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CONSTRUCTION SCHEDULE OF WORKS AS COMBINATORIAL OPTIMIZATION PROBLEM

HARMONOGRAMOWANIE PRZEDSIĘWZIĘCIA JAKO ZAGADNIENIE KOMBINATORYCZNE

A b s t r a c t

Construction schedule optimization generally deals with the identification of the feasible sequence of activities and allocation of modes that provide the most efficient construction performance according to the assumed evaluation criteria. The specific technological order of activities results in the numerous feasible sequences of activities and availability of alternative modes results in the numerous mode combinations. Construction scheduling becomes, therefore, a difficult combinatorial problem that is usually underestimated by the planners. This is the main reason for obtaining schedules that result in construction implementation lasting too long and costing too much. It is, nevertheless, possible to identify the construction schedules that provide excellent results. A simple, simulation-based approach is presented in this paper. Its originality results from nature of applied model and a way the calculations are made. The effectiveness of the approach is illustrated by means of a sample analysis based on data provided by [1]. The approach also proved useful for solving scheduling problems in engineering practice.

Keywords: construction, schedule, optimization, decision, support, activity sequence, mode, allocation, combinatorial problem

S t r e s z c z e n i e

Optymalizacja harmonogramu przedsięwzięcia polega na doborze takiej dopuszczalnej kolejności wykonywania prac oraz przydzieleniu takich sposobów wykonania operacjom, które zapewnią najlepsze możliwe, zgodnie z przyjętymi kryteriami oceny harmonogramu, wykonanie przedsięwzięcia. Z technologicznego porządku prac wynika zwykle duża, a często nawet astronomiczna liczba ich dopuszczalnych uporządkowań. Złożoność procesu harmonogramowania dodatkowo podwyższa dostępność alternatywnych sposobów wykonania prac. Harmonogramowanie przedsięwzięć stanowi więc w istocie trudne, a przy tym niedoceniane przez planistów, zagadnienie kombinatoryczne. W rezultacie otrzymujemy harmonogramy odpowiadające nadmiernie czaso- i kosztochłonnej realizacji przedsięwzięcia. Przy odrobinie wysiłku można jednak uzyskać harmonogramy, które odpowiadają krótkiej i taniej realizacji przedsięwzięcia. Temu celowi służy również autorskie narzędzie symulacyjne przedstawione w pracy. Stanowi ono oryginalne podejście zarówno w zakresie modelu, jak i sposobu wykonywania obliczeń, do których dane zaczerpnięto z pracy [1]. Wyniki pracy zostały także zastosowane w praktyce inżynierskiej.

Słowa kluczowe: przedsięwzięcie budowlane, harmonogram, optymalizacja, decyzja, wspomaganie, kolejność prac, sposób wykonania, przydział, zagadnienie kombinatoryczne

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1. Introduction

Construction projects deal with the execution of many different activities associated with the various building work being carried out. Sound project implementation requires the careful preparation. Construction schedules are applied in this regard. Construction scheduling deals with three primary decisions related to the choice of an appropriate feasible sequence of activities, the project start date and the allocation of available modes of construction activities. A specific schedule for a construction project results from a set of such decisions. The effects of these decisions are evaluated by means of schedule evaluation criteria. Project makespan T and total cost C are usually applied in this regard.

Note, that the technological order of activities is applied in numerous feasible sequences of activities, however, the availability of alternative modes for activities cause such scheduling to be problematic. Construction scheduling can then become a severe combinatorial optimization problem. The problem is hard to solve in acceptable time even in the case of construction projects that consist of average or small numbers of activities. Due to this, we are usually forced to rely solely on approximation schemes while scheduling real construction projects. Note, that mainly heuristic and metaheuristic approaches are applied in this regard [2]. The application of such approaches brings advantages in the case of the complex projects. This is because efficient “evolutionary” mechanisms allows for feasible solutions to the scheduling problem. This approach also deals with several drawbacks. At first, they comprise a kind of a black box. This fact discourages the conscious planners from using such approaches which risk losing control over the accuracy of analysis results. Secondly, the successful application of this approach relies on conducting time-consuming introductory numerical experiments. These experiments provide appropriate values vital to those parameters which influence the performance of the approaches.

