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GRAPHS WITH EVERY PATH OF LENGTH k IN A HAMILTONIAN CYCLE

GRAFY Z DOWOLNĄ ŚCIEŻKĄ DŁUGOŚCI k ZAWARTĄ W PEWNYM CYKLU HAMILTONOWSKIM

A b s t r a c t

In this paper we prove that if G is a $(k+2)$-connected graph on $n \geqslant 3$ vertices satisfying $\mathrm{P}(n+k)$:

$$\mathrm{d}_G(x,y) = 2 \ \Rightarrow \ \max\{\mathrm{d}(x), \mathrm{d}(y)\} \geqslant \frac{n+k}{2}$$

for each pair of vertices x and y in G, then any path $S \subset G$ of length k is contained in a hamiltonian cycle of G.

Keywords: cycle, graph, hamiltonian cycle, matching, path

S t r e s z c z e n i e

W pracy udowodniono, że W $(k+2)$-spójnym grafie G o $n \geqslant 3$ wierzchołkach, który spełnia warunek $\mathrm{P}(n+k)$:

$$\mathrm{d}(x,y) = 2 \ \Rightarrow \ \max\{\mathrm{d}(x), \mathrm{d}(y)\} \geqslant \frac{n+k}{2}$$

dla dowolnej pary wierzchołków x i y, każda ścieżka $S \subset G$ długości k jest zawrta w pewnym cyklu hamiltonowskim grafu G.

Słowa kluczowe: cykl, cykl hamiltonowski, graf, skojarzenie, ścieżka

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1. Introduction

We consider only finite graphs without loops and multiple edges. By V or $\mathrm{V}(G)$ we denote the vertex set of the graph G and respectively by E or $\mathrm{E}(G)$, the edge set of G. By $\mathrm{d}_x(G)$ or $\mathrm{d}(x)$, we denote *the degree of a vertex x in the graph G* and by $\mathrm{d}(x,y)$ or $\mathrm{d}_G(x,y)$, *the distance between x and y in G.*

Definition 1.1 (cf [10])**.** *Let $k, s_1, \dots s_\ell$ be positive integers. We call S a path system of length k, if the connected components of S are paths:*

$$\begin{array}{ll} P^1: & x_0^1 x_1^1 \dots x_{s_1}^1, \\ & \vdots \\ P^l: & x_0^\ell x_1^l \dots x_{s_\ell}^\ell \end{array}$$

and $\sum_{i=1}^{\ell} s_i = k$.

Let S be a path system of length k and let $x \in \mathrm{V}(S)$. We shall call x *an internal vertex* if x is an internal vertex (cf [3]) in one of the paths P^1 , … , P^ℓ .

If q denotes the number of internal vertices in a path system S of length k then $0 \leqslant q \leqslant k-1$. If $q = 0$, then S is *a k-matching* (i.e. a set of k independent edges).

Let H be a subgraph of G. By $G \setminus H$ we denote the graph obtained from G by the deletion of the edges of H.

Definition 1.2. *The graph F is said to be an H-edge cut-set of G if $F \subset \mathrm{E}(H)$ and $G \setminus F$ is not connected.*

Definition 1.3. *The graph F is said to be a minimal H-edge cut-set of G if F is an H-edge cut-set of G which has no proper subset being an edge cut-set of G.*

Definition 1.4 (cf [7])**.** *Let G be a graph on $n \geqslant 3$ vertices and $k \geqslant 0$. G is k-edge-hamiltonian if for every path system P of length at most k there exists a hamiltonian cycle of G containing P.*

Let G be a graph and $H \subset G$ a subgraph of G. For a vertex $x \in \mathrm{V}(G)$, we define the set $N_H(x) = \{y \in \mathrm{V}(H) : xy \in \mathrm{E}(G)\}$. Let H and D be two subgraphs of G. $\mathrm{E}(D,H) = \{xy \in \mathrm{E}(G) : x \in \mathrm{V}(D) \text{ and } y \in \mathrm{V}(H)\}$. For a set of vertices A of a graph G, we define the graph $G(A)$ as the subgraph induced in G by A. In the proof, we will only use oriented cycles and paths. Let C be a cycle and $x \in V(C)$, then x^- is *the predecessor of x* and x^+ is its *successor.*

Definition 1.5 (cf [2])**.** *Let W be a property defined for all graphs of order n and let k be a non-negative integer. The property W is said to be k-stable if whenever $G + xy$ has property W and $\mathrm{d}(x) + \mathrm{d}(y) \geqslant k$ then G itself has property W.*

