very difficult. Current students of descriptive geometry at universities would probably have problems with similar exercises.

---

[27] We will use the term ‘Realschule’ (from German) as singular and ‘Realschules’ as plural.

[28] In the Figure 19 there are drawings of this exercises [15]:

1. Orthogonal projections of a triangle *abc* are given; rotate it on the side *ab* about 60°.
2. Draw a hyperbolic section of a cone surface of revolution with a directing circle in the first plane of projection. The plane of section is in general position.
3. Draw the shadow of a parabolic solid whose axis is perpendicular to the first plane of projection. The rays come out from one point.

For secondary school students new textbooks and collections of exercises on descriptive geometry were also written. One of them, the Czech book *Deskriptivní geometrie pro střední školy reálné* (Prague, the first edition: 1875–1877, the other editions: 1887, 1893, 1900, 1905) by Vincenc Jarolímek[29] was translated into Bulgarian. Bulgarian edition was published in 1895 in Plovdiv (Fig. 20). This action was only one of many attempts to spread descriptive geometry to countries of Eastern Europe[30].

6.2. Lectures on descriptive geometry at polytechnics and universities

Descriptive geometry has been taught at polytechnics in Cisleithania[31] from the first half of the 19th century (e.g. at Prague Polytechnic School since the 1830s). At some of them the professorship of descriptive geometry was established (e.g. in Vienna in 1842, in Prague and Brno in 1850, in Graz in 1861)[32].

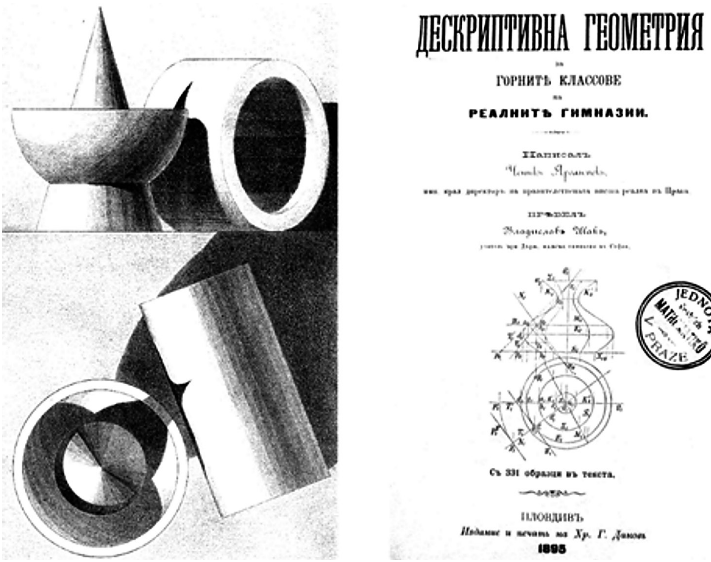


Fig. 20. Illustration from the first edition of Jarolimek’s secondary textbook and the title page from Jarolimek’s textbook translated into Bulgarian

---

29 Vincenc Jarolímek (1846–1921) was a teacher of mathematics and descriptive geometry at Realschules. In 1907 he became a professor of descriptive geometry at Prague Polytechnic School. For more about the first Czech textbooks on descriptive geometry see [2].

30 Regarding influence of Czech mathematics in Bulgaria see [1].

31 Polytechnics (Fig. 21) were established in Prague (1806), Graz (1811), Vienna (1815), Brno (1850) and Lemberk (now Lviv, 1871). Similar schools were established also in other countries of Europe, the firsts of them were in Italy (Neapol, Roma), Germany (Berlin, Karlsruhe, München, Dresden), Great Britain (London), etc. All polytechnics were established following the model of École Polytechnique.

32 For more about systemization of descriptive geometry professorships at technical schools in Cisleithania see [10].

A syllabus of descriptive geometry at Prague Polytechnic in 1852 included various kinds of methods of projections (orthogonal, oblique, perspective), theory of curves and surfaces and other topics. The lectures were provided by Rudolf Skuherský[33], who lectured according to Hönig's[34] work and his own works. Number of lessons a week was high – about 12 hours. The lectures were obligatory for students of building constructions at first, later for students of engineering, architecture, forestry and other areas of study as well.

At the other polytechnics the syllabi of descriptive geometry were similar. Sometimes extra lectures on projective geometry, perspective or stereotomy were provided. The quality of lectures depended mainly on the lecturer, but we can say that it was generally very high. Besides Hönig's textbook the above-mentioned work by F.A. Leroy was often used. In the second half of the 19th century and in the first third of the 20th century new scientific works and textbooks on descriptive geometry for polytechnic students were published. Students also prepared litographed notes of some lectures.

Fig. 21. Map of polytechnics in Cisleithania

A growing number of polytechnic students caused growing necessity for teachers of descriptive geometry not only at polytechnics but also at secondary schools. These teachers graduated from polytechnics at first, but studying at a university was a better way of preparation for teaching career. For example, the future teachers of descriptive geometry

[33] Rudolf Skuherský (1828–1863) studied at Prague Polytechnic School and at Viennese Polytechnic School. In Vienna he was the student of the professor Johann Hönig. In 1854 he became the first professor of descriptive geometry at Prague Polytechnic School.

[34] Johan Hönig (1810–1886) was a professor of descriptive geometry at Viennese Polytechnic School between 1843–1870. He wrote a textbook on descriptive geometry *Anleitung zum Studium der darstellenden Geometrie* (Wien, 1845). This book was used by students of polytechnics in Cisleithania for many years.

could study this subject at Czech University in Prague from the 1910s courtesy of Jan Sobotka. At German University in Prague students could attend periodical lectures on descriptive geometry only a few years later.

6.3. Next personalities of descriptive geometry in Cisleithania

Regarding the textbooks and lectures on descriptive geometry we mentioned only a few personalities who made descriptive geometry in the Central Europe famous. But there were many other persons who contributed to the development of descriptive geometry. Let us recall their names at least: František Tilšer (1825–1913), Čeněk Hausmann (1826––1896), Gustav Adolf Viktor Peschka (1830–1903), Josef Schlesinger (1831–1901), Rudolf Niemtschik (1831–1876), Wilhelm Otto Fiedler (1831–1912), Rudolf Staudigl (1838–1891), Emil Koutný (1843–1880), Karel Pelz (1845–1908)), Emil Müller (1861–1927), František Kadeřávek (1885–1961) and others. For more about their work and life see [6] or [10].

7. Conclusion

The article showed a short summary of history of descriptive geometry and its coming to the Central Europe. In the 19th century and in the first half of the 20th century this geometry together with projective geometry was at its peak. In technical literature we can find in this connection the terms "Czech geometrical school" or "Viennese geometrical school".

This trend lasted until the World War II. Afterwards descriptive geometry never had such a big role in education. Currently computer programs are usually used for constructions of objects in geometry and only few of people with high knowledge of descriptive geometry are needed as employees in industry or as teachers.

References

[1] M. Bečvářová, *České kořeny bulharské matematiky*, Matfyzpress, Praha 2009.

[2] V. Chmelíková, *History of Descriptive Geometry Teaching at Secondary Schools*, [in:] J. Šafránková, J. Pavlů (ed.), *WDS'09 Proceedings of Contributed Papers*, Matfyzpress, Praha 2009, 171-175.

[3] J. Folta, *Geometria i sztuka* (*Widzenie i tworzenie obrazów*), [in:] *Matematyka przełomu XIX i XX wieku*, Materiały z IV Ogólnopolskiej Szkoły Historii Matematyki pod red. S. Fudalego, Szczecin 1990, 141-190.

[4] F. Kadeřávek, *Geometrie a umění v dobách minulých,* Jan Štenc, Praha 1925 (also German edition, *Geometrie und Kunst in Früherer Zeit*, Teubner, 1992).

[5] S. Lawrence, *History of Descriptive Geometry in England*, Proceedings of the First International Congress on Construction History, Madrid, 20th–24th January 2003, 1269-1281.

[6] G. Loria, *Storia della geometria descrittiva dalle origini sino ai giorni nostri*, Ulrico Hoepli, Milano 1921.

[7] G. del Monte, *Perspectivaelibri sex*, Apud Hieronymum Concordiam, Pesaro 1600, on-line: <http://echo.mpiwg-berlin.mpg.de/MPIWG:KOVBCYU7>.

[8] F.J. Obenrauch, *Geschichte der darstellenden und projectiven Geometrie mit besonderer Berücksichtigung ihrer Begründung in Frankreich und Deutschland und ihrer wissenschaftlichen Pflege in Österreich,* Druck und Verlag der k. u. k. Hof-Buchhandlung Carl Winiker, Brünn 1897.

[9] M.P. Vitruvius, *De architectura libri decem*, commentated edition by D. Barbaro, Venezia 1567.
[10] Z. Sklenáriková, *Z dejín deskriptívnej geometrie v Rakúsko-Uhorsku*, [in:] J. Bečvář, E. Fuchs (ed.), *Matematika v proměnách věků II*, Prometheus, Praha 2001, 14-45.
[11] Ch. Wiener, *Lehrbuch der darstellenden Geometrie I*, Teubner, Leipzig 1884.
[12] K. Wojciechowski, *Wstępna ocena merytoryczna I księgi Optyki Witelona*, Wiadomości Matematyczne – II" 11 (1969–1970), 235-246.
[13] *Plan of St. Roch church*, fund *Old plans* collection, sign. 178/6, Archive of the Prague Castle.
[14] *Plan of the water pipeline of Prague Old Town*, fund *Pictorial collection*, sign. B 346, Archive of the Prague water pipeline and sewage system.
[15] *School leaving exams 1877*, fund *Real Gymnasium in Hradec Králové*, b.n. 43, inv.n. 430, State District Archive in Hradec Králové.
[16] *Witelonis Perspectivae Liber Primus*, An English Translation with Introduction and Commentary and Latin Edition of the Mathematical Book of Witelo's Perspectiva by Sabetai Unguru, Studia Copernicana 15, Ossolineum, Wrocław 1977.

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE
1-NP/2014

KATALIN MUNKÁCSY*

REMARKS ON NON-EUCLIDEAN GEOMETRY IN THE AUSTRO-HUNGARIAN EMPIRE

UWAGI O GEOMETRII NIEEUKLIDESOWEJ W MONARCHII AUSTRO-WĘGIERSKIEJ

Abstract

Since 1800s, Central European mathematicians have achieved great results in hyperbolic geometry. The paper is devoted to brief description of the background as well as history of these results.

Keywords: *Austro-Hungarian Empire*, *history of hyperbolic geometry*

Streszczenie

Od XIX w. matematycy w Europie Środkowo-Wschodniej osiągali znaczące wyniki w geometrii hiperbolicznej. Niniejszy artykuł zarysowuje tło i historię tych wyników.

Słowa kluczowe: *Monarchia Austro-Węgierska*, *historia geometrii hiperbolicznej*

* Katalin Munkácsy, ELTE TTK Matematikatanítási és Módszertani Központ, Budapest.
This paper was prepared for publication by S. Domoradzki and M. Stawiska-Friedland on the basis of the author's draft.

1. Introduction

The Bolyai geometry is an important historical phenomenon in mathematics, and a timely research topic with potential applications. I will say a few words about these topics here.

I would like to say first something about the expression “Bolyai geometry”.

Officially the hyperbolic geometry is called B-L geometry, but this form is not really used anywhere. In 1894 Poincaré was the chairman of the committee that compiled the bibliography of hyperbolic geometry. The title was originally Lobachevsky’s Geometrie. However, it was changed to Geometrie de Bolyai et Lobachevsky – as a result of Hungarian mathematicians’ argumentations (see, e.g. [16]).

The most common name is “hyperbolic geometry”; sometimes “Bolyai-Lobachevsky-Gauss” is used. In the Russian-speaking world the common name is ‘Lobachevski’s geometry’, while in Hungary it is called “Bolyai geometry”. However, this is not only inaccurate, but can also be confusing, because “Bolyai geometry” in mathematics has a special meaning: it is the name of absolute geometry discovered by Bolyai János. Bolyai worked out a geometry where both the Euclidean and the hyperbolic geometry are possible, depending on a parameter k. Three elementary geometries exist: hyperbolic, parabolic and elliptic geometry. The names refer to the Greek names of the sum of the three angles of a triangle.

However, in this presentation I will call the hyperbolic geometry “Bolyai geometry”, as we frequently do in Hungary.

The hyperbolic geometry is of importance in philosophy of mathematics, and also in mathematics education. According to Wikipedia (https://en.wikipedia.org/wiki/Non-Euclidean_geometry).

„The existence of non-Euclidean geometries impacted the ‹‹intellectual life›› of Victorian England in many ways and in particular was one of the leading factors that caused a re--examination of the teaching of geometry based on Euclid’s Elements. This curriculum issue was hotly debated at the time and was even the subject of a play, Euclid and his Modern Rivals, written by Lewis Carroll, the author of Alice in Wonderland.”

The hyperbolic geometry slowly entered the public consciousness; M.C. Escher’s graphics played a large role in this process. Computer technology offers additional options. The model of the elliptical geometry is the spherical geometry. The hyperbolic geometry does not have a model in Euclidean space, hence the importance of the hyperbolic computer drawing programs. There have been teaching experiments concerning the role of hyperbolic geometry in the school curriculum (see [19]).

2. Backgrounds of hyperbolic geometry

Hyperbolic, or Bolyai, geometry has ancient antecedents. The oldest maps such as Hecateus indicate Earth to have the shape of a disc, in accordance with the tree-of-life picture of the myths. Our lives happen in a disc-shaped world lying at the foot of the tree of life.

According to the researches of Imre Tóth, the thought of possible multifarious geometry emerged already in the mind of the ancient Greek. The Euclidean geometry was built on the parallel axiom. This geometry, formulated by Euclid, was a choice among possible geometries [18]. This view disappeared later on, and the Euclidean geometry seemed to be only possible geometry. The parallel axiom seemed to be a theorem, which many mathematicians tried to prove. The attempts at an indirect proof did not lead to a paradox, so the possibility emerged that the negation of the statement could be true. We can read in detail about this procedure, and also about the activity of Saccheri and Lambert, e.g., in [13].

The multifarious geometry was discovered separately by Bolyai and Lobachevski. The question of who was first is very hard to decide because the writing of the manuscript, the first publications and the reaction of Gauss were a process in which Bolyai or Lobachevski alternately had priority. Gauss knew about both discoveries and recognized their significance, but he did not let them be published for the wider readership. The first studies about foundation of geometries came out after Gauss' death. The emphasis is now on the plural: there is not 'a geometry', but there are 'geometries'. The results of Bolyai and Lobachevski can be found in some subsequent works without references, e.g. in the great work of Riemann (see [12]).

But this theory was not generally appreciated. Beltrami thought that hyperbolic geometry was not an independent, new theory, but a part of differential geometry. Hyperbolic plane was a special kind of a surface with constant curvature. What he indicated was that J. Bolyai and N.I. Lobachevsky had not really introduced new concepts at all, and so there was no alternative to Euclidean geometry (see [2, 3]).

The turn of the previous century was the era of great development in Central and Eastern Europe.

The Compromise of 1867, which created the Dual Monarchy of Austria-Hungary, caused quick economic development and at the same time accelerated the Hungarian cultural development. Hungarian became the language of instruction from elementary schools to universities. (Earlier, it was first Latin, then German.) The Hungarian mathematical research was integrated into the international scientific world. Many articles written by Hungarian authors appeared: in Comptes Rendus nearly 20, in German journals 100 articles. The majority of the articles were first published in Hungarian. In that period two Hungarian mathematics journals were founded: *Mathematikai és Természettudományi Értesítő* (1882–1941) and *Mathematikai és Physikai Lapok,* (1891–1944).

Famous scholars were elected as the HAS (Hungarian Academy of Science, Magyar Tudományos Akadémia) members, for example Arthur Cayley, Charles Hermite, Hermann Helmholtz, Hugo Kronecker, Paul du Bois-Reymond, Felix Klein, Gaston Darboux and Gösta Mittag-Leffler.

3. The perception in the Austrian-Hungarian Monarchy

The international acknowledgement began in Göttingen, and French, Italian, American translations were completed afterwards. This process is well documented, we can read about it for example in [15, 9]. In Budapest András Benedek (see [5]) and János Tanács (see [17]), in Transylvania Tibor Weszely (see [20, 21]) work on this period, i.e., the end of 19th century and the beginning of 20th century in Hungary. Emil Molnár (see [10, 11]) investigates the history and the modern applications of hyperbolic geometry. I consulted their works while preparing this article. There are also earlier Hungarian books on mathematics history, mainly by Szénássy (see [15, 16]), which contain information about perception of Bolyai's geometry.

Baltzer's book titled "Elemente der Mathematik" (1860) was the first university course book, and a popular one, which mentioned some results of the two Bolyais. The name of Bolyai became known after Gauss' death, when his legacy, including his correspondence, was analysed. In 1867 Hoüel asked for the Appendix from Cluj Napoca. He got a printed copy presumably, and he had it published in French: János Bolyai, *La science absolute de l'espace*,1867, Bordeaux. Hoüel [8] translated it (see [16]). Almost at the same time (1869) an Italian historian of mathematics also asked for it from Boncompagni.

And what happened in Austria, Hungary and the neighbouring countries? The following information, gathered thanks to many colleagues at conferences and by Internet, comes from my lecture given at the International Congress of History of Science and Technology in Budapest, 2009 [1].

Frischauf held a lecture in Graz about non-Euclidean geometry in 1871/72 and the material of the lecture was published, as well. Frischauf: *Absolute Geometrie nach Johann Bolyai*, Leipzig, 1872 (see [16]).

Until 1900 almost nothing about non-Euclidean elementary geometry was taught, except some differential geometry, theory of surfaces, projective geometry and spherical trigonometry.

Gustav Kohn's lecture was the first on non-Euclidean geometry in 1905 (G. Kohn was in Berlin as a „student" of Otto Stolz, 1870–1871)[1].

The hyperbolic geometry came to Prague from Russia.

Eduard Weyr (1852–1903) was the first Czech professor of mathematics who wrote on the non-Euclidean geometry in the Czech lands. In 1896, he published two short articles which gave account of Lobachevsky's centenary celebration in Russia and which contained the first analysis of his works in Czech. Eduard Weyr translated some interesting and important parts from the proceedings which were published by University in Kazan (i.e. the parts form the lectures of F. M. Suvorov (1845–1911) and A.V. Vasiljev (1853––1929)). See [6, 7, 22–25][2].

University of Belgrade was established in 1905. Until 1946 there were no lectures on geometry. Research and lectures on hyperbolic geometry started after 1946[3].

There is a famous school of differential geometry in Belgrade, working on surfaces of constant curvature.

First seminar on Bolyai geometry was in Kolozsvár -Cluj

Gyula Vályi (1855–1913) was a mathematician at the University of Kolozsvár. He held a course on Bolyai geometry in 1891–1892.

What were the origins of his seminar?

The scientific source: Vályi saw the role of new theories of geometry in contemporary mathematics during his scholarship in Berlin, 1878–1880.

---

1 Thanks to Hellmuth Stachel and Christa Binder for references.

2 Thanks to Martina Bečvářová for numerous references.

3 Thanks to Mileva Pranovic for this information.

The personal source: He had a copy of Tentamen (1. edition, 1832), dedicated by Farkas Bolyai to his father, Károly Vályi, who was a student of Farkas Bolyai. This book was available neither in the libraries, nor in the book shops. Luckily the Tentamen, the book with Appendix was preserved as a relic by the Vályi family. (We know all this from a personal letter of a university professor of the name of Réthy Mór)[4].

There was a chain of teachers and their students between Bolyai and Szénássy, who was a great Hungarian historian of mathematics. David Lajos (University professor of mathematics in Kolozsvar and in Debrecen), was a student of Gyula Vályi, and Barna Szénássy (University professor of mathematics in Debrecen) was the student of David Lajos. This chain explains the mystery how the information was transmitted when neither the book nor the manuscript was available.

The research on history of mathematics started early, but the non-Euclidean geometries became a part of university curriculum only later. Only in 1930s did Béla Kerékjártó write his books. The Foundations of Geometry, Foudation of Projective geometry, 1937, 1944.

4. Modern applications

It is possible that crystallography can be expressed more easily with non-Euclidean than Euclidean geometry. There are a lot of articles by Emil Molnár, some of which are intended for secondary mathematics teachers (see [10]).

A new type of the Internet browser was built on a hyperbolic tree (see [14]). We can read on this topic in the broader context of dynamic visualization and hyperbolic mappings.

5. Conclusion

Since 1800s, Central European mathematicians have achieved great results in hyperbolic geometry. However, these achievements (and other elements of modern mathematics) are still absent from the school curricula. This presents a challenge for mathematics education.

References

[1] M. Bečvářová, Ch. Binder, *Mathematics in the Austrian-Hungarian Empire*, Proceedings of a Symposium Held in Budapest on August 1, 2009 During the XXIII ICHST, Matfyzpress, Prague 2011.

[2] E. Beltrami, *Saggio di interpretazione della geometria non-euclidea*, Giornale di Mathematiche VI, 1868, 285-315.

[3] E. Beltrami, *Teoria fondamentale degli spazii di curvatura costante*, Annali. Di Mat., ser. II 2, 1868, 232-255.

[4] A. Benedek, *Problématörténeti látásmód és tudásreprezentáció*, www.phil-inst.hu/recepcio/htm/6/606_belso.htm, [in:] Recepció és kreativitás – Nyitott magyar kultúra, HAS research project.

4 Tibor Weszely [20] helped me to collect these pieces of information.

[5] A. Benedek, *A 20. századi magyar matematika közvetlen előzményei* (Immediate antecedents of the 20th century Hungarian mathematics), Műszaki és természettudományok, [in:] Fábry György (ed): Magyarország a XX. században, Babits Kiadó, Szekszárd 1996–2000.

[6] V. Hauner, *Geometrie neeuklidovská a její poměr k teorii poznání* (Non-Euclidean geometry and its relation to the theory of recognition), Česká Mysl, vol. IV, 1903.

[7] V. Hauner, *Geometrie neeuklidovská: Theorie Riemannova* (Non-Eucliedean geometry: Riemann's theory), Česká Mysl, vol. IX, 1908.

[8] J. Hoüel, *Essai critique sur les principes fondamentaux de la géométrie élémentaire*, ou, Commentaire sur les XXIII premiéres propositions d'Euclide, Paris, Gauthier–Villars, 1897.

[9] J. Kürschák, *The last hundred years in the history of Hungarian mathematics, in The first century of the Hungarian Academy of Sciences*, Budapest 1926, 451-459.

[10] E. Molnár, *Nice tiling, nice geometry!?!*, Teaching Mathematics and Computer Science, Debrecen 2012, 269-280.

[11] E. Molnár, A. Prékopa, *Non-Euclidean Geometries: János Bolyai Memorial Volume*, Mathematics and Its Applications, Springer 2005.

[12] B. Riemann, *On the hypotheses which lie at the foundation of geometry* (1868) translated by W.K.Clifford, Nature 8, 1873, Nos 183, 184 – reprinted in Clifford's Collected Mathematical Papers, London 1882 (MacMillan); New York 1968 (Chelsea); see: http://www.emis.de/classics/Riemann/.

[13] B.A. Rozenfel'd, *A History of Non-Euclidean Geometry: Evolution of the Concept of a Geometric Space* (Abe Shenitzer translation ed.), Springer, 1988, 65.

[14] C. Shirky, *Information Visualization: Graphical Tools for Thinking about Data*, Edventure.com, 2002.

[15] B. Szénássy, *A magyarországi matematika története* (*A legrégibb időktől a 20. század elejéig*), Akadémiai Kiadó, 1970.

[16] B. Szénássy, J. Bognár, *History of Mathematics in Hungary*, Until the 20th Century, Springer--Verlag GmbH, 1992.

[17] J. Tanács, *Ami hiányzik Bolyai János Appendixéből – és ami nem. A Bolyai-féle „parallela" rekonstrukciója* (What is missing from Bolyai's Appendix?), L' Harmattan Kiadó, Budapest 2009.

[18] I. Toth, "*Gott und Geometrie: Eine viktorianische Kontroverse*", Evolutionstheorie und ihre Evolution, Dieter Henrich, ed., Schriftenreihe der Universität Regensburg, band 7, 1982, 141--204.

[19] Z. Trepszker, *Nem euklideszi geometriák az iskolában* (Non-Euclidean geometries in the schools), Iskolakúltúra, 2002/12, 97-108.

[20] T. Weszely, *150 éve született Vályi Gyula* (Vályi Gyula was born 150 years ago), Természet Világa, 2005/1.

[21] T. Weszely, *Bolyai János matematikai munkássága* (Mathematics works of Bolyai János), Kriterion, Bukarest 1981.

[22] E. Weyr, *Oslava stoleté ročnice dne narození N.I. Lobačevského cís. Kazaňskou universitou* (The 100 anniversary of the birth of N.I. Lobachewsky organized by University in Kazan), Časopis pro pěstování mathematiky a fysiky 25, 1896, 1-38 – the article and its review (Jahrbuch ueber die Fortschrite der Mathematik und Physik 27, 1896, 15) are on www pages of EMS.

[23] E. Weyr, *Oslava stoleté ročnice dne narození N.I. Lobačevského cís. Kazaňskou universitou* (The 100 anniversary of the birth of N.I. Lobachewsky organized by University in Kazan), Živa 6, 1896, 6-10 (This is a short extract from [22]).

[24] E. Weyr, *Slavnostní odkrytí pomníku N.I. Lobačevskému v Kazani 1. září 1896* (The ceremonial unveiling of N.I. Lobachewsky's monument in Kazan on September 1, 1896), Časopis pro pěstování mathematiky a fysiky 26, 1897, 249-254.

[25] E. Weyr, *Založení ceny na počet Lobačevského a odhalení jeho pomníku* (The foundation of Lobachewsky's Prize and the ceremonial unveiling of his monument), Časopis pro pěstování mathematiky a fysiky 26, 1897, 31-32.

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE
1-NP/2014

ANDREW SCHUMANN*

ON THE HISTORY OF LOGIC IN THE RUSSIAN EMPIRE (1850–1917)

O HISTORII LOGIKI W IMPERIUM ROSYJSKIM (1850–1917)

A b s t r a c t

In 1850 a very important decision for the whole history of humanities and social sciences in Russia was made by Nicholas I, the Emperor of Russia: to eliminate the teaching of philosophy in public universities in order to protect the regime from the Enlightenment ideas. Only logic and experimental psychology were permitted, but only if taught by theology professors. On the one hand, this decision caused the development of the Russian theistic philosophy enhanced by modern methodology represented by logic and psychology of that time. On the other hand, investigations in symbolic logic performed mainly at the Kazan University and the Odessa University were a bit marginal. Because of the theistic nature of general logic, from 1850 to 1917 in Russia there was a gap between philosophical and mathematical logics.

Keywords: *Russian Empire; Emperor's command of 1850*, *psychologism*, *philosophical logic*, *mathematical logic*

S t r e s z c z e n i e

W 1850 r. car Rosji Mikołaj I wydał ważny dla nauk humanistycznych w Rosji edykt: wyeliminować nauczanie filozofii w uczelniach publicznych w celu ochrony systemu naukowego od idei Oświecenia. Tylko logika i psychologia eksperymentalna były dozwolone, jeśli prowadzili je profesorowie teologii. Z jednej strony, taka decyzja spowodowała rozwój rosyjskiej filozofii teistycznej wzmocnionej przez nowoczesne metodologie reprezentowane przez logikę i psychologię tamtych czasów. Z drugiej strony, badania w logice symbolicznej prowadzone głównie na uniwersytetach w Kazaniu i Odessie miały charakter marginalny. Ze względu na ogólny charakter teistyczny logiki, w Rosji w latach 1850–1917 nie było związków między logiką filozoficzną i matematyczną.

Słowa kluczowe: *Imperium Rosyjskie*, *edykt Imperatora z roku 1850*, *psychologizm*, *logika filozoficzna*, *logika matematyczna*

* Andrew Schumann, Head of Department of Social Sciences, University of Information Technology and Management in Rzeszow, Poland.

The Russian Empire, which existed from 1721 until the February Revolution of 1917, was the predecessor of the Soviet Union. At one point in 1866, it stretched from Eastern Europe across Asia and into North America. The Russian Empire was a Christian successor to the Mongol Empire; thus it inherited the political type of government with hard centralism and absolutism from the Mongol Empire (however, after Europe-oriented emperors, Peter the Great, Peter III, Catherine II, etc., the Russian Empire became quite westernized). It is necessary to notice that the Mongol Empire was the largest contiguous empire in the history of the world. Formally, the Russian Empire was the successor to the Tsardom of Russia. It became the second largest contiguous empire in the world. At the beginning of the 19th century, Russia extended from the Arctic Ocean in the north to the Black Sea in the south, from the Baltic Sea in the west to the Pacific Ocean in the east. It had the third largest population of the world at the time, after China and British Empire. Ruled by the Emperor, it was one of the last absolute monarchies left in Europe. Accordingly, the political system was the least liberal in Europe, with very high social stratification between the very poor and the very rich.

Nevertheless, in the decade from 1810 to 1820 the Enlightenment philosophy expressed by promising ideas of natural law, social contract, and natural religion became very popular in Russia. Social and political philosophy of western thinkers like Hobbes, Montesquieu, Rousseau, and Voltaire were adopted and developed by progressive domestic authors, such as Aleksandr Radishchev. As a result, noble army officers who had been raised on those Enlightenment ideas organized the Decembrist revolt of 1825 to implement the first constitution in Russia. This uprising was suppressed by Nicholas I, the Emperor of Russia, who since that event was afraid of any expression of political thought that could be associated with the Enlightenment ideas. The news of revolutions in Western Europe in 1848 scared him again. All talk of reform and political philosophy was banned, and travel beyond the Empire's borders was forbidden. The culmination of Emperor's commands of this kind took place in 1850, when the minister of education prepared the Emperor's command to eliminate the teaching of philosophy in public universities in order to protect the regime from the Enlightenment ideas. Notice that some restrictions on the teaching of philosophy persisted until 1889. The best-known appropriate motto of Nicholas I was "The profit of philosophy is not proven, but a damage caused by it is possible" ('Польза философии не доказана, а вред от нее возможен'). Instead of general philosophy (especially social and political philosophy) only logic and psychology were permitted, but only if taught by theology professors:

> „С упразднением преподавания философии светскими профессорами в университетах Санкт-Петербурга, Московском, Св. Владимира, Харьковском и Казанском, а также в главном Педагогическом институте и Ришельевском лицее, возложить чтение логики и опытной психологии на профессоров богословия или законоучителей, назначенных к этой должности по сношению Министерства Народного Просвещения с духовным ведомством Православного исповедания.
>
> Профессоров богословия и философии из лиц духовного сана в означенных выше университетах и главном педагогическом институте сравнить в окладах жалованья с ординарными профессорами,

> присовокупив к тому и производство квартирных денег, определенных по этому званию, если они не живут в церковных домах или не имеют казенного помещения...
> Программы преподавания логики и опытной психологии утвердить по соглашению духовного православного ведомства с Министерством Народного Просвещения” [40, p. 1414].
>
> “After the elimination of teaching philosophy by secular professors at the universities of St. Petersburg, Moscow, St. Vladimir, Kharkov and Kazan, and also at the main Pedagogical Institute and Lycée Richelieu, assign the teaching of logic and experimental psychology to theology professors or catechists, nominated to this position after the coordination of the Ministry of National Education with the Ecclesiastic Department of the Orthodox Confession.
> Theology and philosophy professors from clergy at the universities mentioned above and the main pedagogical institute should be equated in salaries with ordinary professors, adding to that accommodation money according to their position if they do not live in church houses or have no state-issued room...
> Syllabi of logic and experimental psychology should be approved after the coordination of Ecclesiastic Orthodox Department with the Ministry of National Education”.