The simple, yet powerful approach that addresses the drawbacks of the more advanced optimization methods, is presented in this paper. The approach is based on simple numerical simulation experiments [3] and provides near Pareto-efficient schedule for a construction project in terms of simultaneous minimization of make span and total cost.

2. Schedule optimization model

The following assumptions are made while structuring the model. A construction project consist of m activities, denoted by o(1), o(2)...o(m). The activities represent the consecutive construction works. There are o_i alternative modes available for the activity o(i), where i = 1, 2...m. It is assumed that each activity is executed by means of a single selected mode only. Each alternative mode requires the application of a specific resource – manpower, equipment and building materials. Building materials are considered to be a non-renewable resource because they undergo continuous exhaustion in the course of the construction process. Note, that a given set of structural and material solutions is considered. The same building materials are then applied while executing a given construction activity, regardless of the selected mode. It is also assumed that the necessary building materials are always available when required.

Manpower and equipment are the renewable resources because they become available again as soon as an activity ends. It is assumed that the renewable resources are available in sets corresponding with different modes available for the activities. Such sets are called the technical means sets (TMSs) and are denoted by z(1), z(2)...z(Z), where Z is the number of available TMSs. The number of available items for z(k) is expressed by l(k). Note, that peculiarity of construction processes results in the possibility of using the same renewable resources for executing different activities. A TMS may also consist of other, less complex TMS. If an item of a given TMS is utilised while executing a construction activity then it is unavailable for other activities at the same time. This assumption is important for the cases when different activities are concurrently executed and they may use the same TMS. The resource-related conflicts then occur due to the limited availability of the required TMS.

The acyclic, asymmetric and joined digraph $G(V, E)$ is applied to represent a feasible sequence of construction activities. In this paper this sequence is known as a project structure. Digraph vertices V express project events labelled by 0, 1...n, while digraph arcs E express the activities This paper describes the technological order of activities as the precedence structure. It is also represented by the acyclic, asymmetric and joined digraph $\Gamma(V, E)$. The digraph arcs correspond with construction activities while the digraph arcs express the relations of direct precedence for the activities. The sample precedence structure for a sample 10-activity project is presented in Fig. 1, while a corresponding sample project structure is presented in Fig. 2.

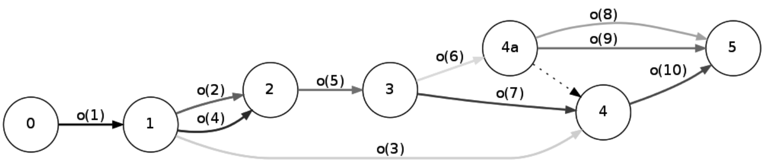

Fig. 1. A precedence structure for a sample construction project

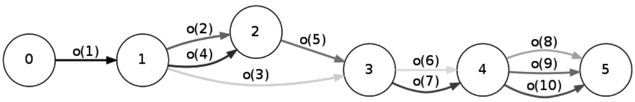

Fig. 2. A feasible project structure for a sample construction project

The model that corresponds with the presented assumptions is given in Eqns. (1–6). The meaning of the applied symbols is expressed immediately below.

$$\min_{G\in\bar{G}}\left\{\min_{\mathbf{x}} F = w_1\frac{T_n(G,\mathbf{x},\boldsymbol{\tau},T_0)-T_0}{\bar{T}} + w_2\frac{C(G,\mathbf{x},\boldsymbol{\kappa})}{\bar{C}}\right\}, \tag{1}$$

$$\underset{i\in\{1,\dots m\}}{\forall} \sum_{j=1}^{o_i} x_{ij}=1, \quad \underset{i\in\{1,\dots m\}}{\forall}\ \underset{j\in\{1,\dots o_i\}}{\forall} x_{ij}\in\{0,1\}, \tag{2}$$