J.A. Bondy and V. Chvátal [2] proved the following theorem, which we shall need in the proof of our main result:

Theorem 1.1. *Let n and k be positive integers with $k \leqslant n-3$. Then the property of being k-edge-hamiltonian is $(n+k)$-stable.*

In 1960, O. Ore [9] proved the following:

Theorem 1.2. *Let G be a graph on $n \geqslant 3$ vertices. If for all nonadjacent vertices $x, y \in \mathrm{V}(G)$ we have*

$$\mathrm{d}(x) + \mathrm{d}(y) \geqslant n$$

then G is hamiltonian.

Geng-Hua Fan [4] has shown:

Theorem 1.3. *Let G be a 2-connected graph on $n \geqslant 3$ vertices. If G satisfies*

$$\mathrm{P}(n): \quad \mathrm{d}(x,y) = 2 \Rightarrow \max\{\mathrm{d}(x), \mathrm{d}(y)\} \geqslant \frac{n}{2}$$

for each pair of vertices x and y in G, then G is hamiltonian.

The condition for degree sum in Theorem 1.2 is called *an Ore condition* or *an Ore type condition for graph G* and the condition $\mathrm{P}(k)$ is called *a Fan condition* or *a Fan type condition for graph G*.

Later, many *Fan type theorems* and *Ore type theorems* are shown.

Now we shall present Las Vergnas [8] condition $\mathcal{L}_{n,s}$.

Definition 1.6. *Let G be graph on $n \geqslant 2$ vertices and let s be an integer such that $0 \leqslant s \leqslant n$. G satisfies Las Vergnas condition $\mathcal{L}_{n,s}$ if there is an arrangement $x_1, \ldots, x_n$ of vertices of G such that for all i, j if*

$$1 \leqslant i < j \leqslant n\,,\ i + j \geqslant n - s\,,\ x_i x_j \notin \mathrm{E}(G)\,,$$

$$\mathrm{d}(x_i) \leqslant i + s \text{ and } \mathrm{d}(x_j) \leqslant j + s - 1$$

then $\mathrm{d}(x_i) + \mathrm{d}(x_j) \geqslant n + s$.

Las Vergnas [8] proved the following theorem:

Theorem 1.4. *Let G be a graph on $n \geqslant 3$ vertices and let $0 \leqslant s \leqslant n-1$. If G satisfies $\mathcal{L}_{n,s}$ then G is s-edge hamiltonian.*

Note that condition $\mathcal{L}_{n,s}$ is weaker than Ore condition.

Later Skupień and Wojda proved that the condition $\mathcal{L}_{n,s}$ is sufficient for a graph to have a stronger property (for details see [10]). Wojda [11] proved the following Ore type theorem:

Theorem 1.5. *Let G be a graph on $n \geqslant 3$ vertices, such that for every pair of nonadjacent vertices x and y*

$$\mathrm{d}(x) + \mathrm{d}(y) > \frac{4n-4}{3}\,.$$

Then every matching of G lies in a hamiltonian cycle.

In 1996, G. Gancarzewicz and A. P. Wojda proved the following Fan type theorem:

Theorem 1.6. *Let G be a 3-connected graph of order $n \geqslant 3$ and let M be a k-matching in G. If G satisfies* $\mathrm{P}(n+k)$:

$$\mathrm{d}(x,y) = 2 \;\Rightarrow\; \max\{\mathrm{d}(x), \mathrm{d}(y)\} \geqslant \frac{n+k}{2}$$

for each pair of vertices x and y in G, then M lies in a hamiltonian cycle of G or G has a minimal odd M-edge cut-set.

In this paper we find a Fan type condition under which every path of length k in a graph G lies in a hamiltonian cycle.

For notation and terminology not defined above a good reference should be [3].

2. Result

Theorem 2.1. *Let G be a graph on $n \geqslant 3$ vertices and let S be a path of length k in G. If the graph G is l-connected, where $l = \min\{k+2\,,\; n-1\}$ and satisfies* $\mathrm{P}(n+k)$:

$$\mathrm{d}(x,y) = 2 \;\Rightarrow\; \max\{\mathrm{d}(x), \mathrm{d}(y)\} \geqslant \frac{n+k}{2} \tag{2.1}$$

for each pair of vertices x and $y \in \mathrm{V}(G)$, then S lies in a hamiltonian cycle of G.

