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JUSTYNA FERENC*

THE RANDOM VARIABILITY ANALYSIS OF THE MECHANICAL PROPERTIES OF THE SELECTED ALUMINUM ALLOYS

ANALIZA ZMIENNOŚCI LOSOWEJ CECH MECHANICZNYCH WYBRANYCH STOPÓW ALUMINIUM

A b s t r a c t

This paper presents the results of the experimental research on mechanical properties of selected aluminum alloys and also statistical methods for the analysis of the random variability. Results of laboratory tests describe the local changes of the strength characteristics of Re, Rm and Young's modulus E.

Keywords: *aluminum, testing, homogeneity, variability, analysis,*

S t r e s z c z e n i e

W artykule zaprezentowano wyniki własnych badań cech mechanicznych wybranych stopów aluminium oraz metody analizy statystycznej ich zmienności losowej. Otrzymane rezultaty badań opisują przebieg zmian lokalnych cech wytrzymałościowych R_e, R_m i E wzdłuż osi badanych prętów.

Słowa kluczowe*: aluminium, badania, jednorodność, zmienność*

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1. Introduction

The main goal of the study is the analysis of variability and identification structure for the sparklines of the mechanical properties of selected aluminum alloys. The mechanical properties of aluminum alloys depend on the content of alloying elements, as well as on the type of thermal or mechanical treatment. Effect of tempering the strength of the alloy is a specific feature of the aluminum alloy, which can significantly affect the uniformity of strength properties along the axis of the bar.

Studies of variability of the mechanical properties along the axis of bars have been carried out only on specimens of steel. Difference in the cooling rate of steel rods stored in circles, according to a study [1] and also [2] resulted in differentiation of the rods strength through the length, as demonstrated by the occurrence of a trend. Metallurgical products used in the steel or aluminium structures have lengths of a few to several meters, the assessment of the variability of local strength properties of steel such St_3S was carried out in [3]. Obtained series of values f_y and f_u – *c*onsidered as realizations of a random function, were characterized by random noise. Based on knowledge of production technology aluminum alloys, cannot be excluded that it is non-stationary process. Random variability of material properties affects the variability of the whole structure, and therefore, the statistical distribution, found in the researches of the material, has a significant impact on the ability of the structure to carry loads.

2. The issue of stochastic homogeneity of the material

The concept of stochastic homogeneity is introduced when all the elements of the set (aluminium product) correspond to random variables or random functions Y (f_y, f_u, E) with the same distribution. However a diversity distribution of random variables Y and non-stationarity random function Y is stochastic heterogeneity. The stochastic process $F(t)$ is called a stationary process if its properties do not change when moving the timeline, the mean and variance is constant and the correlation between the cross-sections of the process $F(t)$ and $F(t + \Delta t)$ depend only on the distance Δt, and not t. With regard to the local characteristics of strength over the length of the rod, the time t is replaced by a measure of length. The non-stationarity can provide signals such as deterministic component of an unknown course and difficult to determine the analytical, the presence of a trend – deterministic component linearly variable along the length of the rod, or a harmonic of the period.

To test the strength characteristics of local changes in the length of the rod, carried out the static tensile test, the results are attributed to the longitudinal axis of the rod points. The local yield strength R_e, identified with the result of the tensile test section of the rod of length L_0, is determined for the area Ω (metallographic grain size) on the order of a few centimeters, in the central plane of the bar [3]. After bonding to yield a succession of n sections of the rod are obtained argument continuous random function $R(x)$ for $x \in \langle 0,L \rangle$. $R_j(x)$ as a single realization, characterized by a hypothetical general population associated with a single rod, coming from one production cycle [2, 3].

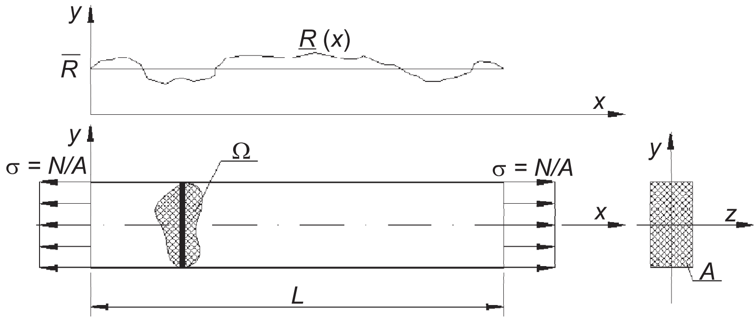

Fig. 1. The concept continual model of bar tensile strength (source: [3] fig. 4.16)

Verification of uniform sets of results is based on statistical tests. The study of statistical features is performed in two stages: putting statistical hypothesis for a single implementation and its verification by an independent statistical material.

3. Research of variation of mechanical properties of aluminum alloys

Experimental research on random variation of mechanical properties of aluminum bars included two parts of aluminium alloys in series 6xxx (durability class B) and 5xxx (durability class A) in a single product.

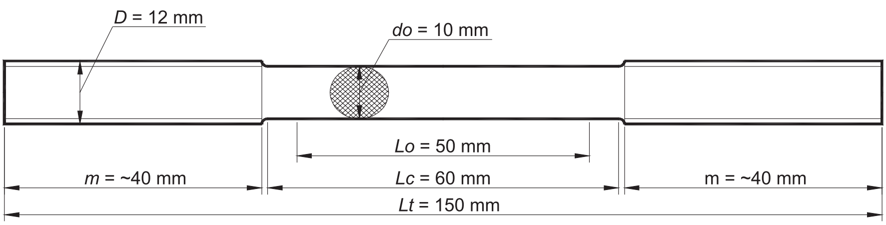

Fig. 2. The proportional round specimen

The figure Fig.1 shows specimen, which was adopted to tensile test. The specimens were cut from round bars with a diameter D = 12mm aluminum alloy AW-6060 T6 (3 bars in length 6m, signed as A, B and D) and AW-5754 H14 (4 bars in length 3m, signed as E,F,G,H).

3.1. The static tensile test

Mechanical properties of aluminium alloys were performed by static tensile test using electro-mechanical testing machine Zwick, equipped with extensometer, with a measuring range 2,4 kN to 1200 kN for Class 1 (see Fig. 3), using computer-aided measuring system.

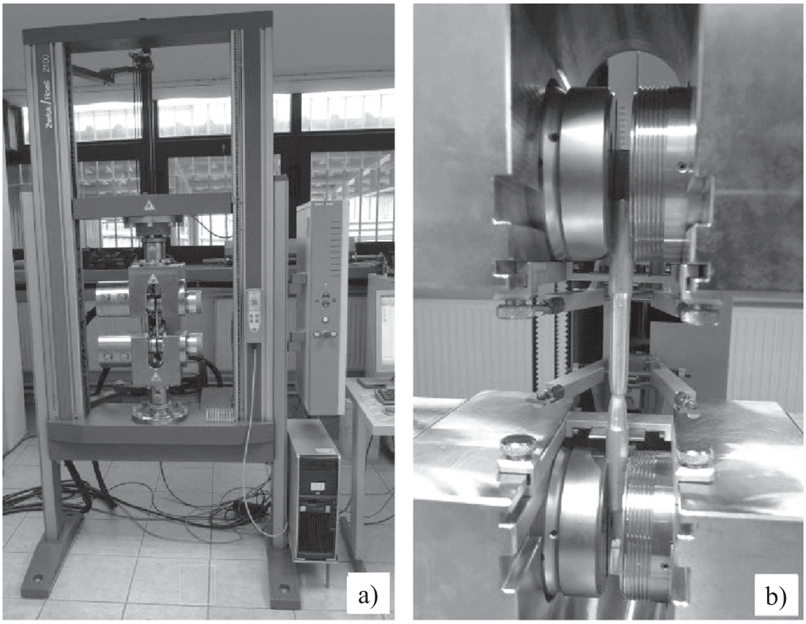

Fig. 3. The static tensile test: a) the testing machine Zwick Roell Z100, b) proper placement of the specimen in the grips (source: [11])

The study was conducted at ambient temperature and relative elongation rate of permanent deformation of specimens and the measuring range of the testing machine determined according to the standard EN 10002–1:2002 [4].

3.2.Test results

As a result of the static tensile test results were obtained for the following sequences of specimens: diameter measurements, yield strength, ultimate strength and Young's modulus. The diameters of the specimens were measured in two perpendicular directions. The diameters (Table 1) used for the analysis was the average of the two measurements in three sections (in two perpendicular directions) on the base length of each sample.

Table 1

Summary of the results diameter measurements for bars in alloy EN-AW 6060

Bar	n	Mean $\bar{x}$	Variance $s2$	Standard deviation s	Coeff. of variation v [%]
Diameter D [mm]					
A	37	10,015	0,013	0,011	0,11
B	38	10,015	0,020	0,014	0,14
D	36	10,013	0,010	0,010	0,10

Diameter measurements were performed to assess the uniformity of the control bars batch by analysis of variance. Analysis of results of measurements of diameters showed a slight variation, therefore, further analysis does not take into account the effect of the diameter of the specimen on the results of static tensile test.

The graphs (Fig. 4) shows the realizations of yield strength $f_{0,2}$ for 38 specimens cut from the bar A, 38 from bar B and 36 from the bar D and (Fig. 5) 19 specimens cut from the bar E, 17 from the bar F,18 from the bar G and 19 from bar H. Red circles denote values considered as outliers in carrying out the appropriate statistical tests.