$$T_0=0, \quad \underset{k\in\{1,2\dots n\}}{\forall}\ \underset{i\in\Gamma_k^-}{\forall} T_k\left(G,\mathbf{x},\boldsymbol{\tau},T_0\right)\ge t_i^{(s)}\left(G,\mathbf{x},\boldsymbol{\tau},T_0\right)+\sum_{j=1}^{o_i}x_{ij}\boldsymbol{\tau}_{ij} \tag{3}$$

$$\underset{k\in\{1,2\ldots n\}}{\forall} T_k \geq T_{k-1}\,, \tag{4}$$

$$C(G,\mathbf{x},\boldsymbol{\kappa}) = \sum_{i=1}^{m}\sum_{j=1}^{o_i} x_{ij}\kappa_{ij}, \tag{5}$$

$$\underset{k\in\Xi(G)}{\forall} \sum_{(i,j)\in\zeta^{(k)}} x_{ij} \leq L_{l(k)}\,. \tag{6}$$

The goal function presented by Eqn. (1) uses the simultaneous minimization of T and C. Note, that to make T and C commensurable, they are divided by the reference values $\overline{T}$ and $\overline{C}$, respectively. Normalized weights w_1 and w_2 express the relative importance of T and C. Note that the goal function provides the appropriate project structure G out of the set of all feasible project structures $\overline{G}$. The corresponding allocation of available modes to activities is also identified. The selected modes are represented by the binary decision variables x_{ij}, where: $i = 1, 2\ldots m$ and $j = 1, 2\ldots o_i$. The variables comprise the matrix of decision values $\mathbf{x}$.

The makespan is expressed by means of the difference between time of terminal project event occurrence T_n and the assumed time of starting project event occurrence T_0. It depends on the assumed project structure G, the selected modes x and the regular duration of activities corresponding with the available modes. Note, that the application of the j-th mode applicable in the case of the activity o(i) results in the regular activity duration τ_{ij}. Regular activity durations corresponding with all modes comprise the matrix $\boldsymbol{\tau}$. The total project cost C depends on the applied project structure G, and the chosen modes $\mathbf{x}$. Regular cost for available modes is denoted by κ_{ij}. Let us observe that matrices $\mathbf{x}$, $\boldsymbol{\tau}$ and $\boldsymbol{\kappa}$ have the same sizes.

Eqn. (2) assures that a single application mode is applied in the case of each activity and Eqn. (3) enforces the natural occurrence of the consecutive project events: T_k, where $k = 1, 2\ldots n$. Time of the occurrence of project events depends on the assumed project structure G selected modes $\mathbf{x}$, the regular duration of the activities $\boldsymbol{\kappa}$, and the assumed time of occurrence of the starting project event T_0. Note, that Γ_k^- expresses the set of activities terminated by the k-th project event, where: $k = 1, 2...n.$, and $t_i^{(s)}$ is the time of the occurrence of the starting event for the activity o(i).

The assumed sequence of the activities G is enforced by Eqn. (4). The following sum: $\sum_{j=1}^{o_i} x_{ij}\kappa_{ij}$ denotes the actual cost of the execution of the activity o(i), where: $i = 1, 2...m$. The total project cost is given in Eqn. (5). The actual duration of the activity o(i) is equal to: $\sum^{o_i} x_{ij}\tau_{ij}$.

The Eqn. (6) deals with the possible competition for renewable resources between different activities. It is assumed that the k-th conflict deals with a specific TMS. The number of available items of that TMS is denoted by $L_{l(k)}$. Note, that the number of possible conflicts is denoted by Ξ and depends on the assumed project structure G. The set of modes invoλved in the κ-th conflict, where: $k = 1, 2...\ \Xi$ is denoted by $\zeta^{(k)}$. The involved modes are described by the pairs of indices (i, j), where i and j express the number for the activity and the mode, respectively ($i = 1, 2...m$; $j = 1, 2...o_i$).