That year was the crucial point in the whole history of humanities and social sciences in Russia from 1850 to 1917. On the one hand, social and political philosophy was banned as such. Therefore there were no reflections on the future of societies which would find some effective solutions for social conflicts and inconsistencies in the Russian Empire. Instead of academic social and political reflections the radical Marxist ideas became popular. As a consequence, the unsolved inconsistencies caused the February Revolution of 1917, which occurred March 8–12 (February 23–7, Old Style). The revolution was accompanied by the abdication of Tsar Nicholas II, the collapse of Imperial Russia and the end of the Romanov dynasty. On the other hand, the teaching of logic and psychology was not forbidden between 1850 and 1917. It was in safe hands of theology professors. The Orthodox journals such as ‘Faith and Mind’ (‘Вера и разум’), ‘Orthodox Review’ (‘Православное обозрение’), ‘Orthodox Interlocutor’ (‘Православный собеседник’), etc. very often published papers devoted to different logical subjects.

One of the most noteworthy of theology professors in the Nicholaevan years was *Fiodor Golubinsky* (1798–1854) [12, 13, 14], who is recognized as the founder of the Moscow School of Theistic Philosophy. The School’s main feature was subordination of philosophy to theology and epistemology to ontology. In fact, the Emperor’s command eliminating the teaching of western philosophy entailed the development of original Russian philosophy, from the Vladimir Soloviev’s theistic philosophy of total unity to the semi-theistic philosophy of Russian cosmists. Probably, it was true intention of the minister of education to stimulate Russian own philosophy. In any case, logic and psychology as a part of theology initiated development of the original Russian philosophy as a whole.

At Russian universities and academies there was an original approach to logic within the world trends [2, 3, 4, 41, 43, 46]. For example, *Ivan Skvortsov* (1795–1863) from the Kyiv Ecclesiastic Academy proposed the division of logic into the following three parts: (1) the logic of reason or theory of thinking (notion, proposition, inference); (2) the logic of mind or theory of cognition (analytics of feelings, analytics of common sense and analytics of reason); (3) methodology or the doctrine of application of laws and forms of thinking in the process of cognition. Along with German logicians from Kant to Hegel, the theology professors teaching logic like Skvortsov tended to follow psychologism, a theory of reducing logic to a psychology of thinking. *Mikhail Vladislavev* (1840–1890), *Nikolai Grot* (1852–1899), *Leonid Rutkovski* (1858–1920) were other psychologists. However, their psychologism was not so much empirical but rather of speculative or even theological nature and it had a religious basis [44].

Vasily Karpov (1798–1867), the founder of Russian academic philosophy [20–23], e.g. he translated Plato's main works into Russian for the first time, and wrote one of the first logical handbooks, after the educational reforms of Nicholas I. This handbook was entitled 'Systematic Survey of Logic' ('Систематическое изложение логики' [19]). He argued for the substantial unity of the Self or I, which makes experience possible. This unity is the first obvious fact, which is not epistemological as in Kant's philosophy, but ontological in the Platonic sense as logos creating the world. Developing these ideas, *Alexey Kozlov* (1831–1901) [24–28] from the Kyiv University rejected the independent existence of space and time, assuming that they possessed being only in relation to thinking and sensing creatures. The ontological interpretation of the substantial unity allowed Kozlov to state that all judgments were analytic.

Another Russian philosopher, Mikhail Karinsky (1840–1917) from the St. Petersburg Ecclesiastic Academy continued argumentations against Kant and western philosophy [15, 16]. His main argumentation is that inner experience, unlike outer, makes no distinction between reality and appearance. The ultimate improvable of inner experience, i.e. truths, is called by him "self-evident" [18, 19]. This self-evident should play role of the first premises for all legitimate conclusions [17]. In his opinion, German Idealism is irrationalistic because of the assumption that the reflective self (self-evident) is just subjective and has nothing objective in itself.

After studying the fundamental work in mathematical logic 'Principia Mathematica' written by Alfred North Whitehead and Bertrand Russell, *Pavel Florensky* (1882–1937) proposed to construct a formal logic of antinomies [11] that could be applied in studying the self-evident of the Russian theistic philosophy. For him, this self-evident is presented in dogmas of the Orthodox Church. He believed that Orthodox Christianity was an inconsistent but non-trivial theory and a formal logic of antinomies allowed him to explicate the inconsistent content of Christian dogmas. So, Florensky could be called one of the founders of present-day paraconsistent logic or logic of antinomies.

Thus, logical investigations in Russia since 1850 were inspired by the critical reviews of German transcendental philosophy, first of all by the Kantian one, but in details these investigations have focused rather on the Orthodox theology which had accepted and supported the Platonic tradition of subordinating epistemology to ontology. This feature of Russian theistic philosophy became possible just due to eliminating the teaching of western social and political philosophy from public universities.

The teaching of logic and psychology by theology professors provided theology and theistic philosophy with modern methodology and made them more rationalistic. Many theistic reflections developed later in Russian philosophy were included in the Syllabus of Logic 1850 written for all universities and academies by the scholars of the Moscow Ecclesiastic Academy (the whole text of the Syllabus is contained in the research paper [1]). This Syllabus was accepted by the Holy Synod of the Russian Orthodox Church. It was divided into the following sections: Introduction, On Principles of Reasoning ('О началах мышления'), On Laws of Reasoning ('О законах мышления'), On Forms of Reasoning ('О формах мышления'), On Experienced Cognition ('Об опытном познании'), On Mental Cognition ('О познании умозрительном').

In the Introduction the subject of logic was defined and its relations to other sciences, first of all to psychology, were considered. In the section 'On Principles of Reasoning' it was claimed that the human reflexive self was finite and it had its origin in God as infinite being. Logic was a main tool of the human reflexive self and it should be subordinated to the Revelation that opens the higher substantial unity of the Self. In the section 'On Laws of Reasoning' the following three logical laws were considered: (i) the law of identity, (ii) the law of contradiction or the law of excluded middle, and (iii) the law of sufficient reason. The section 'On Forms of Reasoning' was devoted to concepts, judgements, and conclusions. The section 'On Experienced Cognition' was about forms of experience (observation, experiment, and testimony) and probabilistic reasoning (induction, analogy, and hypothesis) and their connection with the Revelation. In the section 'On Mental Cognition' the relationships between faith and knowledge were considered.

As we see, the Syllabus suggested some theistic reflections which were advanced later by some philosophers. As an example of the theistic nature of this Syllabus, let us quote some passages from the section 'On Principles of Reasoning':

> „Понятие о начале вообще; различие между началом и первоначальным обнаружением, или исходной точкой. Мышление, как деятельность духовная, должно иметь начало внутреннее — в самой природе человеческого духа, оно есть видоизменение его самосознания; посему за коренное начало его должно быть признано то, что есть в самосознающем духе человеческом глубочайшего, деятельнейшего, всеобщего и несомненно истинного.
> Глубже всего человеческий дух сознает, что он небезначален, но имеет начало от Существа Бесконечного (действительное бытие идей и Бог в человеческом духе). Идея о Боге и есть именно: а) нечто высшее в нашем духе, — не собственно силою его мышления она производится, но врожденна ему свыше, и по необъятности своего содержания безмерно превосходит все другие представления и мысли наши; б) нечто деятельнейшее в духе, чему единственно обязаны мы непреодолимым стремлением к знанию или истине, которое удовлетворяется только в познании последней, Бесконечной причины всего; в) нечто общее всем людям, хотя различно ими понимаемые; наконец е) есть нечто такое, что не только истинно само в себе, но и составляет единственное условие,

по которому возможно для человека истинное познание предметов, единственное ручательство в согласии законов и форм человеческого мышления с действительным бытием вещей — что могло бы уверить нас в сем согласии, если бы не нашли опоры в Единого истинного Виновника и бытия и мышления?
Таким образом, как удовлетворяющая всем показанным условиям врожденная идея о Боге должна быть признана коренным началом мышления" [1].

"The notion of reason as a whole; the distinction between the reason and the ultimate reason, or a starting point. The thinking as spiritual activity should have an internal reason – in the very nature of human spirit, it is a modification of human consciousness; therefore the deepest, most active, most general, and undoubtedly true in the self-conscious human spirit should be recognised as its fundamental reason.
The human spirit understands most deeply that it has a reason and originates from the Endless Being (the actual being of ideas and God in the human spirit). The idea of God is namely: (a) something higher in our spirit, it cannot be inferred by thinking, but it is innate from above, and by the immensity of its content it immensely surpasses all other images and our thoughts; (b) something most active in the spirit that causes our insuperable aspiration for knowledge or truth which is satisfied only in knowledge of the latter, i.e. in the infinite reason of all; (c) something common for all people, though it can be understood by them differently; and finally (d) it is something that is not only true in itself, but also constitutes the only condition for our true knowledge of things, the unique guarantee of the agreement of laws and forms of human thinking with the actual being of things – what could assure us of this agreement if we did not find a support in the Absolute true Reason of both being and thinking?
Thus, the innate idea of God, satisfying all conditions shown above, should be recognised as the fundamental reason of thinking".

Thus, in spite of the social problems undermining the Russian society from within, in the Russian Empire one can detect a well developed logical tradition that is linked with the theistic philosophy. Meanwhile, for many years logic was out of interest for mathematicians and pure philosophers. Logical investigations in the strict sense were performed mainly at the Kazan University and the Odessa University. These investigations were quite marginal, although they were carried out by well-qualified mathematicians. In Saint Petersburg and Moscow these investigations were not regarded as prestigious because of the fact that logic was considered as too metaphysic and theistic. For example, *Andrei Markov* (1856–1922), the leader of Saint Petersburg mathematicians, considered mathematical logic as unimportant for mathematics at all, in the same way as H. Poincaré did.

Platon Poretsky (1846–1907), the professor of the Kazan University was one of the most known Russian founders of modern logic [29–39]. For example, Louis Couturat

[10] evaluated Poretsky's methods as a culmination in the development of algebra of logic for that period. Poretsky was a mathematician who graduated from the Kharkov University. Then he worked in Astrakhan and Pulkovo. After that he found a position as an astronomer at the Kazan University, but he began to study the works of George Boole [5, 6] and was fascinated by algebra of logic. As a result of these studies, he developed some modern logical calculi with their applications to probability theory.

Evgenie Bunitsky (1874–1952), a professor of the Odessa University, was a known Russian logician specializing in algebra of logic, too [7, 8]. His research interest was in applying some results of algebra of logic into arithmetic, and also in determining the number of terms in logical polynomials. He spent two years (1906–1907) in Göttingen at Hilbert's laboratory, the best laboratory of mathematical logic of that time. In 1922 he immigrated to Prague. Since 1923 he worked at the Russian Free University in Prague.

Another prominent logician who carried out highly rated investigations in mathematical logic in Russia was *Jan Śleszyński* (Ivan Sleshinsky) (1854–1931) [45], a professor in Odessa, then in Cracow; in fact, he became the first professor of mathematical logic in Poland. Some other logicians of that period, like *Ivan Zhegalkin* (1869–1947) [53], a professor of mathematics at the Moscow State University, continued their investigations later after the February Revolution of 1917. Zhegalkin was best known for his formulation of Boolean algebra as the theory of the ring of integers *mod* 2 (the so-called Zhegalkin polynomials). Zhegalkin can be recognized as one of the founders of the mathematical logic group of Moscow State University, which became the Department of Mathematical Logic established by Sofia Janovskaja in 1959. The mathematicians from Moscow, such as I. Zhegalkin, D. Egorov, N. Lusin, started to study mathematical logic from the point of view of set theory and theory of functions of a real variable.

The career of some logicians, like that of *Samuil Shatunovsky* (1859–1929), [42] was quite hard. He was born in Velyka Znamianka, Ukraine, in a poor Jewish family as the 9th child. He completed secondary education in Kherson. He lived in small Russian towns, supporting himself by private lessons. Because of his mathematical papers sent to the Odessa University, he was admitted to the university, received financial support, obtained a degree and was appointed a staff member in 1905. In 1917 he became a professor. Shatunovsky focused on several topics in mathematical analysis and algebra, such as group theory, number theory and geometry, trying to develop axiomatic theories.

Because of the theistic nature of general logic, in Russia from 1850 to 1917 there was a gap between philosophical and mathematical logics. The first was too metaphysic and speculative. The second was too symbolic and without any philosophical reflections. The same situation took place in the USSR: on the one hand, there was philosophical logic called dialectic taught at departments of humanities or social sciences, on the other hand, there was mathematical logic taught at departments of engineering sciences or mathematics. And they had no relationship with each other at all. One of the rare attempts to find out some connections between philosophical and mathematical logics before 1917 was made by *Nicolai Vasiliev* (1880–1940) who proposed for the first time the idea of non-Aristotelian logic, free of the laws of excluded middle and contradiction [47, 49, 50]. Reasoning of that logic was called by him 'imaginary,' by analogy with the 'imaginary' geometry of Lobachevsky. He was also the first to distinguish levels of logical reasoning, and introduced the notion of metalogic [48].

Russian textbooks on logic were of good quality. In many neighbouring countries they were translated into national languages. For example, the book ‘Logic as a Part of Theory of Knowledge’ [51] written by a prominent Russian philosopher and psychologist, *Alexander Vvedensky* (1856–1925) was one of the most popular Russian logical textbooks. It was translated into Latvian in 1921. In Latvia this translation became the first textbook on logic. The ‘Handbook of Logic’ written by *Georgy Chelpanov* (1862–1936) had many editions not only before 1917, but also in the USSR and was recently reprinted in Russia as well. Some textbooks like ‘Logic’ by *Kallistrat Zhakov* (1866–1926) contained references to symbolic logic.

Thus, Emperor’s commands of 1850, eliminating the teaching of western social and political philosophy in public universities and permitting logic and psychology to be taught only by theology professors, intensified the development of the original Russian theistic philosophy and weakened any social and political reflections in the Russian society. This feature of Russian humanities and social sciences caused the gap between philosophical and mathematical logics. Hence, the educational policy governs development not only of sciences, but also of societies.

References

[1] Bazhanov V., *Logic in Russia and the Orthodox Church*, Logical Investigations, vol. 18, 2012, 5-25 (in Russian).

[2] Bazhanov V., *The Interrupted Flight. The History of “University” Philosophy and Logic in Russia*, Moscow University Press, Moscow 1995 (in Russian).

[3] Bazhanov V., *History of Logic in Russia and the USSR. Conceptual Context of University Philosophy*, Canon+, Moscow 2007 (in Russian).

[4] Bazhanov V., *Essays on the Social History of Logic in Russia*, Mid Volga Research Center, Simbirsk–Ulyanovsk 2002 (in Russian).

[5] Boole G., *Studies in Logic and Probability*, Vol. 1, London 1952.

[6] Boole G., *An Investigation of the Laws of Thought*, London 1854.

[7] Bunitsky E., *Some Applications of Mathematical Logic to Theory OND and NOK*, Vestnik opytnoj fiziki i elementarnoj matematiki, Nr 274, Odessa 1899, 249-253.

[8] Bunitsky E., *The Number of Terms in the Logical Polynomial*, Vestnik opytnoj fiziki i elementarnoj matematiki, Nr 249, Odessa 1896, 241-246.

[9] Chelpanov G., *Handbook of Logic*, Vol. 2, *Logic*, 1897, 6-th edition, 1911 (in Russian).

[10] Couturat L., *Algebra of Logic*, Odessa 1909 (in Russian).

[11] Florensky P., *The Pillar and Ground of the Truth: An Essay in Orthodox Theodicy in Twelve Letters*, Moscow 1914 (in Russian).

[12] Golubinsky F., *Lectures on Philosophy*, vol. 4, 1884–1886 (in Russian).

[13] Golubinsky F., *Lectures on Philosophy. Theoretic Psychology*, Moscow 1898 (in Russian).

[14] Golubinsky F., *On Finite Causes, Letter 1*, Pribavlenija k izdaniju Tvorenij svjatykh ottsov, vol. 5, Moscow 1847.

[15] Karinsky M., *Toward the Question about Positivism*, Pravoslavnoe obozrenie, 1875.

[16] Karinsky M., *The Critical Review of the Latest Period of German Philosophy*, Saint Petersburg 1873 (in Russian).

[17] Karinsky M., *Classification of Conclusions*, Saint Petersburg 1880 (in Russian).

[18] Karinsky M., *On Self-Evident Truths*, vol. 1, Saint Petersburg 1893 (in Russian).

[19] Karinsky M., *Phenomenon and Reality*, Pravoslavnoe obozrenie, 1878.
[20] Karpov V., *Introducing Lecture on Psychology*, Khristianskoe chtenie, vol. 2, 1868.
[21] Karpov V., *Introduction to Philosophy*, Saint Petersburg 1840 (in Russian).
[22] Karpov V., *Systematic Survey of Logic*, Saint Petersburg 1856 (in Russian).
[23] Karpov V., *Philosophical Rationalism of the Newest Time*, Khristianskoe chtenie, vol. 3–6, 12, 1860.
[24] Kozlov A., *V. Soloviev as Philosopher*, Znanie, Nr 1–2, 1875.
[25] Kozlov A., *Philosophy as Science*, Kyiv 1877 (in Russian).
[26] Kozlov A., *The Newest Research on Plato*, Voprosy filosofii i psihologii, vol. 11, 1892.
[27] Kozlov A., *Positivism of O. Kont*, Voprosy filosofii i psihologii, vol. 15–16, 1893.
[28] Kozlov A., *French Positivism*, Voprosy filosofii i psihologii, vol. 19, 22, 1894.
[29] Poretsky P., *Appendice. Sur mon nouvel travail "Theorie des non-egalites logiques"*, Izvestija Fiziko-matematicheskogo obshhestva pri imperatorskom Kazanskom universitete, 2nd series, vol. 14, Nr 2, 1904, 118–131.
[30] Poretsky P., Exposé élémentaire de la théorie des égalités logiques à deux termes a et b, Revue de Métaphysique et de Morale, vol. 8, 1900.
[31] Poretsky P., *Quelques lois ulteuieures de la théorie des égalités logiques*, Izvestija Fiziko--matematicheskogo obshhestva pri imperatorskom Kazanskom universitete, 2nd series, vol. 10, Nr 1, 1900, 50-84; Nr 2, 1900, 132-180; Nr 3, 1900, 191-230; vol. 11, Nr 2, 1901, 17-63.
[32] Poretsky P., *Sept lois fondamentales de la théorié de égalités logiques*, Izvestija Fiziko--matematicheskogo obshhestva pri imperatorskom Kazanskom universitete, 2nd series, vol. 8, 1899, 33-103, 129-181, 183-216.
[33] Poretsky P., *Théorie conjointe des égalités et des non-égalités logiques*, Izvestija Fiziko--matematicheskogo obshhestva pri imperatorskom Kazanskom universitete, 2nd series, vol. 16, Nr 1–2, 1908, 9-41.
[34] Poretsky P., *Theorié des non-égalités logiques*, Izvestija Fiziko-matematicheskogo obshhestva pri imperatorskom Kazanskom universitete, 2nd series, vol. 13, Nr 3, 1908, 80-119; Nr 4, 1908, 127-184.
[35] Poretsky P., *The Law of roots in Logic*, Nauchnoe obozrenie, Nr 19, 1896, 538-593.
[36] Poretsky P., *From the Area of Logic*, Fiziko-matematicheskij ezhegodnik, posvjashhennyj voprosam matematiki, fiziki, khimii i astronomii v jelementarnom izlozhenii, 2nd year, Nr 2, Moscow 1902.
[37] Poretsky P., *Statement of Basic Principles of Mathematical Logic in the Most Onstensive and Popular Form*, Sobranie protokolov zasedanij sekcii fiziko-matematicheskih nauk obshhestva estestvoispytatelej pri Kazanskom universitete, vol. 1, Kazan 1881, 2-31.
[38] Poretsky P., *On the Tools of Solutions of Logical Equations and On the Inverse Tool of Mathematical Logic*, Sobranie protokolov zasedanij sekcii fiziko-matematicheskih nauk obshhestva estestvoispytatelej pri Kazanskom universitete, vol. 2, Kazan 1884, XXIV.
[39] Poretsky P., *The Solution of General Problem of Probability Theory by Means of Mathematical Logic*, Sobranie protokolov zasedanij sekcii fiziko-matematicheskih nauk obshhestva estestvoispytatelej pri Kazanskom universitete, vol. 5, Kazan 1887, 83-116.
[40] *Proceedings of Prescripts of the Ministry of National Education*, vol. III, Saint Petersburg 1855 (in Russian).
[41] Schumann A. (Ed.), *Logic in Central and Eastern Europe: History, Science and Discourse*, University Press of America, 2012.
[42] Shatunovsky S., *Algebra as Doctrine on Comparison by Functional Modules*, Odessa 1917 (in Russian).

[43] Silakov V.D., Stiazhkin N.I., *The Brief Survey of the History of General and Mathematical Logic in Russia*, Moscow 1962 (in Russian).

[44] Skvortsov I., *The Christian Using of Philosophy, or Philosophy of Gregory of Nyssa*, Trudy Kievskoj Duhovnoj Akademii, Nr 10, 1863, 129-160.

[45] Śleszyński J., *Teoria dowodu*, vol. 1, 1925; vol. 2, 1929.

[46] Stiazhkin N.I., *The Formation of Mathematical Logic*, Moscow 1967 (in Russian).

[47] Vasiliev N., *On Partial Judgments, on the Triangle of Opposites, on the Law of Excluded Third*, Uchenye zapiski imperatorskogo Kazanskogo universiteta, October 1910.

[48] Vasiliev N., *Logic and Metalogic*, Logos, vol. 1/2, 1912–1913.

[49] Vasiliev N., *Imaginary (non-Aristotelian) Logic*, Zhurnal ministerstva narodnogo prosveshhenija, Newest Series, August 1912.

[50] Vasiliev N., *The Report of Privat-Docent at the Department of Philosophy*, Nauchnaja Biblioteka KGU, ORRK. Manuscript, Ruk. 6217.

[51] Vvedensky A., *Logic as a Part of Theory of Knowledge*, Tipografija M.M. Stasjulevicha, 1917 (in Russian).

[52] Zhakov K., *Logic*, Saint Petersburg 1912 (in Russian).

[53] Zhegalkin I., *On Tools of Computing Propositions in Symbolic Logic*, Matematicheskij sbornik, 1927, 9-28.

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE
1-NP/2014

GALINA I. SINKEVICH*

THE NOTION OF CONNECTEDNESS IN MATHEMATICAL ANALYSIS OF XIX CENTURY

POJĘCIE SPÓJNOŚCI W ANALIZIE MATEMATYCZNEJ XIX WIEKU

A b s t r a c t

The notion of connectedness was introduced by Listing in 1847 and was further developed by Riemann, Jordan and Poincaré. The notion and rigorous definition of metric and topological space were formed in Frechet's works in 1906, and in Hausdorff's works in 1914. The notion of continuum could be traced back to antiquity, but its mathematical definition was formed in XIX century, in the works of Cantor and Dedekind, later of Hausdorff and Riesz. Karl Weierstrass (1815–1897) brought mathematical analysis to a rigorous form; also, the notions of future areas of mathematics – functional analysis and topology – were formed in his reasoning. Weierstrass's works were not translated into Russian, and his lectures were not published even in Germany. In 1989, synopses of his lectures devoted to additional chapters of the theory of functions were published. Their material served as the basis for this article.

Keywords: *connectedness*, *Weierstrass*, *Cantor*

S t r e s z c z e n i e

Pojęcie spójności zostało wprowadzone w 1847 roku przez Listinga, a dalej zostało opracowane przez Riemanna, Jordana i Poincarégo. Pojęcie i rygorystyczna definicja przestrzeni metrycznej i topologicznej pojawiły się w pracach Frécheta w 1906 roku i Hausdorffa w 1914 roku. Pojęcie kontinuum sięga starożytności, ale jego matematyczna definicja powstała w XIX wieku, w pracach Cantora i Dedekinda, później Hausdorffa i Riesza. Karl Weierstrass (1815–1897) przedstawił analizę matematyczną w rygorystycznej formie; również pojęcia przyszłych dziedzin matematyki – analizy funkcjonalnej i topologii – zostały zapoczątkowane w jego rozumowaniu. Prace Weierstrassa nie zostały przetłumaczone na język rosyjski, a jego wykłady nie były publikowane nawet w Niemczech. W 1989 roku ukazały się streszczenia jego wykładów poświęconych dodatkowym rozdziałom teorii funkcji. Opublikowany w nich materiał jest podstawą tego artykułu.

Słowa kluczowe: *spójność*, *Weierstrass*, *Cantor*

* Galina Sinkevich, Department of Mathematics, St. Petersburg State University of Architecture and Civil Engineering, St. Petersburg, Russia.

1. Introduction

The history of topology dates to the Königsberg bridge problem, formulated and solved by L. Euler in 1736 [1].

The first work from which topology got its name was written by Listing in 1848 [2]. J.B. Listing (1808–1882) was a professor at the University of Göttingen, where Gauss was a lecturer, and where Riemann was a student. Like Riemann, Listing paid attention mainly to combinatorial properties of transformations, and he did not have an idea of a domain as of a set of points yet. In 1862 Listing continued combinatorial topological topics in his work "Description of spatial manifold, or generalization of the Euler's polyhedron theorem" [3]. That was an early period of development of topology, still before the works of G. Cantor (1845–1918) on the set theory.

For the first time, the notion of connectedness was used by B. Riemann (1826–1866) in his dissertation titled "Fundamentals of the Theory of Functions of a Complex Variable" (1851), in his report "On the Hypotheses which lie at the Foundation of Geometry" (1854) and in the "Theory of Abelian functions" (1857). Riemann considered the space as having a real physical meaning, but not as a set of points; he considered a surface as a sheet spread out on a plane, or over the plane [4, p. 52]. He applied the notion of '1-connected' to a "piece" of surface bounded by a closed non-self-intersecting curve. In 1851 in the "Fundamentals of the theory of functions of a complex variable", Riemann wrote: "We shall consider two parts of a surface to be connected if a curve belonging to the surface can connect a point of one part to a point of the other part; otherwise, two parts of the surface shall be defined as disconnected, or separately located" [4, p. 54].

2. The notion of connectedness in the works of Georg Cantor

Cantor started from the analysis of convergence of trigonometric series and analysis of points on a straight line, and in his initial works he created the theory of point domains. He introduced the notion of real number on the basis of a fundamental sequence; he developed the notion of accumulation point proposed by Weierstrass in 1865; on its basis he formulated the notions of derived set and uniform convergence. Then he constructed a hierarchy of infinite sets, which led him to transfinite numbers. Cantor was interested in the nature of continuum, and many of his investigations gave topological results, for example, the issue of the possibility of a one-to-one mapping of a two-dimensional continuum on the domain of real numbers (1878). During the period from 1879 till 1884, Cantor published a cycle of six articles "On the infinite linear point manifolds" [5, p. 40-139], which contained his main results on the set theory. Cantor defined the sets of the first kind, which had empty n-derived set, and all the others – sets of the second kind. He introduced the notion of density within an interval, and demonstrated that the sets of the first kind were not dense anywhere within the interval; he demonstrated denumerability of the sets of first kind and some of the sets of second kind. He introduced the notion of an isolated set, as a set not containing its accumulation points, and proved denumerability of isolated sets in R^n. In his fifth work, Cantor introduced transfinite ordinals, and formulated the hypothesis of continuum, and also considered an issue of when a subset of R^n could be defined as a "continuum". For that purpose, he defined the notion of a perfect set and a connected point set. A perfect set coincides with its derived set by definition. The set T would be connected by definition if for any $\varepsilon > 0$ and any t and t' in T, a finite number of points $t_1, t_2, \ldots, t_n$, exist in T, so that all the distances $tt_1, t_1t_2, t_2t_3, \ldots, t_{n-1}t_n, t_nt'$ do not exceed ε. If a subset R^n is perfect and connected, then it would be called a 'continuum'.

In 1880s, many new results and notions appeared in German mathematicians' works devoted to mathematical analysis. These notions needed to be uniformized and made rigorous. Ketsier wrote: "Cauchy has created a new conceptual apparatus to ensure a strong basis for the existing analysis, and in his mathematics a function would always be linked to a formula. In the second half of XIX century the conceptual apparatus itself became an object of research. This happened due to generalization of the notion of function: it started to mean an arbitrary correspondence between numbers" [6, p. 3]. At the same time, the theory of real numbers was still insufficiently developed – though there were different definitions of irrational number, it was not known how many irrational numbers exist, in comparison with rational numbers, or whether there are other numbers, non-definable with the help of sequences of number, and, above all, how the irrational numbers are distributed on the complete number scale. The uniform continuity theorem was formulated for a function ranging between two rational limits. Weierstrass realized that construction of real numbers takes place speculatively, "in the world of our thoughts", and tried to harmonize arithmetical conception of a number with general conception of a value as a result of measurement of a geometrical of physical object. Alongside with the growth of importance of the notion of an irrational number, criticism was also growing, regarding extension of the notion of a real number. A colleague of Weierstrass at the University of Berlin, L. Kronecker (1823––1891), came out strongly against the theories of Weierstrass and Cantor, and asserted that all numbers should be expressible through natural numbers and their relations. His harsh words, both in his publications and in conversations among the circle of his colleagues, as well as in his lectures delivered to students, were meant to prove that Weierstrass's theory of functions was groundless [7, p. 327]. Klein related emotional experiences of Weierstrass, as described in the letter from Weierstrass to S. Kovalevskaya, on March, 24, 1885, regarding the malicious attacks of Kronecker. Probably it was an aspiration to defend himself and to demonstrate validity of the theory of functions in the light of new concept of real numbers that resulted in Weierstrass's intention to deliver an additional course of lectures devoted to the grounds of mathematical analysis. He implemented that in 1886.

3. Lectures of Weierstrass delivered in 1886

Karl Weierstrass (1815–1897) delivered lectures at the Königlichen Gewerbeinstitut of Berlin and the University of Berlin, starting from 1856. He systematized the course of mathematical analysis and introduced the notion of continuous function in the language of "ε – δ". He did a lot for the theory of real numbers. It was he who ensured a rigorously substantiated form of mathematical analysis. He aspired to arrange in order new discoveries made during the 1870s by Charles Méray [8], Edward Heine [9], Richard Dedekind and Georg

Cantor [10], while striving to put them in classical terms and harmonize with traditional notion of a number as a ratio of values.

Especially for that purpose, he delivered a special course of lectures devoted to basic notions of mathematical analysis during the summer semester of 1886. The lectures were delivered thrice a week, in May and June. Synopsis of these lectures prepared by his students was published relatively recently, in 1989 [11].

Weierstrass started the course of lectures with the words: “These lectures have been compiled in order to supplement the lectures on the theory of analytical functions delivered during the winter semester of 1884/1885. The intended aim was achieved, using however a more synthetic method, and for some of the results no desirable generalization was attained; the quality of proof was not fully satisfactory. Hence after delivering those lectures it appears useful to recount in detail the various methods underlying the theory of functions, outline them historically and critically, in order to demonstrate the various points of views, and make an attempt to reconcile them. In short, to demonstrate the tendency of historical development of mathematical science, especially in the field of mathematical analysis, and thus, to explain the fundamental notions of science. Our aim will be to demonstrate that the principles of mathematical science are based on an actually solid foundation” [11, p. 20].

For the purpose of substantiation of representability of a function, Weierstrass used the notion of real number, including his own theory about a number as an aggregate (visible totality), i.e. finite or infinite decimal (or other) record, which in the infinite case would represent an absolutely convergent series satisfying the relation of equality (equivalence for infinite representations) and order.

Like Dedekind, Weierstrass separated physical reality and “the world of our thoughts”, in which an idea of number is formed for the description of numbers and functions; and therefore, a set of numbers can be extended by means of passage to the limit. Hence, any lengths could be represented by numbers; however, the length would not correspond to any number.