Note that $1 \leqslant k \leqslant n-1$ and since an $(n-1)$-connected graph of order n is a complete graph K_n, which is obviously k-edge hamiltonian for any k the result is interesting when $k < n-3$.

For $k = 1$, the path S is a 1-matching and we have a special case of Theorem 1.6 (the graph is 3-connected, so in this case we can not have a minimal S-edge cut set).

Unfortunately, in Theorem 2.1 we can not decrease the connectivity of graph G. We can consider a vertex x and a complete graph K_m , $m \geqslant 3$. In the complete graph K_m we choose a path $S : s_1 \dots s_{k+1}$ of length $k = m-2$. There is only one vertex $y \in K_m$ not contained in S.

Let G be a graph of order $n = m+1$ obtained from two complete graphs $K_1 = \{x\}$ and K_m by adding edges xs_i , for $i \in \{1, \dots, k+1\}$ The path S is a path of length k contained in G which is not contained in any hamiltonian cycle of the $(k+1)$-connected graph G, see Figure (1).

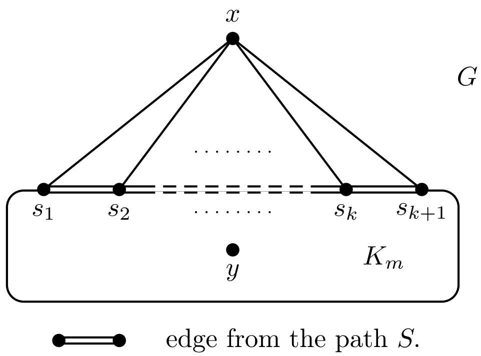

Fig. 1: A $(k+1)$-connected graph G with no hamiltonian cycle through the path S : $s_1 \dots s_{k+1}$.

Note that we can replace the vertex x by a complete graph K_ℓ, $\ell \geqslant k+1$. Let $\{x_1, \dots, x_{k+1}\} \subset \mathrm{V}(K_\ell)$ and let G be a graph of order $n = m + l$ obtained from two complete graphs K_ℓ and K_m by adding edges $x_i s_i$, for $i \in \{1, \dots, k+1\}$ The path S is a path of length k contained in G which is not contained in any hamiltonian cycle of the $(k+1)$-connected graph G, see Figure (2).

3. Proof

Proof of Theorem 2.1:

Take G and S as in the assumptions of Theorem 2.1.

Consider the nonempty set

$$A = \{x \in V(G) : \mathrm{d}_x(G) \geqslant \frac{n+k}{2}\}.$$

Note that if x and y are nonadjacent vertices of A, then the graph obtained from G by the addition of the edge xy also satisfies the assumptions of the theorem. Therefore, and by Theorem 1.1 we may assume that:

$$xy \in E(G) \quad \text{for any} \quad x, y \in A \quad \text{and } x \neq y. \tag{3.1}$$

By (3.1), A induces a complete subgraph $G(A)$ of the graph G.

In fact, since the property of being k-edge-hamiltonian is $(n+k)$-stable, we can replace G with its $(n+k)$-closure.

Let G_A be a graph obtained from G by deletion of vertices of the graph $G(A)$ (i.e. vertices from the set A).

Now take D, a connected component of the graph G_A.

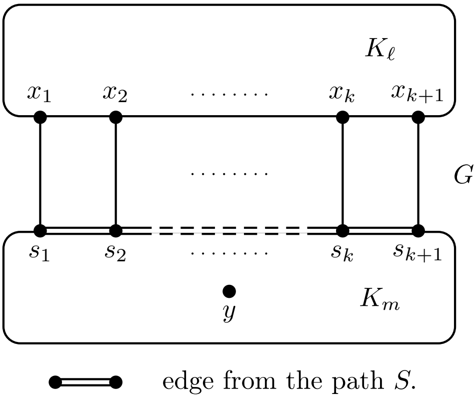

Fig. 2: A $(k+1)$-connected graph G with no hamiltonian cycle through the path S : $s_1 \dots s_{k+1}$.

Suppose that there exist two nonadjacent vertices in D. Since D is connected, we have two vertices x and y in D such that $\mathrm{d}_G(x,y) = 2$ and by the assumption that G satisfies $\mathrm{P}(n+k)$, we have $x \in A$ or $y \in A$, which is a contradiction.

