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TEMPERATURE PROFILES IN GROUND HEAT EXCHANGERS

PROFILE TEMPERATUR W GRUNTOWYCH WYMIENNIKACH CIEPŁA

Abstract

A mathematical model of a ground heat exchanger cooperating with a heat pump was presented. The model is based on a one-dimensional heat conduction equation with an internal heat source. On the basis of the solutions to the model equations, both the temperature distribution of different types of ground at different times of year as well as the time courses of temperature at different depths are shown. Calculations based on the presented model can be useful for the simulation and design of this apparatus.

Keywords: transient heat conduction, renewable heat sources, ground heat exchangers

Streszczenie

W artykule przedstawiono model matematyczny gruntowego wymiennika ciepła. Model opiera się na jednowymiarowym równaniu przewodzenia ciepła z wewnętrznym źródłem ciepła. Na podstawie rozwiązań równań modelu uzyskano profile temperatur w gruncie w zależności od czasu i rodzaju gruntu, a także czasowe przebiegi temperatur na różnych głębokościach. Obliczenia oparte na zaprezentowanym modelu mogą być wykorzystane do symulacji i projektowania tych aparatów.

Słowa kluczowe: nieustalone przewodzenie ciepła, odnawialne źródła energii, gruntowe wymienniki ciepła

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1. Introduction

Geothermal and solar energy systems are renewable heat sources. Geothermal heat pumps with vertical heat exchangers, also called ground-coupled heat pump (GCHP) systems, which use the ground as a heat source, have been gaining increasing popularity, especially for heating or cooling buildings, thanks to their high energy efficiency. Heat pumps allow the use of natural energy resources, which may come from atmospheric air, soil, surface water or groundwater. As the cost of energy continues to rise, the heat pump is becoming a key element in an energy recovery system with great potential for energy saving.

Ground heat exchangers are essential parts of ground-source heat pumps and the accurate prediction of their performance is of fundamental importance. The ground heat exchanger (GHE) is designed for extraction and injection of heat from/into the ground by a heat transfer fluid which is circulating in a closed cycle. The heat transfer fluid is water or an organic liquid with a low freezing point (e.g., an aqueous solution of ethylene glycol).

In a zone of a specific depth (6 m to approx. 50 m), the ground has a constant temperature throughout the year which is approximately equal to the mean annual air temperature. This is due to complex interactions between the heat coming from the surface and that which is rising from the depths of the Earth. At a depth below 6 m, the ground temperature directly corresponds to the climatic conditions. The heat is most intensively received by the geothermal collector during the winter season, and the ground regenerates, especially in warmer periods such as spring and summer. Ground regeneration follows mainly due to solar radiation and atmospheric rainfall, which ensures that the ground again accumulates heat for the next heating season.

The value of the temperature deviations from the annual average depends on the properties of the ground. When the ground is more saturated with water and when it contains more minerals, then the accumulation properties and thermal conductivity have the greater values. The range of power one unit can collect from the ground is between 10–35 W/m² ground surface (sandy ground, dry: 10–15 W/m²; sandy, wet: 15–20 W/m²; ground loam, dry: 20–25 W/m²; loamy, wet: 25–30 W/m²; water-bearing ground: 30–35 W/m²).

Most embedded heat exchangers for houses are installed either horizontally or vertically in the ground. Horizontal ground heat exchangers can be made in various configurations – in series or coil systems. The exchangers are made mostly of plastic pipes (PVC, polyethylene, polypropylene) arranged at a depth of 1.2–2.0 m. The temperature increase of the heat carrier in the ground is 3 to 4 K. The average temperature of fluid medium depends on the type of the ground along with the depth at pipes are arrangement.

The modeling of ground heat exchanger occupies many researchers. The ground heat exchanger layout follows a spiral pattern characterized by three parameters – length, depth, and spacing. In [1], the influence of each parameter on the amount of heat extracted from the ground and on the ground temperature at the control point is assessed.

In research [2], a numerical model of heat transfer in the ground was developed for determining the temperature distribution in the vicinity of the pipe. The finite difference approximation is used for numerical analysis.

The aim of the study [3], was to validate the effects of parameters such as the depth of the buried earth coupled heat exchanger, the mass flow rate of the water-antifreeze

solution on the performance of a horizontal ground-source heat pump (GSHP) system used for space heating experimentally.