Note, that the considered scheduling problem is a kind of the Multi-mode Resource Constrained Project Scheduling Problem (RCPSP) [5] and is formulated as the mixed integer linear programme (MILP). Both the exact and the approximate [2] approaches can be applied to solve it. For example, the following exact approaches are applicable in this regard: Mixed Integer programming, Dynamic Programming Constraint Programming, Branch and Bound, and Benders Decomposition. The approaches mentioned provide exact optimization results but become less efficient in the case of the construction projects consisting of the numerous activities.

3. The applied approach

The applied approach addresses the drawbacks of the exact and approximate methods. It is based on the decomposition of the original problem into 2 levels:

1. The upper level deals which provides the appropriate feasible project structure G^* and the corresponding selection of modes $\mathbf{x}^*$ – note, that they define the near Pareto-efficient schedule.
1. The lower level deals with the (lower level) tasks – the MILP problems, obtained for the representative project structures $G \in \overline{G}$ and the following goal function which replaces the goal function given in Eqn. (1):

$$\min_{\mathbf{x}} F = w_1 \frac{T_n(G, \mathbf{x}, \boldsymbol{\tau}, T_0) - T_0}{\overline{T}} + w_2 \frac{C(G, \mathbf{x}, \boldsymbol{\tau})}{\overline{C}}. \tag{7}$$

Note, that the solution of the upper level problem is identical with the solution of the original problem given in Eqns. (1–6). The simple ranking of the locally Pareto-efficient solutions corresponding with the lower level tasks is enough in this regard. The ranking corresponds with the decreasing values of the goal function given in Eqn. (7).

The simulations are applied to generate the representative project structures G. The redundant representation of all feasible structures for a project is applied in this regard [3]. The representation is expressed by the acyclic, asymmetric and joined digraph $\overline{G}(\overline{V}, \overline{E})$. It consists of the vertices mapping all possible project events from the starting event labelled 0 to the latest possible terminating project event labelled m. The digraph arcs correspond with all alternative locations of the activities in feasible project structures. Note, that the choice of a single arc for each activity is enough to generate a feasible project structure.

Two different proposed approaches are finally applied [5]. They differ in the methods applied to solve a lower level problem. The first detailed approach is called MC-LP and combines random generation of representative project structures with linear programming to solve the lower level tasks. The second detailed approach, called MC-MC, applies the random generation for both the representative project structures G and the selection of modes $\mathbf{x}$. The both approaches complement each other. MC-LP is capable of providing more accurate results, while MC-MC is capable of performing better in the case of projects with a considerable number of activities.

Note, that the definition of the number of the generated feasible structures N' is required in order to apply the MC-LP and MC-MC approaches. The number of generated allocations of modes to activities (N'') should be defined to make the MC-MC application possible. The simple introductory simulation experiments are utilised to provide the required values for N' and N''. The introductory experiment for estimating N' deals with a number of lower level tasks solved by the means of the MC-LP approach while the introductory experiment for the MC-MC deals with a lower level task obtained for a single project structure. The following Formulae are applied in this regard:

$$N' \geq \frac{Z_{\frac{\alpha}{2}}^{2}\, \sigma(F)^{2}}{d'^{2}}, \quad N'' \geq \frac{Z_{\frac{\alpha}{2}}^{2}\, \sigma(F)^{2}}{d''^{2}} \tag{8}$$

where: $Z_{\frac{\alpha}{2}}$ denotes the parameter corresponding with the assumed possibility distribution and the confidence level α, while generating project structures (N') or mode allocations (N''), $\sigma(F)$ is the standard deviation in F values obtained during an introductory experiment and d', d'' denote the assumed absolute accuracy level while generating the feasible project structures and the mode allocations, respectively.