We have proved that every component of $G_{\overline{A}}$ is a complete graph K_ι , $\iota \in I$, joined with $G(A)$ by at least $k+2$ edges.

Claim 3.1. *If $K_{\iota_0}, K_{\iota_1} \in \{K_\iota\}_{\iota \in I}$ are such that $\iota_0 \neq \iota_1$, then:*

$$N(K_{\iota_0}) \cap N(K_{\iota_1}) = \emptyset. \tag{3.2}$$

Proof of Claim 3.1:

Suppose that $N(K_{\iota_0}) \cap N(K_{\iota_1}) \neq \emptyset$. Then we have a vertex $y \in K_{\iota_0}$ and a vertex $y' \in K_{\iota_1}$ such that $\mathrm{d}_G(y,y') = 2$ and by $\mathrm{P}(n+k)$ either $y \in A$ or $y' \in A$. This contradicts the fact that K_{ι_0} and K_{ι_1} are two connected components of $G_{\overline{A}}$.

□

We have shown that the graph G consists of a complete graph $G(A)$ and of a family of complete components $\{K_\iota\}_{\iota \in I}$, of $G_{\overline{A}}$, which do not have common neighbors in $G(A)$.

Since G is $(k+2)$-connected, we have the following:

Claim 3.2. *Every component $\{K_\iota\}_{\iota \in I}$, is joined with $G(A)$ by at least three edges such that end vertices of these edges are not internal vertices of the path S.*

We label vertices of path $S : s_1 s_2 \dots s_k s_{k+1}$.

Graph G consists of complete graph $G(A)$ and disjoined complete graphs $\{K_\iota\}_{\iota \in I}$, joined with $G(A)$ by at least three edges such that end vertices of these edges are not internal vertices of path S.

Firstly we consider the case when path S is contained in one complete graph (i.e. $G(A)$ or one graph $K_{\iota_0} \in \{K_\iota\}_{\iota \in I}$). In this case, by Claim3.2 we have a hamiltonian cycle through S.

Now we assume that S is not contained in the complete graph $G(A)$ or one graph $K_{\iota_0} \in \{K_\iota\}_{\iota \in I}$ and we can now define a cycle $C \subset G$ containing the path S and all vertices of $G(A)$.

We shall consider four cases:

1. Both end vertices of S are in $G(A)$ i.e. s_1 , $s_{k+1} \in G(A)$.
2. Both end vertices of S are in the same component K_ι of G_A i.e. s_1 , $s_{k+1} \in K_\iota$.
3. End vertices of S are in different components of G_A i.e. $s_1 \in K_{\iota_1}$, $s_{k+1} \in K_{\iota_2}$, K_{ι_1} , $K_{\iota_2} \in \{K_\iota\}_{\iota \in I}$ are such that $\iota_1 \neq \iota_2$.
4. One end vertex of S is in $G(A)$ and the other end vertex is in a component K_ι of G_A. In this case, we can assume without loss of generality that $s_1 \in G(A)$ and $s_{k+1} \in K_\iota$.

If $C \subset G$ is a cycle in G, then by $G_{V \setminus C}$ we denote a graph obtained from G by deletion of vertices of cycle C.

Case 1: Both end vertices of S are in $G(A)$ i.e. s_1 , $s_{k+1} \in G(A)$.

Note that even in this case, path S may pass through some components K_ι creating a kind of ears of the complete graph $G(A)$, on every incident graph K_ι. We can find an example of such ears on Figure (3).

Since $G(A)$ is a complete graph we have a cycle C containing the path S and all vertices of $G(A)$ performing the following conditions:

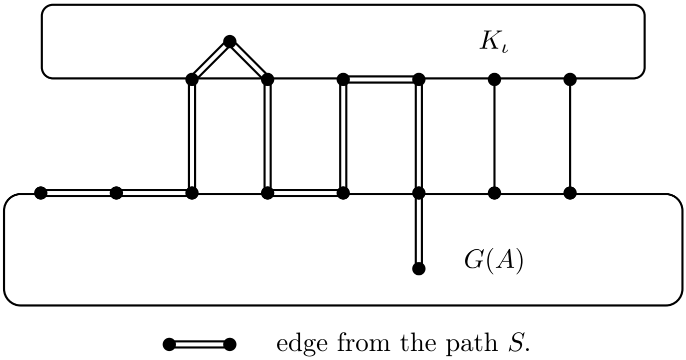

Fig. 3: Example of ears of the graph $G(A)$.