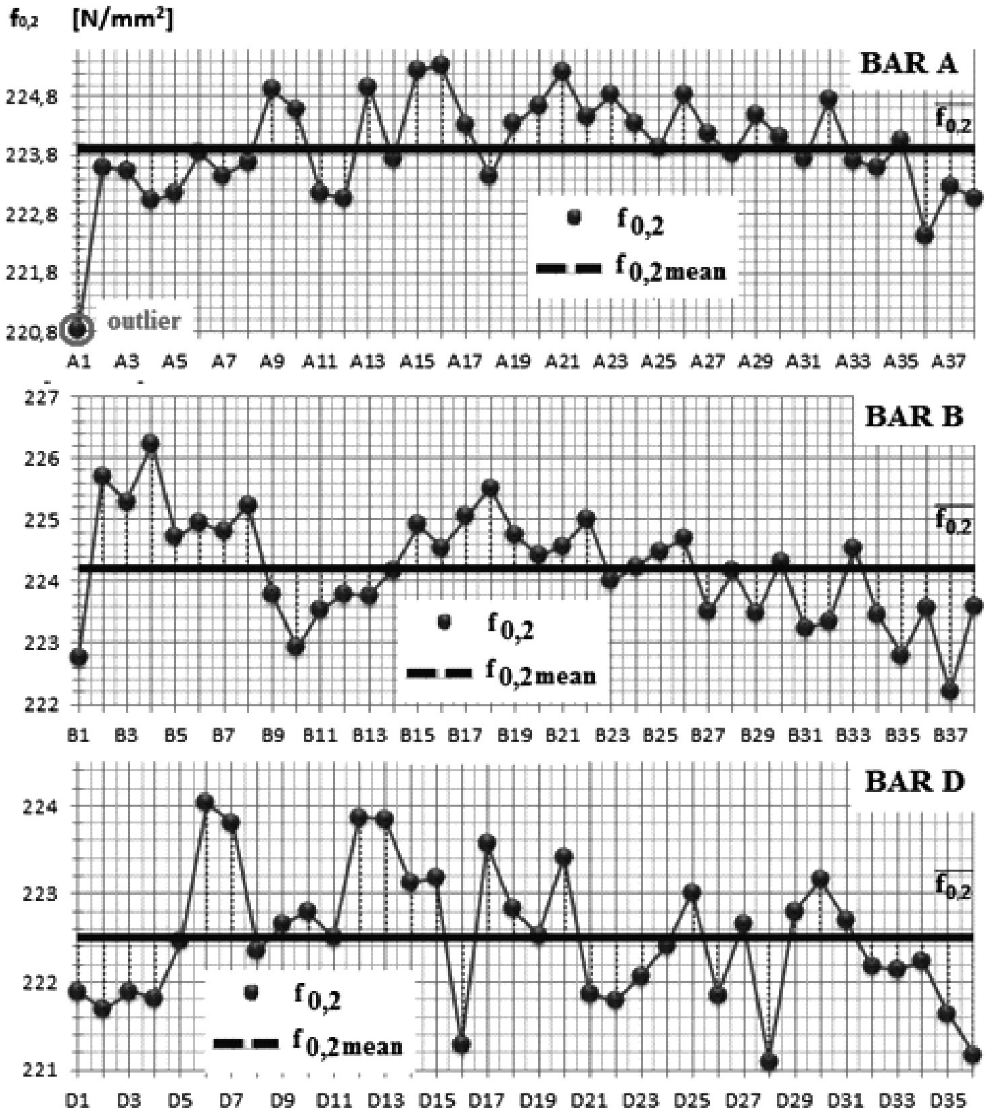

Fig. 4. The sparklines for yield strength – A, B, D bars

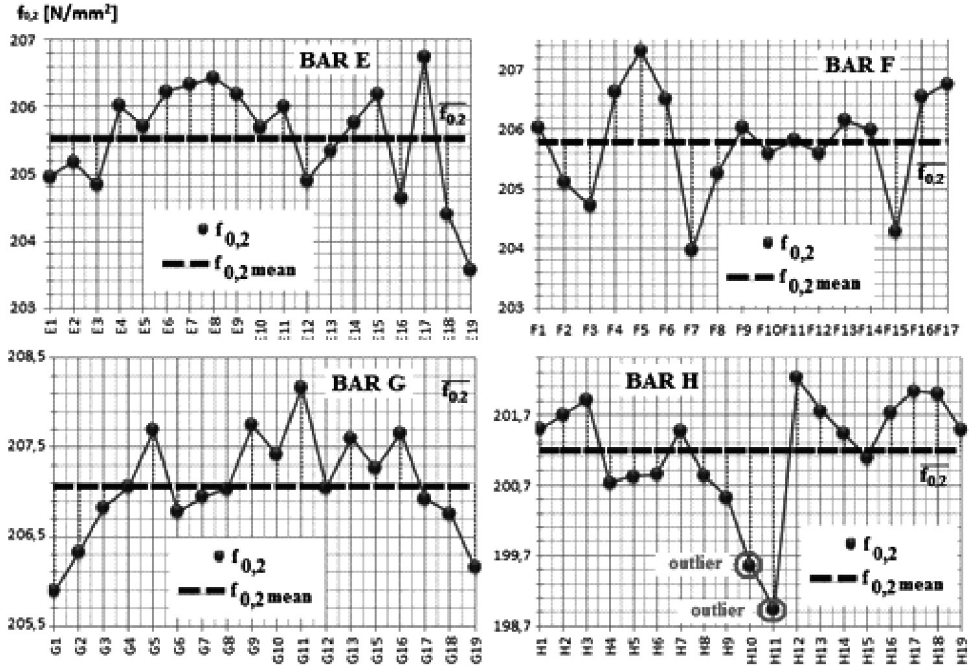

Fig. 5. The sparklines for yield strength – E, F, G, H bars

The graphs (Fig. 6) shows the realizations of ultimate strength f_u for 38 specimens cut from the bar A, 38 from bar B and 36 from the bar D and (Fig. 7) 19 specimens cut from the bar E, 17 from the bar F, 18 from the bar G and 19 from bar H.

The modulus of elasticity was estimated using a computer program (operating testing machine on which the tensile test was carried out) for the stress in the range of from about 25% to 50% of the yield strength. For the calculation of the elastic modulus E was used the linear correlation model. The sparklines estimated in this way, the values of E are shown in figure (Fig. 8) for 38 specimens cut from the bar A, 38 from bar B and 36 from the bar D and 19 specimens cut from the bar E, 17 from the bar F , 18 from the bar G and 19 from bar H.