In research [4] on the optimum design of slinky-coil horizontal ground heat exchangers, a commercial finite-element simulator was used to simulate the performance of slinky-coil horizontal ground heat exchangers taking into account the energy balance at the ground surface.

Because of the complexity of the boundary conditions, the heat conduction equation has been solved numerically using an alternating direction implicit finite difference formulation and the effects of manipulating the solution parameters on the results were investigated in the work carried out in [5].

In most studies, a mathematical model and its verification was presented. However, few of them present the effect of soil properties on the performance of the ground as a heat source. The influence of this factor is presented in this paper. The aim of this paper is to present a mathematical model of a horizontal ground heat exchanger, and the results obtained on the basis of the solutions to equations of the model as well as its qualitative verification. In this model, the ground is treated as a semi-infinite body with an internal heat source.

2. Mathematical model

A system which consists of a lower and upper heat exchanger where a working fluid (glycol solution) circulates between them has been considered. The lower heat exchanger is located under the ground, while the upper exchanger is a part of the heat pump. In winter conditions, in the lower exchanger, the liquid absorbs heat from the ground and transfers it to the boiling thermodynamic medium in the heat pump. In summer conditions, the working fluid transfers the heat to the ground, thus cooling off at the same time, and then absorbs the heat from the condensing thermodynamic medium in operative heat pump (in this case as the air conditioner). Therefore, due to the season of the year, one is faced with a cyclical process consisting of alternating heating and cooling of the ground.

For the upper heat exchanger the temperature of the working fluid, which receiving (or collecting) the heat from the operating fluid is unchangeable. Through the surface of the ground, heat is exchanged with the environment. The ground surface has a temperature periodically changing in time (on an annual basis).

Fig. 1 presents the location and orientation of the ground heat exchanger, where x is position coordinate and H is the depth at which the heat exchanger is located. In this model, the flow through the parallel arrangement of heat exchanger pipes was replaced by a flow through a horizontal cuboid channel of very small thickness. The heat is transferred into and from the ground symmetrically by both the lower and the upper surface of the heat exchanger. It is assumed that the temperature of liquid in the exchanger is constant with the position (not in time) and thus, the flow through the channel is equivalent to the flow through a perfect mixing tank.

In the upper heat exchanger, the temperature of the thermodynamic medium receiving (or collecting) heat from the operating fluid is constant. This is due to phase transition of the thermodynamic medium in the heat pump. Temperature T_v , however, varies for

different periods of operation. For condensation (summer months), the temperature of the thermodynamic medium T_v is higher than in winter conditions in which medium evaporation is present.

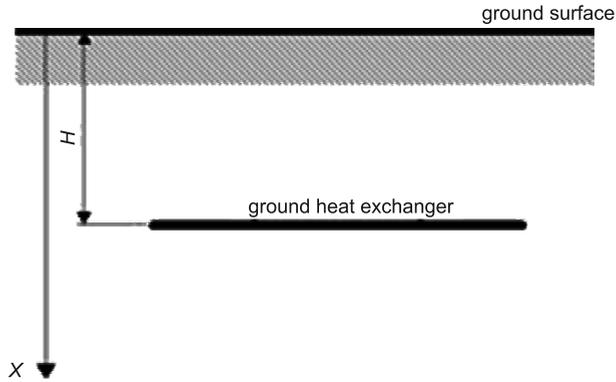


Fig. 1. The location and orientation of GHE

The operating fluid of specific heat c_L flows through the ground at a flow rate \dot{m}_L . The liquid temperature varies from T_{in} at the inlet to the heat exchanger to T_{out} at the outlet (Fig. 2). In winter conditions, the liquid is heated $T_{out} > T_{in}$, opposite to summer conditions.