- C contains all edges of $\mathrm{E}(S) \cap \mathrm{E}(G(A))$ and all vertices of A. (3.3)
- If K_{ι_0} and K_{ι_1} are two different components of $G_{V \setminus C}$ then (3.4)

  $N(K_{\iota_0}) \cap N(K_{\iota_1}) = \emptyset$.
- Let $x \notin V(C)$, $y \in V(C)$ and $xy \in E(G)$ then: (3.5)

  if y is not an internal vertex of S, then $y \in A$,

  if y^- is not an internal vertex of S, then $y^- \in A$,

  if y^+ is not an internal vertex of S, then $y^+ \in A$.

Properties (3.3 — 3.5) will allow us to extend C to a hamiltonian cycle.
Note that this cycle C may not be contained in $G(A)$.

Subcase 1.1: Extending the cycle C through components K_ι incident with ears

Since graph G is $(k+2)$-connected, we have at least $k+2$ edges joining K_ι with $G(A)$, so at least one of these edges say uc_i, $u \in \mathrm{V}(K_\iota)$, $c_i \in \mathrm{V}(G(A)) \setminus S$, is not incident with S. If a component K_ι is incident with an ear, at least one interior vertex of path S is contained in this ear, and we have an additional edge say $u'c_j$, $u' \in \mathrm{V}(K_\iota)$, $c_j \in \mathrm{V}(G(A)) \setminus S$, not incident with S joining K_ι with $G(A)$. Using these two edges uc_i and $u'c_j$, we can extend the cycle C through the remaining vertices of K_ι.

Without loss of generality, we can assume that on cycle C, the vertices are ordered in the following way: $s_1 \ldots s_{k+1} c_{k+2} \ldots c_i \ldots c_j \ldots s_1$.

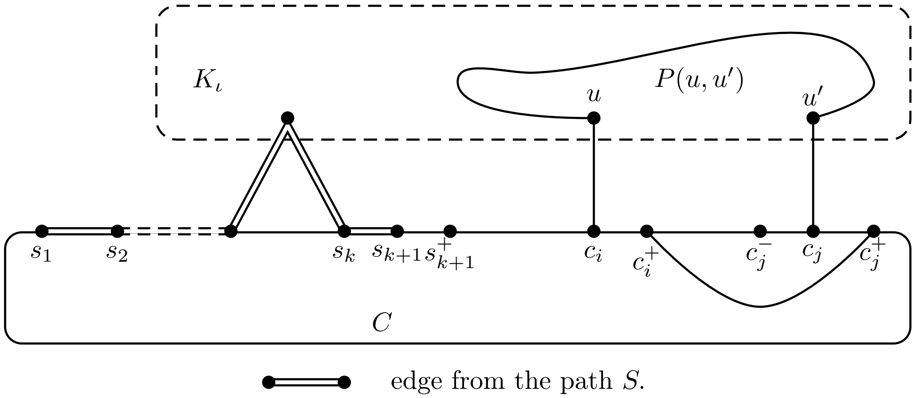

Fig. 4: Extension of the cycle C through component K_ι incident with an ear.

We can replace cycle C, by the following cycle C'

$$C': \quad s_1 \dots s_{k+1} s_{k+1}^+ \dots c_i P(u,u') c_j c_j^- \dots c_i^+ c_j^+ \dots s_1\,,$$

where $P(u,u') \subset K_\iota$ is a path joining u with u' containing all vertices of $K_\iota \setminus S$. Note that this cycle C' satisfies conditions (3.3 — 3.5). We can find an example of the cycle C' on Figure 4.

Case 2: Both end vertices of S are in the same component K_ι of G_A i.e. $s_1\,,\ s_{k+1} \in K_\iota$.

Since the graph G is $(k+2)$-connected, we have at least $k+2$ edges joining K_ι with $G(A)$. In this case, at least two edges from the path S : $s_i s_{i+1}$ and $s_j s_{j+1}$ are joining K_ι with $G(A)$, so at least two independent edges uv, $u'v'$, u, $u' \in \mathrm{V}(K_\iota)$, v, $v' \in \mathrm{V}(G(A))$, not incident with S joining K_ι with $G(A)$.