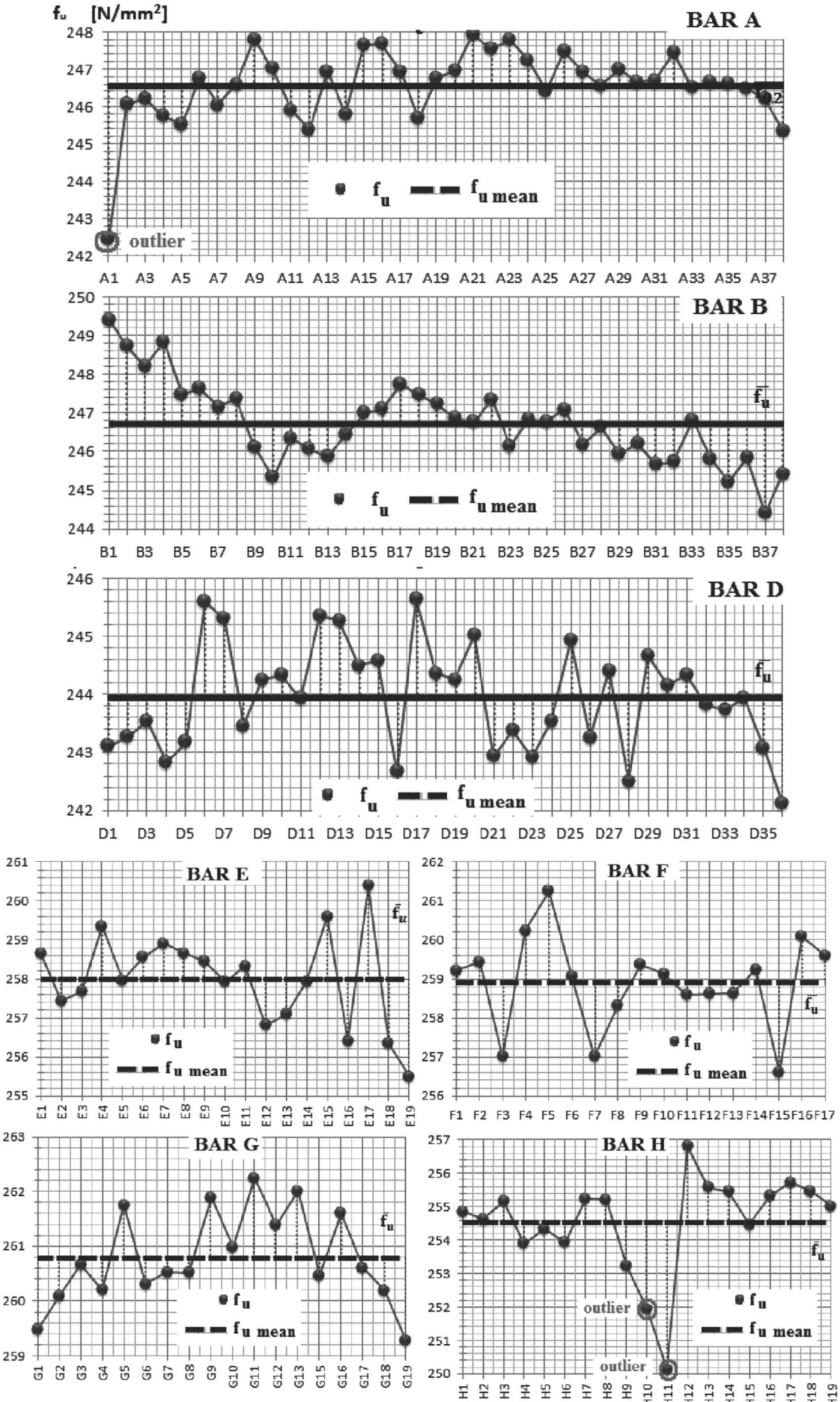

Fig. 6. The sparklines for ultimate strength – E, F, G, H bars

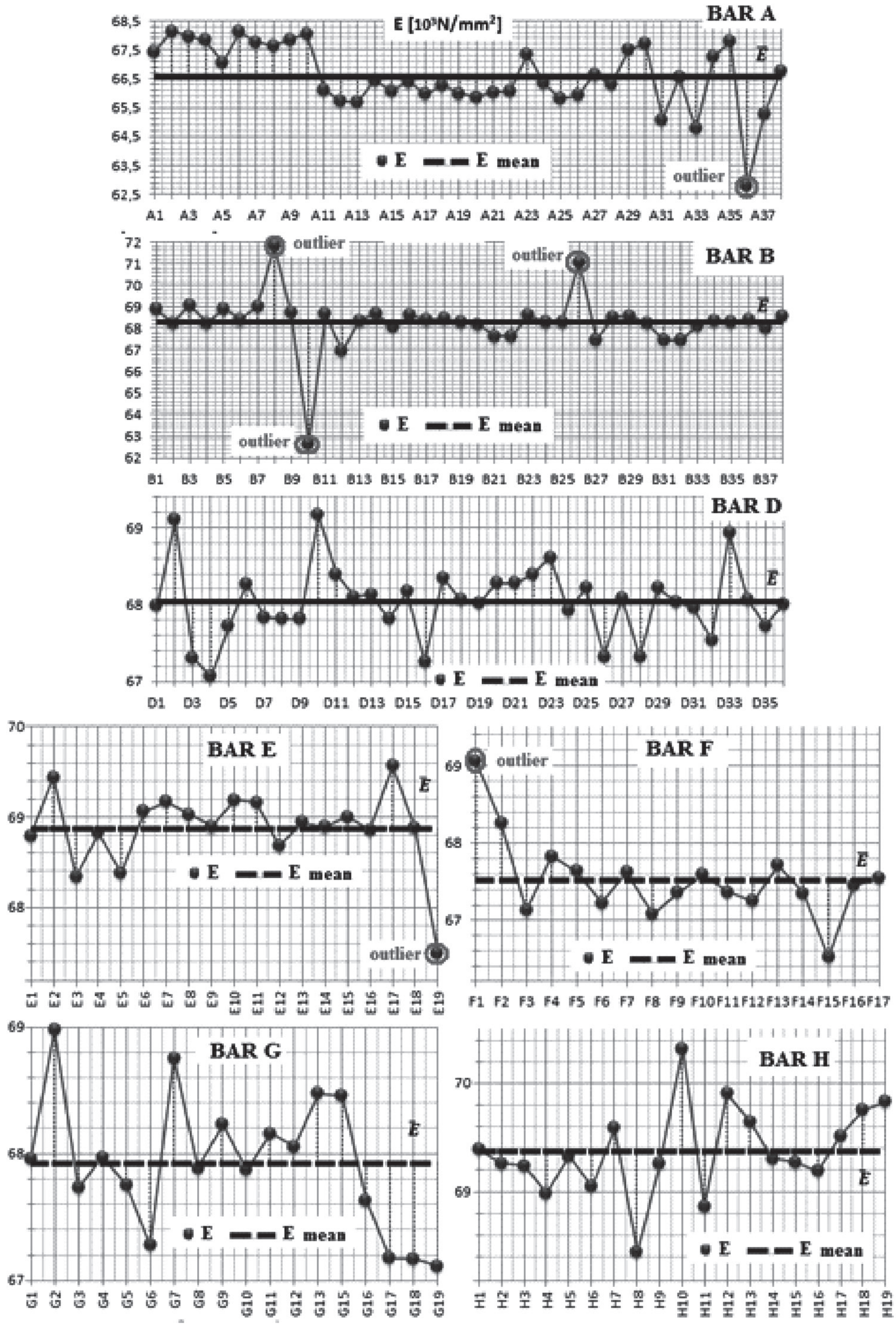

Fig. 7. The sparklines for Young's modulus E – E, F, G, H bars

4. Analysis of test results

The main objective of the study is to estimate the variability of mechanical properties of selected aluminium alloys. Static tensile test gives information about the characteristics of the local minimum at a length of measuring base sample. The first step prior to the analysis of the results obtained to identify and reject outliers in a data set.

4.1. Detection of outliers

In a data set outliers can occur [5, 6], which may be due to [7] approximation error, omission, bias or error thick (mistakes, mistakes in reading and writing performance, improper preparation of the sample or its attachment in the jaws of the testing machine). Typically, these results identified as an extreme value in the results obtained under the same conditions (obtained by the same method in the same laboratory using the same equipment, at short intervals) [8] Results of the study including outliers are characterized by statistical heterogeneity. In order to exclude them from further analysis of the results of appropriate tests statistics. In the literature you can find many methods to verify the results questionable, such as: Q-Dixon [9, 10] or Grubbs test [5]. In Table 2 and Table 3 are presented summaries of the results after the rejection of outliers.

Table 2

Summary of the results after the rejection of outliers for: yield strength $f_{0,2}$, tensile strength f_u and Young's modulus *E* measurements, for bars in alloy EN-AW 6060

Bar	n	Mean $\bar{x}$	Variance s^2	Standard deviation s	Coeff. of variation v [%]
yield strength $f_{0,2}$ [MPa]					
A	37	224.023	0.521	0.721	0.32
B	38	224.215	0.789	0.888	0.40
D	36	222.508	0.612	0.783	0.35
tensile strength f_y [MPa]					
A	37	246.677	0.504	0.710	0.29
B	38	246.720	1.104	1.050	0.43%
D	36	243.951	0.852	0.923	0.38%
Young's modulus E measurements					
A	37	66.692	0.894	0.946	1.42%
B	35	68.293	0.230	0.480	0.70%
D	36	68.037	0.229	0.478	0.70%

Table 3

Summary of the results after the rejection of outliers for: yield strength $f_{0,2}$, tensile strength f_u and Young's modulus *E* measurements, for bars in alloy EN-AW 5754

Bar	n	Mean $\bar{x}$	Variance s^2	Standard deviation s	Coeff. of variation v [%]
yield strength $f_{0,2}$ [MPa]					
E	19	205.535	0.677	0.801	0.39%
F	17	205.786	0.807	0.898	0.44%
G	18	207.076	0.349	0.591	0.29%
H	17	201.421	0.269	0.519	0.26%
tensile strength f_y [MPa]					
E	19	258.002	1.479	1.216	0.38%
F	17	258.909	1.446	1.202	0.46%
G	18	260.785	0.742	0.861	0.33%
H	17	254.958	0.695	0.834	0.33%
Young's modulus E measurements					
E	18	68.951	0.096	0.309	0.45%
F	16	67.432	0.144	0.379	0.56%
G	18	67.924	0.289	0.538	0.79%
H	19	69.376	0.166	0.418	0.60%

4.2. Statistical analysis of the homogeneity of selected mechanical properties

The condition for the applicability of the Analysis of Variance is normality and homogeneity of variance of the variable for all the compared populations [12]. In order to verify the normal distribution of compatibility tests performed Shapiro-Wilk's test and Pearson's Chi-square test. Both tests gave no reason to reject the hypothesis of normal distribution of yield strength, tensile strength and modulus of elasticity for all bars in 5xxx and 6xxx series at the given level of significance $\alpha = 0.05$. The hypothesis of homogeneity of variance was tested using the Bartlett's test. The test results are summarized in Table 4.

Only for 6xxx series (bars A, B and D) must be rejected the hypothesis of equality of variances at the given level of significance. To be able to use the ANOVA procedure must be converted using the so-called stabilization of the variance.

Verification of the hypothesis of equality of means using Analysis of Variance carried out on the assumption that the distribution of the dependent variable results in each group is similar to the normal group compared to a similar size, the individual observations are independent and the variances in the groups are similar. The ANOVA test results are summarized in Table 5.

Table 4

Results of Bartlett's test for bars in alloy EN-AW 6060 and EN-AW 5754

The Bartlett's Test															
	Yield strength $f_{0,2}$ [MPa]					Tensile strength f_y [MPa]					Young's modulus E measurements				
Bar	n	s^2	χ^2	$\chi^2_{0,05}$		n	s^2	χ^2	$\chi^2_{0,05}$		n	s^2	χ^2	$\chi^2_{0,05}$	
A	37	0.521	1.60	5.99	+	37	0.504	5.42	5.99	+	37	0.894	23.20	5.99	-
B	38	0.789				38	1.104				35	0.230			
D	36	0.612				36	0.852				36	0.229			
E	19	0.677	6.25	7.82	+	19	1.479	4.00	7,82	+	18	0.096	5.22	7.82	+
F	17	0.807				17	1.446				16	0.144			
G	18	0.349				18	0.742				18	0.289			
H	17	0.269				17	0.695				19	0.166			

Table 5

Results of ANOVA test for bars in alloy EN-AW 6060 and EN-AW 5754

ANOVA											
	k	N	SSb	SSw	MSb	MSw	F	DF k–1	DF N-k	$F_{crit(0,05)}$	H0
Yield strength $f_{0,2}$ [MPa]											
A+B+D	3	111	63.979	69.360	31.989	0.642	49.810	2	108	3.09	–
E+F+G+H	4	71	311.230	35.318	103.74	0.527	196.81	3	67	2.742	–
Tensile strength f_y [MPa]											
A+B+D	3	111	183.688	88.766	91.844	0.822	111.745	2	108	3.09	–
E+F+G+H	4	71	308.27	73.489	102.76	1.097	93.683	3	67	2.742	–
Young's modulus E measurements											
A+B+D	3	108	53.829	48.017	26.914	0.457	58.854	2	105	3.08	–
E+F+G+H	4	71	42.375	11.845	14.125	0.177	79.9	3	67	2.742	–
SS (total sum of squares) = SSb (between) +SSw (within) DF (degree of freedom) MS (mean square)											

The hypothesis H0 was rejected in the Analysis of Variance, so must be carried out further tests (post hoc tests), involved multiple comparisons. After the completion of the group obtained average values, which do not significantly differ. These tests showed that only mean values for pairs of bars A+B and E+F for yield strength are not significantly different.

4.3. Stationarity analysis

The production process can introduce harmonic components with a period longer than the length of the bar, so may be impossible to detections its along the short bars. Statistical tests are performed separately for each bar and the implementation of sparklines, present the general shape of the variation in mechanical properties, could help to exclude potential presence of trend and periodic components. Independence results for specimens can be seen from the graph of autocorrelation function. An example of the stationary process is “white noise” process.

Identification of the trend has influence on the appropriate method of statistical analysis. There are no proven techniques allow for the identification of the components of the trend, however can be seen if it is constantly growing/increasing. In the literature, [13–15] are described stationary statistical analysis through the use of such unit root tests, so it is possible to identify the type of non-stationarity in the data, which can be used to remove any trend in order to bring the data into a stationary process . The pre-existence of a trend, it is possible to detect by visual diagrams and simple regression analysis. Based on the calculated regression coefficients determined the probability test, which compared to the accepted level of significance.

In the analysis of one-dimensional random function uses two basic features that characterize this variability: the autocorrelation function or function autocovariance. According to [16] field autocorrelation function should be limited up to one third the length of the measured execution. The function was calculated for the step Δx = 15 cm corresponding to the length of the specimen, and the maximum delay equal to half the length of the bar. In the case where the test process is stationary, autocorrelation function values should be close to zero, which means that it is uncorrelated sequence of random variables with a fixed variance and null mean value. For this purpose, the analysis of autocorrelation functions performed for centralized.

To verify the hypothesis of no first-order autocorrelation of the random component is used Durbin–Watson’s test, under conditions of normal distribution of the random component. Durbin – Watson test indicates a positive first-order autocorrelation yield for bars: A, B, E and F. For the rod D, and G, H, there are no autocorrelation. The figure shows the graph autocorrelation function (Fig. 8) for the yield of bars A, B and D.