For him, all the values expressed through proportions (ratios) are limited; an infinite numerical value is called defined when each element of its underlying convergent series is specified. Definite points correspond to each of the summands of a well ordered series, but only in case of absolute convergence of the series. Condition for that consists in uniform convergence. Then, with any rearrangement of terms in series, the limit would be the same numerical value, extending the notion of a number.

Weierstrass introduced the notion of a variable, associating the notion of accumulation point with it, and, following Cantor, he defined an irrational number as an accumulation point of rational numbers. He gave an example: “the number e, consisting of the elements $1, \frac{1}{2}, \frac{1}{6}, \ldots, \frac{1}{n!}, \ldots,$ is a well ordered series, which defines a particular numerical value; however, it can be shown that there is no rational numerical value equal to it by definition, which shows that numerical domain of rational numbers is not full” [11, p. 58]. Weierstrass solved this problem in relation to analytical continuation of a function.

Weierstrass referred to Bolzano’s theorem of the least upper bound of a variable, but he doubted whether a numerical value always corresponds to a point. For him, the entire totality of positive numerical values is wider than the entire totality of all possible segments between A and B, up to AB. If a numerical value is defined, i.e. expressed by absolutely converging series, it would undoubtedly be in correspondence with, e.g., a geometrical decimal segment, in the form of $a_0 + \frac{a_1}{10} + \frac{a_2}{10^2} + \ldots$, where $0 \leq a_k \leq 10$, when $k \geq 1$; a series can represent any numerical value on the segment, and such values appear in the above expression of the series as the first, the second (subtotal) and so on. By interrupting separation of elements, we shall thus define infinitely many numerical values in whose neighborhood infinitely many definite points are condensed, and a continuous sequence can be formed without definite points. In that way, one may define position of the point D, subdividing the segment into two continuous segments, "so that it would correspond somehow to our innate, natural notion of limit, whereupon we could imagine that a straight line is not limited by anything except points, so one can suppose that D represents a definite value" [11, p. 63].

4. Cut sets and their extension to a plane and multidimensional domains. Neighborhood

Hence, there should be one and only one point of separation of two segments from each other, and this point is the numerical value under consideration. Having defined cut set on segment, as exact upper bound of the subtotals of convergent series, Weierstrass passed to determination of a neighborhood and a bound in the space of points (x_1, x_2, …, x_n), having defined the point of n-multiple manifold as (a_1, a_2, …, a_n):

"Let us assume that in an n-multiple point manifold defined in an arbitrary way, however in the way that infinitely many points comply with this definition, at least one point exists in whose direct neighborhood infinitely many definite points are concentrated. Let us consider the totality of all the points created in case where x_1 can possess all the values from $a_1 - d$ to $a_1 + d$, x_2 can possess all the values from $a_2 - d$ to $a_2 + d$, and so on, and assume that, in that way, one can form all possible numerical combinations of n values in the **neighborhood** of the point (a_1, a_2, …, a_n); thus we affirm, that if d is arbitrarily small, then in every arbitrarily small neighborhood of at least one point, infinitely many points exist which comply with the definition.

Now it is possible to define a neighborhood of the point (a_1, a_2, …, a_n) in the domain of n-multiple manifold of real variables through inequalities $\sqrt{(x_1 - a_1)^2 + \ldots + (x_n - a_n)^2} \leq d$, in that way, for n-multiple complex manifold of the values in the form of $x_k = \xi_k + \eta_k i,\ k = 1, 2, \ldots, n$ there will be $\sqrt{\sum_1^n (\xi_k - a_k)^2 + \sum_1^n (\eta_k - b_k)^2} \leq d.$

The value d is referred to as the radius of neighborhood of the point under consideration (a_1, a_2, …, a_n); the expression for the space is applicable to arbitrary n-multiple manifolds.

Now, two approaches are possible: either we relate a point whose each neighborhood contains infinitely many definite points, or we do not relate such a point, to definite points. In the latter case, the point shall be referred to as a frontier point (limit point, Grenzstelle)" [11, p. 65].

5. Definition of continuum

This Section is devoted to Weierstrass's attempt to accomplish connectedness and separation of plane domain with the help of removable sets. Weierstrass defined continuum in an *n*-multiple manifold. Initially, he considered a function of two independent variables whose domain of definition was a part of a plane with some excluded points. "Passage from one non-excluded point to another similar point is possible through a continuously connected path. One could always separate a part of a plane which connects the first point to the second. This is possible with the help of a sequence of the circles for which the center of the next circle is located within the previous circle, and radii are selected so that all discarded points would be left outside. If the number of excluded points is infinitely large, there is no need in constructing a line. E.g., the excluded points are located on a circumference, so that an arbitrary initial point is projected onto the arc u' towards some direction; this circle has a unit radius for $u' = 2\xi\pi$, where ξ runs through all rational values from 0 to 1, despite the fact that the set of removable points is not a continuous line (thus, a certain part of the plane without points located continuously on the line is removed). No excluded points are located inside the circle; we shall take any point as the center of a circle whose all points are to be defined; one can make sure that its radius does not exceed a certain limit; again, we shall take a new point in the circle and circumscribe a circle around it in a similar way, like around the first point. One can draw a conclusion that if we continue such a procedure for an unlimited number of times, we could never get out from the interior through discarded points bounding the circle; just as by analogy, from beyond the bounds of the circle, a point from outside would never be able to get into the circle. Hence, we see that a continuous sequence of points is insufficient for separation of a two-dimensional manifold into parts. As we see from the example, it is not possible, even a priori, to define the kinds of separation of the plane into parts" [11, p. 66].

Professor E. Mioduszewski kindly commented on this place in the work of Weierstrass: "Prehistory of Weierstrass's connectedness is rather interesting. That notion for modern topologists is a pure property of figures and spaces, but in case of Weierstrass, his research was motivated by the tasks facing him. The sets considered by Weierstrass were the sets of points *excluded* from definitional domain of a function (singularities of function), or their complements. In this example, the set of points $u' = 2\xi\pi$ of unit circumference with rational ξ means the set of excluded points. It's countable, but sufficient to prevent analytical extension from the interior of a circle to exterior, since analytical extension is made with the help of finite chain of open discs, each of which has a common point with the previous one. This is a very strong condition of connectedness, considerably stronger that the Cantor's condition, where we only require connectedness for any ε, of a finite sequence of *points*, each of which is remote at a distance ε from the next one. By means of such sequences, we can pass through from the interior of a circle to the exterior. In order to prevent such free passage from definitional domain into the complement of the set of discarded points, a set would be required, containing nontrivial continuums. This is just the case described by Phragmen. It is not known, whether somebody considered the notion of connectedness as applicable to the function theory. Perhaps, only Mittag-Leffler was able to consider connectedness as an autonomous tool in mathematical study of functions".

Weierstrass; "We shall now pass to consideration of an n-multiple manifold; we can define it as a set of definite points, and discuss the main theorem, a detailed substantiation of which we saw in the case of a prime manifold. A point set is referred to as **closed** if any neighborhood of each of its definite points contains infinitely many definite points. If we define, for example, all points of a circular domain, then each point of the circumference would be defined, and simultaneously it would be a frontier point, whereas outside the circular domain not a single point would be found about which it could be said that any of its small neighborhood would have definite points. Now, we can make any point set P closed, by adding its accumulation points P' to it. Assuming that the point belongs neither to P nor to P', we could in any case circumscribe a circle of finite radius around it, the circle not containing P; otherwise, it would contain P'. But it cannot contain P' either, because by definition, P' contains infinitely many points of P.

With the help of such closed set of points, we shall now separate either a **single** continuum from an n-multiple manifold, **or** a **number** of continua, located at a distance from each other. Let the point A be given, not belonging to the set of points; we could circumscribe it by a neighborhood of a radius ρ such that no definite (definierten) points would fall within it; ρ is a variable having an upper bound, to the effect that it possesses arbitrarily smaller values. **An absolute neighborhood** of A would be a neighborhood whose radius ρ_0 just presents this upper bound. Starting from the point A, we should construct a sequence of points $A_1, A_2, \ldots, A_n$ in such a manner that every next one would be contained within the neighborhood of the previous one. Then, two cases could be possible: either we would pass from A to any point that does not belong to the point set P, or we would not. In the first case, all those points not included in the point set P would form a continuum; in the second case, because of our assumption, the point A would only define the part of n-multiple manifold remaining after removal of P. Now, in the remaining part of n-multiple manifold, we shall select the point B, again, in order to define a new continuum in that manifold with the help of this point. Thus, we see that by virtue of the definition of the point set, an n-multiple manifold can be separated into infinitely many parts".

6. Connectedness of continuum

"Now, a question arises whether two points initially belonging to the same defined continuum always result in that same continuum or not. To answer this question, let us imagine that between A and B there is a sequence of points such that we can pass from A to B via these points, whereas each successive point is located within the neighborhood of the previous one. Then it appears immediately that from A we could reach all the points which we can get from B, because we simply can get at A after passing B. It is not quite clear how we can reach from B all the points reachable from A. To show that, it would be necessary to prove that it would be possible to return to A from B, and namely, that it would be possible to pass through all the points reachable from A after leaving B for A. To prove that, we shall connect A and B using a sequence of points, which could provide passage from A to B. We shall connect these points using a broken line; i.e. $A_{n-1}A_n$, completely in the neighborhood of A_{n-1}, and so on. Let this line go through a point in such a manner that each position be in correspondence with **absolute radius of neighborhood** ρ, which is a variable quantity having a lower bound not equal to zero; and if we use the principle of classification which we applied in the proof of the main theorem, then, first of all, we would be certain that it would be necessary to get at one or a number of **particular** locations for which the lower bound is actually attained. But now, at any point of the broken line, ρ would have a finite value, because otherwise the mentioned location (point) would be an accumulation point for A, i.e. it would belong to the point set P, so that it could be used as an extension of the continuum.

We would like to present another proof, which is based on the following argument. If two points A and B are located at a distance δ, and ρ is the radius of neighborhood of A, then between $\rho - \delta$ and $\rho + \delta$ a radius ρ' of a neighborhood of B would exist. Now it is possible to select ρ small enough so that the difference between $\rho - \delta$ and $\rho + \delta$ would be arbitrarily small. Let a point run along the segment from A_1 to A_n; then the respective value of ρ would attain its smallest value, which is not equal to zero, as we mentioned above. Let the point run along the segment backwards, A_n, A_{n-1}, …, A_1, and we shall select the distance $A_{\nu-1}A_\nu$ to be small enough so that A_ν would be placed within the neighborhood $A_{\nu-1}$, also, $A_{\nu-1}$ would be placed within the neighborhood of A_ν, which could be achieved if $A_{\nu-1}A_\nu < \frac{1}{2}\rho$; this is the same radius of the neighborhood of A_ν that we have referred to as being $> \frac{1}{2}\rho$; thus, $A_{\nu-1}$ would be inside the circumference circumscribed around A_ν. If we select intermediate points so that all distances between them be $< \frac{1}{2}\rho_0$, where ρ_0 is the lowest bound of ρ, then we would be certain that it could be possible to return to A from A_n as mentioned above, i.e., from any point we could only get to the points of continuum whose points belong to the same one, and it is actually so; or, in other words, a point can only belong to the continuum, which has been proved completely".

"Wednesday, 6.23.1886.

This theorem is applicable, first of all, to a plane, i.e., to a two-dimensional manifold; and now, our task is to extend the proof onto an arbitrary n-multiple manifold. Here we shall use geometrical expressions reduced to the following form: let $x_k = a_k + t\,(b_k - a_k)$, $k = 1, …, n$, where t is an unlimited real variable. Then we shall give the name of a **line** in the respective manifold, to a totality of points $(x_1, x_2, …, x_n)$ complying with this expression. The grounds for such name do not require comments. The totality of values of the system $(x_1, x_2, …, x_n)$, which follow from this expression, where t possesses the values from 0 to 1, has the name of **segment** ab. By giving t the values > 1, we would obtain an extension of the segment ab beyond the bounds of b; by giving t negative values, we would obtain an extension beyond the bounds of a. The expression $\sqrt{\sum_{1}^{n}(b_k - a_k)^2}$ has the name of distance from the point b to the point a. If the distance = r, then we understand the neighborhood of the point a, having the radius r, as a totality of all the system's values for which $\sqrt{\sum_{1}^{n}(b_k - a_k)^2} < r$

7. Triangle inequality, calculus of variation

"Now we shall prove the theorem, which has been a direct generalization of the following statement for a space: If a, b, c are three points within a space, then $ac < ab + bc$, unless the points are collinear. Now, let c vary, but always in the way that bc would have the same value, so that the positions could exist where ac reaches its maximum, and those where ac becomes minimal. For an n-multiple manifold this theorem is formulated as follows: If a, b, x are three points, then $ax < ab + bx$, or $\sqrt{\Sigma(x_\lambda - a_\lambda)^2} < \sqrt{\Sigma(b_\lambda - a_\lambda)^2} + \sqrt{\Sigma(x_\lambda - b_\lambda)^2}$, except for those cases where $x_\lambda = a_\lambda + t\,(b_\lambda - a_\lambda)$. In the latter case, the above inequality reduces to equality. Now we can use a purely algebraic proof, but first we shall do the following. Let a, b be fixed, x be a variable, however, such that bx has constant value ρ, so that at first it would be necessary to show that a point exists for which $\sqrt{\Sigma(x_\lambda - a_\lambda)^2} = R$ takes the maximum, and another point, for which R is minimal. It shall also satisfy $\Sigma(b_\lambda - a_\lambda)^2 = r^2$. We shall apply a common approach and form the following expression: $\Sigma(x_\lambda - a_\lambda)^2 - \varepsilon\Sigma(x_\lambda - b_\lambda)^2$; this should possess minimal or maximal value, and in addition we multiply the item by an indefinite constant ε, because this problem is related to **calculus of variations**. In this way, we combine equations $(x_\lambda - a_\lambda) - \varepsilon\,(x_\lambda - b_\lambda) = 0$, $(\lambda = 1, \ldots, n)$ or $x_\lambda - a_\lambda = -\dfrac{\varepsilon}{1-\varepsilon}(b_\lambda - a_\lambda)$, which proves that the desired points are actually on a line. Then we would obtain: $R = \pm\varepsilon\rho = \pm\dfrac{\varepsilon}{\varepsilon - 1}r$, and also, $\varepsilon = 1 \pm \dfrac{r}{\rho}$.

Thus we obtain two points, and not more, satisfying the requirement. Then, in the first case, $R = \left(1 + \dfrac{r}{\rho}\right)\rho = r + \rho$, in the second case, $R = \left(\dfrac{r}{\rho} - 1\right)\rho = r - \rho$. In effect, in one case we have maximum, in another case we have minimum, so in fact in other cases $ax < ab + bx$. This theorem will serve as a basis for the proof of our theorems for n-multiple manifolds. Let a closed point set be given in a closed manifold, but so that there exist points not belonging to the set. Then, for each point not belonging to the set, a neighborhood will certainly be defined. Definitely, the points of the set P are not in an arbitrary proximity to the said points, i.e., there may be a point, denoted by a, with a neighborhood having radius ρ, inside of which there are no points of P. Undoubtedly, this radius doesn't exceed the finite bound r, i.e., the neighborhood of r is defined so that with a small increase the point P would fall inside the neighborhood. By marking some point b within the neighborhood of the point a, one could show that in case where $ab = d$, the radius of the neighborhood of b would not be less than $r – d$, and would not exceed $r + d$. If c is a point from the neighborhood of b, then, by virtue of the theorem just proved, first of all $ab + bc > ac$ or $bc > r – d$, and also, $bc < ab + ac$, $bc < r + d$, which was to be proved. In the second part, c stands for a point on the boundary of the neighborhood of b, located within the neighborhood of a. So, we can formulate the following definition: if the point b exists within the neighborhood of the point a, the point c exists within the neighborhood of the point b, and so on, then any point s through which we could move from a to c would be called a point **connected** (adjacent) to the point a. We now show that if s is connected to a, then a is also connected to s. Then it would be proved, obviously, that having started from any point of a continuum, we would always stay within the same.

Now, let the point b exist within the neighborhood of the point a, so that $ab < \frac{1}{2}r,$ where r is the neighborhood (radius of the neighborhood) of the point a; then, the neighborhood (radius of the neighborhood) of the point b, by virtue of the theorem proved, will be $> \frac{1}{2}r.$ Let us connect ab by a segment of straight line, then, as soon as $ab = d$, a neighborhood could always be defined for any point, such neighborhood would be of radius $> r – d$; we only need – which is always possible – to connect a and b by a sequence of points of such a kind that at passage from a to b every point would be within the neighborhood of the previous one, to be sure that it could be possible to return back from b to a; however, it would always be possible, since the distance between the points is $< r – d$. But a is connected to b, and starting from b, we can arrive at any point at which we can also get if we start moving from a; therefore, it would be necessary to depart from b through a. This theorem is true for any n-multiple manifold".

8. Bounds of continuum

"Friday, 6. 25. 1886. Now we shall put the question about creation of the concept of the **bounds of a continuum**;_that is, about whether all the points of manifold belong to continuum, or not. In the latter case, there should exist points in whose arbitrary proximity both the points belonging to the continuum and those not belonging to the continuum would exist. We shall refer to the totality of these points as the **bound (limit)** of the continuum. This represents just the largest possible manifold. For example, a single point, an infinite number of points located discretely, and finally, a continuous contour can represent a delimiter on a plane. We have to prove two things:

1) that with the exception of the above simple case, bounds exist in a continuum;
2) that all this can be found in a closed point set, with the help of which we defined continuum from the very beginning.

It is absolutely unnecessary for all the points belonging to a closed set of points to be limit points (frontier points); for example, we shall define a closed set in a space with the help of points existing in a sphere, or on its surface; in this regard, if we separate the continuum of frontier points from the sphere, then the remaining internal points of the sphere would not represent a closed point set anymore.

Therefore, in the first place, it would be necessary to prove the existence of accumulation points. Let a be a point within a continuum, b would then be any other point; according to the definition given earlier for an n-multiple manifold: $x_\lambda = a_\lambda + t\,(b_\lambda - a_\lambda)$, $(\lambda = 1, \ldots, n)$, where t takes all positive and negative real values; then a and b could be connected by a straight line. Now, let τ be an arbitrary positive value, i.e. such a value that, when used as a substitute for t in the expression for coordinates of the segment, would result in the point P on the segment, located within the continuum, and it would be such a point that all the points from a to this point would belong to the segment (part) of the continuum; then the distance (segment) aP would be a variable quantity, and as such, it would have an upper bound. Should it be infinitely large, then all the points of the straight line would belong to the continuum; however, this is not the case, and hence, for P_0, there should be an extreme location for the given P; thus, P_0 would simply be a bound for P, as it follows from the definition of the bound. The segments for which these frontier points P_0 actually exist should be available in any case; indeed, one could mark any point for which a point set connected to a has been defined in the continuum; of course, this point has been a point of the class of P_0, otherwise, there would be no points on the segment preceding the point in connection to the point a, i.e., those corresponding to a smaller value of t.

And finally, one can easily see that all the frontier points belong to a closed set; because any point which does not belong to the point set belongs to the continuum; otherwise, if the manifold is located within point sets in a number of continua, then it belongs to one of these continua.

A continuum like one that has just been described shall be referred to as an **open continuum**; we think that in case when all accumulation points are added to such a continuum, we could obtain a structure which could rightfully be referred to as a **closed continuum**. In fact, the continuum that we defined before actually represents a closed set" [11, p. 63-73].

In her thesis about Mittag-Leffler, Laura Turner describes two different definitions of the continuum in the following way: "Central to Weierstrassian analysis was Weierstrass' notion of a continuum. These are unions of open disks in the complex plane which arise as domains in which the power series representation of a function is uniformly convergent. Weierstrass' continua are thereby closely connected to his concept of monogenicity; the domain of a function is divided into continua, and if there is only one such continuum the function is monogenic. However, if singularities block analytic continuation, the domain will be divided into multiple continua at the boundary they form. Cantor, on the other hand, in connection with his advocacy of a general shift from functions – and in particular the representations of functions – to dealing with set of points of their domain and range, had a different definition. For him, a continuum was a perfect connected point set.

Mittag-Leffler claimed to have found both notions important, and Weierstrass' in particular for its connections to his own work. On 27 February 1883 he wrote to Cantor: "I very much agree with your definition of continuum, and would however like to refer to what Weierstrass calls a continuum as a "completely connected point set" (...). It will follow sufficiently from my work that such completely connected point sets have their necessary place in the theory of analytic functions and can't be replaced by your continua.

Mittag-Leffler was interested, moreover, in the relative relationship between the two definitions within a three dimensional space, and he took this issue as a potential starting point for some of Phragmén's earliest work. This is the first of two instances for which there exists evidence that Mittag-Leffler explicitly requested that Phragmén consider a particular topic. In a letter to Phragmén dated 7 December 1883 Mittag-Leffler indicated that a line in the plane can be defined as a perfect connected point set — a Cantorian continuum – of which no part comprises a continuum in the plane according to Weierstrass' definition. However, if one considers this definition within a three dimensional space, the distinguishing characteristics between a surface and a line are not immediately obvious. Mittag-Leffler added: "It is well worth speculating over this issue. I believe that it is closely connected to some of the most important questions in the theory of functions". The next documented instance of Mittag-Leffler's proposal of a specific problem to Phragmén concerns work on Weierstrass' notion of the continuum. A letter written by Phragmén to Mittag-Leffler dated 29 December 1883 concerned the relationship between Weierstrassian continua and particular points sets in the plane. Phragmén published this work in January of 1884 in an article entitled «En ny sats inom teorien för punktmängder» ("A new theorem within the theory of point sets") [24, p. 114–115].

9. Further development of the ideas of Weierstrass

As we can see, the notions of measurable sets, metric and topological space have already been stipulated in the lectured delivered by Weierstrass. The theorem stipulating that each bounded infinite subset in R^n has an accumulation point was discussed by Weierstrass for $n = 2$ in his course of lectures delivered in 1865; a general proof was proposed by him in 1874. However, Weierstrass did not distinguish the notion of accumulation point as a basic one: this was done in 1872 by Cantor , who constructed the hierarchy of derived sets. The lectures delivered by Weierstrass were not published; however, his ideas were disseminated by his audience: German, Italian and Russian students. In Italy, his ideas were developed by G. Ascoli (1843–1896), C. Arzela (1847–1912), and U. Dini (1845–1918).

10. Weierstrass's ideas in Italy

The courses of mathematical analysis in Italy were brought into accord with achievements of German and French mathematicians. For that purpose, F. Brioschi, E. Betti, F. Casorati (more than once) visited Germany to meet Weierstrass, Kummer, Kronecker and other mathematicians [15, p. 221]. Due to an intensive exchange of letters between H. Schwarz, Casorati and U. Dini, Italian mathematicians were well informed about all German mathematical news. The course of lectures by Ulisse Dini, "Fundamentals of the theory of functions of real variable" [12, 16], which he delivered at the University of Pisa from 1871 till 1915, was considered to be the best in Italy. New results obtained by Weierstrass, Cantor and Dedekind were included in that course of lectures [14]. During the years of 1877–1878, S. Pincherle (1853–1936) was among Weierstrass's students, and in 1880 Pincherle started to deliver the course of lectures "Theory of analytical functions according to Weierstrass" (Teoria delle funzioni analitiche secondo Weierstrass), at the University of Pavia, and later on he published the course [17]. In this way, the ideas of Weierstrass were disseminated in Italy.

In 1883 Volterra started creating the theory of functionals, or "functions of lines with real values". These functions were considered as the elements of a set for which the notion of neighborhood and limit of a sequence can be defined. Volterra gave the definition of continuity and derivative of a line function, and made an attempt to construct the theory of line functions by analogy with Riemann's theory of complex functions.

In 1884 Giulio Ascoli (1843–1896) extended Bolzano–Weierstrass theorem to sets of functions. In 1889 C. Arzelà generalized the theorem and proved that an equicontinuous set F of uniformly bounded functions on $[a, b]$ has a limit function.

Then Arzelà discussed continuous real-valued functionals defined on an equicontinuous set of functions F, and demonstrated that, if F is closed, i.e., it contains all its limit functions, with the lower bound of the set of values of the functionals, then the upper bound would be reachable, along with all the intermediate values.

Now the Ascoli-Arzelà fundamental theorem in mathematical analysis is formulated in terms of compactness, in the language created by Fréchet in 1904.

11. M. Fréchet, F. Riesz and F. Hausdorff

In 1906, Maurice R. Fréchet (1878–1973), in his dissertation "On some questions of functional calculus" [18] introduced the notion of a metric domain [18, p. 30], formalized by Hausdorff in 1914, with identity, symmetry axioms, and triangle inequality.

In 1908, F. Riesz (1880–1956), in his report "Continuity and abstract theory of sets" [19] delivered at IV International mathematical congress in Rome, characterized continuum using the notion of accumulation point satisfying the three basic axioms: each element representing an accumulation point of a set M also represents accumulation point of any set that contains M; if a set is divided into two subsets, then each accumulation point would be an accumulation point of at least one of these subsets; a subset that consists of one element only does not have an accumulation point. For the purpose of strengthening, Riesz added the fourth axiom: each accumulation point of a set is defined uniquely through a totality of all its subsets whose accumulation point it is.

Felix Hausdorff (1868–1942), who graduated from the University of Leipzig in 1891, attended the lectures at the University of Freiburg and Berlin. Probably, he was aware of the Weierstrass's lectures delivered in 1886. Yet in 1912, Hausdorff, while delivering lectures on the set theory at the University of Bonn, introduced the notion of neighborhood U_x of the point x as a set of all the points y for which $xy < \rho$, where ρ is a positive number:

the interior of a sphere having the radius ρ, $xy = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \ldots + (x_n - y_n)^2} \geq 0$ is the distance between the points x and y. A neighborhood, according to him, possesses the following properties: Each U_x contains x and is contained within r (where r is any n-dimensional space, e.g., a plane). For two neighborhoods of the same point: $U'_x \supseteq U_x$ or $U_x \supseteq U'_x$. If there exists y within U_x, then a neighborhood U_y exists which is contained within $U_x (U_x \supseteq U_y)$. If $x \neq y$, then two neighborhoods, U_x, U_y, having no common points would exist [20]. In 1914, Hausdorff wrote one of first methodical accounts of the set theory and the theory of topological spaces "Grundzüge der Mengenlehre" (Essentials of Set Theory) [21], where he introduced the notion of topological space. In that book, Hausdorff used the notion of upper bound, introduced by Weierstrass.

12. Conclusion

The notion of connectedness is primarily used in topology [22]. Weierstrass, while developing his method of analytic extension, created his own notion of connectedness for the purpose of the theory of analytic functions; the notion that proved to be stronger than the one of Cantor's. The intention of Weierstrass to substantiate and systematize the contemporaneous mathematical analysis resulted in his creation of new trends and notions in analysis and topology. As Weierstrass used to say, "even an introduction to mathematical sciences requires study of various problems, which, in the first place, shows us significance and consistency of science. One should bear in mind that the ultimate aim of the study of foundations of science is the striving for confidence in objectivity of science" [11, p. 20]. Weierstrass analyzed the methods and notions of the classical analysis so deeply that his constructions resulted in the notion of metric and topological space, on the basis of which functional analysis was formed.

We know the early history of the notion of connectedness because of a trace left by B. Bolzano (1817) – G. Cantor (1879–1883) – C. Jordan (1893) – N.J. Lennes (1905– 1911) – A. Schoenflies (1904) – F. Riesz (1906) – W.H. and G.C. Young (1906) – F. Hausdorff (1914) [23, 24, 25]. In [23, 25] there are nothing about Weierstrass` ideas. At present, Weierstrass occupies a deserved place in the history of evolution of the notion of connectedness.

The author would like to thank Professor Mioduszewski for his comments.

References

[1] Jejler L., *Pis'ma k uchenym*, Izd-vo AN SSSR, Moskva–Leningrad 1963, 336-340.
[2] Listing J.B., *Vorstudien zur Topologie*, J.B. Listing, Göttingen 1848.
[3] Listing J.B., *Der Census raumlicher Complexe oder Verallgemeinerung des Euler'schen Satzes von den Polyeder*, J.B. Listing, 1862.
[4] Riman B., *Sochinenija*, Moskva–Leningrad 1948, 52.
[5] Kantor G., *Trudy po teorii mnozhestv*, Moskva 1985, 40–139.

[6] Ketsier T., Mill J. van, *By their fruits ye shall know them: some remarks on the interaction of general topology with other areas of Mathematics* (http://www.math.vu.nl/~vanmill/papers/papers1999/teun.pdf), 3.
[7] Klein F., *Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert*, Teil I, Berlin 1926, 327.
[8] Sinkevich G.I., *Razvitie ponjatija nepreryvnosti u Sharlja Mere*, Trudy X Mezhdunarodnyh Kolmogorovskih chtenij: sbornik statej, Izd-vo JaGPU, Jaroslavl' 2012, 180-185.
[9] Heine E., *Die Elemente der Functionenlehre*, J. Reine Angew. Math., 74, 1872, 172-188.
[10] Sinkevich G.I., *Georg Kantor & Pol'skaja shkola teorii mnozhestv*, Izd-vo SPbGASU, 2012, 356.
[11] Weierstrass K., *Ausgewählte Kapitel aus der Funktionenlehre. Vorlesung gehalten in Berlin 1886 mit der Akademischen Antrittsrede, Berlin 1857, und drei weiteren Originalarbeiten von K. Weierstrass aus den Jahren 1870 bis 1880/86*, Teubner-Archiv zur Mathematic, Band 9, 272, Reprint 1989.
[12] Dini U., *Fondamenti per la teoria delle funzioni di variabili reali*, Nistri, Pisa 1878, VIII+407.
[13] Kol'man Je., *Bernard Bol'cano*, Moskva 1955, 192.
[14] Sinkevich G.I., *Uliss Dini i ponjatie nepreryvnosti*, Istorija Nauki i Tehniki, 10, 2012, 3-11.
[15] Borgato M.T., *Continuity and discontinuity in Italian mathematics after unification: from Brioschi to Peano*, Organon, 41, 2009, 219-232.
[16] Dini U., *Grundlagen für eine Theorie der Funktionen einer veränderlichen relleen Grösse*, Leipzig 1892, XVIII+554.
[17] Pincherle S., *Saggio di una introduzione alla teoria delle funzioni analitiche secondo i principi del prof. C Weierstrass*, Napoli 1880, 124.
[18] Fréchet M., *Sur quelques points du calcul fonctionnel*, Rend. Circ. Mat. Palermo, 22, 1906, 1-74.
[19] Riesz F., *Stetigkeit und Abstrakte Mengenlehre*, Atti del IV Congresso Internationale dei Matematici, vol. 2, Rome 1908, 18-24.
[20] Hausdorf F., *By their fruits ye shall know them: some remarks on the interaction of general topology with other areas of Mathematics* (http://www.math.vu.nl/~vanmill/papers/papers1999/teun.pdf), 16.
[21] Hausdorff F., *Grundzüge der Mengenlehre*, von Veit, Leipzig 1914.
[22] Mioduszewski E., *Connectedness*, [in:] *Encyclopedia of General Topology*, Elsevier, Amsterdam 2003, 223-226.
[23] Wilder R.L., *Evolution of the topological concept of "connected"*, Amer. Math. Monthly, 85, 1978, 720-726.
[24] Turner L., *Gösta Mittag-Leffler in the Development and Internationalization of Mathematics in Sweden and Beyond, 1880–1920* (http://css.au.dk/fileadmin/www.ivs.au.dk/css.au.dk/Turner_PhD_Thesis_2012.pdf).
[25] Bottazzini U., Gray J., *Hidden Harmony – Geometric Fantasies*, Springer, New York 2013, 848.