Consider the following path:

$$P: \quad vus_1 \dots s_k P(s_{k+1}, u') v'\,,$$

where $P(s_{k+1}, u') \subset K_\iota$ is a path joining s_{k+1} with u' containing all vertices of $K_\iota \setminus \{\mathrm{V}(S) \cup \{u\}\}$. We can find an example of the path P on Figure 5.

The graph $G(A)$ is complete, so we can extend P to a cycle C containing all vertices of $G(A)$ and satisfying (3.3 — 3.5).

Note that as in Case 1 the path S may pass through some components K_i creating the kind of ears of the complete graph $G(A)$. Using the same argument as in the

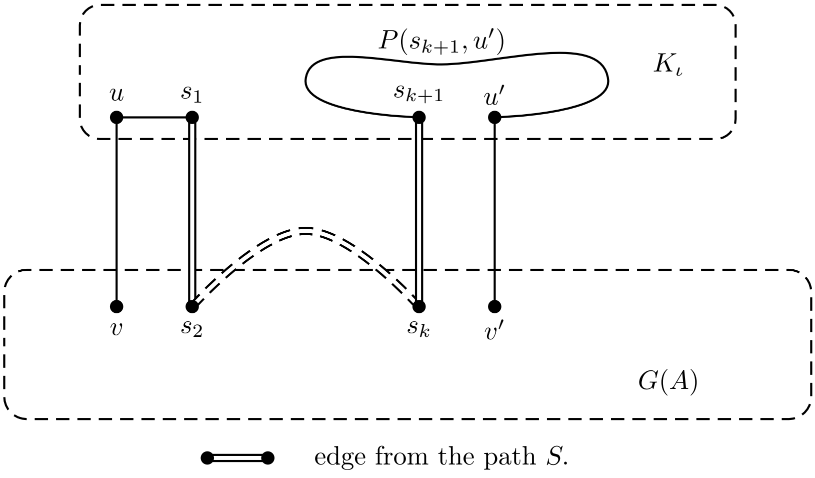

Fig. 5: Path P containing S with both end vertices in $G(A)$.

Subcase 1.1 we can extend the cycle C through components K_i incident with ears preserving the properties (3.3 — 3.5).

Case 3: End vertices of S are in different components of G_A i.e. $s_1 \in K_{\iota_1}$, $s_{k+1} \in K_{\iota_2}$, K_{ι_1} , $K_{\iota_2} \in \{K_\iota\}_{\iota \in I}$ **are such that** $\iota_1 \neq \iota_2$.

Again, since graph G is $(k+2)$-connected we have at least $k+2$ edges joining every component K_i with $G(A)$. In this case, for $i = \iota_1$ and $i = \iota_2$ at least one edge from the path S is joining K_i with $G(A)$, so we have at least two independent edges uv , $u \in \mathrm{V}(K_{\iota_1})$, $v \in \mathrm{V}(G(A))$, $u'v'$, $u' \in \mathrm{V}(K_{\iota_2})$, $v' \in \mathrm{V}(G(A))$, not incident with S joining respectively K_{ι_1} and K_{ι_2} with $G(A)$.

Consider the following path:

$$P: \quad vP_1(u, s_1)s_2 \dots s_k P_2(s_{k+1}, u')v' ,$$

where $P_1(u, s_1) \subset K_{\iota_1}$ is a path joining u with s_1 containing all vertices of $K_{\iota_1} \setminus \{\mathrm{V}(S) \cup \{u\}\}$ and $P_2(s_{k+1}, u') \subset K_{\iota_2}$ is a path joining s_{k+1} with u' containing all vertices of $K_{\iota_2} \setminus \{\mathrm{V}(S) \cup \{u'\}\}$. See Figure 6.

The graph $G(A)$ is complete so we can extend P to a cycle C containing all vertices of $G(A)$ and satisfying (3.3 — 3.5).

Note that as in Case 1 path S may pass through several components K_i creating the kind of ears of the complete graph $G(A)$. Using the same argument as in Subcase

