**TECHNICAL TRANSACTIONS** | **CZASOPISMO TECHNICZNE**
FUNDAMENTAL SCIENCES | NAUKI PODSTAWOWE
1-NP/2014

ROMAN DUDA*

# SOME REMARKS CONCERNING RECEPTION OF MATHEMATICS IN CENTRAL-EASTERN EUROPE IN THE YEARS 1850–1920

# KILKA UWAG ODNOŚNIE DO RECEPCJI MATEMATYKI W EUROPIE ŚRODKOWO-WSCHODNIEJ W LATACH 1850–1920

Abstract

In the flow of mathematical ideas from the West to Central-Eastern Europe one can distinguish several typical forms: 1) foreign mathematicians, invited to cultivate mathematics upon new ground (e.g. Euler in Russia); 2) domestic mathematicians who completed their studies abroad and continued research after returning home (e.g. W. Buniakowski or M. Ostrogradski in Russia); 3) domestic mathematicians who dared developing new directions, thus initiating original schools of mathematics (e.g. N. N. Lusin in Russia). A separate phenomenon was a startling discovery of non-euclidean geometry (N. N. Lobatchevsky in Russia, J. Bolyai in Hungary).

*Keywords*: *cultivation of mathematics*, *continuation of research*, *mathematical journal*, *founding a school in mathematics*

Streszczenie

W przepływie idei z Zachodu do Europy Środkowo-Wschodniej można wyróżnić kilka typowych form: 1) matematycy obcy, zapraszani do wdrażania matematyki na nowej glebie (np. L. Euler w Rosji); 2) matematycy rodzimi, którzy po studiach za granicą kontynuowali badania w zakresie tamtejszej problematyki (np. W. Buniakowski I M. Ostrogradski w Rosji); 3) matematycy rodzimi, którzy odważyli się na rozwijanie nowych kierunków, kładąc w ten sposób podwaliny pod oryginalne szkoły (np. N. Łuzin w Rosji). Osobnym zaskakującym wydarzeniem było odkrycie geometrii nieeuklidesowej (N. N. Łobaczewski w Rosji, J. Bolyai na Węgrzech).

*Słowa kluczowe*: *uprawianie matematyki*, *kontynuacja badań*, *czasopismo matematyczne*, *założenie szkoły matematycznej*

* Roman Duda, Professor emeritus, Institute of Mathematics, University of Wrocław.

In the period 1850–1920 there were considerable differences in the development of mathematics (and of other exact sciences as well) between Western and Central-Eastern Europe. The West was then further ahead and so the dominant flow of ideas was from the West to the East. At that time Central-Eastern Europe had been still on the acquiring side.

Domination of the direction from the West to the East does not mean, however, that there was no flow in the opposite direction. In our part of Europe some new ideas of great scientific value also occurred, just to mention Mendeleyev's periodic table, Lobachevski's or Bolyai's geometry, Smoluchowski's statistical physics. Phenomena of such a kind, however, were rare and rather exceptional.

Being on the acquiring side means a transfer of scientific ideas from the outside, and their reception (assimilation) on the so far alien soil. Although the aim of this note is to describe this process, its course and its effects in the specific area of mathematics, it seems, however, that analogous processes can be also observed with respect to other exact sciences.

Ideas live in people and move with them. For this note particularly important are people who possessed some mathematical ideas and worked actively upon their development and transmission. Quite often such a role was played by foreigners like Leonhard Euler (1707–1783) or Jakob Bernoulli (1759–1789) in Petersburg, Christian Martin Bartels (1769–1826) in Kasan, Simone Antoine Lhuillier (1750–1840) in Poland, Czech mathematicians in Bulgaria [2], Otto Volk (1892-1989) in Lithuania [1]. All of them were people from abroad who came to their new destination in our part of Europe as men with already established reputation and who successfully ingrained their ideas into minds of local people. Their role was to initiate development and their influence has been of great value.