Statistical significance of further investigating the correlation coefficient is the Ljung-Box statistics form. Autocorrelation coefficients are individually significant for delays R = 1 and R = 18 for the bar A. For the bar B, first few rows of autocorrelation is important – there is a trend , confirms that visual analysis of scatter plot points of the empirical and the course of the auto-regression function . There are also random fluctuations and no periodic fluctuations. In the case of the bar D, the autocorrelation is not observed.

Fourier analysis (spectral) [17] is used to study the harmonic structure of the time series (random function). The purpose of this analysis is to determine the number of the power spectrum versus frequency or period (Fig. 9).

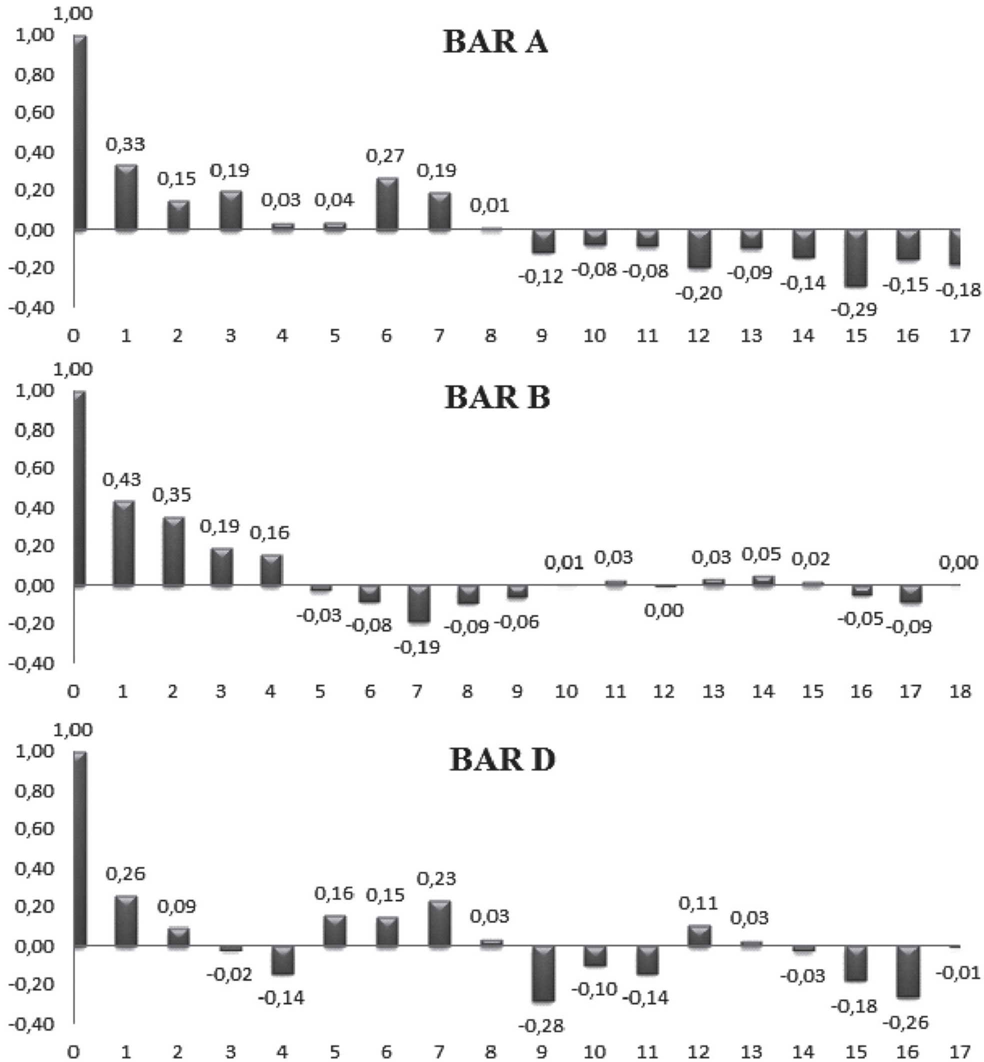

Fig. 8. Autocorrelation function graph for the yield strength

Decomposition of time series, looking like random noise, allows discovering some periodic cycles of different lengths. Prior to spectral analysis, potential trends must be removed, and the average should be subtracted to receive stationary process.

The process can be considered as white noise if its components are normally distributed and the value of the periodogram (Fig. 10) will have an exponential distribution. For this purpose, compliance testing is carried out, f.eg. Kolmogorov – Smirnov's test [18].

If the harmonic period is less than the length of realization (bar), the periodic component should be distinguished by the occurrence the peak in a function of spectral density (Fig. 9).

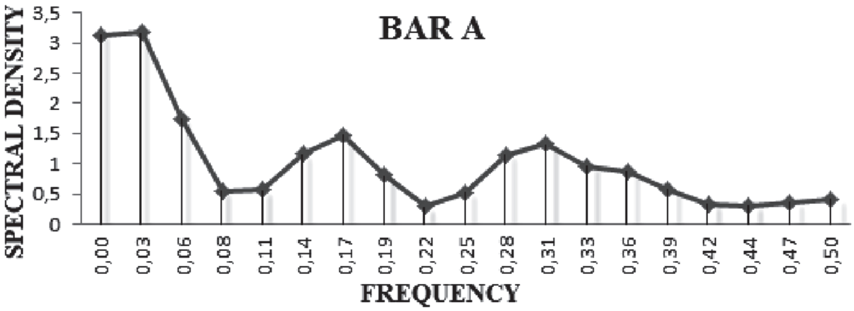

Fig. 9. Spectral density graph for yield strength

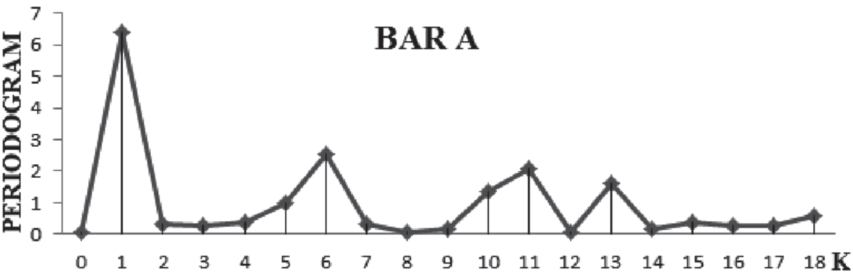

Fig. 10. Periodogram for yield strength

5. Conclusions

The study of the mechanical properties of selected aluminum alloys, are shown graphically in the following figures (Fig. 4–7) as realizations of random function corresponding to a hypothetical general population bars meet the requirements of homogeneity. Summaries of the results for: Young's modulus E, yield strength $f_{0,2}$, tensile strength f_u for bars in alloy EN-AW 6060 and EN-AW 5754 are shown in the Table 2 and Table 3. An initial inspection of the implementation of graphs, simple regression analysis and autocorrelation analysis helped to identify the structure of the outputs as a stochastic process. After elimination of possible trends and bringing the process into a stationary spectral analysis was carried out to demonstrate the absence of significant periodic signals nonstationarity on the length of the rods. Therefore, to describe the variability of strength for both parties of bars from 5xxx and 6xxx series aluminum alloys can be assumed the stationary model similar to the white gaussian noise.

The obtained results show a slight heterogeneity of mechanical stochastic local features. In particular, the coefficients of variation of the yield strength and ultimate strength are less than 1% (see variance according to Table 2 and Table 3). For comparison, for flats made of steel *St3S* [19], where he obtained values of the coefficients of variation v = 2.4–2.8%. Differences in the size and the measurement of displacement in the distribution of the values of the outputs of the coefficient of variation described aluminum and steel products have also been observed in [20]. Based on the results of this analysis can be pre-concluded that the stochastic variance [21–23], which is one of the three components of variance (beside statistic variance and probability variance) is so small that there is no significant effect on the coefficient of variation γ_M=1,1 proposed by Eurocode 9 [24]. It should also be noted that the strength test was carried out at that time, the strength of the older generation machines, which were not equipped with electronic measurement recording systems, individual embodiments are therefore less accurate than contemporary recorded.

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KRZYSZTOF KUCHTA, IZABELA TYLEK*

APPLICATION OF STAINLESS STEELS IN BUILDING STRUCTURES

ZASTOSOWANIE STALI NIERDZEWNYCH W KONSTRUKCJACH BUDOWLANYCH

A b s t r a c t

In the paper structural stainless steels consistent with EN 10088 were characterized. Authors described stainless steel categories, production process, basic properties and element joining techniques. Some examples of stainless steel applications in structural building, realized in the last few years in Poland and in the world were presented.

Keywords: stainless steel in building structures

S t r e s z c z e n i e

W artykule scharakteryzowano konstrukcyjne stale nierdzewne zgodne z EN 10088. Opisano ich rodzaje, proces produkcji, podstawowe właściwości oraz technikę łączenia elementów. Podano również przykłady zastosowania stali nierdzewnych w konstrukcjach budowlanych zrealizowanych w ostatnich latach na świecie i w Polsce.

Słowa kluczowe: stale nierdzewne w konstrukcjach budowlanych

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1. Introduction

Stainless steel large-scale production began over 100 years ago in France and Germany. In 1912, after four years of researches in Krupp's Concern laboratory in Essen, conducted by professor Benno Strauss and doctor Eduard Maurer, patent for austenitic stainless steel was obtained. This steel is known all over the word as Nirosta (acronym from German language – ***ni****cht****ro****stender* ***Sta****hl*). Development of stainless steel production technology was continued mainly in United States and Great Britain. In spite of long production history and many spectacular applications in building industry, stainless steel began to be perceived as constructional building material about15–20 years ago.

Currently stainless steel is more frequently used not only by architects but also by structural engineers. Apart from its high corrosion resistance, stainless steel has other advantages: high durability, availability of many grades with different mechanical and physical properties, easiness of machining, forming and joining of elements, products diversity (sheets, plates, coils, tubes, bars, profiles, fittings etc.), many types of surface finishing (bright finishes, deco and patterned, fingerprint resistant, polished and brushed, primed and painted etc.), easiness of keeping clean. This is also proper material to work both in low and high temperature. Another reason that stainless steel becomes more popular is ecology. It is estimated that almost 60% of material used for stainless steel production comes from recycling (data from The European Stainless Steel Development Association Euro Inox), addition of expensive chromium and nickel makes profitable to recycle of all waste products. Long life cycle of stainless steel products enables to safe energy as a result of manufacturing process limiting. Very important for environmental protection is the fact that stainless steel elements do not need additional anti-corrosive chemical coatings.