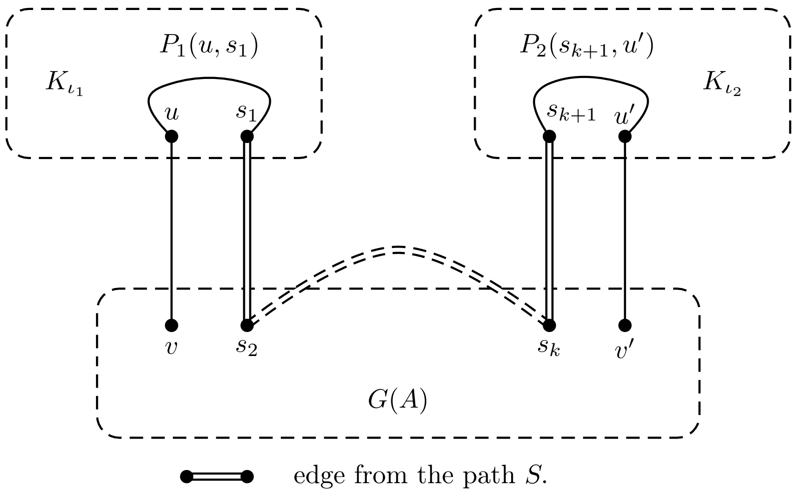

Fig. 6: Path P containing S with both end vertices in $G(A)$.

1.1, we can extend cycle C through components K_i incident with ears preserving the properties (3.3 — 3.5).

Case 4: One end vertex of s is in $G(A)$ and the other end vertex is in a component K_ι of G_A. In this case, we can assume without loss of generality, that $s_1 \in G(A)$ and $s_{k+1} \in K_\iota$.

Since graph G is $(k+2)$-connected, we have at least $k+2$ edges joining the component K_ι with $G(A)$. In this case, at least one edge from the path S is joining K_ι with $G(A)$, so we have at least one edge uv, $u \in \mathrm{V}(K_\iota)$, $v \in \mathrm{V}(G(A))$, not incident with S joining K_ι with $G(A)$.

Consider the following path:

$$P: \quad s_1 s_2 \ldots s_k P(s_{k+1}, u) v\,,$$

where $P(s_{k+1}, u) \subset K_{\iota_0}$ is a path joining s_{k+1} with u containing all vertices of $K_\iota \setminus \{\mathrm{V}(S) \cup \{u\}\}$, see Figure 7.

Both v and s_1 are in $G(A)$ and the graph $G(A)$ is complete, so we can extend P to a cycle C containing all vertices of $G(A)$ and satisfying (3.3 — 3.5).

Note that as in Case 1, path S may pass through several components K_i creating the kind of ears of the complete graph $G(A)$. Using the same argument as in Subcase

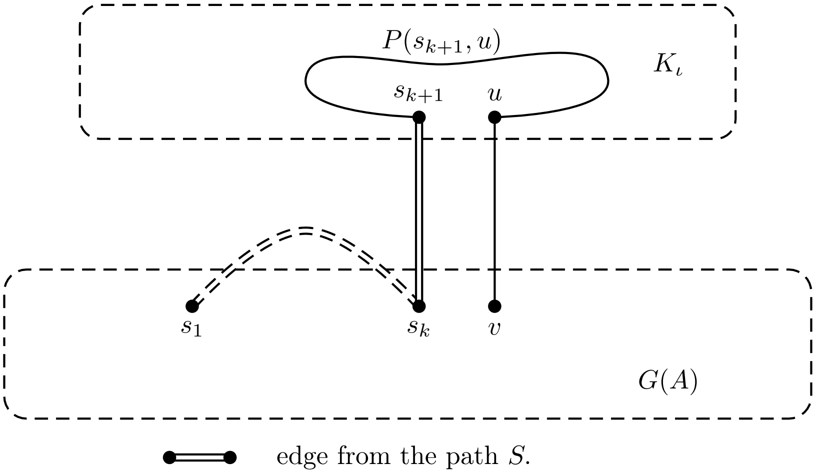

Fig. 7: Path P containing S with both end vertices in $G(A)$.

1.1, we can extend the cycle C through components K_i incident with ears preserving the properties (3.3 — 3.5).

In all cases we have defined a cycle C containing S and now we shall extend this cycle to a hamiltonian cycle.

Extending the cycle C to a hamiltonian cycle

We have already a cycle C satisfying conditions (3.3 — 3.5) and containing the path S, all vertices of $G(A)$, all vertices from components K_ι containing vertices of the path S.

Consider component K_ι not included in cycle C. This component does not contain any edge from S and since the graph G is $(k+2)$-connected we have at least $k+2$ edges joining K_ι with $G(A)$, so at least one of these edges say uc_i , $u \in \mathrm{V}(K_\iota)$, $c_i \in \mathrm{V}(G(A))$, is not incident with S and at least one edge say $u'c_j$, $u' \in \mathrm{V}(K_\iota)$, $c_j \in \mathrm{V}(G(A))\setminus$, not incident with internal vertices of S , joining K_ι with $G(A)$. In the worst case $c_j = s_1$ or $c_j = s_{k+1}$.