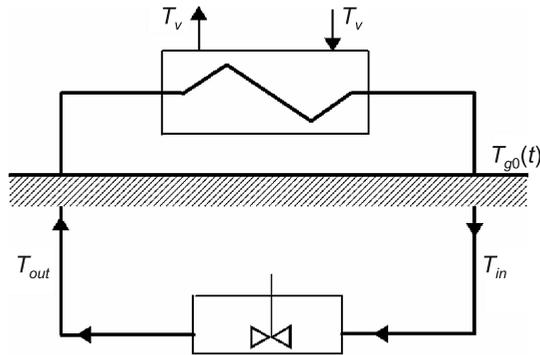


Fig. 2. The circulation of working fluid between the lower and upper heat exchangers

The heating surface area of the upper heat exchanger is A_{up} and the overall heat transfer coefficient in the heat exchanger is U . Instantaneous heat flux transferred in the upper and the lower heat exchanger is equal to:

$$\dot{Q} = \dot{m}_L c_L (T_{out} - T_{in}) \quad (1)$$

The heat flux is positive when the heat is received from the ground. Therefore, for the winter, \dot{Q} is positive, while for the summer - negative. Heat transfer equation for the upper heat exchanger takes the form:

$$\dot{Q} = UA_{up} \frac{T_{out} - T_{in}}{\ln \frac{T_v - T_{out}}{T_v - T_{in}}} \quad (2)$$

From the above equation results the relationship between the temperatures T_{in} and T_{out} :

$$T_{in} = T_v - (T_v - T_{out}) \exp\left(-\frac{UA_{up}}{\dot{m}_L c_L}\right) \quad (3)$$

Heat is transferred through the ground surface with the surroundings. The atmospheric air in contact with the ground has a temperature T_0 periodically changing in time (on an annual basis) $t_0 = 24 \cdot 365.24$ h:

$$T_0 = 10.0 + 11.0 \cdot \cos\left(\frac{2\pi}{t_0} t\right) \quad (4)$$

where:

t – time, h.

In this model, the ground is treated as a semi-infinite body. The heat conduction equation for the plate with an internal heat source placed at H distance from the ground surface (Fig. 1) was used. The model includes only vertical heat transfer due to the small thickness of the heat exchanger. For transient conduction in an infinite plate with an internal heat source, the following relationship is valid [6]:

$$\frac{\partial T_g}{\partial t} = \frac{\partial}{\partial x} \left(a_g \frac{\partial T_g}{\partial x} \right) + \frac{q_v}{\rho_g c_g} \quad (5)$$

where:

T_g – ground temperature,

t – time,

a_g – thermal diffusivity of ground, m^2/s ,

x – distance from the ground surface,

q_v – performance of the heat source, W/m^3 ,

ρ_g – density of the ground,

c_g – specific heat of the ground.

The performance of the heat source is the rate of heat generation per unit volume of the plate. This value is related to the transport of heat between the working fluid flowing through the ground heat exchanger and the ground. Heat performance is:

$$q_v = -\frac{\dot{Q}}{V} \quad (6)$$

where:

V – the volume of the horizontal cuboid with a very small thickness.

The following initial condition was assumed:

$$t = 0, \quad T_g = T_b \quad (7)$$

where:

T_b – ground temperature at great depth.

The boundary condition for the ground surface takes the form:

$$x = 0, \quad T_g = T_0 \quad (8)$$

where:

$T_0(t)$ – the air temperature, varying periodically and it is defined by the formula (4).

In numerical calculations, for the distance from the ground surface H_{inf} , it must be assumed that the temperature will remain constant with the position (in theory of the apply to $x \rightarrow \infty$). The second boundary condition is therefore:

$$x = H_{inf}, \quad T = T_b \quad (9)$$

The finite difference method (Crank-Nicolson scheme) was used for solving model equations.

3. Simulation of the process

The figures presented below show the results of calculations made on the basis of the mathematical model described above.

Ground heat exchangers usually work periodically to allow for partial thermal regeneration of the ground. The working conditions of the ground heat exchanger are as follows:

- when the energy supply to the heat pump: $T_0 < 10^\circ\text{C}$ and $T_{out} > T_v + 7$ (heating season)
- when receiving energy from the air conditioner $T_0 > 20^\circ\text{C}$ (hot summer months).

When neither the first nor the second condition is realized, the ground heat exchanger is turned off.

There are various types of ground. Each of these has different characteristics such as density, thermal conductivity and specific heat – these are summarized in the Tab. 1. As one can see, the individual parameters of the various ground types have very different values. In this study, the layers of sand/gravel and grained ground have been considered.