The impulse which strongly stimulated propagation of stainless steel structures was, in last twenty years, appearance of new generation of Standards pertaining to design and execution structures made of stainless steel. Until the European Standards has been introduced in Poland, only metallurgical Standards applicable to stainless steels: PN-71/H-86020 "Stal odporna na korozję (nierdzewna i kwasoodporna). Gatunki", PN-71/H-86022 "Stal żaroodporna. Gatunki", BN-77/0631-11 "Nowe stale odporne na korozję (nierdzewne, kwasoodporne, utwardzalne wydzieleniowo, żaroodporne). Gatunki" were established.

2. Categories and grades of stainless steel

Basic criterion of dividing stainless steel into categories is their microstructure at room temperature, which has a decisive effect on steel physical and mechanical properties. European Standard EN 10088-1 [14] be distinguish following categories of stainless steel: martensitic, ferritic, austenitic, austenitic-ferritic (duplex) and precipitation hardening. Steels from the last category are hardened in a special way which caused formation of precipitates within the microstructure. According to EN 1993-1-4 [8] in building structures only ferritic, austenitic and austenitic-ferritic (duplex) stainless steels can be used. Martensitic steels due to their low toughness and weldability are not permitted.

Selected properties of structural stainless steel categories are compared in Table 1 [3].

Table 1

Comparison of selected stainless steel properties [3]

Micro-structure	Magnetic response[1)]	Work hardening rate	Corrosion resistance[2)]	Harde-nable	Ductility	High temperature resistance	Low temperature resistance
Ferritic	Yes	Medium	Medium	No	Medium	High	Low
Austenitic	Generally: No	Very high	High	By cold work	Very high	Very high	Very high
Austenitic-ferritic (Duplex)	Yes	Medium	Very high	No	Medium	Low	Medium

1) Attraction of the steel to a magnet. Note same austenitic grades can be attracted to a magnet if cold worked, cast or welded.
2) Varies significantly between grades within each group.
3) Measured by toughness or ductility at sub-zero temperatures. Austenitic grades retain ductility to cryogenic temperatures.

European Standards, currently in force, introduce two systems of steel grades designation: by mean of symbolic letters and numbers according to EN 10027-1 [12] and numbering system according to EN 10027-2 [13].

Designation of stainless steel grade according to [12] begins with letter *X* and a number, which are followed by sequence of letters and ended by sequence of numbers separated by a hyphen, for example: X2CrNiN18-10. Letter *X* symbolizes stainless and other alloyed steels, where the mean alloying content of at least one element is above 5%. The first number after X represents 100-times the mean percentage value of the carbon content. The following letters are the symbols for the most important alloying elements, in descending order by content. Numbers separated by a hyphen specify the approximate content of the main alloying elements. If there is no number, it means that content of this element is lower than 1%. As an example: X2CrNiN18-10 it is stainless steel (X) with the mean value of carbon content 0,02% (2/100), containing 18% of chromium (Cr, 18), 10% of nickel (Ni, 10) and less than 1% of nitrogen (N, lack of number).

Steel grades marked according to second of European designation systems [13] have fixed number of digits which refers only to one steel grade. The structure of steel number is as follows: 1.*xxyy*, where: 1 – material group number (steel), *xx* – steel group number, *yy* – sequential number of steel grade. In Table 2.1 EN 1993-1-4 [8] were classified four groups of structural stainless steel:

1) 1.40*yy* – grades with less than 2.5% nickel, without molybdenum, without special additions,
2) 1.43*yy* – grades with more than 2.5% nickel, without molybdenum, without special additions,
3) 1.44*yy* – grades with more than 2.5% nickel, with molybdenum, without special additions,
4) 1.45*yy* – grades with special additions, such as titanium, niobium or copper.

Chemical composition (cast analysis) of stainless steel grades included in Table 2.1 EN 1993-1-4 [8] is compared in Tables 2 to 4.

Table 2

Chemical composition of ferritic stainless steels [14]

Steel grade	% by mass								
	C max.	Si max.	Mn max.	P max	S max.	N max	Cr	Ni	Ti
1.4003	0.030	1.00	1.50	0.040	0.015	0.030	10.50–12.50	0.30–1.00	6x(C+N) to 0,65
1.4512	0.030	1.00	1.00	0.040	0.015		10.50–12.50		
1.4016	0.080	1.00	1.00	0.040	0.015[1)]		16.00–18.00		

[1)] For bars, rods, sections and the relevant semi-finished products, a maximum content of 0.030% S applies.
For any product to be machined, a controlled sulfur content of 0.015% to 0.030% is recommended and permitted.

Table 3

Chemical composition of austenitic stainless steels [14]

Steel grade	% by mass									
	C max.	Si max.	Mn max.	P max	S max.	N	Cr	Cu	Mo	Ni	Ti
1.4318	0.030	1.00	2.00	0.045	0.015	0.10-0.20	16.50–18.50			6.00–8.00	
1.4307	0.030	1.00	2.00	0.045	0.015[1)]	$\leq$ 0.11	17.50–19.50			8.00–10.00	
1.4306	0.030	1.00	2.00	0.045	0.015[1)]	$\leq$ 0.11	18.00–20.00			10.00–12.002)	
1.4311	0.030	1.00	2.00	0.045	0.015[1)]	0.12-0.22	17.00–19.50			8.50–11.50	
1.4301	0.070	1.00	2.00	0.045	0.015[1)]	$\leq$ 0.11	17.00–19.50			8.00–10.50	
1.4541	0.080	1.00	2.00	0.045	0.015[1)]		17.00–19.00			9.00–12.00[2)]	5xC to 0.70
1.4404	0.030	1.00	2.00	0.045	0.015[1)]	$\leq$ 0.11	16.50–18.50		2.00–2.50	10.00–13.002)	
1.4406	0.030	1.00	2.00	0.045	0.015[1)]	0.12-0.22	16.50–18.50		2.00–2.50	10.00–12.00[2)]	
1.4401	0.070	1.00	2.00	0.045	0.015[1)]	$\leq$ 0.11	16.50–18.50		2.00–2.50	10.00–13.00	
1.4571	0.080	1.00	2.00	0.045	0.015[1)]		16.50–18.50		2.00–2.50	10.50–13.50[2)]	5xC to 0.70
1.4432	0.030	1.00	2.00	0.045	0.015[1)]	$\leq$ 0.11	16.50–18.50		2.50–3.00	10.50–13.00	

Steel grade	% by mass										
	C max.	Si max.	Mn max.	P max	S max.	N	Cr	Cu	Mo	Ni	Ti
1.4435	0.030	1.00	2.00	0.045	0.015[1)]	≤ 0.11	17.00–19.00		2.50–3.00	12.50–15.00	
1.4439	0.030	1.00	2.00	0.045	0.015	0.12-0.22	16.50–18.50		4.00–5.00	12.50–14.50	
1.4539	0.020	0.70	2.00	0.030	0.010	≤ 0.15	19.00–21.00	1.20–2.00	4.00–5.00	24.00–26.00	
1.4547*)	0.020	0.70	1.00	0.030	0.010	0.18-0.25	19.50–20.50	0.50–1.00	6.00–7.00	17.50–18.50	
1.4529	0.020	0.50	1.00	0.030	0.010	0.15-0.25	19.00–21.00	0.50–1.50	6.00–7.00	24.00–26.00	

[1)] For bars, rods, sections and the relevant semi-finished products, a maximum content of 0.030% S applies.
For any product to be machined, a controlled sulfur content of 0.015% to 0.030% is recommended and permitted.
[2)] Where for special reasons, e.g. hot workability for the fabrication of seamless tubes where it is necessary to minimize the delta ferrite content, or with the aim of low permeability, the maximum Ni content may be increased by the following amounts:
0.50% (m/m): 1.4571;
1.00% (m/m): 1.4306, 1.4406, 1.4541;
1.50% (m/m): 1.4404.
[*)] Patented steel grade (see: EN 10027–2 [13])

Table 4

Chemical composition of austenitic-ferritic stainless steels [14]

Steel grade	% by mass									
	C max.	Si max.	Mn max.	P max	S max.	N	Cr	Cu	Mo	Ni
1.4362[*)]	0.030	1.00	2.00	0.035	0.015	0.05–0.20	22.00–24.00	0.10–0.60	0.10–0.60	3.50–5.50
1.4462	0.030	1.00	2.00	0.035	0.015	0.10–0.20	21.00–23.00		2.50–3.50	4.50–6.50

[*)] Patented steel grade (see: EN 10027-2 [13])

Elements not quoted in Tables 2 to 4 may not be intentionally added to the steel without the agreement of the purchaser except for finishing the cast. All appropriate precautions are to be taken to avoid the addition of such elements from scrap and other materials used in production which would impair mechanical properties and the suitability of the steel [14].

3. Basic physical and mechanical properties of stainless steel

Diversity of chemical composition and structures of stainless steels is the reason of significant quantitative and qualitative differences of physical and mechanical properties in comparison to other metal alloys used for fabrication of building load-bearing structures. In Table 5 some basic parameters of metal alloys used in building structural elements are compared.

Table 5

Comparison of basic metal alloys for building structures

Metal alloy	ρ [kg/m^3]	f_y [MPa]	E [GPa]	α_T [10^{-6}/K]	λ [W/m K]	C [J/kg K]	$A_5/A_{50}/A_{80}$ [2)] [%]	KV [J]
Ferritic stainless steel according to EN 10088 [14–18] [1)]	7.70	210–280	220	10,0–10.5	25.0	430–460	18.0–25.0	-
Austenitic stainless steel according to EN 10088 [14–18] [1)]	7.90–8.10	175–350	195–200	15.8–16.5	12.0–15.0	450–500	30.0–45.0	90.0–100
Austenitic-ferritic stainless steel according to EN 10088 [14–18] [1)]	7.80	400–480	200	13.0	15.0	500	20.0–25.0	90.0–100
Structural carbon steel according to EN 10025-2 [11]	7.85	215–440	210	12.0	53.3	440	16.0–26.0	55.0–63.0
Structural aluminium	2.70	35.0–280	70.0	23.0	142–191	911	1–16	-

1) Applies to steel grades listed in Table 2.1. EN 1993-1-4 [8].
2) Alternatively: A_5 or A_{50} or A_{80}.

where

- ρ – specific gravity,
- f_y – characteristic value of yield strength,
- E – longitudinal modulus of elasticity at 20ºC,
- α_T – average coefficient of thermal expansion at temperature between 20ºC and 100ºC,
- λ – thermal conductivity at 20ºC,
- C – unitary thermal capacity at 20ºC,
- $A_5/A_{50}/A_{80}$ – ultimate elongation,
- KV – impact energy at 20ºC.

It can be seen that range of yield strength for austenitic stainless steels corresponds to the most popular structural carbon steels S235 and S355. Stainless steels with austenitic-ferritic structure (duplex steels) are characterized by the highest value of yield strength (480 MPa).