Using these two edges uc_i and $u'c_j$, we can extend cycle C through the remaining vertices of K_ι.

We consider the case $c_j = s_1$ and without loss of generality we can, assume that on cycle C, the vertices are ordered in the following way:
$s_1 \ldots s_{k+1} c_{k+2} \ldots c_i \ldots s_1$.

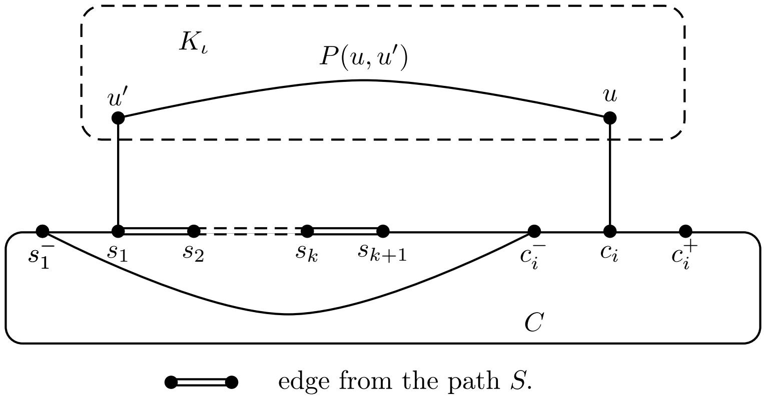

Fig. 8: Extension of cycle C through a component K_ι not incident with S.

Note that since uc_i is not incident with S we have $c_i^- c_i$, $c_i c_i^+ \notin \mathrm{E}(S)$ and we can replace cycle C with the following cycle C'

$$C' : \quad u' s_1 \dots s_{k+1} s_{k+1}^+ \dots c_i^- s_1^- \dots c_i^+ c_i P(u,u') \,,$$

where $P(u,u') \subset K_\iota$ is a path joining u with u' containing all vertices of K_ι, see Figure 8.

Note that this cycle C' satisfies conditions (3.3 — 3.5), $\mathrm{V}(C) \subset \mathrm{V}(C')$ and $\mathrm{E}(S) \subset \mathrm{E}(C')$.

The case $c_j = s_{k+1}$ is similar.

Applying this argument for all other components K_ι we can extend C to a hamiltonian cycle containing the path S and the proof is complete.

□

References

[1] A. Benhocine and A.P. Wojda, *The Geng-Hua Fan conditions for pancyclic or hamiltonian-connected graphs,* J. Combin. Theory Ser. B 42, 1985, 167—180.

[2] J.A. Bondy and V. Chvátal, *A method in graph theory,* Discrete Math. 15, 1976, 111—135.

[3] J.A. Bondy and U.S.R. Murty, *Graph theory with applications,* The Macmillan Press LTD London 1976.

[4] G. Fan, *New sufficient conditions for cycles in graphs,* J. Combin. Theory Ser. B 37, 1984, 221—227.

[5] G. Gancarzewicz and A.P. Wojda, *Graphs with every k-matching in a hamiltonian cycle,* Discrete Math. 213, 2000, 141 — 151.

[6] R. Hăggkvist, *On F-hamiltonian graphs* in Graph Theory and Related Topics, ed. J.A. Bondy and U.S.R. Murty, Academic Press N.Y. 1979, 219—231.

[7] H.V. Kronk, *Variations of a theorem of Pósa* in The Many Facets of Graph Theory, ed. G. Chartrand and S.F. Kapoor, Lect. Notes Math. 110, Springer Verlag 1969, 193—197.

[8] M. Las Vergnas, *Sur une propriété des arbres maximaux dans un graphe,* C. R. Axad. Sci. Paris, Sér. A, 272, 1971, 1297—1300.

[9] O. Ore, *Note on hamiltonian circuits,* Amer. Math. Monthly 67, 1960, 55.

[10] Z. Skupień and A.P. Wojda, *On highly hamiltonian graphs,* Bull. Acad. Polon. Sci., Sér. Sci. Math. Astronom. Phys. 22, 1974, 463—471.

[11] A.P. Wojda, *Hamiltonian cycles through matchings,* Demonstratio Mathematica XXI 2, 1983, 547—553.