Abstract
The paper shows the modelling process of a 3D digital model of the human radius bone based on its external and internal geometry. The obtained numeral characteristics of the model of the chosen mathematical functions can be used for further static and dynamic analysis of the bone subjected to external forces of different magnitudes.

Keywords: geometric properties of radius bone, digital modelling of 3D structures, analysis of cross sections of bone

Streszczenie
Artykuł przedstawia proces modelowania cyfrowego 3D kości promieniowej człowieka na podstawie jego geometrii zewnętrznej i wewnętrznej. Otrzymane charakterystyki liczbowe modelu wybranych funkcji matematycznych mogą być stosowane do dalszej statycznej i dynamicznej analizy kości poddanej różnym obciążeniom zewnętrznym.

Słowa kluczowe: właściwości geometryczne kości promieniowej, cyfrowe modelowanie struktur 3D, analiza przekrojów kości

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1. Introduction

Biomechanics is an evolving field that has recently witnessed rapid growth and increase of interest. So far, very few models showing the geometry of the bone have been constructed. This article presents the study of the geometrical properties of one of the arm bones – the radius bone. The aim of this work is to create a digital 3D model of a bone which contains all geometric properties without any simplifications.

The created model shows both, inner and outer geometry. The model was opened in Pro Engineer Wildfire 4.0, a CAD program with a highly developed cross-sections creation. Collected data was used to obtain graphs of functions and thereby, equations describing the examined parameter. In order to exemplify the possible use of the formula, the article presents the calculation of critical force of the radius during buckling.

There are no articles describing the problems included in this work. Bone models are usually simplified, with no internal geometry included. There are bone models obtained from computed tomography, however, the modelling presented in the article gives much more control over the end result.

Geometrical properties of bones haven’t been described extensively either. Hence the findings presented in this article concerning this issue hold a great value for other works.

2. The radius

The radius is a long bone located on the thumb side of the forearm. It constitutes to the wrist and elbow joints. The radius consists of a body and two extremities – proximal and distal. The body of the radius has three surfaces – anterior, posterior and medial, and

Fig. 1. The bones of the forearm, the ulna (top), the radius (below) [1]
three edges – frontal, external and internal (where the interosseous membrane connects the radius and the ulna). The proximal extremity – the head of the radius – is a surface that articulates with the ulna and the humerus. Around the head runs the circumference of the radial head (the surface which is in contact with the proximal end of the ulna). The head of the radius and the body connect the neck, below which, on the ulna side, is the radial tuberosity. On the distal, medial side of the radial body is the ulna notch (the surface which is in contact with the distal part of the ulna). The dorsal tubercle is located on the external surface of the distal end [1, 8].

3. Construction of the model

The scanned model was moulded out of plastic from the real bone. The bone was scanned with the use of reversed engineering technology thanks to the courtesy of the Coordinate Metrology Laboratory at the Cracow University of Technology. The outcome model of the bone had perfect external geometry parameters. The description of the formation of the model and the technology used can be found in references [2, 5]. In order obtain data necessary to reproduce the internal structure of the bone (the bone marrow cavity), a subject was submitted to a CAT scan at Jagiellonian University Medical College. The subject’s right forearm was placed in the anatomical position, while the CT scan made a series of images in planes perpendicular to the axis of the bone adopted. The CT scanning enabled the reconstruction of the marrow cavity shape and its location within the bone. The marrow cavity modelling and its integration with the external geometry of the model was made with the Solid Works 2013 software (Fig. 2). Finally, the position of the coordinate system was set in accordance with the previous model of the scanned arm. The Z axis was taken along the bone and was coinciding with the line connecting the characteristic points on both joint surfaces. The X and Y axes were adopted in order to set the radius in the anatomical position (Fig. 3).

Fig. 2. Digital model of the bone: internal structure (top), external geometry (bottom)
4. Cross-section planes analysis

The model prepared in accordance with the procedure presented in the previous chapter was imported into ProEngineer Wildfire 4.0. This program was chosen because of its very extensive module for the analysis section. The first step was to generate a number of planes intersecting the model. The planes were parallel to the XY plane and spaced apart by 1 [mm] each. Subsequently, all planes were tested with a tool for analysing cross-sections. Each cross-section made a base for a separate report. All reports were compiled in Microsoft Office Excel. The last step was to create a chart with all the characteristic properties of the model.

5. Selection of mathematical formulas

There are various programs designed to determine the formula of a function on the basis of its chart data, among those the most popular is Microsoft Office Excel. Excel was chosen for the study because of its trendline option. The graphs below present the changes in the cross-sectional area of the bone and the changes of the moment of inertia of the centre of gravity. Each chart had a trendline with the describing formula generated. The trendline presented was chosen cause of its simplicity (polynomial of the lowest degree possible) in order not to adopt the negative values. This means that the polynomials with lower degrees were taking negative values within the range studied, which is incorrect, since none of the tested properties cannot take those values (Fig. 4–7).

![Fig. 3. Location of the coordinate system for the 3D model](image)

![Fig. 4. Changes within the cross-section area](image)
6. Application of the formulas

The formulas obtained were used to assess the buckling resistance of the bone. It was determined that the distal joint surface (the wrist joint) occupies most of the surface from the forearm’s side, consequently most of the compression load would be transmitted on the radius. Figure 8 presents the buckling model adopted.
The bone in the elbow joint, where the radius is in contact with the humerus, best represents the support model of the non-sliding articulated joint. The presence of the distal part of the ulna at the wrist joint can best be described as a sliding joint. The described characteristics are presented in Figure 1.

The energy method was used with this case study. The internal energy of the flexing rod was compared with the compression force being at work at the rod end movement, which resulted in the energy formula for critical force (1).

\[
P_{cr} = \frac{\int_0^L E J (y^*(x))^2 \, dx}{\int_0^L (y'(x))^2 \, dx}
\]  

(1)

where:

- \( E \) – Young modulus,
- \( J \) – moment of inertia,
- \( L \) – length of the radius.

Adopted by the approximate form of the function of deflection (2):

\[
y(x) = a \left( -\frac{x^4}{12} + \frac{x^3 L}{6} - \frac{L^3 x}{12} \right)
\]

(2)

where:

- \( L \) – length of the radius,
- \( a \) – coefficient = 1.
Young’s modulus was adopted on the basis of the available literature concerning the living bones. By applying the formulas describing the function properties to the formula [7] the value of the critical force was obtained.

\[ P_1 = 4683.87 \text{ [N]} \]
\[ P_2 = 1575.52 \text{ [N]} \]

7. Conclusions and further research

The study presents the latest techniques that allow the description of extremely complex structures – a level of description which until recently, was impossible to achieve. Extremely accurate waveforms of the characteristic properties of the bone. Moreover, it was possible to present these properties in a graphical form and with mathematical formulas that allowed the most accurate description. One of the most important issues is that received results were adapted to classical theories of mechanics and strength of materials. Further research will be continued in two ways. Firstly, the best model describing the features presented in the study should be chosen, independently from those few that are currently available in the most popular programs. The aim of this study would be to produce a formula describing the function matched as closely as possible to the actual conduct of the property. Secondly a digital model for numerical simulations should be used that could provide a wider view of the behaviour of bones under certain external influences.

References