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ADAM SZMAGLIŃSKI, GRZEGORZ CZAJKOWSKI*

OPTIMAL INVESTMENT HORIZONS FOR THE MAIN INDICES OF THE WARSAW STOCK EXCHANGE

OPTYMALNE HORYZONTY INWESTYCYJNE DLA GŁÓWNYCH INDEKSÓW WARSZAWSKIEJ GIEŁDY PAPIERÓW WARTOŚCIOWYCH

A b s t r a c t

The investment horizon is the smallest time interval when an asset crosses a fixed value of the return level. For a given return level, the investment horizon distribution is created by putting the investment horizons into a histogram. We fit probability distribution function to the histogram. The maximum of the function is called the optimal investment horizon. We performed the analysis of some indices of the Warsaw Stock Exchange for WIG, WIG20, mWIG40 and shares of KGHM and MBK. For these assets, we found the coefficients of linear proportion between the optimal investment horizons and the logarithm of their return levels.

Keywords: econophysics, financial markets, inverse statistics

S t r e s z c z e n i e

Horyzont inwestycyjny jest najmniejszym odcinkiem czasu, w którym dana inwestycja przekroczyła ustalony poziom zwrotu. Dla danego poziomu zwrotu tworzymy rozkład horyzontu inwestycyjnego, składając horyzonty inwestycyjne w histogram. Maksymalna wartość dopasowanej funkcji rozkładu prawdopodobieństwa jest optymalnym horyzontem inwestycyjnym. Przeprowadziliśmy analizę dla niektórych indeksów Warszawskiej Giełdy Papierów Wartościowych WIG, WIG20, mWIG40, sWIG80 oraz akcji KGHM i MBK. Dla wymienionych instrumentów finansowych wyznaczyliśmy współczynniki proporcji liniowej pomiędzy optymalnymi horyzontami inwestycyjnymi i logarytmami poziomów zwrotu.

Słowa kluczowe: ekonofizyka, rynki finansowe, odwrócona statystyka

* Institute of Physics, Faculty of Physics, Mathematics and Computer Science, Cracow University of Technology, Poland; aszmag@op.pl, g.czajkowski@yahoo.pl.

1. Introduction

The character of price movements was described quantitatively by the random walk hypothesis proposed by Bachelier [1]. However, the nature of these movements better reflects the random walk hypothesis for the logarithm of the price $s(t) = \ln(S(t))$ [2]. According to this assumption, the distribution of the returns of an asset is effectively described by a Gaussian distribution [3–5].

A large amount of financial data is recorded for financial studies and benchmarking. An important and common task in studying the data is calculating the distribution of returns over a fixed time period Δt. The distribution measures gains or losses at time $t + \Delta t$ produced by the investment made at time t.

Many empirical studies for small values of Δt argue that the price changes are much larger than expected from the Gaussian distribution. The distributions have so-called fat tails [3–6]. For larger values of Δt the distribution of returns converges to the Gaussian distribution [7–10]. The analogue distribution is found for turbulence in air and fluids [11]. The statistics of financial markets were compared with turbulent fluids [4, 12–14].

The inversion of the standard return-distribution problem was proposed by Simonsen, Jensen and Johansen [15–17]. They studied the probability distribution of waiting times needed to reach a fixed return value ρ for the first time [11].

Another kind of investigation on waiting times and price movements was proposed in [18]. There were studied the frequency of occurrences of subsequent movements' proportions in price and time. Their proportions are effectively described by the generalized Gamma probability distribution.

2. Investment horizons

The first passage time problem was described in [19]. The solution to the Brownian motion problem is analytically provided by the Gamma distribution in [20, 21]:

$$p(t) = \frac{1}{\sqrt{\pi}} \frac{|a|}{t^{3/2}} e^{-a^2/t}, \tag{1}$$

where a is proportional to the return level ρ.

The overall growth of the economy modulated with times of recession influences financial time series $s(t)$ with a positive drift over long time scales. In the presence of such a drift, we cannot use the Brownian motion model to describe these series. For this reason, we should use so-called deflated asset prices $\tilde{s}(t)$ for reducing the effect of this drift. The drift $d(t)$ we describe with a 1000-day moving average for stock indices and a 250-day moving average for shares due to their higher volatility. These two periods naturally correspond to four calendar years and one calendar year respectively. For our analysis, we use logarithmic prices with subtracted drift $\tilde{s}(t) = s(t) - d(t)$. The prepared data for the WIG index and the MBK shares is depicted in Fig. 1. and Fig. 2.