Emerging scientific institutions in Central-Eastern Europe required new personnel and so it seemed reasonable to send talented people abroad with the task of gaining education there and to share their knowledge with local students after return. In the period considered it was a common practice and some of those who returned became quite influential at home. Such was the case of Jan Śniadecki (1756–1830) in Poland, who after return from Paris had reformed university in Cracow, wrote some manuals and taught extensively. Later he became the rector of Vilnius university and continued his reforms and teaching there. He deserves much credit for the raising of (then-low) mathematical culture in Poland.

Such local people who first studied in leading mathematical centres in Western Europe and then returned filled with knowledge, ideas and enthusiasm, and who as a rule kept in contact with their teachers, were essential for further development. They were important not only because of their number (in that period there were many of them) but mainly because of their close contact with the local territory. After their return home, they could more easily (than their foreign predecessors) recognize local needs and evaluate local chances. In consequence they were usually more effective in training new adepts and creating new trends – sometimes even original schools in themselves.

Such native bearers of knowledge (acquired elsewhere) were taking two distinct courses of action at home: continuation or starting something new. These two kinds will be now described and exemplified.

More often there was a continuation. This was a case of a man who mastered a skill in a specific area of mathematics abroad and distinguished himself there by a Ph.D. thesis and/or original publications, and who successfully continued research in that area after

return. Acquaintances he struck up made it for him relatively easy to publish results obtained at home and to gain in that way some appreciation abroad. In such a way the man could become known in the community and his mathematics could achieve wider circulation.

Some early examples are from Russia:

Wiktor Buniakowski (1804–1889). He studied in Paris, defending his Ph.D. thesis in 1825. After return to Russia, he had spent his life in Petersburg where he worked both at the university and in the Academy. He was a professor of the university in the years 1826–1864 and became an adjunct to the Academy in 1826, since 1830 its ordinary member and from 1864 until the end – its deputy president. His interests embraced number theory, mathematical analysis and calculus of probability, and his best known result was the Buniakowski-Schwarz inequality, proved independently by both named mathematicians (Buniakowski in 1859, Schwarz in 1884).

Michail Ostrogradski (1801–1861). A similar course of life. He studied 1816–1820 in Kharkov and then 1822–1828 in Paris. After return to Russia, he also had spent his life in Petersburg where he was teaching in several institutes of higher education and at the same he became an adjunct to the Academy in 1828 and ordinary member since 1830. His interests in mathematics were diverse, including mathematical analysis, mechanics and mathematical physics, but those most important were concerned with the propagation of heat. He discovered the well-known Ostrogradski formula relating triple integral with respect to a volume and double integral with respect to the surface of that volume.

The two men, Buniakowski and Ostrogradski, were the first Russian-born mathematicians who gained an international status. There was, however, a delay in their recognition due to the fact that Russia was then at the periphery of scientific world and many of their results were published in Russian, a language then hardly known outside the Russian empire.

Two other examples are from Poland:

Stanisław Zaremba (1863–1942). Born in Ukraine, he studied in Petersburg and Paris, receiving Ph.D. degree in 1889 in Paris. In 1900 he accepted an invitation from the Jagiellonian university and spent the rest of his life in Cracow. Zaremba was interested in problems of mathematical analysis related to physics, particularly in partial differential equations. He was highly valued by H. Poincaré [14] and by H. Lebesgue, among others [7].

Kazimierz Żorawski (1866–1953). After studies at the Russian university in Warsaw he got a scholarship to continue abroad. He went to Göttingen and Leipzig. Sophus Lie became his master in Leipzig and there Żorawski got his Ph.D. After return to the home country he stayed briefly in Lvov and from 1893 in Cracow, where some years later he was joined by Zaremba. Żorawski worked in the theory of Lie groups, publishing in Polish and German. Highly evaluated by S.Lie and E. Cartan, he was the only Polish mathematician mentioned by F. Klein in his account of mathematics in the XIX century [9].