Differences between mechanical and physical properties of carbon and stainless steels, important for structural safety, were taken into account when preparing the European Standard EN 1993-1-4 [8], which gives supplementary provisions for the design of buildings and civil engineering works that extend and modify the application of EN 1993-1-1 [6], EN 1993-1-3 [7], EN 1993-1-5 [9] and EN 1993-1-8 [10] to austenitic, austenitic-ferritic and ferritic stainless steels.

According to EN 1993-1-4 [8] in structure global analysis and to determine the resistances of members and cross-sections it may be assumed that longitudinal modulus of elasticity (Young's modulus) E is equal to 200 GPa for the austenitic and austenitic-ferritic (duplex) grades specified in Table 2.1 [8] excluding grades 1.4539, 1.4529 and 1.4547, for which Young's modulus is 195 GPa. Ferritic stainless steels are characterized by modulus of elasticity E = 220 GPa. Shear modulus (Kirchhoff's modulus) is determined, depending on Young's modulus value, based on a formula $G = E/[2(1 + v)]$, where v – Poisson's ratio in elastic range equal to 0.3.

Deflections of individual member may be calculated using the secant modulus appropriate to the stress in the member at the serviceability limit state. Its value should be determined according to 4.2(5) of EN 1993-1-4 [8].

4. Durability and corrosion of stainless steels

Criteria of building structures durability are defined by the requirements that structure shall satisfy according to designer and investor's intentions. European Standard EN 1993-1-4 [8] distinguish two basic types of stainless steel application in structural buildings:

– **cosmetic applications** in which the prime consideration in the choice of material is to maintain the appearance during the life of the structure. In this case it is also necessary to distinguish between indoor and outdoor applications because elements located under shelter have to be cleaned more often than outdoor element that have possibility of natural cleaning by weather agents;
– **structural applications** in which the mechanical properties of stainless steel are essential. In this case most natural atmospheres have no detrimental effects on stainless steels.

Stainless steels can corrode. In particularly corrosively aggressive environment, e.g. hydrochloric acid, stainless steel will corrode not very slower than structural carbon steel. Corrosion, as is the case of ordinary steel, is then the main factor determining durability of stainless steel structures. Expected durability of stainless steel structures is determined by selection of materials, the design process and the fabrication procedures and by the environmental conditions.

Selection of appropriate stainless steel grade for design structure should be preceded by analysis of steel corrosion resistance, its mechanical and physical properties and ability of structure maintenance.

The first step of this analysis is to establish general corrosion characteristic of environment including influence of corrosively active elements and substances which may permanently or periodically come into contact with stainless steels. Important parameters of the analysis can be surface condition, steel temperature and the anticipated service stress. Necessary may occur quantity evaluation of the effects of cyclic heating and cooling of the structure.

Influence of final decision of steel grade selection has also ease of fabrication, availability of product forms, surface finish and expected costs. Proper selection of stainless steel grade ensure long-term use of structure without any problems and also increase its economical effectiveness.

Assessing the suitability of grades is best approached by referring to experience of stainless steels in similar applications and environments. For atmospheric environments, Table A.1 of EN 1993-1-4 [8] (shown below as Table 6) gives guidance for selecting suitable grades from a corrosion point of view.

Table 6

Suggested grades of stainless steel for atmospheric applications [8]

Steel grade according to EN 10088 [14–18]	Type of environment and corrosion category											
	Rural			Urban			Industrial			Marine		
	Low	Mid	High	Low	Mid	High	Low	Mid	High	Low	Mid	High
1.4003 1.4016	Y[1]	X	X	Y[1]	X	X	X	X	X	X	X	X
1.4301 1.4311 1.4541 1.4318	Y	Y	Y	Y	Y	(Y)	(Y)	(Y)	X	Y	(Y)	X
1.4362 1.4401 1.4404 1.4406 1.4571	O	O	O	O	Y	Y	Y	Y	(Y)	Y	Y	(Y)
1.4439 1.4462 1.4529 1.4539	O	O	O	O	O	O	O	O	Y	O	O	Y

Corrosion conditions (aggressiveness of environment):
Low: Least corrosive conditions for that type of environment, e.g. low humidity or low temperatures.
Mid: Fairly typical for that type of environment.
High: Corrosion likely to be higher than typical for that type of environment, e.g. increased by persistent high humidity, high ambient temperatures or particularly aggressive air pollutants.

Symbols (assessment of steel usability):
O Potential over-specification from a corrosion point of view.
Y Probably the best choice for corrosion resistance and cost.
Y[1] Indoor applications only. The use of ferritic stainless steels for cosmetic applications should be avoided.
X Likely to suffer excessive corrosion.
(Y) Worth considering provided that suitable precautions are taken (i.e. specify a relatively smooth surface and then carry out regular washing).

It should be noticed that corrosion resistance depends on its alloying components what implies that different grades with the same microstructure may behave differently in the same environment.

Stainless steels show significantly higher durability resulting from higher corrosion resistance than carbon steels. Durability of paint coatings protecting elements from carbon steel from corrosion is defined in EN ISO 12944-1 [24] as expected life of protective paint system to the first major maintenance, in case of "high" durability it should be more than 15 years. In low and medium corrosively aggressive environments, after standard 50 years of use, structures made of properly selected stainless steel grade may need only maintenance for aesthetic reasons.

Stainless steel corrosive resistance is caused by presence of thin (about 5×10^{-6} mm) film formed on steel surface that prevents steel from reacting with an atmosphere. Behaviour of this so called "passive film" depends on steel composition, its surface treatment and the corrosive nature of environment. Unlike carbon steel which for passivation require additional treatments (e.g. oxidizing), passivation of stainless steel surface takes place spontaneously if alloy content of chromium exceeds 12–18%. Corrosion of stainless steels may occur if passive film is broken down and can't self-repair.

Stainless steel corrosion resistance contrary to expectations may be caused by [8]:

– incorrect assessment of the environment or exposure to unexpected conditions (e.g. contamination by chloride ions),
– introduction of a state not envisaged in the initial assessment, by the way in which the stainless steel has been worked or treated (e.g. using the same tools for stainless and carbon steels).

Very important in preventing corrosion is also appropriate structure detailing (e.g. avoiding moisture and dirt entrapment).

European Standard EN 1993-1-4 [8] describes six types of corrosion, but only three of them are likely to occur in buildings:

– **pitting** – localized form of corrosion that can occurs as a result of exposure to specific environments, especially those containing chloride ions. Pitting occurs because chloride ions penetrate the passive film in weak spots. In most structural applications intensity of superficial pitting is low and acceptable because the reduction in the section of the component will be negligible. More restrictive requirements are taken for ducts, piping and containment structures. Products of corrosion may also spoil architectural effects;
– **crevice** – localized form of corrosion, initiated by the differentials in oxygen levels between the creviced and exposed regions. This type of corrosion usually is not a problem, except in solutions where permanent threat of chlorides can occur. The severity of crevice corrosion is very dependent on the geometry of the crevice; the narrower and deeper the crevice, the more severe the corrosion. Crevices typically occur between nuts and washers or around the thread of a screw or the shank of a bolt. They can also occur in welds that fail to penetrate and under deposits on the steel surface;
– **bimetallic** – occurring when dissimilar metals are in electrical contact in any electrolyte (rainwater, condensation etc.). The less noble metal (the anode) corrodes faster than would have occurred if the metals were not in contact. Bimetallic corrosion may be prevented by excluding water from the detail (for example by painting or taping over the assembled joint) or, preferably, by electrically isolating the metals from each other (for example by painting the contact surfaces of the dissimilar metals).

Besides of material loss, effect of corrosion can occur as discolouration and staining (often due to carbon steel contamination) but they can be removed by mechanical and chemical cleaning methods.

Products made of stainless steel are usually subjected to surface finishing operations such as: pickling, skin passing, grinding, brushing, mechanical or electrochemical polishing, colouring etc. These finishing processes, apart from aesthetic effect, also increase uniform corrosion resistance (the smoother surface the higher corrosion resistance). The exception is stainless steel susceptible to stress corrosion cracking. Stresses caused by mechanical polishing of the surface can initiate corrosion cracking in structure elements situated in environment containing chloride ions.

5. Fabrication of stainless steels

Stainless steels can be fabricated in the same conventional way as carbon steels but due to high strength of material and very high hardening rate in stainless steel production process heavier machines which can generate larger forces are required. In first step of stainless steel production scrap and ferroalloys are melting and then refining to adjust the carbon content and remove impurities. Most often stock is melting in an electric-arc furnace and followed by refining by argon oxygen decarburization. Molten stainless steel is casting continuously or into ingots. During the final stages of producing basic mill forms (sheets, strips, plates and bars) the material is subjected to hot reduction with or without subsequent cold rolling operations, annealing, and cleaning. Further steps are required to produce other mill forms, such as wire and tube [3].

Most of stainless steels are subjected to heat treatment. In some of those cases removal of contamination is required, for example: scale formed on stainless steel surface during annealing must be removed e.g. by pickling.

Important characteristic of stainless steel element is surface finishing, especially in the case when, besides of mechanical properties, visual values are also significant. Some types of surface finish form during the production process, the others are result of additional treatments, e.g. polishing, brushing, colouring. Choosing type of surface finish it should be noted that it has influence on element corrosion resistance, e.g. rough surface will decrease effective corrosion resistance in comparison with smooth surface finishing.

It was attempted to define the most common types of surface finish (e.g. EN 10088-2 [15]) but due to fact, that some producers reserve the right to their solutions, full standardization is rather unlikely.

6. Joints in stainless steel structural elements

Stainless steel elements can be join by bolt-and-nuts or by welding. Both bolt-and-nuts and welding materials, besides of appropriate ultimate strength should enable to make joint with corrosion resistance not less than corrosion resistance of connected material. Glue joints are also in use but gluing process is not covered by EN 1993-1-4 [8].

Stainless steel bolts and nuts should conform with EN ISO 3506-1,2,3 [19–21], but grade A1, because of significant sulphur content and connected with it reduction of corrosion resistance, should not be used for bolts. Washers should be of stainless steel and should conform with EN ISO 7089 [22] or EN ISO 7090 [23]. Additionally, according to [8] high strength bolts made of stainless steel should not be used as preloaded bolts designed for a specific slip resistance, unless their acceptability in a particular application can be demonstrated from test results.

In case of welding materials, additionally to EN 1993-1-8 [10], welding electrodes should be capable of producing a weld with a corrosion resistance that is adequate for the service environment, provided that the correct welding procedure is used. The welding electrodes may be assumed to be adequate if the corrosion resistance of the deposited metal and weld metal is not less than that of the material to be welded [8].

