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JOLANTA GINTOWT, PAULINA ŁYSIK*

COMPARATIVE ANALYSIS OF THE CRITERIA USED TO SELECT THE OPTIMAL ENERGY SAVING VARIANTS IN BUILDINGS. SELECTED ISSUES

ANALIZA PORÓWNAWCZA KRYTERIÓW STOSOWANYCH DO WYBORU OPTYMALNYCH WARIANTÓW ENERGOOSZCZĘDNYCH W BUDOWNICTWIE. WYBRANE ZAGADNIENIA

A b s t r a c t

Comparative analysis of multi-criteria Pareto and SPBT optimization, for a single-family, detached residential building used as an example. Main elements of interest include minimum power and minimum cost of investment.

Keywords: multi-criteria optimization, Pareto, Kuhn-Tucker, SPBT

S t r e s z c z e n i e

Analiza porównawcza dla kryterium optymalizacji wielokryterialnej w sensie Pareto i SPBT na przykładzie budynku mieszkalnego jednorodzinnego, wolnostojącego. Funkcjami kryterialnymi są minimum energii i minimum kosztów inwestycyjnych, zmiennymi decyzyjnymi – izolacyjność termiczna przegród zewnętrznych, wielkość przeszklenia, orientacja budynku względem stron świata.

Słowa kluczowe: optymalizacja wielokryterialna, Pareto, Kuhn-Tucker, SPBT

* M.Sc. Eng. Jolanta Gintowt, Institute of Building Materials and Structures, Faculty of Civil Engineering, Cracow University of Technology.
** Paulina Łysik, student, Cracow University of Technology.

Designations

$f(x)$	–	vector of objective function
x	–	vector of decision variables
r	–	positive multiplier, controls the magnitude of the penalty terms
$h(x)$	–	vector of equality constraints
Gj	–	Heaviside operator
$g_j(x)$	–	vector of inequality constraints,
ρ_i	–	random number <0;1>
$P[f(x)]$	–	preference function (substitute function)
F_1	–	criterion function, energy [kWh]
$F2$	–	criterion function, investment cost [zł]
$g(i)$	–	boundary conditions
$x(5)$	–	thermal resistance [m^{2}K/W]
$x(6)$	–	simplex sides of the base of the building

1. Optimization method to determine reasonably low energy consumption

1.1. Unconstrained Minimization

One of the most developed groups of numerical optimization methods is the iterative type [1]. For this method, a point is established on the basis of the previously obtained results, which indicates where the minimum is likely to be, or the general direction in which it is likely to lie. This approach includes, without limitation, the following methods: Direct Search Method of Hooker and Jeeves, Simplex Method of Nelder and Mead, Variable Metric Method of Davidon-Fletcher-Powell. These methods however, should not be used to evaluate energy-efficient buildings, as they primarily determine the technical requirements, such as maximum heat transfer coefficient. In this case it is possible finding a local optima.

1.2. Constrained Minimization

The most common approach to solving constrained minimization problems involves the use of penalty functions to convert these problems into unconstrained problems. The most popular penalty function is the one, which associates a penalty – which is proportional to the square of a violation – as in the following formula (1).

$$\min_{x \in R^n} \phi(x,r) = f(x) + r\sum_{j=1}^{p}[h_j(x)]^2 + r\sum_{j=1}^{m} Gj[g_j(x)]^2 \tag{1}$$

where:

Gj – Heaviside operator such that $Gj = 0$ for $g_j(x) \geq 0$ and $Gj = 1$ for $g_j(x) < 0$.

One of the widely used formulations of the transformed interior function is the one shown in formula (2):

$$\phi(x,r) = f(x) + r\sum_{j=1}^{n}\frac{1}{g_j(x)} \tag{2}$$

If any of constraint functions $g_j(x)$ approaches 0, the penalty term increases very rapidly. In this method, it is necessary to start the search from an interior, feasible starting point.

Flexible Tolerance Method. The flexible tolerance method was developed by Himmelblau. In this method, $T(x)$ is defined as a positive square root of the sum of squared values of all violated inequalities or/and equality constraints. Formula (3) describes the dependency.

$$T(x) = \left\{\sum_{j=1}^{p}[hj(x)]^2 + \sum_{j=1}^{n} Gj[g_j(x)]^2\right\}^{\frac{1}{2}} \tag{3}$$

A small value of the $T(x^t)$ implies that x^t is relatively far from the feasible region.

Exploratory Methods. The accuracy of this method depends on he density of the grid, that is why we set up the grid with points spaced together close enough to define a minimum, as determined by the inspection of each point (Combinational Method).

Random Search Method (Monte Carlo Method, Modal Method). The value of objective function is evaluated for each point and the best result is taken as the minimum. This method offers two approaches for dealing with constraints. First method: a penalty is used for violating the solution outside the feasible region. In this case, the objective function is evaluated for all generated points. Second method: each generated point is tested for violation and discarded, if it is not a feasible solution. In this case, the objective function is evaluated only for a feasible solution. We select values of x_i – vector of decision variables. Used formula (4):

$$x_i = x_i^k + \rho_i(x_i^u - x_i^k) \tag{4}$$

Usually, this method will locate the solution in the neighborhood of the global minimum.

Method for Discrete and Integer Variables. Solving optimization problems with discrete variables directly is much more difficult than solving similar problems with continuous variables.