The two men, Zaremba and Żorawski, were the first Polish mathematicians who gained an international status in modern times [16] (earlier there were some Polish mathematicians enjoying international recognition, but it happened only in the XV–XVI centuries).

Because of studies abroad and the "continuation" process after return, as in the just--described cases of Petersburg and Cracow, communities appeared which understood modern

mathematics and maintained contacts with leading mathematicians abroad. Some of these communities were supported by newly founded journals (publishing, however, mostly in native languages). The general level of mathematics in Central-Eastern Europe was thus rising and in such conditions a totally new phenomenon could appear: some new domestic leaders could have nourished and then developed new ideas, bringing about their own students, and thus raising "schools" of mathematics which also influenced the development abroad. Particularly influential were two such "schools": in Moscow and in Warsaw.

Bolesław Młodziejewski (1858–1923) was a Russian mathematician of Polish origin. Born in Moscow, he studied there but completed his studies in Zürich, Paris and Göttingen. After return, he became professor of the Moscow university in 1892. His lectures on the theory of real functions and his seminar begun to disseminate ideas of the French school of that theory in Moscow [11, 12]. Młodziejewski has been joined by D.F. Egorow (1969–1931), I.M. Żegałkin (1869–1941) and N.A. Bugajew (1837–1903), and those in turn by their students including P.A. Florenski (1882–1937), N.N. Lusin (1883–1950) and S.P. Nowikow (1883–1964). Of the latter trio the most eminent mathematician was Lusin. He had an opportunity to spend several longer periods in Paris and it was his "Lusitania", as the group of students surrounding him was called, which began the great history of the Moscow school of mathematics [18]. The school was not a direct continuation of the French school, but it started a new domain of mathematics, so-called descriptive set theory [13], and greatly influenced some others, including topology, functional analysis, and probability theory. It included great talents of P.S. Aleksandrov (1896–1982), P.S. Urysohn (1898–1924), S.L. Sobolew (1908–1989), A.N. Kolmogorov (1903–1987) and many others. Despite difficult times (Soviet terror, "Luzinshchina" [3], isolation from the outside world), the Moscow school was an extraordinary phenomenon, soon to become one of the leading mathematical centres in the XX century.

The Warsaw school of mathematics developed along similar lines. Its origins are going back to Lvov (not to Cracow, as one might suppose). In 1908 Wacław Sierpiński (1882–1969) became a docent at the Lvov university and soon thereafter a professor. He taught theory of sets and its applications to real functions and topology, introducing some new ideas of his own. More important, Sierpiński began to gather around himself a group of ambitious young men like Stefan Mazurkiewicz (1888–1945), Stanisław Ruziewicz (1889–1941) or Zygmunt Janiszewski (1888–1920). Members of the group began to publish original results from the area of their interest but this promising seedbed of a new mathematical centre was soon dispersed. After the outbreak of the World War I in 1914 Sierpiński was interned in Russia (where he happened to be on holidays), Janiszewski volunteered to enlist in Polish troops, Mazurkiewicz returned to his native Warsaw, and Ruziewicz was drafted into the Austrian army. The group ceased to exist.

At the end of the war Janiszewski and Mazurkiewicz found themselves in Warsaw. There was a public inquiry on the needs of Polish science and the three men – Zaremba, Janiszewski, Mazurkiewicz – responded. Zaremba proposed to send young men abroad, to secure them teaching positions in secondary schools after return, to encourage their research, and eventually to offer university posts to the best ones among them [17]. It was nothing new, just an old model of "continuation". In contrast to that, Janiszewski proposed a totally new approach, which can be summarized in a few points:

1) to select one specific area of mathematics, possibly a new one (not with a long tradition behind);
2) to concentrate an attention of all young people upon that area;
3) to create within the group an atmosphere of cooperation, exchange of ideas and mutual aid;
4) to support the group with a newly founded journal devoted specifically to the chosen area and in which articles would be published only in internationally recognized languages [6].

While the first three points could be seen as a summary of experiences of the Lvov group, the fourth one was an original idea of Janiszewski himself.