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE
1-NP/2014

GALINA I. SINKEVICH*

CONCEPTS OF A NUMBER OF C. MÉRAY, E. HEINE, G. CANTOR, R. DEDEKIND AND K. WEIERSTRASS

KONCEPCJE LICZBY CH. MÉRAYA, E. HEINEGO, G. CANTORA, R. DEDEKINDA I K. WEIERSTRASSA

A b s t r a c t

The article is devoted to the evolution of concept of a number in XVIII–XIX c. Ch. Méray's, H. Heine's, R. Dedekind`s, G. Cantor's and K. Weierstrass`s constructions of a number are considered. Only original sources were used.

Keywords: *concept of a number*, *Cantor*, *Dedekind*, *Méray*, *Heine*, *Weierstrass*

S t r e s z c z e n i e

Artykuł poświęcony jest rozwojowi koncepcji liczb w XVIII i XIX wieku. Rozważane są konstrukcje liczb Ch. Meraya, E. Heinego, G. Cantora, R. Dedekinda i K. Weierstrassa. W rozważaniach wykorzystano wyłącznie oryginalne źródła.

Słowa kluczowe: *koncepcje liczb*, *Cantor*, *Dedekind*, *Méray*, *Heine*, *Weierstrass*

* Galina Sinkevich, Department of Mathematics, St. Petersburg State University of Architecture and Civil Engineering, St. Petersburg, Russia.

Let us review how the concept of a number developed in the 18th and 19th century.

1707. I. Newton (1642–1727)

"We mean by number not an aggregate of units, but rather a dimensionless ratio of a value to another value of the same nature taken as a unit. There may be three types of a number: an integer, a fraction, and a surd. An integer is something that is measured by a unit; a fraction is a multiple of a part of unit; and a surd is incommensurable with a unit". [1, p. 8].

1758. A. G. Kaestner (1719–1800)

"Fractions are whole numbers, a unit of which is a part of the initially chosen unit; irrational values are fractions, a unit of which is variable and represents an ever reducing part of a whole. Irrational numbers are non-extracted roots. Any such number may be put between two arbitrary close rational approximations. It is a priori assumed that the root $\sqrt[n]{a}$ exists, where a is not an *n-th* power of a rational number, and that arithmetical operations with them are possible" [2].

1821. A. Cauchy (1789–1857)

"If variables keep approximating a certain value, so that finally there is an arbitrary small difference between the variables and this value, the latter value is called a limit. Thus, e.g. the area of a circle is a limit which is approximated by areas of regular inscribed polygons: the greater is the number of their sides, the closer the approximation.

Note that measurements of a line or an arc may represent a numerical value which precisely corresponds to this length, or were obtained as a numerical result of gradual approximations from either side to a fixed point (let's call it an initial point), increasing or decreasing in length as they approach it" [3, p. 349].

Cauchy did not set any rules of procedure for irrational numbers.

1869, 1872. Ch. Méray (1835–1911)

In 1869, Méray laid down two principles of the theory of irrational numbers (immeasurable, incommensurable numbers): "1. Variable v, which sequentially takes value $v_1, v_2, \dots, v_n, \dots$, tending to a certain limit, if its components keep growing or decreasing, however, remaining, in the first case, less, and, in the second case, greater than a certain fixed numerical value. 2. An additional property of variable v is that difference $v_{n+p} - v_n$ tends to zero at n increasing without limit, whatever the relation of n to p may be" [4].

Méray named irrational numbers (whether algebraic or transcendental) as immeasurable numbers.

His reasoning of 1872 was as follows ([5], Méray's italics):

"Let us call numerical value $v_{m,n}$ (whether a whole number or a fraction, positive or negative) the amount whereof depends on the value of integers m, n, …, taken in whatsoever combination of values and numbered with these indices, a variant, e.g.: $v_m = 1 + \frac{1}{2} + \dots + \frac{1}{m-1} + \frac{1}{m}$, $v_{m,n} = \frac{1}{mn}$ is a variant of two indices.

1. If there is a V for which at sufficiently large m, n, …, difference $V - v_{m,n,\dots}$ is arbitrary small in its absolute value for sufficiently large values of the indices, variant $v_{m,n,\dots}$ is said to tend or converge to limit V.

If $V = 0$, variant $v_{m,n,\dots}$ is called infinitely small, as, for example, the difference between the variant and its limit.

Among variants that have no limits, one should mention those the absolute value whereof can become greater than any prescribed number; they are called *infinite* values, while those the numerical value whereof is less than a finite number are called *finite* values.

2. It is easy to assert as follows:

I. A sum and product (or product of powers) of a certain number of finite variants and constant values will be a finite value. This applies to the relation of two similar values if the denominator is not infinitely small.

II. A product of an infinitely small and constant or finite value, a sum of a certain number of such products (positive powers) and an infinitely small value which is opposite to the infinitely large value, will be an infinitely small variant.

III. A power with infinite positive index of a certain constant value or variant will be infinitely large or infinitely small, depending on the final absolute value thereof, i.e. if it exceeds an amount > 1 or is less than < 1.

IV. A sum and product (or product of powers) of a certain number of variants which have limits and a constant value have at the limit a result which would be obtained if the limit of these values is inserted in this calculation. The same applies to the ratio of two similar values, if the denominator is not infinitely small.

Immeasurable numbers

3. Let us call variant $v_{m+n,\dots}$, for which the difference between $v_{m+p,n+q,\dots}$ and $v_{m+n,\dots}$ for arbitrary p and q is less than any infinitely small variant with indices m and n, that is to say, this difference tends to zero for m, n which are infinite regardless of p and q, a convergent variant.

4. Two variants $v_{m,n,\dots}$ and $v'_{m',n',\dots}$ are equivalent if their difference $v_{m,n,\dots} - v'_{m',n',\dots}$ considered as a single variant with indices $m, n, \dots, m', n', \dots,$ is infinitely small.

Having ascertained the above, we will easily prove that:

A sum and a product (or product of powers) of a certain number of convergent variants and fixed values will be a convergent variant equivalent to a variant that would be a replacement of respective equivalents. The same applies to a ratio as well, if the denominator is not infinitely small.

5. This assertion is trivial if limits of the variants are certain numbers. However, if any of them do not converge to any *numerical* limit, this assertion is also true.

Nevertheless, let us admit that, in a figurative sense, this means that an invariant converges to a fictitious *immeasurable* limit if it converges to a point which cannot be accurately determined. If incommensurable limits of two converging variants are equal, such variants will be equivalent; a sum, product, etc. of variants converging to a certain limit, whether real or fictitious, as case may be, is a sum or product, etc. of their real or fictitious limits. If we supplement these conditions, the statements we set forth above are true and correct, as well as the cited theorems.

6. *Converging variant which is not infinitely small is finite, given the certain sign is retained*. According to our hypothesis, there is an infinite number of combinations of m, n, … values, corresponding to $v_{m,n,...}$, the absolute value of which exceeds the fixed number δ. Let us attach sufficiently large values to m, n, …, so that $v_{m+p,n+q,...} - v_{m,n,...}$ would be numerically less than δ, whatever p, q, … might be. Whereas $v_{m+p,n+q,...}$ equals $v_{m,n,...} + (v_{m+p,n+q,...} - v_{m,n,...})$, this equality is correct for all p, q, …, that is to say, for all indices equal to or exceeding $v_{m,n,...}$.

   Moreover, if two variants $v_{m,n,...}$ and $v'_{m',n',...}$ converge to incommensurable limits and are not equivalent, their difference $v_{m,n,...} - v'_{m',n',...}$ is finite and retains the certain sign. Depending on whether it is + or –, we would say that the immeasurable limit of the first one is greater or less than that of the second one.

   In the same way, a measurable number a is said to be greater or less than the immeasurable finite number for variant $v_{m,n,...}$, depending on whether $a - v_{m,n,...}$ is > or < 0.

If the absolute value of this finite difference remains less than ε, we will call it the *value* of an immeasurable number converged in accordance with ε with an excess in the first case and deficiency in the second case.

We will determine all immeasurable numbers, approximating their values with the help of a δ*, however small it might seem*" [5].

1872. H. E. Heine (1821–1881)

"The theory of functions is for the most part developed using elementary fundamental theorems, although insightful research casts some doubt on certain results, as research results are not always well argued. I can explain it by the fact that, although Mr. Weierstrass" principles are set forth directly in his lectures and indirect verbal communications, and in manuscript copies of his lectures, and are quite widely spread, they have not been published as worded by the author, under the author's control, which hampers the uniformity of perception. His statements are based on an incomplete definition of irrational numbers, and the geometric interpretation, where a line is understood as motion, is often misleading. Theorems must be based on the new understanding of real irrational numbers, which have been rightfully founded and do exist, however little they may differ from rational numbers, and the function has been uniquely determined for each value of the variable, whether it is rational or irrational.

Not that I am publishing this work unhesitatingly long since its first and more significant part *About Numbers* has been finished. Apart from complexity of presentation of such a topic, I was hesitant about publishing results of the verbal exchange of ideas which contain earlier ideas of other people and those of Mr. Weierstrass in the first place, so, all that is left to do is to implement these results, which is extremely important so as not to leave any vague issues in my narrative. I am especially thankful to Mr. Cantor from Halle for the discussion which significantly affected my work, as I borrowed his idea of general numbers which form series.

Let us call a numerical sequence a sequence consisting of numbers a_1, a_2, a_3, …, a_n, …, when for each arbitrarily small non-vanishing number η such n number can be found that $a_n - a_{n+v} < \eta$ can be achieved for all whole positive v[1].

Let us assume that for the structure of (rational) numbers a_1, a_2, …, there is such a (rational) number U that $U - a_n$ decreases as n grows. In this case, U is the limit of a.

We will call general numbers, which in particular cases become rational numbers, as first-order irrational numbers. As irrational numbers are formed from first-order rational numbers A, so, in the same way, second-order numbers A' can be obtained from limits of irrational numbers, whereupon, third-order irrational numbers A'' can be obtained from them, and so on. We will let $A^{(m)}$ denote irrational numbers of order $m + 1$" [6].

1872. G. Cantor (1845–1918)

Cantor constructs a set of numerical values currently known as real numbers, supplementing a set of rational values with irrational numbers using sequences of rational numbers he called fundamental, i.e. sequences that meet the Cauchy criteria. Relations of equality, greater, and less are determined for them.

In the same way, it can be asserted, says Cantor, that a sequence can be in one of the three relations to rational number a, which results in $b = a$, $b > a$, $b < a$. Consequently, if b is a limit of the sequence, then $b - a_n$ becomes infinitely small with growing n. Cantor calls the totality of rational numbers domain A and the totality of all numerical b-values domain B. Numerical operations common for rational numbers (addition, subtraction, multiplication, and division, where the divisor is non-vanishing) which are applied a finite number of times can be extended to domain A and B. In this process, the domain A (that of rational numbers) is obtained from the domain B (that of irrational numbers) and together with the latter forms a new domain C. That is to say, if you set a numerical sequence of numbers b_1, b_2, …, b_n, … with numerical values A and B not all of which belong to domain A, if this sequence has such a property that $b_{n+m} - b_n$ becomes infinitely small with growing n and any m, such sequence is said to have a certain limit c. Numerical values c form domain C. Relations of equality, of being greater than, less than, and elementary operations are determined as described above. However, even a recognized equality of two values b and b' from B does not imply their equivalence, but only expresses a certain relation between sequences to which they are compared.

Domain D is similarly obtained from domain C and preceding ranges, and domain E is obtained from all above domains, etc.; having completed λ of such transformations, domain L is obtained. The concept of a number as developed herein comes equipped with a seed of the necessary and absolutely infinite extension. Cantor uses *numerical amount, value*, and *limit* as equivalent.

Further, Cantor considers points on a line, defining the distance between them as a limit of a sequence and introducing relations of being "greater than", "less than", and "equal". He introduces an axiom that, a point on a line corresponds to each numerical value (and vice versa), the coordinate of such point being equal to this numerical value, and moreover, equal in the sense explained in this paragraph. Cantor calls this assertion as axiom, as it is not

---

1 Sic [6, p. 174].

provable in its very nature. Thanks to this axiom, numerical values additionally gain definite objectivity, on which, however, they do not depend at all.

In accordance with the above, Cantor considers a point on a line as definite, if its distance from 0 considered with a definite sign is set as an λ-type numerical amount, value, or limit.

Further, Cantor defines multitude of points or point sets and introduces a concept of an accumulation point of the point set. A neighborhood is understood as any interval which contains this point. Thus, together with a set of points an ensemble of its accumulation points is defined. This set is known as the first derivative point set. If it consists of an infinite number of points, a second derivative point set may be formed of it, and so on [7].

The introduction of the concept of an accumulation point (condensation point) was fruitful. Other mathematicians like H. Schwarz and U. Dini started using it right away.

1872. R. Dedekind (1831–1916)

Dedekind reviews properties of equality, order, density of a multitude of rational numbers *R* (numerical field, a term introduced by Dedekind in appendices to Dirichlet's lectures he published). However, he tries to avoid geometric representations. Having defined the relation "larger" (or "smaller"), Dedekind confirms its transitivity; existence of an infinite multitude of other numbers between two numbers; and, for any number, breaking down a multitude of rational numbers into two infinite classes, so that numbers of one of them are smaller than this number and another one whose numbers are greater than this number; and the number which breaks down the numbers as described above may be assigned either to one class or to the other, in which event it will be either the greatest for the first class or the smallest for the second one.

Further, Dedekind reviews points on a line and sets properties for them in the same way as he has just set for rational numbers, stating that a point on the line corresponds to each rational number.

However, there are infinitely many points on a line which do not correspond to any rational number, e.g., the size of a diagonal line of a square with a unit side. This implies that the multitude of rational numbers needs to be supplemented arithmetically, so that the range of new numbers could become as complete and continuous as a line. Formerly, the concept of irrational numbers was associated with measurement of extended values, i.e. with geometrical representation. Dedekind tends to introduce a new concept by purely arithmetic means, that is, to define irrational numbers through rational numbers:

If the system of all real numbers is split into two classes, so that each number of the first class is less than each number of the second class, there is one, and only one, number which makes this split.

There are infinitely many sections which cannot be made by a rational number. For example, if D is a square-free integer, there is a whole positive number λ, so $\lambda^2 < D < (\lambda + 1)^2$. Therefore, it appears that one class has no greatest, and the other class has no smallest number to make a section, which makes the set of rational numbers incomplete or discontinuous. If that's the case and the section cannot be made by a rational number, let us create a new, irrational, number which will create the section. There is one, and only one, rational or irrational number which corresponds to each fixed section. Two numbers are unequal if they correspond to different sections. Relations "larger than" or "less than" may be found between them.

He defines calculations with real numbers. Herewith, he proves the theorem on continuity of arithmetic operations: "If number λ is a result of calculations which involve numbers $\alpha, \beta, \gamma, \ldots$, and if λ lies in interval L, one can specify such intervals A, B, C (in which numbers $\alpha, \beta, \gamma, \ldots$ lie) that the result of a similar calculation in which, however, numbers $\alpha, \beta, \gamma, \ldots$ are replaced with numbers of respective intervals A, B, $C, \ldots$, will always be a number which lies in interval L" [8].

1886. K. Weierstrass (1815–1897)

Weierstrass delivered his first lecture circuit devoted to immeasurable numbers in the academic year 1861/1862. Records of his lectures from 1878 are also available. In summer term of 1886, in response to reproaches of L. Kronecker to the effect of insufficient justifiability of lectures on theory of analytic functions, Weierstrass read additional chapters devoted to foundations of the theory of functions [9]. By that time, concepts of a number of Cantor, Heine, and Dedekind already appeared. Weierstrass attempts to critically summarize them and align them with the classical concept of a number as a ratio.

Weierstrass notes incompleteness of the field of rational numbers, gives consideration to the difference between concepts of a number and a numerical value. According to Weierstrass, a number is a collection, a finite aggregate, e.g. in the form of a decimal notation. A point on a line corresponds to each number, however, it is not obvious that a number corresponds to each point. Unlike his contemporaries, he defines a real number as a limit of partial sums of absolutely convergent series, noting the need for arithmetization of the concept of a limit. He introduces order and completeness with respect to arithmetic operations.

Weierstrass created his reasoning of the theory of analytic functions. The concepts he introduces are not global in their nature – they are necessary for his constructions only. He introduces his own concepts of a continuum and connectivity which differ from those of Cantor; for analytic continuation, he simultaneously builds up a chain of open discs, which is equivalent to Heine covering lemma. Weierstrass defines a number so that it would be sufficient to define continuous changes in arithmetical values in their mutual dependence, "that is to say, an arithmetic expression is calculated in such detail that for any accuracy requirement for any amount t a function may be represented with any approximation. It is always possible to find a mathematical expression for a strictly defined continuous function as well." However, if a function represents series, this does not narrow down, this rather expands, opportunities for study of this function, but the series must have a uniform convergence. "For any value of x for which a function has been determined, it can in fact be represented".

1886. "There is an arbitrary large number of numerical values in an arbitrary vicinity of each immeasurable number which tend to be arbitrary close to it. Therefore, each immeasurable numerical value is a landmark of measurable (numerical) values defined in this case above. So what kind of a purely arithmetic method of definition of the difference between measurable and immeasurable numerical values should be? If measurable numerical values are assumed to exist, there is no sense in defining immeasurable numbers as exact bounds, as in advance it cannot be clear at all, except for, maybe, measurable and some other numerical values".

This is an expression of criticism of the Méray-Heine-Cantor design of real numbers, although he did not mention any names during his lecture. Further during this lecture, Weierstrass gave his reasons as follows:

"But it was not the numerical value which used to be definite, as a matter of fact, it was understood as a measurable number, however, it also contains other as well. Let us consider number e as an example, this number being represented by order elements $1, \frac{1}{2}, \frac{1}{6}, \ldots, \frac{1}{n!}, \ldots$ which form well determined series. These series unequivocally determine a numerical value which equals them; it can be said that there is no measurable numerical value which equals the represented numerical value (the so-called number e). We therefore conclude that the field of (all) values goes beyond measurable numbers".

"Using the introduced descriptive tool, it is easy to prove that each numerical value corresponds to a certain geometrical length. That is to say, a numerical value can be presented in an arithmetic form, e.g., in a decimal system, as $a_0 + \frac{a_1}{10} + \frac{a_2}{10^2} + \ldots,$ where $0 \le a_k < 10,\ k \ge 1,$ which means that we can present all our (positive) numerical values as segments (of length)" [9].

Cantor. About comparing various methods of introduction of a concept of a number and continuity

Having received Dedekind's work "Continuity and irrational numbers" on 28 April 1872, Cantor wrote to him: "Thank you very much indeed for your work on continuity and irrational numbers. As I could now satisfy myself, the conclusion I came to a couple of years ago proceeding from arithmetic studies, in fact, complies with your viewpoint; the difference is only in the method of introducing the numerical value. I am absolutely sure that you properly defined the essence of continuity".

However, their further correspondence contains polemics regarding the method of defining continuity, and in 1882, Cantor wrote to Dedekind: "I tried to summarize your concept of a section and use it to define the concept of a continuum, but in vain. On the contrary, my point of departure are countable "fundamental sequences" (i.e. sequences whose elements infinitely converge with one another) which seem to suit this attempt".

By 1878, from analysis of point ranges, Cantor proceeds to the concept of power of set, hypothesizes continuum, reviews continuous mappings between multitudes of various dimensions. The more acutely he feels the insufficiency of defining continuity through section. In 1879, he tries to use the Bolzano–Cauchy theorem on roots of a continuous function in an interval to prove that a continuous one-to-one mapping between two different manifolds of different orders is impossible.

In 1883, analyzing various forms of introduction of a number in his cycle of works [10] Cantor wrote: "I would like to briefly and more strictly outline the three basic forms of strictly arithmetic statement of the theory of general real numbers which are known to me and are essentially similar. They, *in the first place,* include the method of introduction Professor Weierstrass used for some years in his lectures on analytical functions and certain resemblance of which can be found in Mr. Kossak's program work (Die Elemente der Arithmetik. Berlin 1872). *In the second place*, in his work "Stetigkeit und irrationale Zahlen" (Braunschweig, 1872), Mr. Dedekind published a kind of a form of a definition. *In the third place*, in 1871, I suggested (Math. Ann. 1872, Bd. 5, S. 123) a form of a definition which has formal resemblance with that of Mr. Weierstrass … I believe this *third one*… is the simplest and most natural of all, and its another advantage is that it is most fit for analytical calculations".

"A definition of any irrational real number would always correspond to the strictly defined set of first-order rational number. This is a common feature for all forms of definitions. The difference is in the point of generation when a set will unite the number it defines and in conditions the set must meet to make a suitable basis for the respective definition of the number.

The *first* form of definition is based on a set of positive numbers a_v which will be denoted as (a_v) and which meets the condition that, whatever is the number and type of these a_v summed up in the finite number, this amount will always remain less than a certain preset threshold. Now, if we have two similar sum-totals (a_v) and (a'_v), it can be rigorously proven that three options may be in place: either each part of $1/n$ unit is always equally frequent in both populations, provided that their elements are summed up in a sufficient amount which can be increased; or $1/n$, starting from a known n, is always more frequent in the first sum-total than in the second one; or, finally, $1/n$, starting from a known n, is always more frequent in the second sum-total than in the first one. Based on these options, denoting the numbers defined by these two sum-totals (a_v) and (a'_v) by b and b', we assume that in the first case, $b = b'$, in the second case, $b > b'$, and in the third case, $b < b'$. If we merge both sum-totals into one new sum-total $(a_v + a'_v)$, this will provide basis for determination of $b + b'$. If a new sum-total $(a_v a'_v)$ is formed out of two sum-totals (a_v) and (a'_v), the elements whereof are products of all (a_v) multiplied by all (a'_v), this new sum-total will be taken as basis for definition of bb'.

We can see here that the point of generation which links a set with the number it incepts constitutes the *generation of sums*. However, it is important to note that only summing of an always *finite* number of rational elements is handled here, and it is *not* assumed in advance, for example, that number b being defined equals the sum Σa_v of infinite series (a_v). This would have been a *logical mistake*, as sum Σa_v can rather be defined only by setting it equal to a predetermined final number b. I believe this logical mistake first avoided by Weierstrass was made nearly by everybody and was not noticed only because it is the rare kind of a mistake which actually cannot do much harm to calculation. Nevertheless, I believe that all those difficulties which lie in the concept of an irrational are associated with the above mistake, while, if this mistake is avoided, an irrational number will lie in place in our soul as definitely, clearly and distinctively, as a rational number.

The form of Mr. Dedekind's definition is based on a *totality of all* rational numbers, however, divided into two groups, so that we will denote numbers of one group U_v, and numbers of the other group through B_μ, and it will always be that $U_v < B_\mu$. Mr. Dedekind calls such division of a multitude of rational numbers "section", denotes it through $(U_v \big| B_\infty)$, and puts in correspondence with number *b*. If you compare these two sections $(U_v \big| B_\infty)$ and $(U'_v \big| B'_\mu)$, as with the *first* form of determination, only *three* options are possible, according to which numbers *b* and *b′* present in sections accordingly are either equated with each other or it is assumed that $b > b'$ or $b < b'$. The first case occurs – if you abstract from certain easily regulated exceptions which arise if the numbers being defined are rational – only where sections are completely identical. This is the definitive and absolute advantage of this form of definition compared to others, that is to say, the advantage is in the fact that there is *only one* section that corresponds to the number *b*. However, this form has a large shortcoming – numbers in the analysis are *never* represented by "sections", and they have to be inserted in this form in a quite artificial and complex way.

And here follow definitions in the form of a sum of $b + b'$ and product of bb' based on new sections obtained from the two preset ones.

A shortcoming associated with the *first* and *third* forms of definition, that is to say, the same, i.e. equal numbers are presented infinitely often and, therefore, all real numbers cannot be directly unequivocally viewed, may be quite easily eliminated by way of specialization of underlying multitudes (a_v), if one of the well-known single-valued systems like the decimal system or simple continued fraction decomposition is considered.

Now, let us proceed to the *third* form of definition of real numbers. This form is based on an infinite set of rational numbers (a_v) of first potency as well, however, now, a different property is attributed to it, not like in Weierstrass' theory. Namely, I demand that, having taken an arbitrary small rational number ε, the finite number of elements of a set could be deleted, so that each two of the remaining ones could have a difference the absolute value where of would be less than ε. I call any such multitude (a_v) which can be characterized as equality $\lim_{v=\infty}(a_{v+\mu} - a_v) = 0$ (with arbitrary μ) a *fundamental sequence* and put it in correspondence with a number *b* it determines, for which it would be advisable even to use the same notation (a_v) as Mr. Heine did, who, after numerous oral discussions rallied to my opinion in these matters (See Crelle's Journal, v. 74, p. 172). Such *fundamental sequence* as may be strictly developed from its concept leads to three options: either its members a_v for sufficiently great values of *v* the absolute value whereof is less than any present number; or they are, starting from a *v*, greater than a definitely predetermined positive rational number ρ, or they are, starting from a known *v*, less than a definitely predetermined negative rational value –ρ. In the first case, I say that *b* is equal to zero; in the second case, that *b* is greater than zero, or positive; and in the third case, that *b* is less than zero, or negative.

Thereafter, we proceed to elementary operations (sum, product, ratio), including those involving rational *a* and irrational number.

And it is only now that we proceed to definition of an equality and both cases of inequality of two numbers b and b' (where b' may also equal a), saying that $b = b'$, $b > b'$, or $b < b'$ – depending on whether the difference $b - b'$ equals zero, is more than zero, or less than zero.

Given these preparatory reasoning, we proceed with the first *strictly provable* theorem which says that if b is a number defined by a fundamental sequence (a_v), then the absolute value of $b - a_v$ with growing v becomes less than any conceivable rational number, or, in other words, $\lim_{v=\infty} a_v = b$.

It should be noted that the following depends on something whose essence can be easily missed: in the case of the third form of definition, the number b is not at all defined as a "limit" of elements a_v of fundamental sequence (a_v). If we accepted this, it would mean to make the same logical mistake as the one we talked about when we considered the *first* form of definition because in that case it is assumed in advance that $\lim_{v=\infty} a_v = b$ exists. However, the situation is rather reversed, that is, thanks to our previous definitions, the concept of the number b is said to have such properties and relationships to rational numbers that it can be with logical clearness concluded as follows: $\lim_{v=\infty} a_v$ exists and equals b. Forgive me all these details. They are justified by the fact that most people miss these indiscernible details and thereafter easily come across contradictions in irrational numbers and doubt them, while, had they observed the above precautions, this would easily prevent such things. In fact, if they observed these precautions, they would clearly understand that due to the *properties assigned to it by our definition*, an irrational number is as real for our spirit as a rational one, even as a whole rational number, and that it need not at all be *obtained* through a limit process. It is rather vice versa, *possessing* these properties, one can generally ascertain the soundness and clearness of limit processes. In fact, the above theorem can be easily summarized as follows: if (b_v) is a multitude of rational or irrational numbers in which $\lim_{v=\infty}(b_{v+\mu} - b_v) = 0$ (whatever μ may be), then there is a number b defined by fundamental sequence (a_v), and $\lim_{v=\infty} b_v = b$.

It therefore turns out that *those* numbers b which were determined on the basis of fundamental sequences (a_v) (I call these fundamental sequences "first-order sequences") so that they turn to be limits a_v, may be set out in different ways and as limits of sequences (b_v), where each b_v is defined with the help of first-order fundamental sequence $(a_\infty^{(v)})$ (at fixed v).

Therefore, if any such multitude (b_v) possesses such property that $\lim_{v=\infty}(b_{v+\mu} - b_v) = 0$ (with arbitrary μ), I use to call it a "*second*-order fundamental sequence".

Similarly, one can form fundamental sequences of the *third*, *fourth*,…, n^{th} order, and fundamental sequences of order α, where α is any number of the second number class.

All fundamental sequences provide the same thing for definition of any real number b as the first-order fundamental sequences. The only difference is but in a more complex extended form of assignment.

Now I use the following way of expressing it: numerical value b is given by a fundamental sequence of the n^{th}, therefore, α, order. If we dare do this, we will thus obtain a remarkably simple and, at the same time, straightforward language to describe the full abundance of diverse, often so complex, constructions of analysis in a most simple and prominent way. This, I believe, will materially contribute to the clearness and transparency of narrative. This way I protest against concerns voiced by Mr. Dedekind in the foreword to his work "Continuity and irrational numbers". It never occurred to me to introduce new numbers with the help of fundamental sequences of the second, third, etc. orders which would not have been already determined with the help of fundamental sequences of the first order: I meant only conceptually different form of an assignment. This is clearly apparent from various parts of my work.

Here I would like to address one remarkable circumstance that orders of fundamental sequences I distinguish with the help of numbers of the first and second number classes absolutely exhaust all forms of regular types of sequences, already found or not yet found, which one can imagine in analysis – exhaust in the meaning that there are no fundamental sequences at all (as I am going to strictly prove in other circumstances) the ordinal whereof could be denoted by any other number, e.g. of third number class".

Conclusions. Change in the type of mathematical definitions

Weierstrass defined a real number as a limit of partial sums of absolutely covergent series, noting the need for arithmetization of the concept of a limit. A point on a line corresponds to each number. However, it is not obvious that a number corresponds to each point. Cantor considered points on a line, defining the distance between them as a limit of a sequence and introducing relations of greater than, less than, and equal to. He introduced an axiom that, vice versa, a point on a line corresponds to each numerical value, the coordinate of such point being equal to this numerical value – equal in such meaning as set forth in this paragraph. Cantor called this statement an axiom, as it is unprovable due to its very nature. Thanks to it, numerical values additionally gain definite objectivity, on which, however, they do not depend at all. Dedekind believed that numbers are subjects of the "world of our thoughts", and it was our right to believe they were related to points. Unlike the above authors, Weierstrass defined a real number as a limit of partial sums of absolutely uniformly convergent series, noting the need for arithmetization of the concept of a limit.

Cantor was developing a perceptual theory of point sets, truly believing that applications were subsidiary issues. Years later, his theory of point sets devised as a summary of contemporary analysis formed the basis of mathematics.

Dedekind developed and arithmetical concept of a number as an algebraist, not being inclined to problems of analysis. Fifteen years later, his design led to creation of Dedekind–Peano arithmetic axiomatics.

Heine pursued educational goals. His narrative on limit and continuity was included in modern courses of analysis. Simultaneously, he set forth a number of fundamental principles: on disregarding of a certain number of points; a covering lemma; the concept of uniform convergence.

Creation of Charles Méray was recognized by his fellow countrymen a century later and is now called a "Méray–Heine" or "Méray–Cantor concept of a number".

After Cantor created the set theory, the language and internal structure of mathematics changed. It did not need anymore a geometrical or physical interpretation and gained a material descriptive component. Language and descriptive forms became the creating tool. The set theory was created as a continuation of arithmetics. However, already ten years later it formed the basis of the theory of a real number. It provided the opportunity to analyze the finest shades of designing mathematical objects and links between them. Many definitions and statements were formed verbally, retaining a high degree of abstraction. This caused discussions among mathematicians devoted to contradictions, many of which were linguistic in their nature. However, a new theory was created as a result, descriptive set theory, the key results of which belong to mathematicians of Warsaw and Moscow schools.

References

[1] Newton I., *Arithmetica Universalis, sive de compositione et resolution Arithmetica liber*, Cantabrigiae 1707.

[2] Kaestner A.G., *Anfangsgründe der Arithmetik, Geometrie, ebenen und sphärischen Trigonometrie und Perspektiv*, 1. Aufl., Göttingen 1758.

[3] Cauchy A.L., *Course d'analyse de l'Ecole royal polytechnique. Premiere partie: Analyse algébrique*, Oeuvres complčtes. Série 2, tome 3, Paris 1882–1974.

[4] Méray Ch., *Remarques sur la nature des quantités définies par la condition de servir de limites ŕ des variables données*, Revue des Sociétés savantes, Sci. Math. Phys. Nat., (2) 4, 1869, P. 280--289.