1.3. Multi-criterion Optimization Methods

In order to solve the problem, we used the Preference Function Method, described by the formula (5):

$$P[f(x^*)] = \min_{x\in X} P[f(x)] \tag{5}$$

as a Weighting Objective Method, Normed Weighting Objective Method, Global criterion Method, Min-Max Method, Weighting Min-Max Method, Method of Ideal Vector Displacement or Constraint Transformation Method.

1.4. Method of Selecting a Set of Pareto Optimal Solution, Kuhn-Tucker method

The Pareto Optimal Solution based on a random search method.
For Kuhna-Tuckera method we formulate the Lagrange function as a

$$L\ (x, \mathrm{l}) = f\,(x) - \Sigma\ \mathrm{li}\ \mathrm{gi}\ (x) \tag{6}$$

The necessary conditions for saddle point of the Lagrange function $L\ (x, \mathrm{l})$, which have to be fulfilled simultaneously, are as follows:

$$\frac{\partial L}{\partial l_i}\begin{cases} \geq 0 & 0 < i \leq u \\ \leq 0 & u < i \leq v \\ = 0 & v < i \leq m \end{cases} \qquad \frac{\partial L}{\partial x_j}\begin{cases} \leq 0 & 0 < j \leq s \\ \geq 0 & s < j \leq t \\ = 0 & t < j \leq n \end{cases}$$

$$l_i \frac{\partial L}{\partial l_i} = 0 \qquad x_j \frac{\partial L}{\partial x_j} = 0 \text{ and for all } i, j \tag{7}$$

1.5. Task description

Criterion functions were: minimum energy consumption for heating F_1, minimum investment costs F_2. Cooling was not analyzed. Such analysis takes several hours [11]. The optimization's decisive variables were: the thickness of wall insulation (thermal resistance of the layer), the size of the glazing on all elevations, the ratio of the sides of the base. Fixed parameters were: thermal resistance on the ground floor, flat roof, building area and ventilation air stream. The analysis' subject was a single-family building; in particular, the ground floor – fixed floor area of 120 m^2. The boundary conditions limiting the insulation were equal to about 10 [m^2 K/W] < R < 3.33 [m^2 K/W].

1.6. Results

Table 1 describes the results of the SPBT [years] and table 2 describes the results from Kuhn-Tucker. Due to the low thickness of the insulation, the SPBT value lies between 15 and 20 cm. This is illustrated by Fig. 1a. The solution method of multi-criteria thickness of the insulation is approx. 36 cm. This is depicted in Fig. 1 b. The difference between the results of both methods is almost 20 cm (one method recommends insulaion twice as thick as the other).

Table 1

SPBT dependence on the type of energy carrier and thickness of insulation

Thickness of insulation [cm]	SPBT [year]		
	coal	gas	electricity
10	122	69	26
20	119	67	25
30	132	74	28

Table 2

Dependence of thickness x (5) and F1 and F2 (method Kuhn–Tucker)

F1		314244	314634.6	315174.8	314950.6	314135.6
F2		5754.035	5739.386	5721.686	5728.87	5724.476
$x(5)$	10–3.33	max	max	max	max	max
$x(6)$	2.5–1	1.119	1.2457	1.23	1.25	1.3069

a) b)

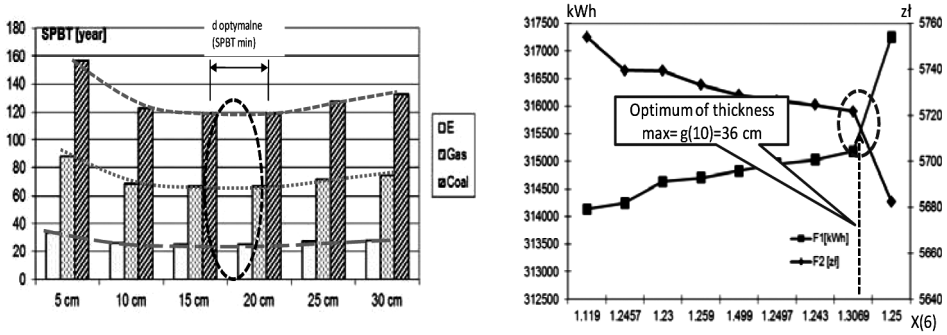

Fig. 1a) The optimal thickness of insulation, SPBT, b) the results of Kuhn–Tucker method

2. Conclusions

1. The SPBT method of choosing the optimum thickness of insulation seems to be only an estimate.
2. The difference in results between the SPBT and multi-criteria optimization methodes is not without significance.
3. It seems that one should apply advanced methods to evaluate the insulation efficiency (as opposed to the simple method which SPBT undoubtedly is).
4. The use of simple and easy ways is not a good choice among the available computational tools.

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[11] Dz. U. 2013 poz. 45, Rozporządzenie Ministra Transportu, Budownictwa i Gospodarki Morskiej z dnia 3 stycznia 2013 r. zmieniające rozporządzenie w sprawie metodologii obliczania charakterystyki energetycznej budynku i lokalu mieszkalnego lub części budynku stanowiącej samodzielną całość techniczno-użytkową oraz sposobu sporządzania i wzorów świadectw ich charakterystyki energetycznej.
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