The log-return over a time interval Δt of an asset of price $S(t)$ at time t reads:

$$r_{\Delta t}(t) = \ln \frac{S(t+\Delta t)}{S(t)}, \tag{2}$$

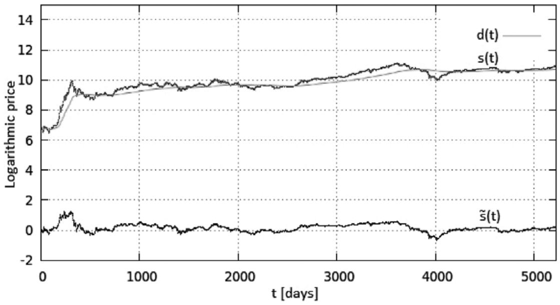

Fig. 1. Daily logarithmic closure prices of the WIG index over years 1991–2013

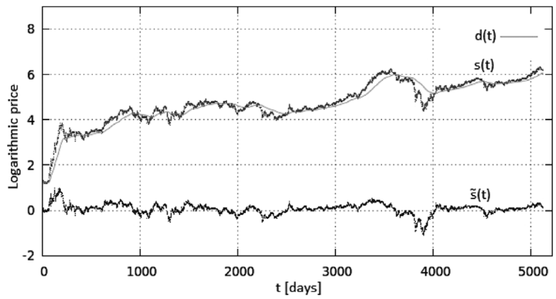

Fig. 2. Daily logarithmic closure prices of the MBK shares over years 1991–2013

For a return level ρ at time t, the *investment horizon* $\tau_p(t)$ is the smallest time interval Δt that satisfies the relation $r_{\Delta t}(t) \geq \rho$. For a fixed return level, we put the investment horizons in the histogram. In this way, we obtain the investment horizon distribution $p(\tau_p)$. Due to the empirical logarithmic stock price process is not the Brownian [3–6], we fit generalized Gamma distribution to the histogram:

$$p(t) = \frac{\nu}{\Gamma(\alpha/\nu)} \frac{|\beta|^{2\alpha}}{(t+t_0)^{\alpha+1}} \exp\left\{-\left(\frac{\beta^2}{t+t_0}\right)^{\nu}\right\}. \tag{3}$$

The distribution (3) reduces to the Gamma distribution (1) for parameters $\alpha = \beta = 0.5$, $\nu = 1$ and $t_0 = 0$.

The maximum of the distribution (3) defines the *optimal investment horizon*:

$$\tau_\rho^* = \beta^2 \left(\frac{\nu}{\alpha+1}\right)^{1/\nu} - t_0 \tag{4}$$

According to (1) for geometric Brownian processes we have relation $\tau_\rho^* \sim \rho^2$. The empirical data generate slightly different dependence, as we will see in the next section.

3. Discussion and results

The analysis was performed originally by Simonsen et al. in [15], it was also used for the WIG and some stock companies quoted in the Warsaw Stock Exchange (WSE) [22–24]. In this paper, we continue the investigation described in [25] for indices and companies quoted in the WSE. For the return level $\rho = 0.10$ in Fig. 3 and Fig. 4, we present $p(\tau_\rho)$ – the probability distribution function (pdf) of the investment horizons measured in trading days τ_ρ. As one can expect from the higher volatility of the share prices of MBK compared to WIG, the optimal investment horizon was $\tau_\rho^* = 4.52$ for MBK and $\tau_\rho^* = 11.33$ for WIG. The probability of reaching the return level for MBK is two times larger than for the WIG index in the area of the optimal investment horizon. The values of the optimal investment horizons for indices and shares we analyzed are placed in Table 1.

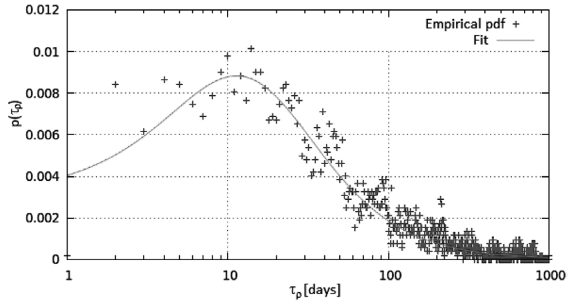

Fig. 3. The probability distribution function of the investment horizons of WIG measured in trading days, for the return level $\rho = 0.10$