[5] Méray Ch., *Nouveau précis d'analyse infinitesimale*, Publication: F. Savy, XXIII, Paris 1872, 310 p.

[6] Heine E., *Die Elemente der Functionenlehre*, J. reine angew. Math., 74, 1872, p. 172-188.

[7] Cantor G., *Ueber die Ausdehnung eines Satzes der Theorie der trigonometrischen Reichen*, Mathem. Annalen von Clebsch und Neumann, Bd. 5, 1872, p. 123-132.

[8] Dedekind R., *Stetigkeit und irrationale Zahlen*, Braunschweig, Vieweg 1872.

[9] Weierstrass K., *Ausgewählte Kapitel aus der Funktionenlehre*, Vorlesung gehalten in Berlin 1886 mit der Akademischen Antrittsrede, Berlin 1857 und drei weiteren Originalarbeiten von K. Weierstrass aus den Jahren 1870 bis 1880/86. Teubner-Archiv zur Mathematik. Band 9, 272 s. Reprint 1989.

[10] Cantor G., *Grundlagen einer allgemeinen Mannichfaltigkeitslehre. Ein mathematisch--philosophischer Versuch in der Lehre des Unendlichen*, Leipzig, Teubner, 1883.

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE

1-NP/2014

MAGDA SIUDA-BOCHENEK*

JAN JĘDRZEJEWICZ – A PROFESSIONAL AMONG AMATEUR ASTRONOMERS

JAN JĘDRZEJEWICZ – PROFESJONALISTA WŚRÓD ASTRONOMÓW AMATORÓW

A b s t r a c t

Jan Jędrzejewicz was an eminent Polish amateur astronomer. He lived and worked as a doctor in a small town of Płońsk, situated 60 km of Warsaw. His great passion was astronomy and he devoted his all free time to it. After gaining essential knowledge, he built observatory, which he professionally equipped with his own funds. The main subject of his work was micrometer measurements of double stars, to which he applied himself with unusual precision and diligence. This was appreciated by an American astronomer S.W. Burnham, who included these results in his catalogue of double stars. Jędrzejewicz also observed the Sun, comets, planets and other sky phenomena, and the results of his works were published in the international journals: "Astronomische Nachrichten" and "Vierteljahrsschrift Astronomischen Ggesellschaft". Noteworthy in his papers are extremely thorough investigation of the subject and a great number of references to papers of contemporaneous professional astronomers. Jędrzejewicz aroused interest of the scientific world, which was demonstrated by the fact that information about him appeared several times in the journal "Nature".

Keywords: *astronomy in Poland*, *nineteen-century astronomy*, *amateur astronomy*

S t r e s z c z e n i e

Jan Jędrzejewicz był wybitnym polskim astronomem amatorem. Mieszkał i pracował jako lekarz w małej miejscowości Płońsk, leżącej 60 km od Warszawy. Jego wielką pasją była astronomia, której poświęcał cały wolny czas. Po zdobyciu niezbędnej wiedzy postanowił zbudować obserwatorium, które z własnych środków profesjonalnie wyposażył. Głównym tematem prac Jędrzejewicza były pomiary mikrometryczne gwiazd podwójnych, którym poświęcał się z niezwykłą precyzją i sumiennością. Zostało to docenione przez amerykańskiego astronoma S.W. Burnhama, który umieścił te wyniki w swoim katalogu gwiazd podwójnych. Jędrzejewicz obserwował również Słońce, komety, planety i wszelkie pojawiające się w owym czasie na niebie zjawiska, a wyniki swoich prac publikował w zagranicznych czasopismach: „Astronomische Nachrichten" i „Vierteljahrsschrift Astronomischen Ggesellschaft". W pracach Jędrzejewicza na uwagę zasługują jego niezwykle dogłębne badanie tematu oraz liczne odnośniki do prac współczesnych mu zawodowych astronomów. Jęrzejewicz wzbudzał zainteresowanie świata naukowego, o czym może świadczyć fakt, że kilkakrotnie pojawiła się o nim informacja w czasopiśmie „Nature".

Słowa kluczowe: *astronomia w Polsce*, *astronomia dziewiętnastowieczna*, *astronomia amatorska*

* Magda Siuda-Bochenek, Institute for the History of Science of the Polish Academy of Science.

1. Introduction

Jan Jędrzejewicz was born in 1835 in Warsaw. His father was an insurgent exiled to Syberia, who after his return worked in the judiciary. Jędrzejewicz attended secondary schools in Warsaw and then began to study architecture at the School of Fine Arts. Due to deteriorating eyesight he dropped out of college but in 1856 he started medical studies at the University of Moscow[1]. In 1862, as a qualified doctor, Jędrzejewicz moved to Płońsk, where he began his medical practice. He lived and worked in Płońsk almost continuously until his untimely death in 1887. He contracted typhus- probably from one of his patients- and, being overworked, he was not able to fight the disease[2].

Outside professional life Jędrzejewicz had a great passion – astronomy. He devoted to it all of his free time and probably all of his savings. But Jędrzejewicz's interest in astronomy was not limited to reading books and articles on the subject and watching the sky on a clear night. He decided to built his own observatory and make his contribution to the development and popularization of that field of science.

2. Observatory at Płońsk

From 1873 to 1875 he built observatory by himself and equipped it with the instruments for astronomical and meteorological observations. In the first building of the observatory there was a transit instrument with an objective diameter of 6.3 cm. It was made in M. Gerlach's workshop in Warsaw. In the second building, covered by a rotating dome, there was a parallactically mounted telescope with an objective diameter 16.2 cm. Jędrzejewicz ordered it from Steinheil's workshop in Munich. The telescope had eyepieces for stellar and solar observations and also a Merz ring micrometer[3]. It was the most important observational instrument in the observatory.

In 1883 Jędrzejewicz bought a Cooke refractor from the widow of Antoni Lewicki, who had built his astronomical observatory in Częstochowa. The telescope with an objective diameter of 14 cm had a clockwork mechanism allowing one to keep up with daily rotation of the sky[4]. The refractor was ordered from a workshop in London and Jędrzejewicz bought it together with a dome, which enlarged his observatory to three buildings[5].

In addition to those instruments, Jędrzejewicz had a Browning solar spectroscope, a small spectroscope, previously a property of Dr. Vogel, a polarimetric helioscope, a wire

---

1 J. Kowalczyk, *Wiadomości o obserwatorium w Płońsku i o pracach Jana Jędrzejewicza w dziedzine astronomii i meteorologii*, Prace matematyczno-fizyczne, t. 1, 1888, p. 113.

2 H. Dobrzycki, *Dr. Jan Jędrzejewicz. Lekarz, astronom i obywatel, założyciel spostrzegalni astronomicznej i stacyi meteorologicznej w Płońsku*, Medycyna, t. 15, nr 52, 1887, p. 845.

3 J. Jędrzejewicz, *Schreiben des Hern Dr. Jedrzejewicz an den Herausgeber*, Astronomische Nachrichten, t. 95, nr 2279, 1879, p. 353-354.

4 J. Jędrzejewicz, *Jahresberichte der Sternwarten fur 1883, Plonsk*, Vierteljahrsschrift der Astronomischen Gesellschaft, t. 19, 1884, p. 129.

5 J. Kowalczyk, *O działalności ś.p. dra Jana Jędrzejewicza w dziedzinie astronomii i meteorologii*, Wszechświat, t. 7, nr 3, 1888, p. 39.

micrometer, a Busch telescope used as a heliograph, a wedge photometer and also some smaller telescopes[6].

Jędrzejewicz's observatory was very well equipped compared to university observatories located on Polish land. During the observatory's activity, Jędrzejewicz had the biggest refractor in Poland, which allowed him to conduct the most accurate observations. University observatories had many more instruments, because most of them functioned for a longer time and had bigger budgets. Nevertheless, Jędrzejewicz's instruments were newer and selected so that they could be fully used by a single observer. His instruments came from reputable workshops and- what is worth mentioning- they were perfectly selected for a small observatory. They were used to make both fundamental measurements (like determining observatory coordinates) and micrometrical observations of solar system bodies and double stars. Furthermore, they were used to conduct spectroscopic observations, fairly new at that time.

3. Jędrzejewicz's astronomical observations

The range of observations which Jędrzejewicz was able to make with his instruments was quite wide. The most extensive and the most important of them were micrometrical measurements of double stars. In his first article dedicated to that subject Jędrzejewicz explained that "micrometrical measurements of double stars are so important in astronomy that it is necessary to increase their number"[7].

A person who suggested that he should observe double stars was Dr. Hermann Vogel, an astronomer from the observatory in Potsdam. According to him, observations of this kind were perfect for instruments possessed by Jędrzejewicz[8]. Using a Steinheil refractor Jędrzejewicz conducted measurements of the position angle and angular distance between two component stars. It is worth mentioning that Jędrzejewicz chose the stars that had not been observed for a long time and then the stars that needed to be observed regularly. From 1876 to his death in 1887 he measured more than 350 double stars. The results of his measurements were published in "Astronomische Nachrichten" in 14 articles.

A lot of observational time Jędrzejewicz devoted to comets. He made his observations with the Steinheil telescope and later with the Cooke telescope, both equipped with the same ring micrometer. From 1881 to 1887 Jędrzejewicz observed 16 comets. In articles published in "Astronomische Nachrichten" he presented tables with the positions of comets at different times together with the position of reference stars.

An important part of Jędrzejewicz's work were observations of the Sun. In his observations of sunspots Jędrzejewicz followed Dr. Gustav Spoerer, an expert on that phenomenon. Using Busch's telescope, equipped with a screen, he made drawings of sun spots and determined their heliographic coordinates. The most interesting spots were drawn separately and on a larger scale. Positions of spots that were close to an edge of the solar

---

6 J. Kowalczyk, *Wiadomości o obserwatorium w Płońsku...*, *op. cit*., p. 115.

7 J. Jędrzejewicz, *Mesures micrometriques des etoiles doubles*, Astronomische Nachrichten, t. 97, nr 2324, 1880, p. 305.

8 J. Jędrzejewicz, *Schreiben des Hern Dr. Jedrzejewicz...*, *op. cit*., p. 355.

disc were measured directly with micrometer mounted on the Steinheil's refractor and were examined with the Browning's solar spectroscope. In Jędrzejewicz's opinion the observations he made could be useful for other observers if there were some accidental gaps in their measurements[9].

Jędrzejewicz also made spectroscopic observations of the Sun but he did not conduct them very often because his spectroscopes were rather small and were used mainly for education. Despite modest equipment he was able to conduct very useful measurements. Jędrzejewicz noticed that in the professional journals there were no sketches of solar spectrum observed with small spectroscope. There were only sketches seen by big instruments, which showed more spectrum lines with bigger precision. This is why he decided to fill the gap[10].

At the end of 1880 and in the beginning of 1881 Jędrzejewicz observed Jupiter. He paid attention to the red spot, which on the grounds of "visible contours could be used to determine the period of rotation of the planet and more specifically of its atmosphere"[11]. Based on the observations of 174 rotations of Jupiter, Jędrzejewicz determined the time of revolution of the red spot around Jupiter and using micrometer he determined the position of the spot.

Moreover, Jędrzejewicz observed the transit of the red spot across the central meridian. He used a method he devised himself for this purpose. He placed a wire of the micrometer in a position that bisects the disc of the planet and then he recorded the times when the eastern edge of the spot touched the wire and when the spot reappeared at the other side of the wire[12].

In addition to the objects already mentioned Jędrzejewicz observed Mercury, Saturn, lunar eclipse and supernova in Andromeda nebula. He attached drawings of some of them to his popular articles published in the Polish magazine "Wszechświat".

On the 19th of August 1887 the total solar eclipse was visible in Western and Central Europe. Like many astronomers, Jędrzejewicz wanted to take this opportunity to observe solar corona and therefore he headed the scientific expedition to Vilnius. He took necessary instruments and carefully prepared the program of observations. Unfortunately, his efforts were wasted because the observations were impossible due to clouds.

Jędrzejewicz's astronomical work gains importance when it is compared to observations made in two major Polish observatories – in Warsaw and Cracow. In Warsaw astronomers made mostly observations of planets, asteroids and comets, but rather occasionally. Only the senior assistant, Jan Kowalczyk, undertook an ambitious program of observations. For 20 years he made positional observations which created a catalogue of stars. Kowalczyk also made calculations in order to determine orbits of planets, asteroids and comets[13]. In Cracow both the director of observatory, Franciszek Karliński, and an assistant, Daniel Wierzbicki, made positional observations of comets and asteroids, but this also happened

[9] J. Jędrzejewicz, *Jahresberichte der Sternwarten fur 1883...*, *op. cit.*, p. 129-130.
[10] J. Jędrzejewicz, *Widma małych spektroskopów*, Wszechświat, t. 4, nr 28, 1885, p. 439-443.
[11] J. Jędrzejewicz, *La tache rouge de Jupiter*, Astronomische Nachrichten, t. 99, nr 2366, 1881, p. 211.
[12] J. Jędrzejewicz, *La tache rouge...*, *op. cit.*, p. 212.
[13] J. Kowalczyk, *Krótki rys dziejów Obserwatorium Warszawskiego od r. 1820–1900*, Wiadomości Matematyczne, t. XI, 1907, p. 94-95.

only occasionally[14]. In both observatories astronomical works were put aside because of other duties. Astronomers had to conduct works in meteorology, geophysics (in Cracow) and also geodesy. It was often difficult to make observations because of bad condition of buildings and frequent renovations.

4. Jędrzejewicz in a mainstream of nineteen-century astronomy

In order to assess whether Jędrzejewicz's astronomical work were in the mainstream of nineteen-century astronomy quantitative analysis of observations was made. Observations published in "Astronomische Nachrichten" during the period of time when Jędrzejewicz's articles appeared there were taken into account.

What is the most striking is a fact that astronomers were mostly interested in solar system bodies. As much as 78 percent of observations were devoted to those objects and among those outnumbered were comets. Jędrzejewicz definitely followed that trend and in the years 1881–1887 he observed 16 comets, 14 of which belong to group of 30 comets the most often observed by astronomers in Europe.

In a group of solar system objects – the Sun, planets, moons and meteors – nearly all of them were observed with uniform frequency and there was no object that dominated others in this respect. Jędrzejewicz observed those objects as well and out of 15 presented objects he observed 8.

Observations of objects outside our solar system, that is nebulae and, generally speaking, stars constituted 18 percent of all observations published in "Astronomische Nachrichten". Among the observations included in the group of stars, double stars – Jędrzejewicz's objects of interest- made up as much as one fifth.

Between 1879 and 1887 Jędrzejewicz published 31 articles, 13 of which were devoted to double stars and 13 to comets. Since 1880, every two years he has sent reports on his observational activity to "Vierteljahrsschrift der Astronomischen Gesellschaft" of German astronomical society.

At that time in "Astronomische Nachrichten" only six articles appeared about observations made in other Polish observatories – 4 from Warsaw and 2 from Cracow. Articles from Warsaw presented results of observations of Jupiter, Mercury, 5 asteroids and 2 comets made by Kowalczyk. In Cracow Wierzbicki focused on 3 comets. These numbers are actually difficult to compare with those of Jędrzejewicz's works. As mentioned before, these observations were made occasionally and they were not a part of regular observational program.

5. Recognition in the West

Results of Jędrzejewicz's observations published in international journals were noticed and used by professional astronomers. High quality of the micrometrical measurements made

[14] J. Mietelski, *Obserwatorium Astronomiczne Uniwersytetu Jagiellońskiego w okresie dyrekcji Franciszka Karlińskiego (1862–1902)*, Zeszyty Naukowe Uniwersytetu Jagiellońskiego DCCCIII, Prace Fizyczne, z. 25, 1986, p. 24-27.

by Jędrzejewicz is proved by a great number of double stars catalogues in which his results are presented. Two of them were catalogues created by S.W. Burnham, an American astronomer and double stars observer of international renown[15],[16], and another one by Thomas Lewis, another double star expert[17].

An interesting publication in which Jędrzejewicz's results are included is "A cycle of celestial objects by George F. Chambers"[18]. In the preface to a revised edition, the author specially thanked Jędrzejewicz for his measurements of double stars that other observers often neglected, which helped him to fill the gaps. Besides Jędrzejewicz, the author thanked only one person – S.W. Burnham. This book, from 1881, is probably the first one in which Jędrzejewicz's results were included and appreciated.

Jędrzejewicz's observations of the red spot on Jupiter were also noticed. In "Monthly Notices of the Royal Astronomical Society" Joseph Gledhill, astronomer from Bermerside Observatory in England, presented different methods of observing the transit of the spot across the central meridian[19]. One of them was Jędrzejewicz's method. After comparing all methods one can come to the conclusion that Jędrzejewicz created his method because it was suitable for his refractor and micrometer. Other methods required a micrometer with more than two movable wires or a refractor with bigger objective. There was another method called "simple eye estimation", which according to the author was often used by amateurs, but apparently Jędrzejewicz was not interested in it.

Some interesting information about Jędrzejewicz can be found in the journal "Nature". In the first note about him, Jędrzejewicz's observatory is described and coordinates of it are presented. There is also information that "number of known observatories of this class (that is, at the level of private observatories in England) upon the continent of Europe is not great"[20]. It is also highlighted that amateur astronomers often observe double stars that were observed many times. They could make better use of their instruments and observe objects neglected by others.

In an article published in 1884[21] there is an abstract of an article from the Russian journal "Novoye Vremya", in which Russian private observatories are presented. Among

---

[15] S.W. Burnham, *A general Catalogue of double stars*, The Carnegie Institution of Washington, Waszyngton 1906.

[16] S.W. Burnham, *A general catalogue of 1290 double stars discovered from 1871 to 1899 by S.W. Burnham. Arranged in order of right ascension with all the micrometrical measures of each pair*, The University of Chicago Press, Chicago 1900.

[17] T. Lewis, *Mesures of the double stars contained in the Mensurae Micrometricae of F.G.W. Struve*, Londyn 1906.

[18] W.H. Smith, G.F. Chambers, *A cycle of celestial objects observed, reduced, and discussed by admiral William Henry Smith. Revised, condensed, and greatly enlarged by George F. Chambers*, The Clarendon Press, Oxford 1881.

[19] J. Gledhill, *On certain phenomena presented by Jupiter's satellites and their shadows during transit, with a note on the red spot; and on some methods of observing the transits of bright and dark spots across the central meridian*, Monthly Notices of the Royal Astronomical Society, t. 56, 1896, p. 494-500.

[20] *Our Astronomical Column*, Nature, t. 20, 1879, p. 629.

[21] *Notes*, Nature, t. 30, 1884, p. 251-253.

them the observatory in Płońsk together with equipment and range of observations is listed with the conclusion: "His observations of double stars are considered most accurate by astronomers".

Noteworthy is the fact that during the time when the information about Jędrzejewicz was published in "Nature", there was only one piece of information about another observatory activity in Poland, that is sunspots observations made in Vilnius.

Jędrzejewicz, despite being an amateur, managed to build his own observatory, which he made into an institution at European level. He definitely stood out among astronomers in Poland. The wide range of observations he made and a great number of articles he published are the best evidence. His observations belonged to the mainstream of nineteenth--century astronomy and his specialty – measurements of double stars – was appreciated by professional astronomers.

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE

1-NP/2014

MAŁGORZATA STAWISKA*

LUCJAN EMIL BÖTTCHER (1872–1937) – THE POLISH PIONEER OF HOLOMORPHIC DYNAMICS

LUCJAN EMIL BÖTTCHER (1872–1937) – POLSKI PIONIER DYNAMIKI HOLOMORFICZNEJ

A b s t r a c t

In this article I present Lucjan Emil Böttcher (1872–1937), a little-known Polish mathematician active in Lwów. I outline his scholarly path and describe briefly his mathematical achievements. In view of later developments in holomorphic dynamics, I argue that, despite some flaws in his work, Böttcher should be regarded not only as a contributor to the area but in fact as one of its founders.

Keywords: *mathematics of the 19th and 20th century*, *holomorphic dynamics*, *iterations of rational functions*

S t r e s z c z e n i e

W artykule tym przedstawiam Lucjana Emila Böttchera (1872–1937), mało znanego matematyka polskiego działającego we Lwowie. Szkicuję jego drogę naukową oraz opisuję pokrótce jego osiągnięcia matematyczne. W świetle późniejszego rozwoju dynamiki holomorficznej dowodzę, że mimo pewnych wad jego prac należy nie tylko docenić wkład Böttchera w tę dziedzinę, ale w istocie zaliczyć go do jej twórców.

Słowa kluczowe: *matematyka XIX i XX wieku, dynamika holomorficzna*, *iteracje funkcji wymiernych*

* Małgorzata Stawiska, Mathematical Reviews, 416 Fourth St., Ann Arbor, MI 48103, USA, email: stawiska@umich.edu.

1. Introduction

Holomorphic dynamics – in particular the study of iteration of rational maps on the Riemann sphere – is an active area of current mathematical research. Among many mathematicians who have worked in it there are several recipients of the Fields medal (the highest honor in mathematics): John Milnor, William Thurston, Curtis McMullen and Jean-Christophe Yoccoz. While the theory requires rather deep knowledge of concepts and methods of various areas of mathematics, including complex function theory, dynamical systems, topology, number theory, etc., its objects have become to some extent present in the popular culture, thanks to amazing computer-generated pictures of Julia sets and the Mandelbrot set. Some problems in holomorphic dynamics can be traced back to Arthur Cayley or even Isaac Newton, but the common view is that it got its start as a systematic and separate area of mathematics around 1918, with works of Pierre Fatou, Gaston Julia, Samuel Lattés and Salvatore Pincherle. The beginnings are discussed in two recent historical monographs: one by Michèle Audin ([Au]) and one by Daniel Alexander, Felice Iavernaro and Alessandro Rosa ([AIR]). These publications, as well as recent textbooks on the subject of holomorphic dynamics, mention Lucjan Emil Böttcher among earlier contributors to the area. His result on the local behavior of a holomorphic function near its superattracting fixed point, referred to as Böttcher's theorem (in which so-called Böttcher's equation and Böttcher's function appear) is a classic one:

Theorem. *Let* $f(z) = a_m z^m + a_{m+1} z^{m+1} + \ldots$, $m \geq 2$, $a_m \neq 0$ *be an analytic function in a neighborhood of* 0. *Then there exists a conformal map* F *of a neighborhood of* 0 *onto the unit disk,* $F(z) = z + bz^2 + \ldots$, *satisfying the equation* $F f(z) = [F(z)]^m$.

This note concerns Lucjan Emil Böttcher, the author of the above result, a Polish mathematician active at the turn of the 19th/20th centuries, who until recently remained a rather obscure figure. Commonly available biographical information about him came from his curriculum vitae which he wrote in Latin and attached to his doctoral thesis published in 1898, so almost nothing was known about his later years[1]. His mathematical output also became largely forgotten. One can point to several possible reasons why this could happen. First, Böttcher wrote mostly in Polish, so his work could not be widely read by international mathematical community (his most cited paper was written in Russian.) Second, he was active in Lwów (later Lvov, now Lviv), an academic center frequently caught in the turbulent history of the 20th century (wars, changing political borders and governments), so many documents of his activity could not be easily retrieved; some might have been dispersed or lost. Third, a reason more interesting for a historian of mathematics, Böttcher's results were not appreciated by his contemporaries from academic establishment. He was an academic teacher, but never made it to the rank of a professor, so he could not disseminate his ideas by guiding doctoral students or holding specialized lectures and seminars (although, as a part of his habilitation proceedings, he proposed a plan of lectures on the general theory of iteration).

[1] Stanisław Domoradzki discovered many materials concerning Böttcher and gave them scholarly treatment in his recent publications [4] and [5]. He and I carried out an extensive analysis of Böttcher's work in our joint article [6].

One should note that Böttcher's work was quite removed from interests of Lwów mathematicians of that time. Worse, it contained flaws: most proofs were only sketched, some conclusions were unjustified and the notions were not always well defined. His critics did not see a wealth of ideas, examples and partial results which amounted to almost complete outline of the theory developed independently only some 20 years later by Fatou, Julia, Lattés and Pincherle. In what follows I will talk about perception of Böttcher's work by his contemporaries. I will go back to his doctoral studies in Leipzig with Sophus Lie and mention a controversy regarding the evaluation of Böttcher's thesis, in which Lie stood for his student against his academic colleagues. I will also discuss Böttcher's later academic career in Lwów, in particular his repeated– but ultimately unsuccesful– attempts to obtain habilitation at the Lwów University. It was only after 1920 that importance of Böttcher's results was realized by other mathematicians (starting with Joseph Fels Ritt, an American who published the first complete proof of Böttcher's theorem). Nowadays Böttcher's theorem is well known to researchers in holomorphic dynamics and functional equations (and was generalized in many ways, see [6]), and he rightly gets the credit for constructing the first example of an everywhere chaotic rational map. But there is more to Lucjan Emil Böttcher's mathematical output that needs to be better known and appreciated.

2. The life of Lucjan Emil Böttcher

Lucjan Emil Böttcher was born in Warsaw on January 7 (21 according to the new style calendar), 1872, in a family of Evangelical-Lutheran denomination. He attended private real schools in Warsaw and passed his maturity exam in 1893 in the (classical) gymnasium in Łomża. The same year he enrolled in the Division of Mathematics and Physics of the Imperial University of Warsaw (where Russian was the language of instruction), attending lectures in mathematics, astronomy, physics and chemistry. In 1894 he was expelled from the university for participating in a Polish patriotic manifestation. He left Warsaw and moved to Lwów. He became a student in the Division of Machine Construction at the Lwów Polytechnic School, where in 1896 he passed a state exam obtaining a certificate with distinction, and got his so-called half-diploma in 1897. Wishing to complete a course of university studies in mathematics, he then moved to Leipzig. He spent three semesters at the university there, attending lectures in mathematics, physics and psychology. He completed his studies getting the degree of doctor of philosophy (under the direction of Sophus Lie, one of the most important mathematicians of the 19th century) in 1898 on the basis of the doctoral thesis "Beiträge zu der Theorie der Iterationsrechnung" as well as examinations in mathematics, geometry and physics.

After finishing his studies Böttcher returned to Lwów and took a job of an assistant in the (Imperial and Royal) Lwów Polytechnic School (initially at the Chair of Mechanical Technology, then at the reactivated Chair of Mathematics). He had his PhD diploma nostrified in 1901. In 1910 he became an *adiunkt* and in 1911 he obtained the license to lecture (*venia legendi*) and habilitation in mathematics in the Lwów Polytechnic School. He lectured on many mathematical subjects in the engineering curriculum as well as on theoretical mechanics. Starting in 1901 he made several attempts to obtain habilitation also at the Lwów University, all of which were unsuccesful.

Besides academic teaching, Böttcher's activities comprised taking part in meetings and conventions of mathematicians and philosophers (he was a member of the Polish Mathematical Society founded in 1917), writitng and publishing papers in mathematics, mathematics education, logic and mechanics. He also wrote lecture notes, textbooks for the use in high schools and booklets on spiritualism and afterlife. The (so far most) complete list of his publications can be found in [6].

Lucjan Emil Böttcher retired from the Lwów Polytechnic School in 1935 and died in Lwów on May 29, 1937.

3. Böttcher in Leipzig

Lucjan Emil Böttcher enrolled in the University of Leipzig on February 1, 1897, in order to study mathematics. He took courses from Sophus Lie (theory of differential invariants, theory of differential equations with known infinitesimal transforms, theory of continuous groups of transformations, seminars in theory of integral invariants and in differential equations), Adolph Mayer (higher analytic mechanics), Friedrich Engel (differential equations, algebraic equations, non-euclidean geometry), Felix Hausdorff (similarity transformations), Paul Drude (electricity and magnetism), Gustaw Wiedemann (exercises in physics) and Wilhelm Wundt (psychology)[2]. On February 7, 1898, Böttcher submitted his dissertation "Beiträge zu der Theorie der Iterationsrechnung". His supervisor was Sophus Lie– the creator of the theory of continuous groups of transformations and one of the most important mathematicians of the 19th century[3]. Like many mathematicians who came into contact with Lie (in Leipzig and elsewhere– e.g., Victor Bäcklund, Élie Cartan, Wilhelm Killing, Emile Picard, Henri Poincaré, Eduard Study and Ernest Vessiot), Böttcher became fascinated with Lie's theories. He wanted to define iterations of maps with arbitrary exponents and then to study relations between iterations and functional equations, and he thought of using the theory of continuous groups of transformations as the framework for his considerations. In Chapter I of his dissertation he expressed some formal relations for iterations by means of one-parameter continuous groups of transformations. He formulated some "fundamental theorems" without proofs, claiming only that they were special cases of some results by Lie. Part II of Böttcher's dissertation was devoted to the study of iterations of rational functions of a complex variable (ranging over the Riemann sphere) and contain results and ideas which can be regarded as foundations of holomorphic dynamics (see further sections or [6] for more information). In part III Böttcher resumed the general study of relations between iteration and functional equations. The first draft of the dissertation was ready in January 1898 and was then presented to Lie.

---

[2] Information about Böttcher's coursework is taken from the curriculum vitae which he wrote in Polish in 1901, now in the archives of Lviv University.

[3] Sophus Lie (1842–1899) was of Norwegian nationality. His work made an impact on the development of modern geometry, algebra and differential equations. His results led to the emergence of new areas of mathematics, e.g., topological groups. They remain significant for today's mathematics and also found applications in quantum physics.

However, on March 1 Böttcher still could not be admitted to doctoral examinations. According to the handwritten note at the bottom of the document confirming the opening of the official proceedings for the degree of doctor of philosophy[4], signed by the pro--chancellor Ferdinand Zirkel, Wilhelm Scheibner refused to submit a report on Böttcher's dissertation, and at the urgent request of Lie a second examiner had to be found. (On the same document it can be seen that Scheibner's name as an examiner was struck out, and the name of Mayer was appended after the name of Lie.) This "urgent request" was in fact made in writing. Here are excerpts from Lie's note:

"At present I cannot recognize that the author has definitely managed to substantiate significant new results. Despite all of this, his considerations, which testify to diligence and talent, have their value (...)".

"In any case, I (as well as Mr. Scheibner) agree that this attempt be accepted as a thesis, and we also agree regarding the evaluation being II. I choose such a good grade because Mr. Böttcher himself chose his topic and developed it independently (...)".

"Under the conditions mentioned above, I support the acceptance of the dissertation with evaluation II and admission to the oral exam".

A further document dated April 27, 1898 records Böttcher's completion of the required examinations in mathematics and physics, along with the examiners' evaluations and overall grade "magna cum laude" (IIa), as well as his promotion to the degree. Sophus Lie's written evaluation was that "[t]he candidate is an intelligent mathematician, possessing good and solid knowledge". There was also a longer examination report by Adolph Mayer.

Lie recognized the failure of Böttcher's initial goal of grounding the theory of iteration in the theory of continuous groups of transformations. Nevertheless, his opinion on Böttcher and his achievements was high. This is especially noteworthy, as out of 56 students in Leipzig who completed their doctorates in mathematics between 1890 and 1898, 26 did so with Lie (including his later collaborator Georg Scheffers, as well as Kazimierz Żorawski). It should also be noted that 1898 was a difficult year for Lie (who was in poor health). On May 22, 1898, he oficially resigned from his position in Leipzig in order to take up a special professorship in Kristiania (now Oslo). He was busy with writing up his research monographs and trying to have an input in naming his successor. In September 1898 Lie returned to Norway and in February 1899 he died of pernicious anemia. (For information on the life and work of Sophus Lie and mathematics at Leipzig, cf. [3, 7, 10]). Lucjan Böttcher, Charles Bouton and Gerhard Kowalewski were his last doctoral students.