The weldability of stainless steel grades depends on their chemical composition and may be different for various grades with the same microstructure [32]:

– ferritic stainless steels can be welded, but they show lower ductility in weld region. All ferrictic stainless steels have tendency to increase in size of grains in heat-affected zone and therefore should be welded with possibly small heat input;
– austenitic stainless steel grades are readily welded but due to high thermal coefficient of expansion they are susceptible to formation of welding residual stresses and strains. Moreover, low thermal conductivity causes higher heat concentration in welding zone but the heat can be easily carrying away by using of cooling copper backing bar;
– austenitic-ferritic (duplex) steels are also well weldable, although not so well as austenitic. It is recommended to use filler with increased content of nickel.

In Table 7 weldability of selected stainless steel grades was compared.

Table 7

Weldability of selected structural stainless steel grades

Stainless steel grade	Microstructure	Weldability
1.4016	Ferritic	Hard to weld, welding not recommended
1.4301	Austenitic	Weldable
1.4306		
1.4307		
1.4401		
1.4404		
1.4435		
1.4439		
1.4529		
1.4539		
1.4541		
1.4571		
1.4362	Austenitic-ferritic (duplex)	Weldable
1.4462		

To weld stainless steel practically all methods can be used, European Standard EN 1993-1-4 [8] recommends professional advice on the selection of welding procedure for joining stainless steels. Proper selection of welding method allows not only to produce a weld with appropriate mechanical properties but also with required visual appearance and corrosion resistance.

Errors occurring during welding are often the results of improper element preparation before welding, e.g. leaving excessive gaps leads to considerable welding strains, contamination of welded element surfaces results degradation of weld mechanical properties and visual appearance. Stainless steels are characterized by large welding shrinkage that why weld area should be minimized.

Post-weld treatment of welded joint includes mechanical and/or chemical processing [5]. The need for surface finishing applies primarily to arc welds, all surface contaminants and irregularities should be removed in order to not act as sites of corrosive attack. In some cases, e.g. where aesthetic or industrial process purity issues are important, it may be necessary to remove surplus of the weld and to polish the weld zone to look like parent material.

Mechanical finishing treatment may be one of the following: hammering, brushing, grinding, polishing and buffing. Irrespective if weld joint was mechanically finished or not, it should always be remembered of weld chemical treatment including acid pickling followed by passivation and washing both after pickling and passivation. Pickling and passivation protecting weld and its surroundings against corrosion due to surface contaminants.

7. Applications of stainless steels

In Europe stainless steel is used mainly in: food and drink industry, household appliances manufacturing, architecture and civil engineering, chemical and pharmaceutical industries, medical equipment manufacturing, pulp and paper manufacturing, water and sewage treatment, transport, energy production and environmental protection.

Stainless steel in civil engineering and architecture is mostly used for balustrades, building wall and roof claddings, elements of building façade, doors and windows, floors, stairs, escalators and lifts and also fastening systems. Wilde field of stainless steel application constitute: installations in e.g. food, chemical and petrochemical industries, chimneys, pipelines, tanks, water seals or steel parts of harbour piers. In Table 8 some examples of typical application of structural stainless steel selected grades of were given.

Among structural applications of stainless steel (especially duplex grades) bridges load bearing elements and reinforcement bars should be mentioned.

One of the first and also the most famous architectural applications of stainless steel is Chrysler Building in New York, which almost whole spire was made of 700 tones of stainless steel produced by American steel mills on Krupp’s license.

Tendency to increase structural applications of stainless steel in civil engineering can be observed recently. This material is chosen for its visual appearance, high corrosion resistance, good mechanical properties and high durability.

Examples of structures build in recent few years, where main elements or entire structure were made of stainless steel [4, 25, 26, 29, 30], are shown in Table 9. In this specification

Table 8

Examples of typical applications of selected stainless steel grades

Grade	Application example
1.4003	Tanks, chimneys, conveyors, footbridges, stairs, balustrades, light lattice girders.
1.4016	Tanks in chemical and food industries, pipelines, fasteners, chimney liners
1.4301	Tanks for acids, liquid oxygen, nitrogen and hydrogen, storage tanks in brewing industry, pipelines, springs, nuts, bolts and screws.
1.4306	In highly oxidizing environments such as nitric acid, e.g. storage tanks for tomato purée.
1.4307	Tanks for acids, pipelines, springs, nuts, bolts and screws.
1.4401	Tanks and pipelines in chemical, pulp and paper, pharmaceutical industries, balustrades, architectural applications.
1.4404	Tanks, pipelines, chimneys in chemical, petrochemical, brewing industries, balustrades, architectural applications.
1.4529	Elements having contact with sea water.
1.4539	Tanks and pipelines in chemical, paper and pulp industries, parts exposed to condensates of combustion gases, buildings fasteners in aggressive environments.
1.4541	Pressure vessels, pipelines in chemical and food industries.
1.4571	Elements that need high corrosion resistance, pipelines, tanks, balustrades, architectural applications.
1.4462	Tanks and pipelines in chemical, pulp and paper industries, elements in environments with high content of chlorides and in marine environment.

grade 1.4162 was also taken into account. It is austenitic-ferritic (duplex) grade with lower (1,5%) than typical stainless steels nickel content (see Table 2–4). This grade is not specified in Table 2.1 EN 1993-1-4 [8] but due to low nickel content and its high price, may be economical alternative to other austenitic stainless steel grades.

Table 9

Examples of structural uses of stainless steel outside Poland [4, 25, 26, 29, 30]

Year	Name	Location	Dimensions [m]	Stainless steel	
				Components	Grades
2001	Millenium Bridge (Footbridge)	York, UK	Length: 150 Span: 80.0	Arch	1.4462
2002	Apate Bridge (Footbridge) – Fig. 3	Stockholm, Sweden	Length: 5.0	Main girder	1.4462
2003	Gas-fired combined heat and power station	Isle of Grain in Kent, UK	Diameter: 0.91	Pipes	1.4306
2003	Padre Arrupe Bridge (Footbridge) – Fig. 3	Bilbao, Spain	Length: 142 Span: 80.0	Box girder with carbon steel internal structure	1.4362
2004	Likholefossen Bridge (Footbridge)	Likholefossen, Norway	Length: 24.0	All except concrete columns	1.4162

Table 9 con.

Year	Name	Location	Dimensions [m]	Stainless steel	
				Components	Grades
2004	17 palm-oil storage tanks	Rotterdam, the Netherlands	Diameter/ Height: 17.0/7.00–9.00	Main structure	LDX2101[*)] (1.4162)
2005	Cala Galdana (Road Bridge)	Menorca	Length: 55.0 Span: 45.0	Main structure	1.4462
2005	Arco di Malizia (Road Viaduct) – Fig. 1	Siena, Italy	Length: 51.5	Arch	1.4362
2005	Storage tank for pure acetic acid – Fig. 4	Tarragona, Spain	Height: 25.0 Diameter: 22.0	Main structure	1.4362
2005	Storage tank for marble slurry	Elnesvågen, Norway	Height: 22.8 Diameter: 15.25	Main structure	LDX2101[*)] (1.4162)
2006	Piove di Sacco Bridge (Road bridge) – Fig. 1	Padua, Italy	Length: 120	Arches, deck and casing	1.4362
2006	Celtic Gateway Bridge (Footbridge)	Holyhead, Wales, UK	Length: 160 Span: 70.0	Load bearing arche	1.4362
2006	Storage tank for honey and edible oils	Barcelona, Spain	Height: 25.0 Diameter: 19.0	Main structure	LDX2101[*)] (1.4162)
2007	40 silos for intermediate storage of atomized clay	Castellón, Spain	–	Main structure	LDX2101[*)] (1.4162)
2007	Al Hidd desalination plant	Bahrain	–	Plates and tube	1.4362 1.4462
2008	Storage tank for biodiesel and edible oils – Fig. 4	Amsterdam, the Netherlands	Height: 20.0 Diameter: 11.0	Main structure	1.4362 LDX2101[*)] (1.4162)
2009	The Helix Bridge (Footbridge)	Marina Bay, Singapore	Length: 280 Span:65.0	Main structure	1.4462
2009	Towers of Stonecutters Bridge (Road Bridge) - Fig. 2	Hong Kong, China	Length: 1596 Span: 1018 Towers height: 300	Outer skin of the towers (the upper 118 m of the towers are composite sections with an outer stainless steel skin and a concrete core rein-forced with stainless steel rebars)	1.4462
2009	KARRATHA Gas Treatment Plant	Australia	Diameter: 0.30	Pipes	1.4301

Table 9 con.

Year	Name	Location	Dimensions [m]	Stainless steel	
				Components	Grades
2009	41 palm-oil storage tanks	Rotterdam, the Netherlands	Diameter/ Height: 17.0/16.5; 17.0/13.0; 10.0/4.00; 17.0/9.50; 17.0/7.00	Main structure	LDX2101*) (1.4162)
2012	Jetty Boil-Off Gas Project (JBOG)	Quatar	Diameter: 0.76–1.57	Pipes	1.4306
2012	The Porsche Pavillon – Fig. 5	Wolfsburg, Germany	Overhanging part: 25.0 x 30.0	Monocoque: structural elements cover panels	1.4301 1.4571
2013	Wheatstone LNG Project (liquefied natural gas and domestic gas development)	Ashburton North, Australia	Diameter: 0.60–1.83	Pipes	1.4306
*) Registered trademark of Outokumpu					

Fig. 1. Structural use of stainless steel in road viaducts and bridges: Arco di Malizia, Siena, Italy (left) and Piove di Sacco Bridge, Pardua, Italy (right) [1]

In structures listed in Table 9 wide range of product forms have been used, including: plates, large diameter tubes, circular, square and rectangular hollow sections, fabricated straight and tapered box sections made from plates. Steel consumption in selected structures was as follows: 110 tones in Piove di Sacco Bridge, 160 tones in Cala Galdana Bridge, in Celtic Gateway Bridge: 220 tones, The Helix Bridge: 400 tones structural pipes and 200 tones other structural parts, Stonecutters Brigde: 1800 tones plates and 200 tones pipes [4]. Very original Porsche Pavillon is the largest construction of this kind, stainless steel consumption in this internally stiffened, doubly curved shell structure was 425 tones [31]. To fabricate 17 storage tanks for palm oil (Rotterdam) 760 tones of stainless steel was needed [29]. 3000 tones of stainless steel rebars was used in main pylon of Stonecutters Bridge (Hong Kong).