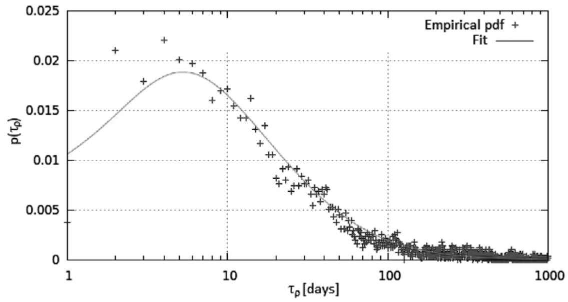

Fig. 4. The probability distribution function of the investment horizons of MBK measured in trading days, for the return level $\rho = 0.10$

We compare results with the DJIA index, which has a few times larger values of τ_ρ^* in comparison with indices quoted in WSE. This higher volatility is a feature of the WSE, rather than of emerging stock markets [22]. Among the main indices on the WSE, mWIG40 has the highest value of the optimal investment horizon. mWIG40 is composed of 40 medium-sized companies.

We also analyzed KGHM, the company with the highest capital in WIG20. Another company is MBK (former BRE Bank), which has been quoted in WIG20 since its beginning. Companies have much shorter values of τ_ρ^* than indices. The reason for this is in their higher volatility than the volatility of indices. The index is a weighted sum of the companies and every price movement of each company is only partially reflected in the change of the index.

Table 1

Optimal investment horizons for return levels ρ = 0.05, 0.10, 0.15 and the exponent of the return level γ

Name	ρ = 0.05	ρ = 0.10	ρ = 0.15	γ
DJIA	10.97	36.15	63.04	1.55
WIG	4.33	11.33	18.23	1.25
WIG20	3.33	9.66	15.27	1.34
mWIG40	6.82	20.27	40.07	1.60
sWIG80	4.82	12.93	22.21	1.35
KGHM	–	5.55	11.51	1.52
MBK	–	4.52	8.60	1.57

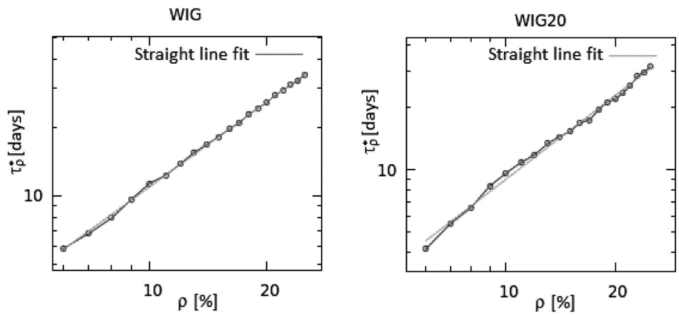

Fig. 5. The optimal investment horizon of two main indices on the WSE as a function of the log-return level ρ

Another important feature we investigate is the proportion:

$$\tau_\rho^* \sim \rho^\gamma \qquad (5)$$

The Brownian motion model with γ = 2 is inconsistent with empirical results [15]. For higher return values ρ, we fitted γ parameter as in Fig. 5. and Fig. 6. We find γ < 2 in the range 1.35–1.60 as shown in Table 1. For smaller return values ρ, we observed higher values of τ_ρ^* (not shown in the picture) than we could expect from (5).

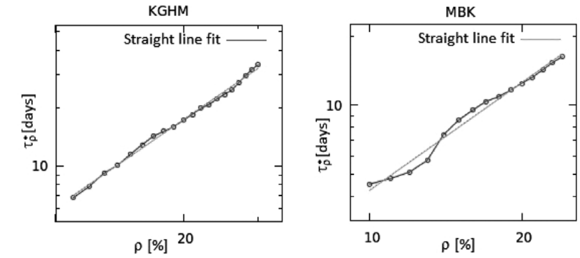

Fig. 6. The optimal investment horizon of two companies as a function of the log-return level ρ.

4. Conclusions

In this work, we analyzed the main indices quoted in the WSE which constitute the benchmark for some capital funds. The distributions and the optimal investment horizons can be applied for estimating the most probable time period for realization of the return level. The analogue passage time distributions are applied in turbulence of fluids, where they are called inverse structure functions.

The lower values of γ parameter indicate that the prices are more stable and connected with the real value of the asset than the expectation of the geometrical Brownian hypothesis which is generally accepted for capital markets [3, 4].

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