4. Böttcher in Lwów

In 1901 Böttcher arrived in Lwów and took a position of an *asystent* in the Lwów Polytechnic School. His PhD diploma from Leipzig was nostrified (i.e., officially recognized) at the Lwów University. The same year (in a letter dated October 17) he applied for admission to habilitation at the Lwów University. Along with the diploma and curriculum vitae, he submitted offprints of two papers: "Principles of iterational calculus,

---

[4] The documents about Böttcher's doctoral proceedings come from the archives of University of Leipzig. All documents cited in this paper were found by S. Domoradzki.

part three" and "On properties of some functional determinants", as well as a plan of lectures for 4 semesters. Böttcher's application was considered by a committee whose members were Józef Puzyna[5], Jan Rajewski[6], Marian Smoluchowski[7] and the dean Ludwik Finkel. The committee's decision, made on February 6, 1902, was not to admit Böttcher to habilitation. His scientific results were deemed correct but insufficient, although the committee also noted that the theory of iteration itself was not yet a well developed area of mathematics. The recommendation was to wait until the official publication of "Principles of iterational calculus, part three" (or of potential works by Böttcher in areas of mathematics other than iteration and functional equations).

In 1911 Böttcher obtained *veniam legendi* in mathematics in the c.k. Polytechnic School in Lwów, where (since 1910) he was employed as an *adiunkt*. In 1911 he requested at the Faculty of Philosophy of the Lwów University that his license to lecture at the c.k. Polytechnic School be also recognized at the university. His request was denied.

Another time he applied for habilitation was in 1918. This time he had more publications to his name and he submitted six of them with the application. Here is the translation of (a fragment of) the committee's decision:

"The Candidate submitted along with the application the following works in mathematics:

1) Major laws of convergence of iterations and their applications in analysis. Two papers in Russian, Kazan, 1903, 1905.
2) A note of solving the functional equation $\Psi f(z) - \Psi(z) = F(z)$, Wiadomości Matematyczne, vol. 13, Warsaw 1909.
3) Principles of iterational calculus, Wektor, 1912, Warsaw.
4) Nouvelle méthode d'intégration d'un système de n équations fonctionelles lineairés du premier ordre de la forme $U_i(z) = \sum_{j=1}^{j=n} A_{i,j}(z) U_j F(z)$, Annales l'Ecole Normale Supérieure, Paris, 1909.
5) A contribution to the calculus of iteration of a rational entire function, Wiadomości Matematyczne, vol.14, Warsaw 1912.
6) Iteration $f_x(z)$ of an algebraic function $f(z)$ metatranscendental in the index x, in Russian, Kazan 1912".

"The paper no. 5 duplicates one written by the author in Polish and self-published already in 1905. (...) After formal deduction of formulas for a solution to the system of equations under investigation the author proceeds to give in §3 'A functional-theoretical discussion of the fundamental law', concluding boundedness of a certain set from finiteness of the numbers in it. Such reasoning is obviously erroneous, and therefore one cannot consider it to be proven that the series given by the author are – under his conditions – convergent".

---

[5] Józef Puzyna (1856–1919) specialized in complex function theory and wrote the first textbook in Polish on the subject.

[6] Jan Rajewski (1857–1906) worked on differential equations and hypergeometric functions.

[7] Marian Smoluchowski (1872–1917) was an outstanding statistical physicist, working on Brownian motions and diffusion.

"The method used by the Candidate in his works cannot be considered scientific. The author works with undefined, or ill-defined, notions (e.g., the notion of an iteration with an arbitrary exponent), and the majority of the results he achieves are transformations of one problem into another, no less difficult. In the proofs there are moreover illegitimate conclusions, or even fundamental mistakes. The author's popular, instructional works, e.g. 'Principles of iterational calculus' (Wektor 1912, no. 9, pp. 501-513, Warsaw), are written in an unclear manner (...)".

"Despite great verve and determination, Dr. Böttcher's works do not yield any positive scientific results. There are many formal manipulations and computations in them; essential difficulties are usually dismissed with a few words without deeper treatment. The content and character diverges significantly from modern research".

"One should also add:

1. The shortcoming, or rather lack of rigor of the definition of iteration with an arbitrary exponent introduced by the candidate met with justified and clearly written criticism by Dr. Stanisław Ruziewicz in Wektor, Warsaw 1912, no. 5 (On a problem concerning commuting functions).

2. Dr. Böttcher applies for a second time for veniam legendi in mathematics. The first time the candidate was advised to withdraw his application because of the faults that the Committee at that time found with the candidate's works. These faults and inadequacies were of the same nature which characterizes the candidate's work also today".

"The Committee's decision passed unanimously on June 21, 1918: Not to admit Dr. Böttcher to further stages of habilitation. (signature illegible)".

Böttcher made his last attempt to obtain habilitation at the University on May 1, 1919, also unsuccessfully. He remained in the position of an *adiunkt* at the Lwów Polytechnic School until his retirement in 1935.

5. The importance of Böttcher's work

There are 19 known mathematical research publications by Lucjan Emil Böttcher. The following are the most important for the development of holomorphic dynamics:

(1) Beiträge zu der Theorie der Iterationsrechnung, published by Oswald Schmidt, Leipzig, pp.78, 1898 (doctoral dissertation).

(2) Zasady rachunku iteracyjnego (część pierwsza i część druga) (Principles of iterational calculus (part one and two)), Prace Matematyczno Fizyczne, vol. X, 1899, 1900, pp. 65--86, 86-101

(3) Zasady rachunku iteracyjnego (część III) (Principles of iterational calculus (part III)), Prace Matematyczno Fizyczne, v. XII, 1901, p. 95-111

(4) Zasady rachunku iteracyjnego (część III, dokończenie) [Principles of iterational calculus (part III, completion)], Prace Matematyczno Fizyczne, v. XIII, 1902, pp. 353-371

(5) Glavn"yshiye zakony skhodimosti iteratsiy i ikh prilozheniya k" analizu (The principal laws of convergence of iterates and their application to analysis), Bulletin de la Societe Physico-Mathematique de Kasan, tome XIII, 1, 1903, p.137, XIV, 2, 1904, p. 155-200, XIV, 3, 1904, p. 201-234.

In his doctoral thesis Böttcher set out to develop a general theory of iteration of functions with an arbitrary (not necessarily integer) exponent, in the newly available framework of Lie groups. He managed to formulate some basic properties and outline the relation with functional and differential equations, but almost halfway through he switched to the study of iteration of rational maps of the Riemann sphere. Unlike many of his predecessors working on iteration, he was interested in global rather than local behavior of maps. Here are his main ideas and results, expressed in modern terminology:

– the study of individual orbits of (iterated) rational maps, of their convergence and the limits that occur;
– the study of "regions of convergence' (later called Fatou components) and their boundaries (Julia sets); determining the boundaries using backward iteration;
– relations between the local behavior of iterates and the magnitude of the derivative at a fixed point of a map;
– an example of an everywhere chaotic map, i.e., a map without regions of convergence constructed by means of elliptic functions;
– some observations about preperiodic points, which Böttcher called "Żorawski points".

These topics re-emerged after 1918 in the works of other mathematicians and have since served as foundations of holomorphic dynamics. Fatou and Julia (independently) took advantage of the theory of normal families, formulated by Paul Montel, to put forward and further study the division of the sphere (or the complex plane) into subsets in which iterates of a map display different behavior. The limit maps of convergent subsequences of iterates and the behavior of their derivatives were studied in detail. An example of an everywhere chaotic map derived from an elliptic function was constructed in 1918 by Samuel Lattés (hence the family of maps coming from such constructions is now known as Lattés examples). The work of Kazimierz Żorawski, preceding Böttcher's, has been nearly forgotten, but preperiodic points nowadays enjoy renewed interest, due to their importance in the study of the parameter spaces of families of rational maps, including the famous Mandelbrot set (in relation to so-called Misiurewicz points, introduced by and named after Michał Misiurewicz, a Polish mathematician active in the USA), as well as in arithmetic dynamics.

As for the relation between Lie's theory and iteration, the problem of defining iterations with an arbitrary exponent by embedding iterations of a function into a one-parameter continuous group of transformation cannot be solved in such generality as Böttcher hoped for. For rational maps, I.N. Baker showed in 1960s that it is impossible to carry out such an embedding in the whole complex plane. Partially defined embeddings can be obtained in some cases. On the other hand, Julia also was interested in continuous groups of transformations, namely Kleinian groups. Many deep parallels between Kleinian groups and iterations of rational maps were observed and systematically explored in 1980s by Dennis Sullivan.

Böttcher himself called the combined papers (2), (3) and (4) a translation of his thesis, but they contain more results on (what is now known as) holomorphic dynamics than (1) and their organization is quite different. The new material, appearing mostly in (2), is the following:

– a different example of an everywhere chaotic map and a sketch of proof of its chaotic behavior;
– determining boundaries of the "regions of convergence" for monomials and Chebyshev polynomials; study of simple dynamical properties of the "boundary curves", e.g. density of periodic points;
– brief attention given to irrationally neutral periodic points (without any conclusions);
– pointing out the role of critical points in the dynamics of rational maps; formulation of an exact upper bound for the number of (periodic) "regions of convergence" in terms of the number of critical points of the map;
– the first formulation of Böttcher's theorem (about the local behavior of a map near a superattracting fixed point; cf. the introduction) and a sketch of its proof.

Again, the properties of "boundary curves" (now known as Julia sets) were later studied in detail by Fatou and Julia (as well as by Salvatore Pincherle). Fatou also looked at irrationally neutral periodic points (pointing out that non-constant maps appear as limits of the sequences of iterates in a neighborhood of such a point) and examined the role of critical points in holomorphic dynamics. He postulated the same upper bound on the number of "regions of convergence" (more precisely, on the number of non-repelling periodic orbits) by the number of critical points as Böttcher did, but he was able to prove only a weaker one. The exact bound was finally proved by Mitsuhiro Shishikura in 1982; in 1999 Adam Epstein gave a different proof of the Fatou-Shishikura inequality.

The paper (5) is the most cited publication by Böttcher. He again formulated his theorem in it and sketched its proof. Joseph Fels Ritt first cited this paper in [8], where he wrote up a complete proof of Böttcher's theorem (unaware of its earlier formulation in (2)), and other scholars have followed suit ever since. The paper emphasizes relations between the theory of iteration and functional equations, and there are few new results in holomorphic dynamics in it, except a detailed analysis of the behavior of the map $z \to z^2$ and the construction of an attracting basin by backward iteration. It is in this paper that the term "chaotic" is introduced to describe the behavior of a map without regions of convergence. It should be noted that "Bulletin de la Societe Physico-Mathematique de Kasan" was considered to be a prestigious journal, since Kazan was the home of the International Lobachevsky Foundation awarding the Lobachevsky prize in geometry.[8] It circulated widely, mainly through exchange for publications of other academic centers and learned societies; the volume X from year 1902 lists 123 institutions (51 in the Russian empire and 72 worldwide) participating in such exchange.

The papers (2), (3), (4) and (5), besides Böttcher's original contributions, contain detailed bibliography of related studies and an exhaustive discussion of other matematicians' results, so Böttcher also comes out as very well versed in the literature of the subject. On the other hand, his papers contain many unjustified conclusions and some false statements, especially concerning iterates with an arbitrary exponent. These shortcomings were noticed

[8] Sophus Lie was the first recipient of this prize in 1897 for his work on groups of transformations. In the years 1951–2000 the Lobachevsky prize continued to be awarded, first by the Soviet Academy of Sciences and since 1992 by the Russian Academy of Sciences. Kazan State University awards the Lobachevsky medal.

the referees of his doctoral thesis in Leipzig and by the members of the habilitation committees in Lwów. But some of inacurracies in Böttcher's paper are typical for an initial phase in the development of a new discipline, when the relevant notions are just being formed. E.g., he refers to the boundaries of regions of convergence as to "boundary curves", while nowadays it is known that they can be totally disconnected – "dust-like" (such a situation occurs e.g. for a map $z \to z^2 + c$ with c lying outside the Mandelbrot set). However, in Böttcher's time set theory and topology were not advanced disciplines, and the notion of a curve was not clearly understood (Peano's example of a space-filling curve from 1890 was considered counterintuitive by some mathematicians).

Böttcher revived his interest in iteration of rational maps around 1903 and subsequently published more papers on this topic:

– Iteracye funkcyi liniowej (Iterations of a linear function), Wiadomości Matematyczne, vol. VIII, 1904, p. 291-307;
– Iteracye funkcyi liniowej (ciąg dalszy i dokończenie) (Iterations of a linear function (continuation and completion)), Wiadomości Matematyczne, vol. IX, 1905, p. 77-86;
– Przyczynek do rachunku iteracyj funkcyi algebraicznej wymiernej całkowitej (A contribution to the calculus of iterations of an algebraic rational entire function), Wiadomości Matematyczne XVI, 1912, p. 201-206;
– Iteracye funkcyi kwadratowej (Iterations of a quadratic function), Wiadomości Matematyczne, XVIII, 1914, p. 83-132.

Even though not always rigorous, Böttcher's mathematical output encompasses many ideas, examples and partial results that were later rediscovered independently by other mathematicians, giving rise to holomorphic dynamics as a new area of mathematics. One should hope that mathematicians and historians of science will recognize Böttcher's pioneering role in the formation of this discipline and will agree with the words of Alessandro Rosa: "Thus, at present, we have this 'four of a kind' for global holomorphic dynamics: Böttcher, Fatou, Julia and Pincherle" [9].

I thank Stanisław Domoradzki for encouraging my involvement in the study of history of Polish mathematics and for constant sharing of ideas and materials related to Lucjan Emil Böttcher. I also thank Alessandro Rosa for his remarks (in lively email exchanges) on the contents of Böttcher's work and its significance. Finally, I thank Terry Czubko for her help with reading handwritten German documents.

Dedication: I dedicate this article to my grade school mathematics teachers: Maria Burek, Janina Ślósarczyk, Maria Kubin and Ewa Dutkiewicz.

References

[1] Alexander D.S., Iavernaro F., Rosa A., *Early days in complex dynamics. A history of complex dynamics in one variable during 1906–1942*, History of Mathematics, vol. 38, American Mathematical Society, Providence, RI, London Mathematical Society, London 2012.

[2] Audin M., *Fatou, Julia, Montel. The great prize of mathematical sciences of 1918, and beyond*, Translated from the 2009 French original by the author. Lecture Notes in Mathematics, vol. 2014, History of Mathematics Subseries, Springer, Heidelberg 2011.

[3] Fritzsche B., *Sophus Lie. A Sketch of his Life and Work*, Journal of Lie Theory, vol. 9, 1999, 1-38.

[4] Domoradzki S., *Mathematics in Lwów before the Lwów Mathematical School*, [in:] Bečvarova, M., Binder, C. (eds), *Mathematics in the Austrian-Hungarian Empire*, Matfyzpress, Praha 2010, pp. 55-73.

[5] Domoradzki S., *The Growth of Mathematical Culture in the Lvov Area in the Autonomy Period (1870–1920)*, History of Mathematics, vol. 47, Matfyzpress, Praha 2011.

[6] Domoradzki S., Stawiska M., *Lucjan Emil Böttcher and his mathematical legacy*, 2012, preprint, http://arxiv.org/abs/1207.2747.

[7] Kowalewski G., *Bestand und Wandel*, Verlag R. Oldenbourg, München 1950.

[8] Ritt J.F., *On the iteration of rational functions*, Trans. Amer. Math. Soc. 21, no. 3, 1920, 348-356.

[9] Rosa A., Private communication with M. Stawiska through email, 11/11/2012.

[10] Stubhaug A., *The mathematician Sophus Lie. It was the audacity of my thinking*, Translated from the 2000 Norwegian original by Richard H. Daly, Springer-Verlag, Berlin 2002.

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE
1-NP/2014

JAN WOLEŃSKI*

THE RECEPTION OF LOGIC IN POLAND: 1870–1920

RECEPCJA LOGIKI W POLSCE: 1870–1920

Abstract

This paper presents the reception of mathematical logic (semantics and methodology of science are entirely omitted, but the foundations of mathematics are included) in Poland in the years 1870–1920. Roughly speaking, Polish logicians, philosophers and mathematicians mainly followed Boole's algebraic ideas in this period. Logic as shaped by works of Gottlob Frege and Bertrand Russell became known in Poland not earlier than about 1905. The foundations for the subsequent rapid development of logic in Poland in the interwar period were laid in the years 1915–1920. The rise of Polish Mathematical School and its program (the Janiszewski program) played the crucial role in this context. Further details can be found in [8]. This paper uses the material published in [20–24].

Keywords: *traditional logic*, *algebraic logic*, *mathematical logic*, *mathematics*, *philosophy*, *the Lvov-Warsaw School*, *Polish Mathematical School*, *logic in Cracow*

Streszczenie

W artykule przedstawiono recepcję logiki matematycznej w Polsce w latach 1870–1920. Polscy logicy, zarówno filozofowie jak i matematycy, kontynuowali algebraiczne idee Boole'a w tym okresie. Logika w stylu Gottloba Fregego i Bertranda Russella stała się znana w Polsce około 1905 r. Podwaliny pod dalszy szybki rozwój logiki w okresie międzywojennym zostały położone w latach 1915–1920. Powstanie Polskiej Szkoły Matematycznej i jej program (program Janiszewskiego) odegrały kluczową rolę w tym kontekście. Dalsze szczegóły można znaleźć w [8, 12]. Niniejszy artykuł korzysta z materiału zawartego w [20–24].

Słowa kluczowe: *logika tradycyjna*, *logika algebraiczna*, *logika matematyczna*, *matematyka*, *filozofia*, *Szkoła Lwowsko-Warszawska*, *Polska Szkoła matematyczna*, *logika w Krakowie*

* Jan Woleński, University of Technology, Management and Information, Rzeszów.

1. General remarks about logic in Poland before the second half of the 19th century

Poland had no major tradition in logic until the interwar period (1918–1939). Jan of Głogów (c. 1445–1507) was perhaps the most interesting Polish logician in the Middle Ages. He, like other Polish logicians of that time, was strongly influenced by the terminist *logica nova* in the Prague style. *Logica* (published in Ingolsdadt in 1618) of Jan Śmiglecki (c. 1562–c. 1619) became a popular textbook in Oxford in the 17th century (it was republished in Oxford in 1634, 1638, 1968). Yet works of both mentioned logicians as well as other Poles working in logic were not particularly original and presented well-known topics elaborated from a typical scholastic point of view (see [5, 9, 17, 19] for further information). Perhaps this link between scholasticism and the style of doing logic decided that the Committee of National Education, acting in Poland in the second half of the 18th century, had serious reservations concerning the place of logic in the general curriculum. Consequently, the Committee asked Étienne Bonnot de Condillac to write a textbook of logic for Polish high schools and universities. His *Logique* (Polish translation 1802) was used in Poland in the first of the 19th century. However, Condillac's textbook presented not formal logic, but rather principles of sensualist epistemology popular among *les philosophes* of the French Enlightenment. In fact, Poland had no reliable textbook of formal logic in the period directly preceding the rise of mathematical (algebraic) logic.

Poland lost its independence at the end of the 18th century (in 1795). The country was partitioned among Russia, Prussia and Austria. The political deterioration caused a crisis in science and education. Polish universities were closed (Warsaw, Vilna) or their academic level essentially decreased (Cracow, Lvov; in fact, the latter was German-speaking). Romanticism considerably dominated Polish philosophy between 1800 and 1850. Although this style of thinking protected national consciousness to some extent, it did not create a sympathetic intellectual atmosphere for logic and related fields. The situation changed in the 1860s and 1870s, mostly in the Russian and Austrian-Hungarian sectors; Polish culture was extremely restricted in the German (Prussian until 1871) sector. Russian authorities allowed the opening the Main School in Warsaw (it was closed in 1869 and replaced by the Imperial (Russian) University). This school, acting as a kind of university, cultivated positivistic philosophy (the Warsaw or Polish positivism). Although scientific achievements of Polish positivists were rather moderate, their activities essentially contributed to the popularization of science. In particular, several important scientific works, popular as well as highly professional, were translated and published in Polish. As far as logic and the foundations of mathematics are concerned, the following books and papers (published as booklets) deserve to be mentioned (the date of Polish edition in brackets; titles are abbreviated in some cases; I also mention some books published after 1900): A. Bain, *Logic* (1878), J.S. Mill, *Logic* (1989), W.S. Jevons, *Logic* (1886), L. Liard, *Logic* (1886), B. Riemann, *On Fundamental Hypotheses in Geometry* (1877), F. Klein, *Lectures on Geometry* (1899), H. Helmholtz, *On Measuring and Counting in Mathematics* (1901), H. Poincaré, *Science and Hypothesis* (1908), *The Value of Science* (1908), *Science and Method* (1911), R. Dedekind, *Continuity and Natural Numbers* (1914), F. Enriques (ed.), *Problems Concerning Geometry* (1914), M. Pieri, *Elementary Geometry* (1915), A.N. Whitehead, *Introduction to Mathematics* (1916) or J.W. Young, *Twelve Lectures on Fundamental Concepts of Algebra and Geometry*

(1917). The importance of this translation enterprise, initiated by mathematicians, particularly by Samuel Dickstein (1851–1939) consisted mainly in the fact that the works mentioned (and other ones) competently informed Polish scientists about the scientific progress obtained in logic, mathematics and philosophical problems of these fields. An important informative role was played by the journal *Wiadomości Matematyczne* (Mathematical News). Let me also mention that the first volume of *Przegląd Filozoficzny* (Philosophical Review) started its life in 1897 and achieved the status of the main Polish philosophical journal very soon. It was open for all directions and problems, including logic and the philosophy of mathematics.

The Danubian Empire became a fairly liberal country after 1870. The authorities gave national provinces wide autonomy. As far as the academic life in Galicia (the Austro-Hungarian part of Poland) is concerned, the universities in Cracow and Lvov became fully Polish. This fact had far-reaching consequences for the development of science in Poland (not only in Galicia, because many Poles coming from other occupied sectors of the country entered these universities). The Jagiellonian University in Cracow was an old (established in 1364) and very respected academic center and, in spite of a temporary regress in the first half of the 19th century, could continue normal academic activities. On the other hand, the University of Lvov (which had the beginnings in 1652, but was finally established in 1818) was practically completely reorganized after 1870, when the last German-speaking professors left its departments. New professors and scholars had to introduce Polish teaching and do science from scratch. Since they were not especially bound by traditional standards and rules, they could freely recommend several novelties for their students. Kazimierz Twardowski (1966–1938), a student of Brentano, became one of the most important university teachers in the entire history of Poland. His idea of doing philosophy in a clear and methodologically responsible way created a very favorable environment for logical investigations. Although Twardowski was not a mathematician or even a formal logician in the customary sense, his role in the development of logic in Poland cannot be overestimated. This is documented by the following words of Alfred Tarski ([18], p. 20):

Almost all researchers, who pursue the philosophy of exact sciences in Poland, are indirectly or directly the disciples of Twardowski, although his own works could be hardly be counted within this domain.

The difference between "all" and "almost" in this quotation refers to logicians in Cracow.

Stanisław Piątkiewicz (1848–?) and Dickstein (see above) can be considered as the very precursors of mathematical logic in Poland. The former, a professor of mathematics in a college (high school, secondary school) in Lvov, published a short report (see [14]) about the algebra of logic. Although this work did not have even a moderate scientific influence, it was (and is) perceived as the first Polish publication in the field of mathematical logic. Dickstein, mentioned earlier as having great merits for the translation program of foreign works into Polish, wrote an extensive treatise (see [4]) on concepts and methods of mathematics. He quoted Bolzano, Cantor, Dedekind, Frege, Grassmann, Hankel, Helmholtz, Peano, Weierstrass and Wundt, that is, leading mathematicians and philosophers involved in investigations of mathematics from the methodological point of view. This list of names indicates that Dickstein was quite well acquainted with the current state of the art in logic and the foundations of mathematics. From a general point of view, Dickstein's book considers mathematics formally on the basis of the theory of mathematical operations. There are several data that this book had some influence in Poland. In particular, it can be viewed of an anticipation of the later style of investigating mathematics which became dominant in the Polish school. According to this perspective, mathematics should be analyzed via its characteristic methods, not from a general philosophical standpoint. In other words, mathematics requires more mathematical foundations, free from strong philosophical assumptions.

2. Cracow

Krakow became the first serious centre of investigations in the field of mathematical logic. Stanisław Zaremba (1863–1942), a distinguished Polish mathematician, had strong interests in logic and the foundations of mathematics (see [25–27]). However, following the attitude of French mathematicians and their style of doing mathematics, he considered logic as a peripheral branch of mathematics, having only a secondary importance, mainly in teaching mathematicians. In particular, he maintained that mathematical logic had no interesting problems, and he rejected set theory as the conceptual basis of the entire mathematics. These views prevailed among mathematicians in Cracow and blocked progress in research. On the other hand, the Jagiellonian University established a professorship in mathematical logic, which was held by Jan Sleszyński (1854–1931). He was the most competent Polish scholar in mathematical logic until the 1920s. Śleszyński's lectures in Krakow were quite advanced and used the ideas of Frege, Peano and *Principia Mathematica* of Whitehead and Russell (see [16]; although they were published in a book form in 1925–1929, the book's content reproduces earlier courses given by Sleszyński). Edward Stamm (1886–1940), more a philosopher than a mathematician, worked on the algebra of logic and various foundational problems in the spirit of Boole. Leon Chwistek (1884-1944), another important figure in Krakow, also came to logic from philosophy, not from mathematics (perhaps it explains why Chwistek considered logic more seriously than did Zaremba and his followers). Very soon he began to work in the style of Russell. A comparison of [11] and [2], two works devoted to a very similar topic, namely the principle of contradiction, gives a good impression of the distance between Krakow and Lvov in doing logic. The latter book uses the framework of the algebra of logic, while Chwistek appealed to Russell's theory of types. Generally speaking, Chwistek tried to combine Russell's logicism and Poincaré's constructivism (he completed his project after 1920). More specifically, Chwistek rejected Platonism and favoured nominalism. His work can be regarded as the first original Polish contribution to mathematical logic. All works of Cracow logicians in the first two decades of the 20th century indicate that their acquaintance with the current (relatively to 1900–1920) state of mathematical logic was fairly high.

3. Lvov

Roughly speaking, Polish logic had two parents, namely philosophy and mathematics. This double pedigree was particularly evident in Lvov. In philosophy (see section 1), practically everything goes to Kazimierz Twardowski. He gave the first course in Poland in which elements of mathematical logic were included. These lectures took place in the academic year 1899/1900, were mostly devoted to Brentano's reform of traditional logic, but also informed about the algebra of logic. Jan Łukasiewicz (1878–1956) participated in this class. He began to study Frege and Russell about 1904; in particular, he was impressed by the latter. Łukasiewicz began systematic courses in advanced algebra of logic. Twardowski was a charismatic teacher. He trained many philosophers with explicit interests in logic, also strongly influenced by Łukasiewicz's teaching of logic, including Kazimierz Ajdukiewicz (1890–1963), Tadeusz Czeżowski (1889–1981), Tadeusz Kotarbiński (1886–1981) and Zygmunt Zawirski (1882–1948); Stanisław Leśniewski (1886–1939) joined this circle in 1910. All Lvov logicians coming from philosophy also studied mathematics, mostly under Wacław Sierpiński (1882–1969), who acquainted his students with set theory (see [15], one of the first textbook in the set theory in the world) and the problems of the foundations of mathematics. Sierpiński's textbook informs about basic mathematical facts concerning sets, but also about difficulties in the foundations of mathematics, for example, it analyses the antinomy of Richard (but does not mention the Russell antinomy). Zygmunt Janiszewski (1988–1920) came to Lvov in 1915 and obtained his habilitation there. He wrote a few popular papers in logic and the foundations of mathematics (see the next section).

Łukasiewicz's book [11] played an important role in Lvov. Although, as I noted earlier, this treatise is very elementary on the purely mathematical level, it successfully popularized formal logic among philosophers in Lvov, due to its Appendix summarizing Couturat's algebra of logic (see [1]). The book was basically devoted to an elaborate analysis of the principle of contradiction in Aristotle and in later philosophy. Łukasiewicz also informed about antinomies of set theory. This problem was discussed by several other authors, for instance, Leśniewski (see [9]) and Czeżowski (see [3]). The former, inspired by various, more or less complicated, attempts to solve set-theoretical paradoxes, offered a completely new solution via mereology considered as a kind of set theory (see [10]). Doubtless'y, Sierpiński's mentioned textbook also essentially contributed to increasing interest in antinomies in Lvov. Yet logical works in Lvov in the period 1900–1920, although based on a relatively solid knowledge of the state of art in logic, could be hardly regarded as systematic or governed by a commonly accepted research project. In fact, logical papers published by logicians from Lvov in 1900–1920 were devoted to various topics and frequently combined formal topics with general philosophical investigations. In many respects, logic in Lvov was similar to that done in Cracow, although the latter centre appears as more advanced from the mathematical point of view.

4. Warsaw and the Polish Mathematical School

Warszawa entered the stage exactly in 1915. The German army very soon took the city in World War I. The German authorities agreed to reopen the (Polish) University of Warsaw in 1915. The academic staff was mainly imported from Lvov. Łukasiewicz was appointed a professor of philosophy. He began lectures in logic (elementary and advanced), and attracted many young mathematicians very soon. Kazimierz Kuratowski (1896–1980) reported ([8], p. 23/24) Łukasiewicz teaching activities in the following way:

“Jan Łukasiewicz was another professor who greatly influenced the interests of young mathematicians. Besides lectures on logic and the history of philosophy, Professor Łukasiewicz conducted more specialized lectures which shed new light on the methodology of the deductive sciences and the foundations of mathematical logic. Although Łukasiewicz was not a mathematician, he had an exceptionally good sense of mathematics and therefore his lectures found a particularly strong response among mathematicians. (...) I remember a lecture of his on the methodology of the deductive sciences in which he analyzed, among other things, the principles which any system of axioms should satisfy (such as consistency and independence of axioms). The independence of axioms in particular was not always observed by writers and even in those days was not always exactly formulated. Łukasiewicz submitted to detailed analysis Stanisław Zaremba’s *Theoretical Arithmetic* (1912) which was well known at that time, questioning a very complicated principle formulated in that work, which was supposed to replace the rule of independence of axioms. The criticism was crushing. Nevertheless, it brought about a polemical debate in which a number of mathematicians and logicians took part in the pages of the *Philosophical Review* (1916–1918). I mention this because a byproduct of Łukasiewicz’s ideas in our country was the exact formulations of such notions as those of quantity, the ordered set, and the ordered pair (the definition of the ordered pair which I proposed during the discussion was to find a place in world literature on the subject). This illustrates the influence brought by Jan Łukasiewicz, philosopher and logician, on the development of mathematical concepts”.
Thus, logic began to play an important role at the inception of the Polish Mathematical School.

Poland recovered its independence in 1918. This also resulted in a great debate about the objectives and prospects of Polish science and culture. Scholars in every field discussed how to develop their disciplines and what to do in order to keep up with the world science. Particularly important was the discussion among mathematicians. In fact, it already started in Lvov, but it was rather personal, involving Sierpiński and Janiszewski. They were strongly disappointed by a lack of a common language and common research interests among Polish mathematicians. Sierpiński and Janiszewski believed that set theory and topology could play a fundamental role in mathematics. The national discussion about science, its needs and perspectives, was a good occasion for manifesting views about the future of mathematics in Poland. Janiszewski became the main exponent of the project, later known as the Janiszewski program (see [8]). Roughly speaking, Polish mathematicians, according to Janiszewski, should concentrate on chosen mathematical fields and work as one strong group. The second point was very soon abandoned, but the first was adopted. Although Janiszewski did not mention any concrete topic to be cultivated in Poland, most Polish mathematicians understood his program as favouring set theory, topology and their applications to other branches of mathematics. Two significant centres of the Polish mathematical school arose in Poland, namely in Warszawa and in Lvov. Krakow remained more traditional, in the spirit of Zaremba (see section 2). Sierpiński, Janiszewski and Stefan Mazurkiewicz (1888–1945) played the main role in Warszawa, while Stefan Banach (1892–1945) and Hugo Steinhaus (1887–1972) became the leaders in the Lvov mathematical community. Yet one important difference between the two centres of modern mathematics in Poland should be noted. Although mathematicians in Lvov worked mainly on applications of set theory and topology, the circle in Warszawa focused more on abstract matters. Janiszewski also postulated that Poland should have a special mathematical journal published in international languages. This idea found its materialization in *Fundamenta Mathematicae* (the first volume appeared in 1920).