The remaining numerical data:

- the number of nodes with respect to the coordinate position of $n = 300$,
- the mass flow rate of the operating fluid $\dot{m}_L = 0.2$ kg/s,
- the product of the heat transfer coefficient and the heating surface area of the upper exchanger (hA_{up}) = 200 W/K,
- heating surface area of the lower heat exchanger $A_{down} = 500$ m²,
- the distance between the lower heat exchanger and the surface of the ground $H = 2$ m,
- the temperature of the thermodynamical medium in winter and summer conditions $T_v = -5^\circ\text{C}$ or $T_v = 40^\circ\text{C}$,
- $H_{inf} = 20$ m,
- $T_b = 10^\circ\text{C}$.

Table 1

Parameters of various types of ground [7]

Type of Ground	Density [kg/m ³]	Thermal conductivity [W/(mK)]	Thermal diffusivity [10 ⁶ m ² /s]	Specific heat [J/(kgK)]
Grounds, humidity 9%	1440	0.98	0.452	1506
Grounds, humidity 13%	1600	1.50	0.521	1799
Humid ground	1800	1.45	0.602	1338
Sand, dry	1650	0.70	0.506	838
Sand, humidity 15%	1780	0.92	0.375	1384
Dry sand	1998	1.60	0.324	2472
Wet sand	1500	1.88	0.105	1199
Sand/gravel	1950	2.00	0.977	1050
Grained ground	2000	0.52	0.141	1844

3.1. Temperature profiles in the ground

The calculations were performed for the ground with varying thermal diffusivity. The ground temperature profiles for different values of thermal diffusivity were shown in Figs. 3–5. The lines in the figures relate to each month. Numbers (1–11) were determined number of months, i.e. 1 – January 2 – February, etc. One can see the differences in the profiles for each month in following years. These differences result from arbitrarily assumed initial condition (7) and subsequent years are disappearing as they reach a cyclic steady state. Profiles relate the case where to/from the ground is not taken or delivered heat (the ground heat exchanger is completely turned off). Fig. 3 concerns the case where the variation of thermal diffusivity is described by formula:

$$\begin{aligned}
 a_g &= 0.977 \cdot 10^{-6} \text{ m}^2/\text{s} & \text{for } x \leq 2 \text{ m} \\
 a_g &= 0.141 \cdot 10^{-6} \text{ m}^2/\text{s} & \text{for } x > 2 \text{ m}
 \end{aligned}
 \tag{10}$$

The above function determines the ground structure: from the ground surface to the depth of 2 m there is sand/gravel, and less than 2 m – grained ground. In Fig. 3 the clearly visible deformation profiles at $x = 2$ m can be seen, i.e. at the point where ground changes its properties by leaps and bounds. The temperatures in subsurface layer with high thermal diffusivity are only slightly changed with the position.

A similar situation occurs when there is a change in the type of soil at a depth of 4 m (Fig. 4), i.e. according to the relationship:

$$\begin{aligned}
 a_g &= 0.977 \cdot 10^{-6} \text{ m}^2/\text{s} & \text{for } x \leq 4 \text{ m} \\
 a_g &= 0.141 \cdot 10^{-6} \text{ m}^2/\text{s} & \text{for } x > 4 \text{ m}
 \end{aligned}
 \tag{11}$$

Then, the profiles deformations occur at a depth of 4 m. The heat conduction rate is significantly higher in sand/gravel layer than in the layer of grained ground. This causes the ground at a depth of 4 m to be at a temperature close to the temperature at the surface.