All points of the Janiszewski's proposal could be disputed. And so they were, both within the country and outside of it. The choice of one specific area of mathematics and concentration of all efforts upon it brought an evident peril of the loss of whole generation of talented people if something went wrong, e.g., if the choice was ineffective. The atmosphere of openness and mutual aid contrasted sharply with the prevailing one of competition. There was no single mathematical journal in the world with the limited scope, while the ban on Polish language could be offensive to many. For instance, H. Lebesgue argued that a journal with a limited scope was doomed because supply of good papers would soon cease [15]. When Lusin pointed out the peril of the domination of one way, Sierpiński responded that it was better to have one than none [5].

Mazurkiewicz was the third to respond the inquiry [10]. He emphasized the necessity of good libraries, of new journals and good books, of scholarships etc. He followed Janiszewski but preferred different development in distinct academic centers.

Janiszewski not only wrote a proposal but also began to collect articles for the first issue of the journal which he founded and named "Fundamenta Mathematicae". The first issue appeared in 1920. He was helped first by Mazurkiewicz and then also by Sierpiński who then just arrived (in 1918) from Moscow. After inviting two logicians, Jan Łukasiewicz (1878–1956) and Stanisław Leśniewski (1886–1939), the five men formed the first Editorial Board of the "Fundamenta Mathematicae" and the article of Janiszewski became the program of the Warsaw school of mathematics, of which Janiszewski, Mazurkiewicz and Sierpiński were obvious leaders. Although Janiszewski soon died, the school became a great success. The jubilee issue of "Fundamenta Mathematicae" 25 (1935) gathered the best world mathematicians of the time, and in 32 issues of it, published in the years 1920–1939, 946 papers appeared, two thirds of which were from Poland and one third from abroad.

Cracow, under the leadership of Zaremba and influenced by him, kept itself apart from the Warsaw movement, but the model of Janiszewski has been duplicated in Lvov. The natural leaders of Lvov mathematics were then Hugo Steinhaus (1887-1972) and Stefan Banach (1892–1945), and the two men founded another journal "Studia Mathematica", limited to functional analysis and supporting the Lvov group. In 9 issues of "Studia Mathematica", published in the years 1929–1940, 161 papers appeared, of which 110 came from the Lvov group.

The Lvov group grew into the Lvov school of mathematics [4] and the Warsaw and Lvov branches were a great success and formed together the so-called Polish School of Mathematics. Its characteristic feature was free use of non-effective methods of proof

(based on the Axiom of Choice, measure theory, Baire category etc.), and it had immense influence upon the development of set theory, point-set topology, functional analysis and mathematical logic.

Each of the two schools, Moscow and Warsaw, was like a springboard to elevate Russian or Polish mathematics, respectively, to the world level and common recognition. The Moscow school was more numerous and lasted more than twice as long as the Polish one did, and its influence was stronger and deeper. In both cases, however, it was politics which had the last word. In Moscow politics forced an isolation of the Russian science, including mathematics, from the outside world in the thirties, and put an abrupt end to the Moscow school of mathematics in the seventies [18]. In Poland it was World War II which nearly annihilated the whole intellectual life in the country.

Putting politics aside, there remains the general picture of emerging science in peripheral countries: reception of modern ideas by people returning from studies abroad, their extended continuation in the home country, and eventual creation of original schools. Some totally original ideas which also arose (N.N. Lobatchevsky in Russia, J. Bolyai in Hungary) did not serve as a real stimulus for the elevation; the history shows that they were fought against and recognized only much later. E.g., Buniakowski and Ostrogradski alike actively fought against geometrical ideas of Lobatchevsky.

## References


[1] J. Banionis, *The famous mathematician of Lithuanian University Otto Theodor Volk (1892–1989)*, this issue. See also: *Matematyka na Litwie 1920–1940* (*Mathematics in Lithuania 1920–1940*), Wiadom. Mat. 42, 2006, 63-72.