Note, that minimal goal function values F' and F'' provided by the introductory experiments can be applied to express the relative accuracy values ε', ε'':

$$\varepsilon' = \frac{d'}{F'}, \quad \varepsilon'' = \frac{d''}{F''} \tag{9}$$

The Eqns. (8, 9) make it then possible to obtain the relations ε' (N') and ε'' (N''). Note, that the MC-MC approach deals with the double simulation experiment, however. The overall relative accuracy ε thus depends on both N' **and** N':

$$\varepsilon = (1+\varepsilon')(1+\varepsilon'') - 1\,. \tag{10}$$

The appropriate combination of N' **and** N'' values can be then estimated for the required level on the basis of a minimal required computational effort expressed by product: $N'\,N''$.

4. The sample analysis

A sample construction project deals with a public garage building [1]. The project consists of 10 activities. Available modes for the activities depend on 31 different TMSs. The precedence structure for the sample project is presented in Fig. 1. The Pareto-efficient schedule is known for the sample project. It results from the exhaustive review of all, almost 10,000 feasible project schedules, and solving the related lower level tasks by the means of the linear programming: $F^* = 0.795$, $T^* = 194$ days and $C^* = 13{,}170{,}000$ PLN. It was obtained while assuming the following parameter values: $w_1 = 0.3$, $w_2 = 0.7$, $\overline{T} = 354$ days and $\overline{C} = 14{,}620{,}000$ PLN. Knowledge about the Pareto-efficient solution is applied to assess the real accuracy of the presented approach.

Uniform possibility distribution is applied while generating feasible project structures and the mode allocations. The introductory experiments deal with the generation of 20 feasible project structures and 20 mode allocations (MC-MC). These results are presented in Fig. 3. The experiments took the fraction of a second of mediocre CPU time. It proves that 13 generated project structures allow for breaking the 1% accuracy ε' limit, and 11 mode allocations should be enough to break the same limit in the case of the ε'' accuracy. Note also, that the Pareto-efficient schedule is identified already after the generation of two project structures only while using the MC-LP approach.

It proves that breaking the 1% ε accuracy level requires at least 150 feasible project structures and 25 mode allocations in the case of MC-MC approach.

A project structure corresponding with the obtained near Pareto-efficient schedule is presented in Fig. 2. The schedule corresponds with the goal function value $F = 0.802$ (+0.9% compared with the Pareto-efficient schedule), $T = 199$ days (+2.4%) and $C = 13.228,000$ PLN. The results are obtained in less than 33 seconds of mediocre CPU time.

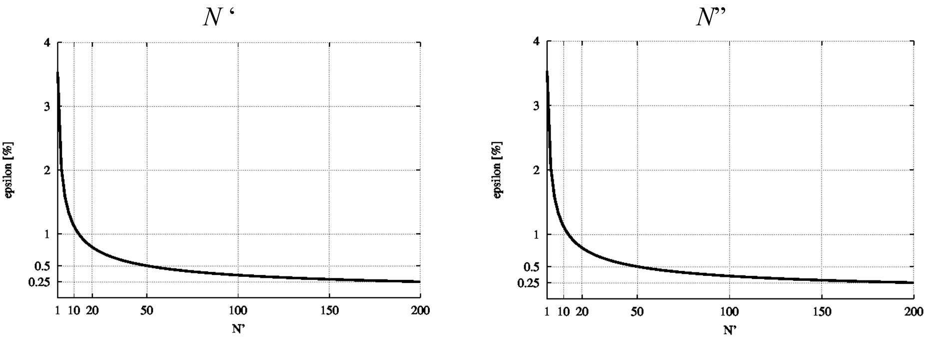

Fig. 3. Introductory analysis results for a sample project

5. Conclusions

The results obtained for the sample construction project confirm usability of the applied approach for the rapid identification of near Pareto-efficient schedules. The schedules provided by the approach are at most slightly worse than the Pareto-efficient schedule. Application of linear programming techniques and Monte Carlo simulations makes the approach reliable both in the case of projects consisting of smaller number of activities and in the case of the more complex projects with a considerable number of activities. MC-MC proves also useful while solving non-linear scheduling problems dealing with the influence of the surrounding environment.

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