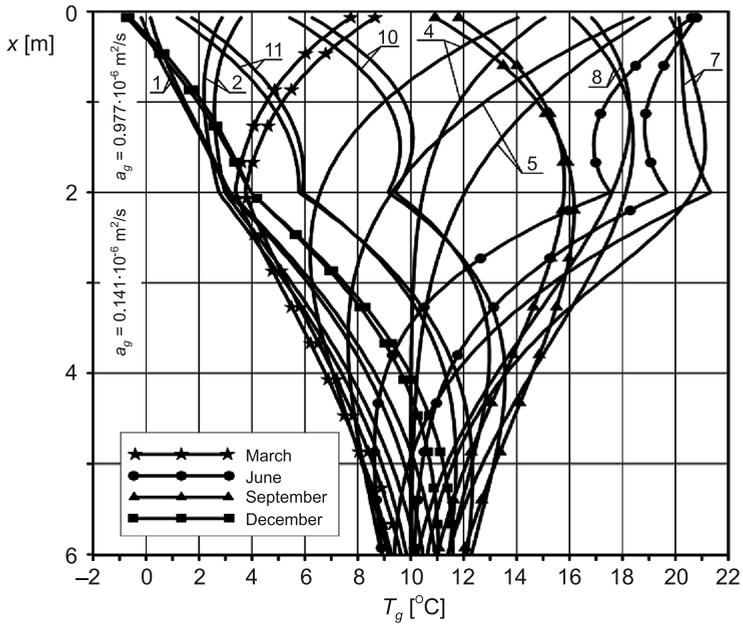


Fig. 3. Temperature profiles of ground with varying thermal diffusivity at a depth of $x = 2$ m. The thermal diffusivity of the upper layer is higher than for the bottom one

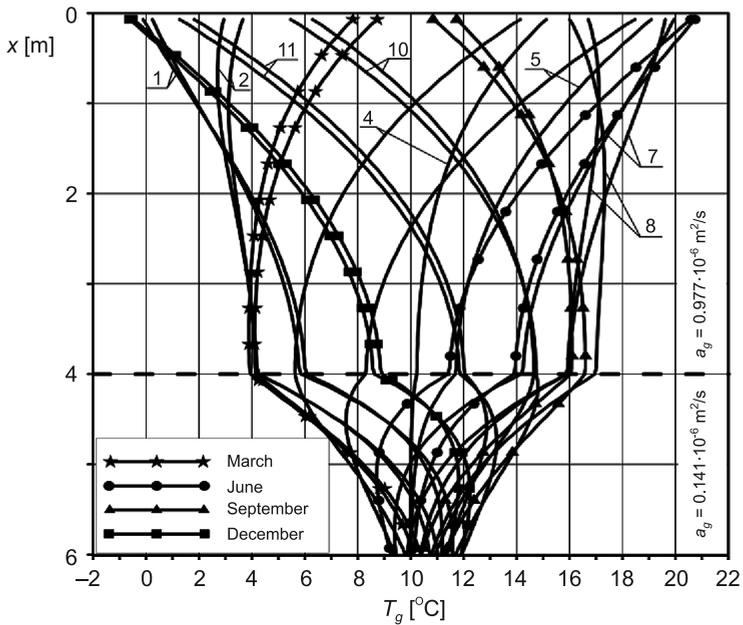


Fig. 4. Temperature profiles of ground with varying thermal diffusivity at a depth of $x = 4$ m. The thermal diffusivity of the upper layer is higher than that of the bottom one

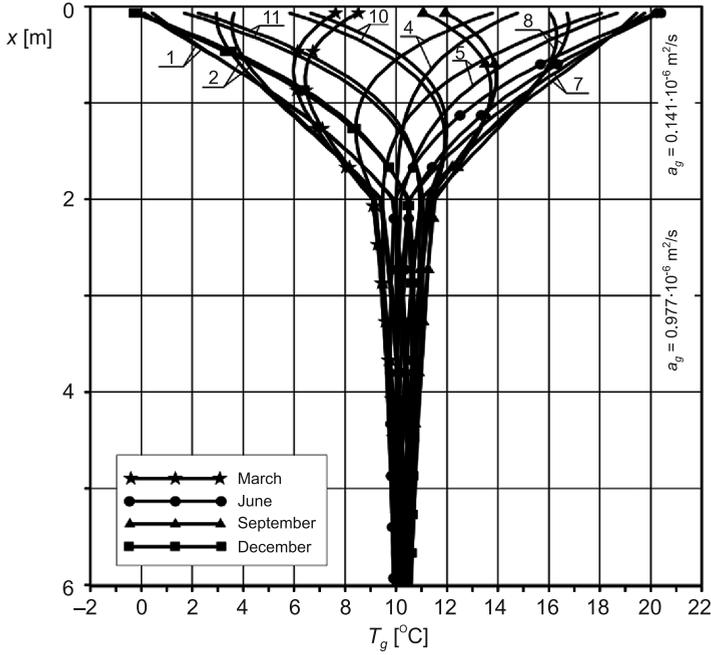


Fig. 5. Temperature profiles of ground with varying thermal diffusivity at a depth of $x = 2$ m. The thermal diffusivity of the upper layer is lower than that of the bottom one