[2] M. Bečvařová, *Czech mathematicians and their role in the development of national mathematics in the Balkans 1850–1951*, History of Mathematics, 41, Prague 2011, 33-42.

[3] S.S. Demidov, B.V. Lefshin (eds.), *Dełło akademika Nikołaja Nikołajewicza Łusina* (*The case of academician Nikolai Nikolaevich Lusin*), Sankt-Petersburg 1999.

[4] R. Duda, *Lwowska Szkoła Matematyczna* (*The Lvov School of Mathematics*), Wrocław 2007 (*English translation in print*).

[5] P. Dugac, *N. Lusin. Lettres à Arnaud Denjoy avec introduction et notes*, Arch. Intern. Hist. Sci. 27, 1977, 179-206; translated fragments (into Polish): Wiadom. Mat. 25.1, 1983, 65-68.

[6] Z. Janiszewski, *Stan i potrzeby matematyki w Polsce* (*State and needs of mathematics in Poland*), Nauka Polska 1, 1918, 11-18; reprint in English in the book: S.M.G. Kuzawa, *Modern Mathematics. The Genesis of a School in Poland*, New Haven 1968, 112-118; reprint in Polish: Wiadom. Mat. 7.1, 1963, 3-8.

[7] *Jubilé scientifique de M. Stanislas Zaremba*, Kraków 1930.

[8] J.-P. Kahane, *Aperçu sur l'influence de l'école mathématique polonaise 1918–1939*, Centre Scientifique de l'Académie Polonaise des Sciences à Paris, 1992; Polish translation: *Próba oceny wpływu polskiej szkoły matematycznej lat 1918–1939*, Wiadom. Mat. 31, 1995, 163-175.

[9] F. Klein, *Die Entwicklung der Mathematik im XIX. Jahrhundert*, 2 volumes, Berlin 1927 (volume 2, 202).

[10] S. Mazurkiewicz, *O potrzebach matematyki w Polsce* (*On the needs of mathematics in Poland*), Nauka Polska 2, 1919, 1-5.

[11] F.A. Medvedev, *Francuzskaja szkoła teorii funkcyj i mnożestw na rubieże XIX–XX ww.* (*The French school of the theory of functions and of sets at the turn of XIX–XX c.*), Moscow 1976.

[12] F.A. Medvedev, *O kurse lekcyj B.K. Młodziejewskogo po tieorii funkcyj dejstwitelnogo peremennego, proczytannych osenju 1902 g. w moskowskom uniwersitete* (*On the course of lectures on theory of functions of a real variable read by B.K. Młodziejewski in the autumn 1902 at the Moscow university*), Istoriko-matem. Issledow. 30, 1986, 223-246.

[13] C. Phili, *La génese et le développment de la théorie descriptive des ensembles*, Istoriko-mat. Issledow., II series, Moscow 2010.

[14] H. Poincaré, *Sur l'équation* $\Delta u + \xi u = 0$. *Analyse d'un mémoire de M. Zaremba*, Bull. Sci. Math. 26(2), 1902, 337-350.

[15] W. Sierpiński, *O polskiej szkole matematycznej* (*On the Polish school of mathematics*), [in:] *Wkład Polaków do nauk. Nauki ścisłe. Wybór artykułów* (*Contribution of Poles to Science. Exact Sciences. A. Selection of Articles*), Biblioteka Problemów 101, Warszawa 1967, 413-434.

[16] W. Ślebodziński, *Wspomnienia matematyka z lat 1903–1968* (*Recollections of a mathematican from the years 1903–1968*), Wiadom. Mat. 12.1, 1969, 17-31.

[17] S. Zaremba, *O najpilniejszych potrzebach nauki w Polsce ze szczególnym uwzględnieniem matematyki* (*On the most pressing needs of science in Poland with a particular attetion to mathematics*), Nauka Polska 1, 1918, 1-10.

[18] S. Zdravkovska, P.L. Duren, *Golden Years of Moscow Mathematics*, American Mathematical Society, 2007.