Janiszewski's program attributed a great role to mathematical logic and the foundations of mathematics. Janiszewski himself wrote a few general papers on logical and foundational matters in 1915–1916. Perhaps the most interesting is [7]), in which he considered mathematical logic as an autonomous branch of mathematics, having its own problems and not dependent on its applications in mathematics or on other practical roles. This was in very deep contrast with the views of Zaremba. The placement of logic and the foundations at the heart of mathematics required certain organizational steps. The University of Warsaw had the Faculty of Mathematical and Natural Sciences. The Department of the Philosophy of Mathematics was organized very soon and Leśniewski became its head. Łukasiewicz left the University in 1918 in order to act as the Minister of Religious Confessions and Education in the government under Ignacy Paderewski, the first Polish Prime Minister. He (Łukasiewicz) returned to the academic staff in 1919 and the University established for him a special position in philosophy at the Faculty of Mathematics and Natural Sciences. Both professors of logic (Łukasiewicz acted more as a logician than a philosopher) began intensive teaching of mathematical logic, mostly to mathematicians but also to philosophers. The first project of *Fundamenta Mathematicae* divided the journal into two series, one devoted to set theory, topology and their applications, and the other to logic and the foundations. This project was finally abandoned, but the significance of mathematical logic in the eyes of the founders of the Polish mathematical school found its impressive manifestation in the composition of the Editorial Board of *Fundamenta*: Mazurkiewicz, Sierpiński, Leśniewski and Łukasiewicz. Polish translation of the mentioned Couturat on the algebra of logic (see [1]) became the first Polish textbook of mathematical logic. However, this book was very soon viewed as too obsolete to be a source of information about modern logic. As Bronisław Knaster (1883–1980), the translator of Couturat's book, remarked (see [1], p. III) in his Preface:

As a deductive theory Couturat's work – when seen in the light of recent requirements of logic and methodology – is not free from certain defects of composition, incorrect formulations, and inexact arrangement.

Clearly, these "recent requirements" were related to works of Frege, Russell and Hilbert and were reported by Łukasiewicz in his lectures mentioned by Kuratowski (see above); Knaster himself attended Łukasiewicz's courses. In particular, Couturat's understanding of the algebra of logic as being interpreted either as propositional calculus or as the algebra of sets became replaced by the arrangement of logical theories in which the former functions as the most basic ingredient of logic. One can say that this step completed the reception of mathematical logic in Poland. The golden period of logic in Poland began just after 1920. Ten years later Heinrich Scholz (see [13], p. 73) called Warsaw one of the world capitals of mathematical logic.

References

[1] L. Couturat, *Algebra logiki* (The Algebra of Logic), Koło Matematyczno-Fizyczne Studentów Uniwsersytetu Warszawskiego, Warszawa 1918.

[2] L. Chwistek, *Zasada sprzeczności w świetle nowszych badań Bertranda Russella* (On the Principle of Contradiction in the Light of New Investigations of Bertrand Russell), Polska Akademia Umiejętności, Kraków 1912.

[3] T. Czeżowski, *Teorya klas* (A Theory of Classes), Towarzystwo dla Popierania Nauki Polskiej, Lwów 1918.

[4] S. Dickstein, *Pojęcia i metody matematyki* (nakładem Prac Matematyczno-Fizycznych), Warszawa 1891.

[5] J.J. Jadacki, *The sources of contemporary Polish Logic*, [in:] J.J. Jadacki, *Polish Analytical Philosophy. Studies on Its Heritage*, Semper, Warszawa 2009, pp. 11-40.

[6] J.J. Jadacki, *Bibliography of the Polish logic from the second half of the 14th century to the first half of the 20th century*, [in:] J.J. Jadacki, *Polish Analytical Philosophy. Studies on Its Heritage*, Semper, Warszawa 2009, pp. 341-371.

[7] Z. Janiszewski, *Logistyka* (Logistics), [in:] S. Michalski (ed.), *Poradnik dla samouków* (A Guide for Autodidacts), v. I, Heflich-Michalski, Warszawa 1915, pp. 449-461.

[8] K. Kuratowski, *A Half Century of Polish Mathematics*, PWN, Warszawa 1980.

[9] S. Leśniewski, *Czy klasa klas nie podporządkowanych sobie jest podporządkowana sobie?* (Is the class of Classes not Subordinated to Themselves Subordinated to Itself?), *Przegląd Filozoficzny* 18, 1914, pp. 63-75; Eng. tr. in S. Leśniewski, *Collected Works*, Kluwer, Dodrecht 1992, pp. 86-114.

[10] S. Leśniewski, *Podstawy ogólnej teorii mnogości I* (Foundations of the General Theory of Sets I), Prace Polskiego Koła Naukowego w Moskwie, Moskva 1915; Eng. tr. in S. Leśniewski, *Collected Works,* Kluwer, Dodrecht 1992, pp. 129-173.

[11] J. Łukasiewicz, *O zasadzie sprzeczności u Arystotelesa* (On the Principle of Contradiction in Aristotle), Polska Akademia Umiejętności, Kraków 1910; Ger. tr. *Über den Satz des Widerspruchs bei Aristoteles*, Olms, Hildesheim 1993.

[12] R. Murawski, *Filozofia logiki i matematyki w Polsce międzywojennej* (Philosophy of Logic and Mathematics in the Interwar Poland), Wydawnictwo Naukowe Uniwersytetu Mikołaja Kopernika, Toruń 2011.

[13] S. Piątkiewicz, *Algebra w logice* (*Algebra in Logic*), IV Gimnazjum we Lwowie, Lwów 1888.

[14] H. Scholz, *Abriss der Geschichte der Logik*, Junker and Dünnhaupt, Berlin 1931.

[15] W. Sierpiński, *Teoria mnogości* (Theory of Sets), Wende, Warszawa 1912.

[16] J. Sleszyński, *Teoria dowodu* (Proof Theory), v. I–II, Kółko Matematyczno-Fizyczne, Uczniów Uniwersytetu Jagiellońskiego, Kraków 1925–1929.

[17] H. Struve, *Historya logiki jako teorii poznania w Polsce poprzedzona zarysem jej rozwoju u obcych* (The History of Logic as Theory of Knowledge in Poland Preceded by An Outline of Its Development Abroad), Nakładem Autora, Warszawa 1911.

[18] A. Tarski, [A Letter to Otto Neurath, 25 IV 1930], German original and English translation by Jan Tarski, *Grazer Philosophische Studien* 43, 1992, pp. 10-12, 20-22.

[19] V. Voisé, Z. Skubiała-Tokarska (eds.), *Z historii polskiej logiki* (From the History of Polish Logic), Ossolineum, Wrocław 1981.

[20] J. Woleński, *Logic and Philosophy in the Lvov-Warsaw School*, Kluwer, Dordrecht 1989.

[21] J. Woleński, *Powstanie logiki matematycznej w Polsce* (The rise of mathematical logic in Poland), in A. Strzałkowski (ed.), *Recepcja w Polsce nowych kierunków i teorii naukowych* (Reception of New Scientific Directions and Theories in Poland), PAU, Kraków 2001, pp. 63-85.

[22] J. Woleński, *Mathematical logic in Poland 1900–1939*: People, circles, institutions, *ideas*, *Modern Logic*, 5, 1995, pp. 363-405; repr. in J. Woleński, *Essays in the History of Logic and Logical Philosophy*, Jagiellonian University Press, Kraków 1999, pp. 59-84.
[23] J. Woleński, *The achievements of Polish School of Logic*, [in:] *The Cambridge History of Philosophy 1870–1945*, ed. by Th. Baldwin, Cambridge University Press, Cambridge 2003, pp. 401-416.
[24] J. Woleński, *Polish logic, Logic Journal of the IGPL 1*, 2004, pp. 399-428; repr. [in:] J. Woleński, *Historico-Logical Essays*, v. 1, Copernicus Center Press, Kraków 2013, pp. 211-261.
[25] S. Zaremba, *Zarys teorii pierwszych zasad teorii liczb całkowitych* (Outline of the Theory of First Principles of Integers), Polska Akademia Umiejętności, Kraków 1907.
[26] S. Zaremba, *Arytmetyka teoretyczna* (Theoretical Arithmetic), PAU, Kraków 1912.
[27] S. Zaremba, *Wstęp do analizy* (Introduction to Analysis), Gebethner i Wolff Warszawa 1915, tom „Wiedzy Matematycznej” wydany z zapomogi Kasy im. Mianowskiego.

TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE

1-NP/2014

ANDRZEJ KAJETAN WRÓBLEWSKI*

POLISH PHYSICISTS AND THE PROGRESS IN PHYSICS (1870–1920)

FIZYCY POLSCY I ROZWÓJ FIZYKI (1870–1920)

Abstract

The Polish-Lithuanian Commonwealth lost independence in 1795 and was partitioned among her three powerful neighbours: Austria, Prussia and Russia. The two old Polish universities in Cracow and Lvov enjoyed relatively liberals laws in the Austrian partition. It was there that Polish physicists (Karol Olszewski, Zygmunt Wróblewski, Marian Smoluchowski, Władysław Natanson, Wojciech Rubinowicz, Czesław Białobrzeski, and others) made most important discoveries and original contributions. There was no possibility of career for Poles living in the oppressive Russian and Prussian partitions where even the use of Polish language was forbidden in schools. Thus many bright Polish students such as e.g. Kazimierz Fajans, Stefan Pieńkowski, Maria Skłodowska, and Mieczysław Wolfke, went abroad to study in foreign universities. In spite of unfavourable conditions under which they had to live and act in the period 1870–1920, Polish scholars were not only passive recipients of new ideas in physics, but made essential contributions to several fields such as e.g. cryogenics, electromagnetism, statistical physics, relativity, radioactivity, quantum physics, and astrophysics.

Keywords: *cryogenics*, *statistical physics*, *electromagnetism*, *relativity*, *radioactivity*, *quantum physics*, *astrophysics*

Streszczenie

Rzeczpospolita Obojga Narodów straciła niepodległość w 1795 r. i została podzielona między trzech potężnych sąsiadów: Austrię, Prusy i Rosję. Dwa stare polskie uniwersytety w Krakowie i Lwowie mogły działać w stosunkowo liberalnych stosunkach w zaborze austriackim. Właśnie tam fizycy polscy (Karol Olszewski, Zygmunt Wróblewski, Marian Smoluchowski, Władysław Natanson, Wojciech Rubinowicz, Czesław Białobrzeski i inni) dokonali największych i najbardziej oryginalnych odkryć. W represyjnych zaborach pruskim i rosyjskim, w których język polski był nawet zabroniony w szkołach, nie było możliwości kariery naukowej dla Polaków. Z tego powodu wielu zdolnych polskich studentów, jak Kazimierz Fajans, Stefan Pieńkowski, Maria Skłodowska czy Mieczysław Wolfke emigrowało, by studiować zagranicą. Mimo niesprzyjających warunków, w jakich przyszło im żyć i działać w okresie 1870–1920, uczeni polscy nie byli tylko biernymi odbiorcami nowych idei w fizyce, ale wnieśli znaczący wkład do wielu dziedzin, jak np. kriogenika, elektromagnetyzm, fizyka statystyczna, teoria względności, promieniotwórczość, fizyka kwantowa i astrofizyka.

Słowa kluczowe: *kriogenika*, *fizyka statystyczna*, *elektromagnetyzm*, *teoria względności*, *promieniotwórczość*, *fizyka kwantowa*, *astrofizyka*

* Andrzej Kajetan Wróblewski, Professor emeritus, Department of Physics, University of Warsaw.

1. Introduction

Poland, or more precisely, the Polish-Lithuanian Commonwealth, once a powerful country, the largest in Europe, became weakened by perpetual wars with Russia and Turkey. After losing independence in 1795 it was partitioned among the three powerful neighbouring empires: Austria, Prussia and Russia.

In the period (1870–1920) considered in this paper Poland still remained an occupied country. Cracow (Kraków) with the Jagellonian University, the oldest in Poland, founded in 1364 by the Polish king Casimir the Great, was included in the Austrian Partition, and so was the university in Lvov (Lwów), founded in 1661 by the Polish king Jan Casimir and later renamed Francis Ist University by the Austrian authorities. There was also Lvov Polytechnic (founded in 1844 as a Technical School).

The revolutionary national movements during the "Springtime of Nations" in the years 1848–1849 were suppressed by the conservative forces. But not long afterwards the Habsburg Empire, weakened by military defeats, became a constitutional monarchy, and the nations which formed parts of it were granted certain freedoms and autonomy. The administration and education system in the Austrian Partition was re-polonized. Teaching and research in universities in Cracow and Lvov, and also at the Lvov Polytechnic could be carried out in the Polish language.

Physics departments existed in all three institutions. The best known physicists at the Jagellonian University at that time were Karol Olszewski, Zygmunt Wróblewski, August Witkowski, Władysław Natanson, and Marian Smoluchowski (after 1913). Those in Lvov were Oskar Fabian and Marian Smoluchowski (until 1913) at the university, and Kazimierz Olearski, Łukasz Bodaszewski, and Tadeusz Godlewski at the Polytechnic.

The Cracow Scientific Society, established in 1816, was transformed in 1873 into the Academy of Sciences (Akademia Umiejętności). In addition to ordinary members from the Austrian Partition it also elected foreign members who were in large part Polish scientists living elsewhere in Europe, including the Russian and Prussian Partitions. The Academy began publishing a number of periodicals in foreign languages, English, French, and German, to report the results of Polish scientists irrespective of where they were doing research. Thus, this institution played a very important role in maintaining links between scholars in the partitioned Polish lands.

Warsaw, the former capital of Poland, was situated in the most oppressive Russian Partition. The Imperial Warsaw University (Cesarski Uniwersytet Warszawski) founded by the Russians in 1869 had all teaching and research conducted in Russian, the use of the Polish language having been banned by law. Polish youth largely boycotted that institution, and so a large part of its students were Russians.

During the revolutionary period 1905–1907 the tsarist regime was forced to make certain concessions in the Russian Partition. For example, in 1907 permission was granted to form the Warsaw Scientific Society (Towarzystwo Naukowe Warszawskie – TNW in short). It began to play a role similar to that of the Academy of Sciences in the Austrian Partition.

No institution of higher education and/or research existed in the Prussian Partition which included Poznań (Posen), Toruń (Thorn), and Gdańsk (Danzig).

Thus there was not much chance for the Poles to pursue a scientific career in the Russian and Prussian Partitions. The number of positions for physicists in the liberal Austrian Partition was small and limited. Therefore many bright young people chose to find education and employment in other countries. Among those émigrés who later excelled in physical sciences were Maria Skłodowska, Czesław Białobrzeski, Jan Czochralski, Kazimierz Fajans, Józef Kowalski-Wierusz, Jakub Laub, Julian Lilienfeld, Stanisław Loria, Stefan Pieńkowski, Wojciech Rubinowicz, Ludwik Silberstein, Ludwik Wertenstein, and Mieczysław Wolfke. Not all of them returned to Poland when it again became an independent country in 1918.

In spite of unfavourable conditions under which they had to live and study, Polish scholars not only maintained close contact with the forefront of research in physics, but made essential contributions to several fields, such as e.g. cryogenics, statistical physics, electromagnetism, relativity, radioactivity, quantum physics, and astrophysics.

2. Cryogenics

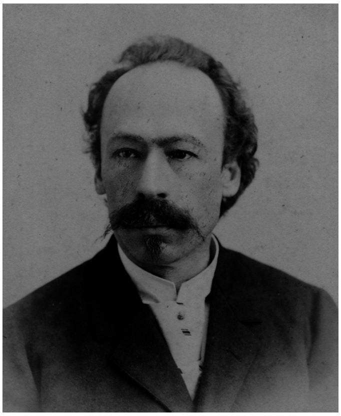


Fig. 1. Karol Olszewski

One of the "hot" subjects in physics in the second half of the XIX century was liquefaction of gases. Many gases could be obtained in a liquid form by simply cooling them and applying high pressure. However, several gases such as oxygen, nitrogen, carbon oxide, and hydrogen resisted liquefaction. Some scientists were even tempted to treat them as "permanent gases" which could not exist in a liquid state. However, in 1877 Louis Cailletet in Paris and Raoul-Pierre Pictet in Geneva, working independently and not knowing about each other, almost simultaneously achieved so-called dynamic liquefaction of oxygen, that is, they momentarily observed a short-lived fog in a container in which the gas, kept under high pressure, underwent rapid expansion caused by a sudden release. The observed fog was of course made up of minute drops of liquid gas which rapidly evaporated, and so there was no possibility for studying the properties of the liquid.

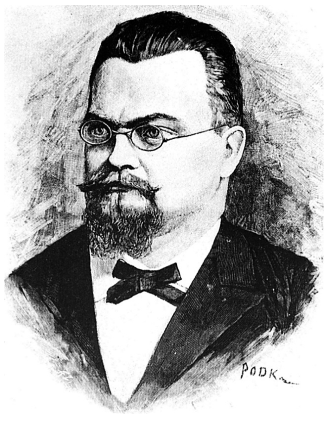


Fig. 2. Zygmunt Wróblewski

The groundbreaking static liquefaction of air and its components was achieved a few years later in Cracow by two scientists of the Jagellonian University, Zygmunt Wróblewski and Karol Olszewski. They made Cracow one of the world centres of low-temperature physics.

Zygmunt Wróblewski [1] (1846–1888) was born in Grodno (now in Belarus), went to schools there and in 1862 entered Kiev University to study physics. He became involved in conspiracy leading to the January Uprising of 1863, which was another desperate attempt by the Poles to regain independence from the Russians. Wróblewski was arrested by the Russian police and spent sixteen months in the Kiev prison. Afterwards he was transported to Siberia to serve the rest of his sentence. He was released in 1869, but as a former political prisoner he was barred from entering university anywhere in the Russian Empire.

Wróblewski went to Germany and studied physics in Berlin, Heidelberg, and Munich, where he obtained doctor's degree, and became *privatdozent* in Strasbourg. His experimental results on diffusion of gases were highly appraised by James Clerk Maxwell [2]. It resulted in an invitation from "Nature" to write a review article on this subject. Wróblewski's article appeared in that prestigious periodical in 1879 [3]. He published the same article also in Polish [4]. It helped to make his name as a scientist.

Wróblewski remained in Strasbourg until 1880. Then, with financial support from the Cracow Academy of Sciences, he visited several important physics laboratories in France and England. This experience helped him to decide on low temperatures as the domain of future research.

In 1882 the Jagellonian University appointed Wróblewski to the chair of experimental physics. In March of that year he came to Cracow bringing with him a Cailletet-type apparatus for attaining low temperatures which at that time had been already commercially produced in France.

Karol Olszewski (1846–1915) was born in Broniszów, a small village east of Cracow. He studied chemistry at the Jagellonian University. Then, in 1872, he went to Heidelberg to enrich his knowledge under Robert Bunsen and Gustav Kirchhoff. After return to Cracow he became first a *privatdozent* and in 1876 a professor of chemistry at the Jagellonian University. He also had in mind research of phenomena at low temperatures.

Wróblewski and Olszewski met and decided to join forces. They made two essential modifications of Cailletet's apparatus. Firstly, they replaced the original capillary tube with a wider one of a different shape, and secondly they used a new cooling container to lower the pressure over the boiling ethylene. In this way they were able to achieve temperatures as low as minus 135 degrees Celsius and finally, on March 29, 1883, succeeded to liquefy oxygen and a few days later also nitrogen and carbon monoxide in a static form, which enabled them to study the properties of these gases in a liquid state. The news about this important achievement were promptly communicated to the Academy of Sciences in Paris [5].

Sadly enough, after only a few months of successful collaboration the two scientists quarelled and separated. Since then they worked independently and later attempted to liquefy hydrogen. They could only claim dynamic liquefaction of that gas. Unfortunately Wróblewski died tragically on April 18, 1888, in consequence of heavy burns inflicted during the fire in his laboratory.

In 1894 William Ramsay and John William Rayleigh surprised the world by discovering a new gas, argon, the first of so-called inert gases. Ramsay did not trust his countryman John Dewar who had a cryogenic laboratory in London, so he sent samples of argon and also newly discovered helium to Olszewski's laboratory in Cracow to be liquefied and studied. Olszewski liquefied argon and examined its properties [6] but he failed to liquefy helium [7].

The successor to Wróblewski in the chair of experimental physics was August Witkowski (1854–1913). He performed important experimental investigations of the properties of gases, especially at low temperatures. Due to his efforts a new larger physics building was constructed in 1911 (this building is now called Witkowski's Collegium). Witkowski is also remembered as the author of an excellent modern physics textbook *Zasady fizyki* (*Principles of physics*, in 3 volumes, 1892–1912).

3. Statistical physics

The chair of theoretical physics at Lvov University was created In 1872. Its first holder was Oskar Fabian (1846-1899). He wrote a textbook on analytical mechanics [8] and a number of papers on mathematics and physics. In 1898 the entire physics department consisting of chairs of experimental and theoretical physics was transferred to the specially constructed building at 8, Długosza Street.

After Fabian's death the chair of theoretical physics was filled by Marian Smoluchowski. He was born in 1872 in Vorderbrühl, a small village near Vienna where his family lived at that time. His father was a Cracow lawyer who became Secretary at the Court of Emperor Franz Joseph. Smoluchowski studied physics and mathematics at the University of Vienna under famous physicists Franz Exner and Joseph Stefan, and also listened to lectures given by Ludwig Boltzmann and Ernst Mach. In 1895 he graduated with honours and after that spent eight months in Paris in the laboratory of Gabriel Lippmann, then several months in Glasgow, where he studied radioactivity under Kelvin, and finally five months in the Berlin laboratory of Emil Warburg.

Fig. 3. Marian Smoluchowski

At the end of 1897 Smoluchowski returned to Vienna to become lecturer in the university of that city. However, already in May, 1899, he was invited to Lvov where at the age of twenty-eight he was promoted to the chair of theoretical physics, thus becoming the youngest professor in the Austro-Hungarian Empire.

During his work at Lvov between 1899 and 1913 Smoluchowski wrote several seminal papers on the theory of the Brownian motion, the kinetic theory of matter, the theory of fluctuations, and the theory of critical opalescence, to name a few.

In May, 1905, Albert Einstein, at that time still an unknown clerk in the Bern patent office, submitted to the "Annalen der Physik" the paper *On the motion of particles suspended in liquids at rest, required by the molecular-kinetic theory of heat* [9]. It contained an analysis of the Brownian motion. The same problem was independently investigated by Marian Smoluchowski. His results, obtained by a somewhat different method, were published in the following year [10].

Einstein and Smoluchowski have proved that irregular movement of particles suspended in a liquid results from their bombardment by molecules of the liquid. One may determine experimentally the mean-square displacement of the suspended particle in a given direction.

This quantity was found to be related to the Avogadro's number and the temperature of the liquid by a fundamental formula – now called the Einstein-Smoluchowski equation – which provided quantitative description of the Brownian motion. The discovery made independently by both scholars was an excellent confirmation of validity of the kinetic theory of matter and contributed to establishing atomistic concepts.

756

4. *Zur kinetischen Theorie der Brownschen Molekularbewegung und der Suspensionen; von M. von Smoluchowski.*

[Bearbeitet nach einer am 9. Juli 1906 der Krakauer Akademie vorgelegten und demnächst in dem Bullet. Int. Crac. erscheinenden Abhandlung.]

§ 1. Die viel umstrittene Frage nach dem Wesen der von dem Botaniker Robert Brown 1827 entdeckten Bewegungserscheinungen, welche an mikroskopisch kleinen, in Flüssigkeiten suspendierten Teilchen auftreten, ist neuerdings durch zwei theoretische Arbeiten von Einstein[1] wieder in Anregung gebracht worden. Die Ergebnisse derselben stimmen nun vollkommen mit einigen Resultaten überein, welche ich vor mehreren Jahren in Verfolgung eines ganz verschiedenen Gedankenganges erhalten hatte, und welche ich seither als gewichtiges Argument für die kinetische Natur dieses Phänomens ansehe. Obwohl es mir bisher nicht möglich war, eine experimentelle Prüfung der Konsequenzen dieser Anschauungsweise vorzunehmen, was ich ursprünglich zu tun beabsichtigte, habe ich mich doch entschlossen, jene Überlegungen nunmehr zu veröffentlichen, da ich damit zur Klärung der Ansichten über diesen interessanten Gegenstand beizutragen hoffe, insbesondere da mir meine Methode direkter, einfacher und darum vielleicht auch überzeugender zu sein scheint als jene Einsteins.

Dem Mangel einer direkten experimentellen Verifikation suche ich teilweise wenigstens durch eine zusammenfassende Übersicht der bisher bekannten Versuchsresultate abzuhelfen, welche im Verein mit einer kritischen Analyse der verschiedenen Erklärungsversuche deutliche Hinweise darauf zu geben scheint, daß das Brownsche Phänomen in der Tat mit den theoretisch vorauszusehenden Molekularbewegungen identisch ist. Den Schluß bilden einige Bemerkungen über die Suspensionen

1) A. Einstein, Ann. d. Phys. 17. p. 549. 1905; 19. p. 371. 1906.

Fig. 4. The beginning of Smoluchowski's paper on the Brownian motion

Some years later the French physicist Jean Perrin performed very precise experimental studies of the Brownian motion. He used a microscope to record successive positions and trace movements of individual particles (of a gum resin) suspended in a liquid. He checked that the mean square displacement in a given direction is indeed proportional to time, as predicted by the Einstein-Smoluchowski formula; from these observations he could calculate the value of the Avogadro's number.

After Witkowski's death in 1913 Smoluchowski was asked to fill the vacant chair and moved from Lvov to Cracow. He was elected rector of the Jagellonian University for the academic year 1917–1918 but unfortunately contracted dysentery and died on September 5, 1917, before taking office. Einstein, Sommerfeld and other eminent physicists of that time wrote commemorative articles expressing grief because of the premature passing away of the great physicist [11].

Marian Smoluchowski was indeed one of the most eminent scientists in Polish history. In addition to his 1906 work on the Brownian motion, he gave the explanation (1908) of critical opalescence, and in 1913 published an important statistical interpretation of the second law of thermodynamics. With the paper published in 1906 Smoluchowski originated the theory of stochastic processes [12].

It is worth to note that the first observation of the Brownian motion in gases has been made by another Lvov physicist, Łukasz Bodaszewski (1849–1908). He published his results in both German [13] and Polish [14]. This important discovery has been cited in several books and articles [15].

The properties of the distribution of colloidal particles have been studied experimentally by an Austrian chemist Richard Zsigmondy and also by a Swedish chemist Theodor Svedberg. They confirmed the formulae derived by Smoluchowski in 1904 [16].

Perrin became the recipient of the Nobel Prize for physics in 1926, whereas Zsigmondy and Svedberg received Nobel Prizes for chemistry in 1925 and 1926, respectively. Had Smoluchowski been alive at that time he would surely be a strong candidate for a Nobel Prize, too.

4. Electromagnetism

In 1864 James Clerk Maxwell announced his revolutionary electromagnetic theory. Its acceptance by physicists had been, however, quite slow.

On the experimental side the exciting discovery was that by Berend Feddersen who had proven in 1862 that the discharge of a Leiden jar (electric condenser) is an oscillatory process and consists of currents travelling in both directions between the plates. A number of authors engaged themselves in elaborating the theory of electric oscillations.

Kazimierz Olearski (1855–1936) was born in a small village near Cracow, went to schools in Cracow, and then studied at the Jagellonian University (1872–1876). Afterwards he completed his education in Leipzig and Berlin. He became *privatdozent* at the Jagellonian University and later went to Lvov Polytechnic where he took the chair of general and technical physics. Olearski published several papers on electromagnetism. His important article [17] on the theory of electrical oscillations published in the "Bulletin of the Cracow Academy of Sciences" has been noticed and cited e.g. by the eminent electrical scientist John Fleming [18].

Another Polish physicist who published papers on electromagnetism was Ludwik Silberstein (1872–1948). He was born in Warsaw, but studied first in Cracow, and afterwards

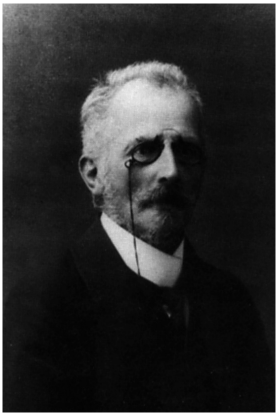


Fig. 5. Kazimierz Olearski

at Heidelberg and Berlin, where he got his Ph.D. In the years 1895–1897 Silberstein was assistant to Olearski at Lvov Polytechnic. That employment did not satisfy him and he returned to Warsaw. For a short period he earned his living by working in a private company. Since 1899 he was *privatdozent* in Bologna and Rome but stayed mostly in Warsaw and was very active in the Warsaw Scientific Society.

Silberstein was interested in mathematical physics, and electromagnetic theory and relativity in particular. Being a prolific author he published a few dozen papers in "Annalen der Physik", "Elektrochemische Zeitschrift", "Philosophical Magazine" and other foreign periodicals. He also published many articles in Polish, and also several books, including an excellent textbook of electric and magnetic phenomena [19]. In one of his papers he introduced a complex vector of the electromagnetic field (Riemann-Silberstein vector) [20]. In other papers he used quaternions to express relativity equations [21].

In 1913 Silberstein left Warsaw for good and lived first in Italy, then in London, and finally settled in the United States. In that later period he wrote several excellent books in English on special and general relativity and also on electromagnetism. According to Abraham Pais, "on several occasions, he was in dogged but intelligent opposition to relativity theory" [22].

5. Relativity

According to Leopold Infeld, a friend and collaborator of Albert Einstein, August Witkowski was the first Polish physicist who understood special relativity theory and saw in it the birth of a new science:

> "My friend Professor Loria told me how his teacher, Professor Witkowski (and a very great teacher he was!), read Einstein's paper and exclaimed to Loria: 'A new Copernicus has been born! Read Einstein's paper'. Later, when Professor Loria met Max Born at a physics meeting, he told him about Einstein and asked Born if he had read the paper. It turned out that neither Born nor anyone else had heard about Einstein. They went to the library, took from the bookshelves the seventeenth volume of Annalen der Physik and started to read Einstein's article. Immediately Born recognized its greatness and also the necessity for formal generalizations" [23].

Witkowski was indeed a critic of ether and a proponent of the principle of relativity which he described in his lectures.

However, the first Polish paper on relativity was published by Jakub Laub (1884–1962). He was born in Rzeszów, studied first at the Jagellonian University in Cracow, and then in Vienna and Göttingen. Afterwards Laub went to the University of Würzburg to study cathode rays under Wilhelm Wien. His Ph.D. thesis (1907) concerned secondary cathode rays. He published his dissertation both in German [24] and in Polish [25].

Wien asked Laub to read Einstein's special relativity paper [26] and give a talk about it at the physics institute colloquium. Laub at once became an ardent adherent of the theory of relativity and presented his calculations concerning the optics of moving bodies in two papers [27] in "Annalen der Physik". In July 1907 he reported his results at the 10^{th} Congress of Polish Physicians and Naturalists in Lvov [28] and later wrote a comprehensive article in Polish [29].