The temperature profiles for a depth in the range 0–2 m, where ground is formed by layer of grained ground, below 2 m – sand/gravel layer, are shown in Fig. 5. Therefore:

$$\begin{aligned} a_g &= 0.141 \cdot 10^{-6} \text{ m}^2/\text{s} & \text{for } x \leq 2 \text{ m} \\ a_g &= 0.977 \cdot 10^{-6} \text{ m}^2/\text{s} & \text{for } x > 2 \text{ m} \end{aligned} \quad (12)$$

In this case, the layer of the ground with a higher thermal diffusivity is below the lower diffusion layer. This causes little change in temperature at a depth of 2 m (temperature profiles are similar to the vertical lines), and thus, their waveforms are only slightly different for each month (ground temperature in the range of 9–11°C). Large temperature changes occur in the upper part of the ground with a low thermal diffusivity.

Comparing the waveforms in Figs. 3 and 5, one should consider the meaning of the upper layer of soil (depth range 0 to 2 m). In Fig. 3, one can see that for the value of the function described a_g , Eq. (10), the upper layer is not significant for the decomposition of a lower temperature because the temperature of the upper layer is similar to the temperature of the ground surface. Totally different case should be interpreted when variation of thermal diffusivity is described by the function (Eq. (12)). Then, the upper layer of the ground is a buffer, which insulates the bottom layer from the impact the ground surface temperature, as shown in Fig. 5.

3.2. Time courses of ground temperatures at different depths

Based on numerical analysis, the time courses of the ground at different depths were determined (Fig. 6a–e). The dotted lines (•) determine temperature courses when the ground heat exchanger (located at a depth of 2 m) is turned on, while the solid lines refer to when the heat exchanger is completely turned off. The value of the heat transfer coefficient between the air and the ground was assumed as $h = 10 \text{ W}/(\text{m}^2\text{K})$.

In Fig. 6a, the temperature time courses on the surface of the ground were shown. In addition, in Fig. 6a, ambient air temperature, periodically variable according to the relationship (4) and varying in the range from -1°C to 21°C , was presented. Courses of all three lines differ slightly. Therefore, the temperature of the ground surface is practically the same regardless of whether the exchanger is enabled or not. This should be interpreted as follows: heat transfer into the ground to a depth of 2 m below the surface causes only slight changes in temperature on the surface. However, small differences in temperature between the ground surface and the air are associated with a small heat flux transferred between the ground and the air. It should be noted that the difference in temperature between the ground and the air must not be zero because the heat is collected (or received) from the ground through the surface. In winter conditions, the ground supplies heat to the environment, while in summer ground extracts heat. Hence, the temperature difference is discussed alternately positive or negative.

At a depth of 1m, significant changes occur in the time courses of the ground temperature (Fig. 6b) as compared to the above discussed passes for $x = 0$. When the heat exchanger is turned off, the ground temperature profile remains periodically changing, but has a lower amplitude than on the ground surface. In summer conditions there is an increase of temperature of the ground, which is related to the supply of heat to the ground – inversely in winter, when there is a noticeable temperature drop. Courses of temperatures for the attached and completely turned off heat exchanger are significant.

A similar situation occurs at a depth of 2 m, which is the depth of the heat exchanger location in Fig. 6c. There is a clear difference between the temperatures when the heat exchanger is turned off and on. Noteworthy is the fact that the temperature deformations (when heat exchanger is turned on) are related to the periodic work (on – off) of this device. When the heat exchanger is completely turned off, the periodical variation of the amplitude of the temperature of the ground is lower than the ground surface ($x = 0$) or to $x = 1$ m.

In Fig. 6d, temperature profiles of the ground at a depth of 4 m are presented. Differences in temperature between the on and off heat exchanger are much smaller, temporary changes in temperature waveforms are clearly flattened.

However, at a depth of 10 m (Fig. 6e), temperature variations do not occur in practice, the work of the heat exchanger has therefore a negligible effect on the temperature of the ground, which is about 10°C .

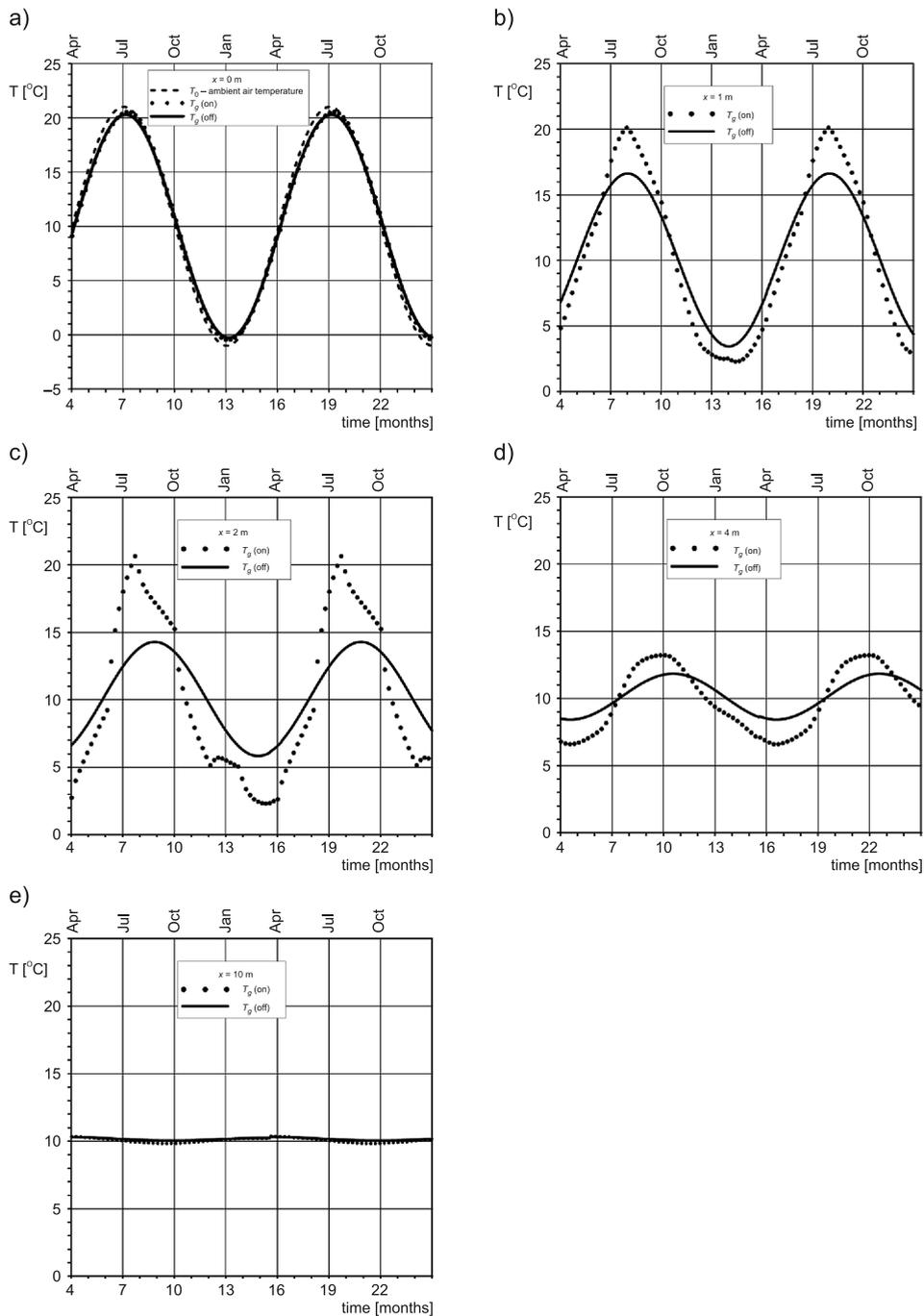


Fig. 6. Time course temperature profiles of ground at depth: a) 0 m, b) 1 m, c) 2 m, d) 4 m, e) 10 m

4. Conclusions

1. The presented model of the ground heat exchanger is based on a one-dimensional heat conduction equation with an internal heat source.
2. The model describes the temperature distributions in the ground correctly depending on the location, time and properties of the ground.
3. The ground temperature is affected by many factors, the presented model takes all of them into account.
4. Calculations based on the presented model can be useful in predicting the impact of different process parameters on the heat pump's ability to heat.
5. The model can be used to simulate the thermal regeneration of the ground after the end of the heating season.

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