In February 1908 Laub wrote a letter to Einstein asking him about possibility of joint work on relativity. At that time Einstein, still an employee of the patent office in Bern, was known only to a few selected people, and he was glad to accept Laub's proposal. Thus Laub became the first collaborator of Albert Einstein. They published three joint papers [30] in "Annalen der Physik".

Following the advice of Einstein, Laub took the post of an assistant to Philipp Lenard in Heidelberg. At that time, however, Lenard became an enemy of Einstein's relativity theory, and ordered Laub to devise experiments which could give a definite proof of the existence of the ether. Laub wrote instead a splendid review of the special relativity theory [31]. Lenard became angry and promptly sacked Laub, who decided to accept the physics chair at the University of La Plata in Argentina. He arrived to Argentina in 1911. Some years later he entered the diplomatic service and was ambassador of Argentina in Germany and then in Poland (1936–1939).

A few other Polish physicists such as Wiktor Biernacki, Kamil Kraft (1873–1945), Henryk Merczyng (1860–1916), Ludwik Silberstein and Czesław Białobrzeski also published papers on the theory of relativity before 1920.

6. Radioactivity

Wiktor Biernacki (1869–1918) studied at the Imperial Warsaw University and was for six years an assistant to Peter Zilov, a Russian professor of physics there. He also simultaneously taught physics in a private Technical School in Warsaw and later at the Warsaw Polytechnic (founded in 1898).

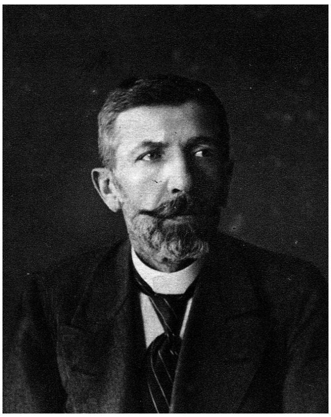


Fig. 6. Wiktor Biernacki

Biernacki published a number of articles on electromagnetism and also an excellent book on the newest discoveries in physics entitled *Nowe dziedziny widma* [32] (*New regions of the spectrum*). Large part of that book was devoted to the newly discovered electromagnetic rays but there was also detailed description of X-rays and "Becquerel rays". The book was published in Warsaw in the middle of 1898, but it had been written much earlier, probably at the time when Maria Skłodowska-Curie decided to study uranium rays.

The discoveries of new radioactive elements, polonium and radium, by Maria Skłodowska-Curie and Pierre Curie aroused great interest everywhere. The groundbreaking works of Maria Skłodowska-Curie, although done entirely in Paris, can be treated also as a part of Polish scientific heritage. The present text is, however, too short to allow for its full and detailed description. Thus we shall only stress that it is because of that great woman that Polish physicists played a significant role in the early history of radioactivity. Robert Lawson of Sheffield University published the following statistics [33]:

> "I have endeavoured to ascertain the numbers of authors in each country who have contributed four or more original papers on this subject. The result is embodied in what follows, the first numbers referring to those authors who have contributed four or more original papers and the numbers in brackets referring to the total number of authors who have made any noteworthy original contribution to radioactivity: British Empire 45 (171); Germany 28 (210); France 18 (70); Austria 10 (76); America 9 (89); Poland 4 (14); Switzerland 3 (19); Sweden 3 (9); Italy 2 (21); Norway 2 (20); Holland 2 (12); Hungary 2 (7); Russia 1 (13); Japan 1 (12); Denmark 1 (4), Roumania 0 (4); Spain 0 (1)".

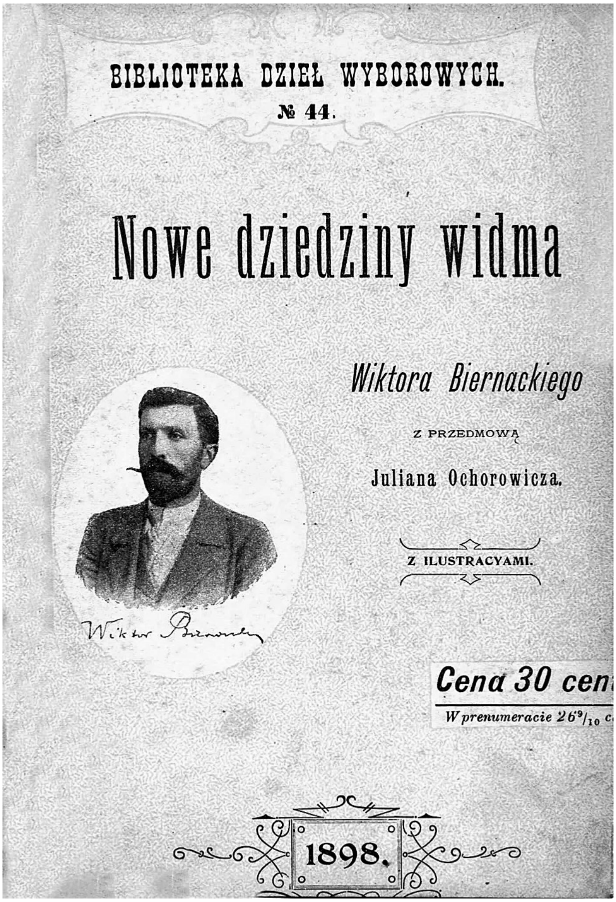


Fig. 7. Title page of Biernacki's book *New Regions of the Spectrum*

Writing in 1921 Lawson apparently took note of the nationality of Polish physicists and not their citizenship imposed by the occupants, because he listed as Polish all those who were doing research in the Austrian and Russian Partitions. A few years later Stefan Meyer and Egon Schweidler [34] quoted as many as thirty Polish researchers, more than Lawson. Let us mention here several most important names.

Fig. 8. Tadeusz Godlewski

Tadeusz Godlewski (1978–1921) was born in Lvov and studied at the Jagellonian University in Cracow. He spent several months (1904–1905) in Rutherford's laboratory in Montreal and in that period published four papers on radioactivity. His most important paper concerned the discovery of a new radioactive substance called AcX (now ^{223}Ra) [35]. After his return to Lvov Godlewski organized the first Polish laboratory devoted to the study of radioactivity. He originally served as an assistant to Olearski and since 1909 was professor of physics at the Lvov Polytechnic.

Kazimierz Fajans (1887–1975) was born in Warsaw and went to schools there. In 1904 he left the Russian Partition to study in Leipzig and Heidelberg. After obtaining his doctorate he spent some time (1910–1911) in Rutherford's laboratory in Manchester. In January 1913 Fajans formulated the ”displacement law”, which connected the type of radioactive decay with the shift of the product in the periodic system [36].

Fig. 9. Kazimierz Fajans

In February, 1913, Frederick Soddy, knowing already Fajans' papers, elaborated on the same topic; hence in the literature one finds the name ”Fajans-Soddy displacement law”. In the same year Soddy introduced the name “isotopes” for different radioactive substances which could not be separated by chemical means. Fajans noticed the same phenomenon but proposed to name such groups ”pleiades”, which was not accepted. For his important contributions Fajans got several nominations for the Nobel Prize in physics and chemistry [37].

In 1927 Fajans was invited to take the chair of physical chemistry at the University of Warsaw but it turned out that the authorities were not able to provide sufficient financial means for his laboratory. Thus he remained professor of chemistry in Munich. However, soon after the Nazi came to power in Germany, Fajans started looking for another employment. In 1935 the ultranationalistic and antisemitic attacks prevented him from accepting the chair of chemistry offered to him at the Lvov University. Finally he emigrated to USA and became professor of physics in Ann Arbor (Michigan).

6.1. Radiological Laboratory in Warsaw

Mirosław Kernbaum (1882–1911) was born in Warsaw, studied first in Charlottenburg and Zurich, and then physics in Geneva. He went to Paris and for three years (1908–1911) worked

Fig. 10. Mirosław Kernbaum

as an assistant to Maria Skłodowska-Curie. He published 10 important papers on radioactivity (and also 4 papers in Polish journals). But then, after returning to Poland, he unexpectedly developed a depression and committed suicide.

His father Józef, a rich Warsaw industrialist, decided to offer a large sum of money to the Warsaw Scientific Society (TNW) for establishing and maintaining a radiological physics laboratory in memory of Mirosław. The Warsaw Scientific Society accepted the offer and decided to invite Maria Skłodowska-Curie to take the direction of that laboratory. To that end a special TNW delegation, including Henryk Sienkiewicz (the 1905 winner of the Nobel Prize for Literature), was sent to Paris carrying that invitation.

However Maria Skłodowska-Curie was not able to return to Warsaw because of her involvement in the setting up of the Radium Institute in Paris. She decided instead to send to Warsaw two of her best Polish assistants, Jan Kazimierz Danysz and Ludwik Wertenstein, who were to run the radiological laboratory on her behalf and with her advice.

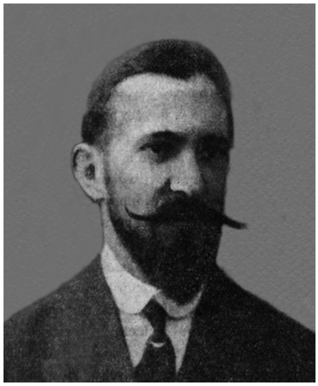


Fig. 11. Jan Kazimierz Danysz

Jan Kazimierz Danysz (1884–1914), known in the French sources as Jean Danysz, was born in Paris as son of Jan Danysz (1860–1928), a well-known Polish biologist who emigrated to France from the Prussian Partition, studied at the Sorbonne and later worked in the Pasteur Institute.

Jan Kazimierz Danysz served as an assistant to Pierre Curie and later to Maria Skłodowska-Curie. He studied radioactivity, in particular the beta radiation, and constructed the first beta spectrometer [38].

Ludwik Wertenstein (1887–1945) was born in Warsaw, finished schools there, and began studying physics at the Imperial Warsaw University. After a short time he was expelled for involvement in the students' protests. He then decided to emigrate to France, studied physics at the Sorbonne, and worked for five years (1908-1913) as an assistant to Maria Curie.

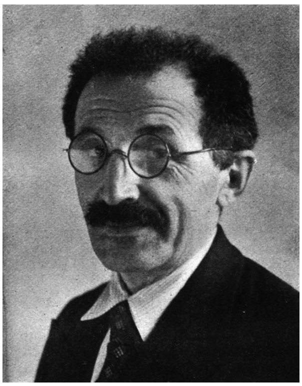


Fig. 12. Ludwik Wertenstein

In the summer of 1913 Jan Kazimierz Danysz and Ludwik Wertenstein arrived in Warsaw to organize and run the Radiological Laboratory of the TNW. The official inauguration of the laboratory took place on November 13, 1913, in presence of Maria Skłodowska-Curie who arrived from Paris. Unfortunately Danysz's stay in Warsaw was short. Being a French citizen, he returned to Paris after the outbreak of World War I, was drafted to the army, and killed in action already in November 1914.

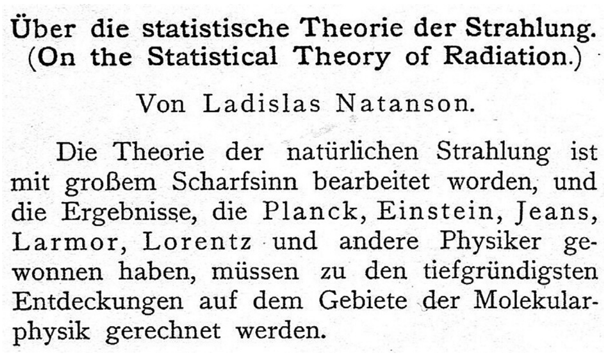
After 1914 the Radiological Laboratory of the TNW was directed by Wertenstein, who became also professor of physics at the Free Polish University (Wolna Wszechnica Polska) in Warsaw. In spite of poor financing the Radiological Laboratory of the TNW became an important research institution. It is worth adding that much later, in 1934, new radioactive isotopes of fluorine and scandium were discovered there by Marian Danysz (the son of Jan Kazimierz Danysz) and Michał Żyw.

7. Quantum physics

Fig. 13. Władysław Natanson

Władysław Natanson (1864–1937) was born in Warsaw, and studied in St. Petersburg, Dorpat and Graz. In 1890 he published the first Polish textbook of theoretical physics *Wstęp do fizyki teoretycznej* [39] (*Introduction to theoretical physics*). In 1891 he became a *privatdozent* at the Jagellonian University and stayed in Cracow ever since. In 1894 he became professor of theoretical physics. Natanson's interest was initially in thermodynamics and the kinetic theory of gases. He published several important papers on irreversible thermodynamics, and later on optical properties of matter. In 1911 he performed a pioneering analysis of radiation and derived the statistics of indistinguishable particles. His paper [40] was much ahead of the time and was not properly recognized. Thirteen years later the statistics of indistinguishable particles was rediscovered by Satyendra Nath Bose and improved by Einstein. Nevertheless, Natanson's priority has been recognized by historians of quantum physics. Thus Friedrich Hund wrote [41]:

> „This method of counting events for indistinguishable particles, which had already been perfectly clearly recognized by Natanson in 1911, was subsequently to be called Bose statistics (Natanson's work had of course been forgotten by 1924)".



Über die statistische Theorie der Strahlung.
(On the Statistical Theory of Radiation.)

Von Ladislas Natanson.

Die Theorie der natürlichen Strahlung ist mit großem Scharfsinn bearbeitet worden, und die Ergebnisse, die Planck, Einstein, Jeans, Larmor, Lorentz und andere Physiker gewonnen haben, müssen zu den tiefgründigsten Entdeckungen auf dem Gebiete der Molekularphysik gerechnet werden.

Fig. 14. The beginning of Natanson's paper *On the statistical theory of radiation*

According to Abraham Pais [42]:

> „Ladislas Natanson from Cracow (1911) was the first to state that distinguishability has to be abandoned in order to arrive at Planck's law".

Similar statements may be found in several other books on the history of quantum physics [43].

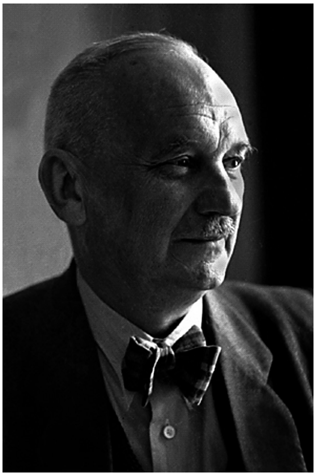


Fig. 15. Wojciech Rubinowicz

Wojciech Rubinowicz [44] (1889–1947) was born in Sadogora, and studied at the University of Czernowitz, where he obtained his doctor's degree. He later spent two years (1916-1918) as an assistant to Arnold Sommerfeld in Munich. It was there that Rubinowicz discovered selection rules for atomic transitions [45].

> „At roughly the same time (1918) A. Rubinowicz in the Sommerfeld school and Bohr gave a selection rule for the angular momentum quantum number, which, following Bohr, was mostly called k and l. Rubinowicz showed that an electromagnetic spherical wave can only transfer an angular momentum of 0 or $\pm h/2\pi$ when it receives or gives up an energy $h\nu$. He deduced from this that the angular momentum quantum number could only vary as between 0 and ±1" [46].

Using modern language we may say that it was the first estimate of the photon spin.

Later Rubinowicz was for a short time professor at the University of Ljublana in Yugoslavia, and since 1922 he was professor of physics at the Lvov Polytechnic. He has published a number of important papers on diffraction of light (1916, 1924, 1938), forbidden transitions in atoms (1929, 1930) and quadrupole radiation (1932). In 1933 he was invited to co-author the *Quantentheorie*, vol. 24/1 of the famous *Handbook der Physik*. The other five authors of that volume were: Wolfgang Pauli, Hans Bethe, Friedrich Hund, Gregor Wentzel, and Nevil Mott – all top physicists of that time (three of them: Pauli, Bethe, and Mott later received the Nobel Prize for physics).

Fig. 16. Mieczysław Wolfke

Another Polish physicist who worked on early quantum theory was Mieczysław Wolfke (1883–1947). He was born near Łódź, and studied in Liège, Paris and Breslau. During World War I he was a *privatdozent* at the Eidgenössische Technische Hochschule (ETH) in Zurich, and in 1921 got the appointment to the chair of experimental physics at the Warsaw Polytechnic. Wolfke published several interesting papers on localized light-atoms [47].

It is worth noting that in 1920 Wolfke proposed the principle of holography. Although his paper [48] was published in a well-known physics journal, the idea was not appreciated and was forgotten.

The principle of holography has been rediscovered much later by Dennis Gabor, who mentioned Wolfke in his Nobel Lecture (11 December, 1971) [49]:

> "(…) I did not know at that time (…) that Mieczislav Wolfke had proposed this method in 1920, but without realising it experimentally".

8. Other areas

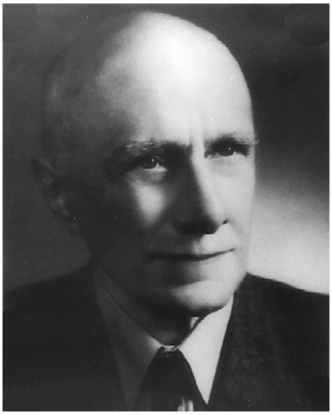


Fig. 17. Czesław Białobrzeski

Czesław Białobrzeski [50] (1878–1953) was born near Yaroslavl in Russia, where his father, a physician, was employed. He went to schools in Kiev and then studied physics at Kiev University. He spent two years (1908–1910) in Paris, working in the laboratory of Paul Langevin. After returning to Kiev we was appointed (1913) professor of physics at the university. When Poland regained independence Białobrzeski left Kiev and after a short stay in Cracow he became professor of theoretical physics at the University of Warsaw (1921).

In 1913 Białobrzeski published a pioneering work [51] on the physics of stars. He gave the proof that radiation pressure was an important factor in maintaining the internal equilibrium within stars. A similar theory was proposed three years later by the English astrophysicist Arthur Eddington [52]. These were World War years, so that he was ignorant of Białobrzeski's paper and did not cite it. Similarly, Białobrzeski had learned about Eddington's paper only after the war had ended. He then sent a copy of his 1913 paper to Eddington, who answered (1922) in a polite letter:

> "I congratulate you on having been apparently the first to point out the large share of radiation pressure in the internal equilibrium of a star".

Unfortunately, it was the only instance of Eddington's acknowledgment of Białobrzeski's paper which he later never cited in his books and articles. There is no doubt that Eddington was by far a better-known and more famous scientist than Białobrzeski. Thus, with years Białobrzeski's priority has been largely overshadowed by Eddington's accomplishment, although he had been quoted in a number of books [53].

The Białobrzeski-Eddington affair is a good example of so-called "Matthew Effect" introduced by R. K. Merton [54]:

> "For unto every one that hath shall be given, and he shall have abundance; but from him that hath not shall be taken away even that which he hath" (St. Matthew 25:29)

O równowadze termodynamicznej kuli gazowej swobodnej. — Sur l'équilibre thermodynamique d'une sphère gazeuse libre.

Mémoire

de M. ***TCHESLAS BIALOBJESKI,***

présenté, dans la séance du 5 Mai 1913, par M. Lad. Natanson m. t.

Je me propose dans ce mémoire de mettre en lumière l'importance de la pression de radiation pour l'équilibre thermodynamique des sphères gazeuses de grandes dimensions. Les corps célestes, le soleil en particulier et les étoiles, peuvent être considérés comme de telles sphères isolées dans l'espace.

§ 1. *Système: gaz, énergie de radiation.*

Examinons un des systèmes les plus simples de la Thermodynamique. Soit un corps de pompe muni d'un piston mobile et ren-

Fig. 1.

fermant un gaz quelconque (fig. 1). On fournit au gaz une quantité de chaleur infiniment petite dQ. Lorsque le gaz se dilate, la

Fig. 18. The beginning of Białobrzeski's paper on radiation pressure

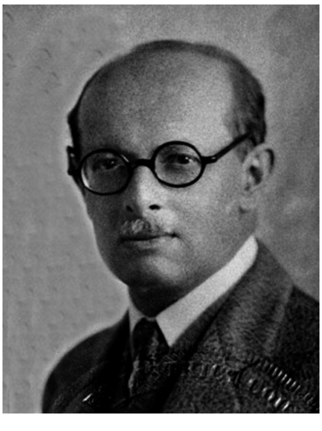


Fig. 19. Julian Lilienfeld

Julius Edgar Lilienfeld (1882–1963) was born in Lvov. In 1899 he began studies at the Technische Hochschule in Charlottenburg, but quickly decided on physics and chemistry at Berlin University; in 1905 he obtained doctor's degree for his studies of application of spectral analysis. He then went to Leipzig, where he was a *privatdozent* and a professor (since 1916). After World War I Lilienfeld paid several visits to the United States and in 1927 settled there for good. In 1930 he obtained a patent for the field--transistor effect [55]. The device was never constructed, but the patent had serious consequences, since because of its existence William Shockley's patent claim (1948) for a field transistor was rejected. Lilienfeld had also several other patents for electronic devices. He always stressed his

Polish background and nationality, and has been remembered as a Polish-American scientist and inventor.

Jan Czochralski (1885–1953) was born in Kcynia, a small town in the Prussian Partition. He initially worked in a pharmacy but in 1901 left to Berlin. He worked there in various industrial firms, and also studied chemical engineering at the Technische Hochschule in Charlottenburg. He specialized in metallurgy and worked in several industrial companies. In 1916 he invented a method of growing large monocrystals (his paper [56] was published only in 1918). Since 1929 Czochralski was professor of metallurgy and physics of metals at the Warsaw Polytechnic. After World War II "the Czochralski method" was found to be the most efficient way for producing crystals required by modern electronics, and Czochralski became the most often cited Polish scientist.

Fig. 20. Jan Czochralski

9. Conclusion

In the period 1870–1920 Polish physicists were not only passive recipients of new ideas in physics, but made essential contributions to several fields, such as e.g. cryogenics, electromagnetism, statistical physics, relativity, radioactivity, quantum physics, and astrophysics.

References

[1] When writing to foreign journals Wróblewski often latinized his name as Sigismund.
[2] J.C. Maxwell, *Diffusion of gases through absorbing substances*, Nature **14**, 1876, 24.
[3] S. Wróblewski, *On the nature of the absorption of gases*, Nature **21**, 1879, 190-192.
[4] Z. Wróblewski, *O istocie pochłaniania gazów*, Kosmos **4**, 1879, 246-249.
[5] S. Wroblewski, K. Olszewski, *Sur la liquėfaction de l'oxygene et de l'azote, et sur la solidification de sulphure du carbone et d'alcool*, Compt. Rend. **96**, 1883, 1140; *Sur la liquėfaction de l'azote*, Compt. Rend. **96**, 1883, 1225.
[6] K. Olszewski, *The liquefaction and solidification of argon*, Philosophical Transactions (A) **186**, 1896, 253-257.
[7] Helium was finally liquefied in 1908 by Heike Kamerlingh-Onnes in his cryogenic laboratory in Leiden.
[8] O. Fabian, *Zarys mechaniki analitycznej jako wstęp do fizyki umiejętnej*, Lwów 1886.
[9] A. Einstein, *Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen*, Ann. Phys. **17**, 1905, 549-560.
[10] M. Smoluchowski, *Zur kinetischen Theorie der Brownschen Molekularbewegung und der Suspensionen*, Ann. Phys. **21**, 1906, 755.
[11] A. Einstein, *Marian v. Smoluchowski*, Naturwissenschaften **5**, 1917, 737-738; A. Sommerfeld, *Zum Andenken an Marian von Smoluchowski*, Phys. Zeit. **18**, 1917, 533-539; L. Lorenz, *M. von Smoluchowski und sein Lebenwerk*, Jahresbericht Phys. Ver. Frankfurt a. M. 1917/1918, p. 3-16.

[12] See Bibliography, pp. 131-137, [in:] Smoluchowski, *His Life and Scientific Work*, Warszawa 2000.
[13] L.J. Bodaszewski, *Rauch und Dampf unter dem Mikroskop*, Dingler's Polytechnisches Journal **239**, 1881, 234.
[14] Ł. Bodaszewski, *Wyniki niektórych doświadczeń fizycznych*, Kosmos **7**, 1882, 177.
[15] G.L. De Haas-Lorentz, *Die Brownsche Bewegung und einige vervandte Erscheinungen*, Braunschweig 1913, p. 9; J. Perrin, *Les atomes*, Paris 1913, p. 122; Robert Andrews Millikan, *The Electron*, Chicago 1917, p. 141; R.A. Millikan, *Electrons, protons, neutrons and cosmic rays*, Cambridge 1935, p. 145, 147.
[16] M. Smoluchowski, *Über Unregelmässigkeiten in der Verteilung von Gasmolekülen und deren Einfluss auf Entropie und Zustandgleichung*, [in:] Boltzmann-Festschrift, Leipzig 1904, pp. 626-641.
[17] K. Olearski, *O elektrycznych oscylacjach*, Pamiętnik Akademii Umiejętności w Krakowie, Wydział Matematyczno-Przyrodniczy, **7**, 1882, p. 141-157.
[18] J. A. Fleming, *The Alternate Current Transformer*, vol. 1, London 1892, p. 375.
[19] L. Silberstein, *Elektryczność i magnetyzm*, vol. 1–3, Warszawa 1908–1910.
[20] L. Silberstein, *Elektromagnetische Gleichungen in bivektorieller Behandlung*, Ann. Phys. **22**, 1907, 579.
[21] L. Silberstein, *The Quaternionic Form of Relativity*, Phil. Mag. **23**, 1912, 790; *Second Memoir on Quaternionic Relativity*, Phil. Mag. **25**, 1913, 135.
[22] A. Pais, *Subtle is the Lord..., The Science and Life of Albert Einstein*, p.305, Oxford–New York 1982.
[23] L. Infeld, *Albert Einstein: His Work and its Influence on our World*, New York 1950, p. 44.
[24] J. Laub, *Über sekundäre Kathodenstrahlen*, Ann. Phys. **23** (**328**), 1907, 285-300.
[25] J. Laub, *O wtórnych promieniach katodowych*, Rozprawy Wydziału Matematyczno-Przyrodniczego Akademii Umiejętności **7B**, 1907, 29-50.
[26] A. Einstein, *Zur Elektrodynamik der bewegter Körper*, Ann. Phys. **17** (**322**), 1905, 891-921.
[27] J. Laub, *Zur Optik der bewegten Körper*, Ann. Phys. **23** (**328**), 1907, 738-742; *Zur Optik der bewegten Körper II*, Ann. Phys. **25** (**330**), 1908, 175-184.
[28] J. Laub, *Optyka ciał ruchomych*, Sprawozdania z posiedzeń naukowych X Zjazdu lekarzy i przyrodników polskich, Lwów 1907–1908.
[29] J. Laub, *Przyczynki do elektrodynamiki ciał poruszających się*, Prace matematyczno-fizyczne **19**, 1908, 63-75.
[30] A. Einstein, J. Laub, *Über die elektromagnetischen Grundglechungen für bewegte Körper*, Ann. Phys. **26** (**331**), 1908, 532-540; *Über die im elektromagnetischen Felde auf ruhende Körper ausgeübten pondermotorischen Kräfte*, Ann. Phys. **26** (**331**), 1908, 541-550; *Berichtung zur Abhandlung "Über die elektromagnetischen Grundglechungen für bewegte Körper"*, Ann. Phys. **27** (**332**), 1908, 232; *Bemerkungen zu unseren Arbeit "Über die elektromagnetischen Grundglechungen für bewegte Körper"*, Ann. Phys. **28** (**333**), 1909, 445-447.
[31] J. Laub, *Über die experimentellen Grundlagen des Relativitätsprinzips*, Jahrbuch der Radioaktivität und Elektronik **7**, 1910, 405-463.
[32] W. Biernacki, *Nowe dziedziny widma*, Warszawa 1898.
[33] R. W. Lawson, *The part played by different countries in the development of the science of radioactivity*, Scientia (Milano), **30**, 1921, 257-270.
[34] S. Meyer, E. Schweidler, *Radioaktivität*, Second edition, Leipzig–Berlin 1927.
[35] T. Godlewski, *Actinum and its successive products*, Philosophical Magazine **10**, 1905, 35-45.
[36] K. Fajans, *Die Stellung der Radioelementen im periodischen System*, Chemiker Zeitschrift **37**, 1913, 151, 242; Phys. Zeit. **14**, 1913, 136.

[37] Elisabeth Crawford, John L. Heilbron, Rebecca Ulrich, *The Nobel population 1901–1937, A Census of the Nominators and Nominees for the Prizes in Physics and Chemistry*, Office for History of Science and Technology, University of California, Berkeley, and Office for History of Science, Uppsala University, Uppsala-Berkeley 1987.

[38] J. Danysz, *Sur les rayons beta de la famille du radium*, Compt. Rend. **153**, 1911, 339-341, 1066-1068; Le Radium **9**, 1912, 1-5.

[39] W. Natanson, *Wstęp do fizyki teoretycznej*, Warszawa 1890.

[40] W. Natanson, *Über die statistische Theorie der Strahlung*, Phys. Zeit. **12**, 1911, 659.

[41] F. Hund, *The History of Quantum Theory*, New York 1974, p. 145.

[42] A. Pais, *Inward Bound*, New York 1986, p. 283.

[43] A. Hermann, *The Genesis of Quantum Theory*, Cambridge (1971); M. Jammer, *The Conceptual Development of Quantum Mechanics*, Chapter 4, Mc Graw Hill, New York 1967; J. Mehra, H. Rechenberg, *The Historical Development of Quantum Theory*, vol. 1, New York–Heidelberg–London 1982.

[44] In early years Rubinowicz often latinized his name and signed his papers as Adalbert, hence the initial A.

[45] A. Rubinowicz, *Bohrsche Frequenzbedingung und Erhaltung des Impulsmomentum*, Phys. Zeit. **19**, 1918, 441-465, 465-474.

[46] F. Hund, *The History of Quantum Theory*, New York 1974, p. 94.

[47] M. Wolfke, *Zur Quantentheorie*, Verh. d. Deutsch. Phys. Ges. **15**, 1915, 1123-1129; 1215-1218; **16**, 1916, 4-6; *Welche Strahlungform folgt aus der Annahme der Lichtatome?*, Phys. Zeit. **15**, 1914, 463-464; *Einsteinische Lichtquanta und räumliche Struktur der Strahlung*, Phys. Zeit. **22**, 1921, 375-379.

[48] M. Wolfke, *Über die Möglichkeit der optischen Abbildung von Molekulargittern*, Phys. Zeit. **21**, 1920, 495.

[49] D. Gabor, *Holography 1948–1971*, Nobel Lecture, [in:] Physics 1971–1980, ed. Stig Lundqvist, World Scientific Publishing Co., Singapore 1992.

[50] Białobrzeski often signed his papers in foreign languages as Tscheslas Bialobjeski to make his name and family name easier to pronounce by foreigners.

[51] T. Bialobjeski, *Sur l'equilibre thermodynamique d'une sphère gazeuse libre*, Bull. Inter. Acad. Sci. Cracoviae, Année 1913, 264-290.

[52] A. Eddington, *On the radiative equilibrium of the stars*, Monthly Notices Royal Astr. Soc. **77**, 1916, 16-35.

[53] see e.g. Georges Thiercy, *L'Équilibre radiatif dans les étoiles*, Chapter V, Paris 1935; Subrahmanian Chandrasekhar, *An Introduction to the Study of Stellar Structure*, Chapter 6, New York 1939.

[54] R.K. Merton, *The Matthew Effect in Science*, Science **159**, 1968, 56.

[55] Michael Riordan and Lilian Hoddeson, *Crystal Fire: the Birth of the Information Age*, New York 1997, p.145, 176-177; John W. Orton, *The Story of Semiconductors*, Oxford 2004, p. 101.

[56] J. Czochralski, *Ein neues Verfahren zur Messung der Kristallisationgeschwindigkeit der Metalle*, Zeitschrift für physikalische Chemie **92**, 1918, 219.

CONTENTS

TREŚĆ