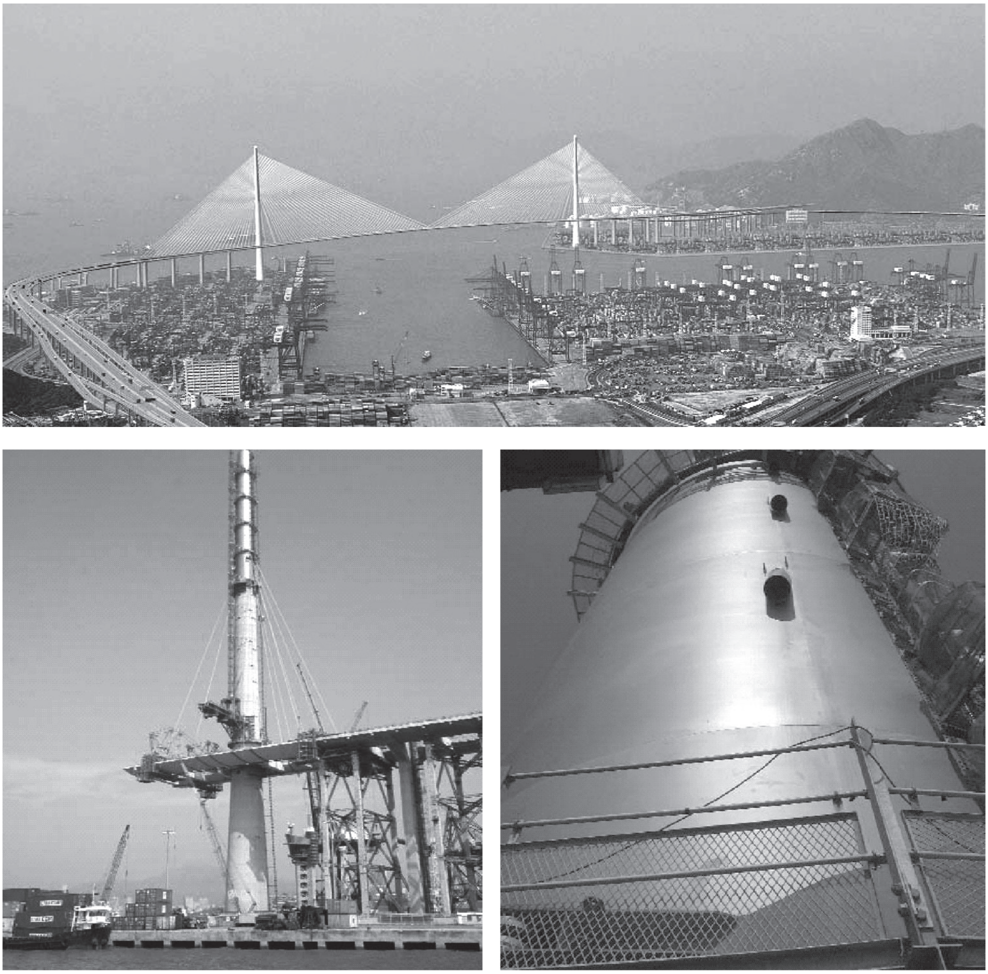

Fig. 2. Structural use of stainless steel in road bridges: towers of Stonecutters Bridge, Hong Kong, China [1]

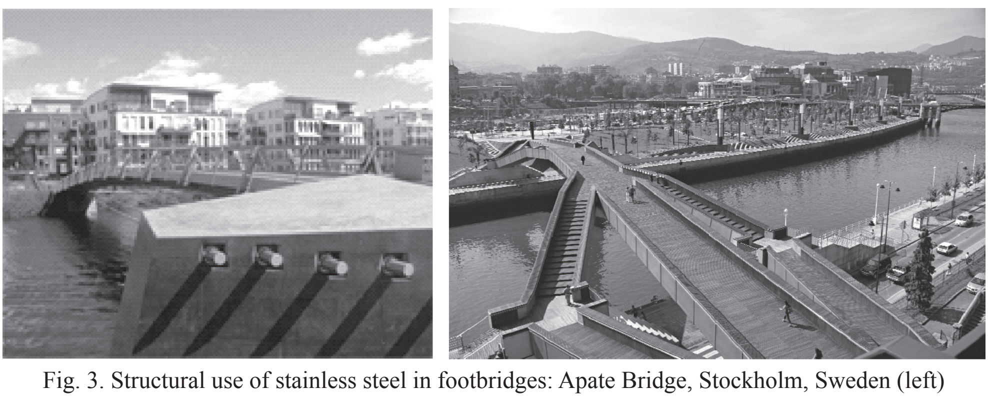

Fig. 3. Structural use of stainless steel in footbridges: Apate Bridge, Stockholm, Sweden (left) (source: www.paintsquare.com) and Padre Arrupe Bridge, Bilbao, Spain (right) [1]

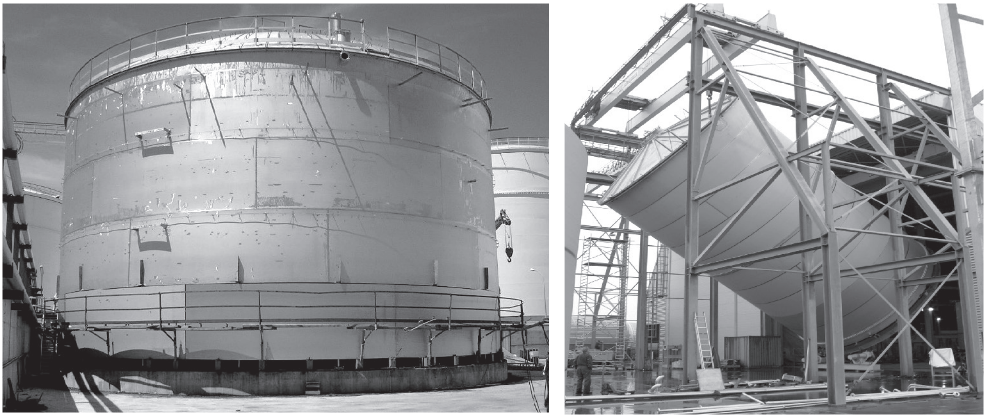

Fig. 4. Structural use of stainless steel in storage tanks: for pure acetic acid, Tarragona, Spain (left) and for biodiesel and edible oils (under construction), Amsterdam, the Netherlands (right) [30]

Fig. 5. The Porsche Pavillon, Wolfsburg, Germany [31]

Fig. 6. Stainless steel façade of ArcelorMittal – Stainless Service Poland Sp. z o. o. building in Siemianowice Śląskie, Poland [2]

Fig. 7. Example of structural use of stainless steel in Poland – tanks (305 m^3) for concentrated fruit juice, Spomasz Zamość S.A., Poland, A. Biczak – author of photography (left) and tanks (490 m^3) in sewage-treatment plant in Targowisko, municipality Kłaj, Poland, P. Żwirek – author of photography (right)

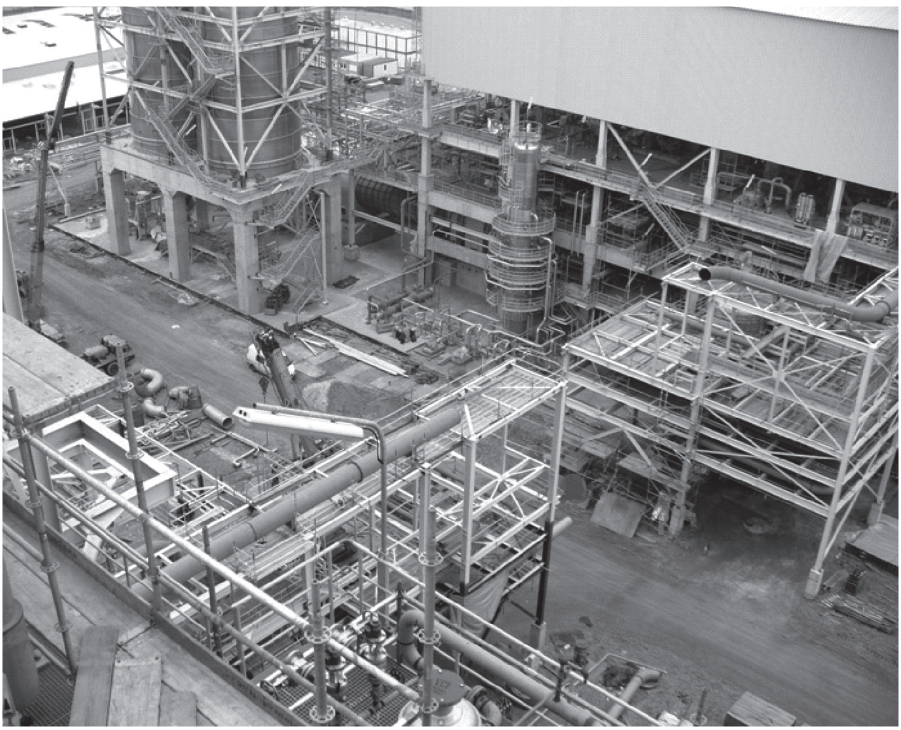

Fig. 8. Example of structural use of stainless steel in Poland – pipelines and tanks for terephthalic acid production line (under construction), ANWIL S.A., Włocławek, Poland (source: www.euroweld.pl)

In Poland stainless steel is still used mainly in architectural applications (Fig. 6). Structural use of stainless steel is still not very common. Stainless steel is utilized mostly in cases where exchange of the structural element will be very expensive due to production process continuity breaking (e.g. pipelines, tanks, chimneys). Selected examples of structural use of stainless steel in Poland are shown in Fig. 7–8.

For many investors low costs of structure erection is still main criterion of building material selection. High price of stainless steel causes that even in case of small structural elements like balustrades, when their number in structure is large, it is worth for investor commissioning design calculations with material and topological optimization of elements.

Fig. 9. Apartment complex "Wiślane Tarasy", Cracow, Poland (contractor: INTER-BUD, structural stainless steel elements design: K. Kuchta, P. Marzec, I. Tylek)

Typical examples of those types of stainless steel applications is apartment complex "Wiślane Tarasy" in Cracow (Fig. 9). In this apartment complex all external steel elements: balustrades, entrance roofs, benches, trash baskets, etc. are made of stainless steel. Original example of stainless steel realization, located in the same apartment complex, where aesthetic aspects were very important was 2800 mm high fountain with internal low-pressure vessel.

It should be expected that in Poland, just like in other countries, more and more often stainless steel will be selected for structures mainly because of its mechanical properties not only visual appearance.

8. Some statistics about stainless steels

Stainless steel as universal material uniting high corrosion resistance, good mechanical properties, high durability and ease of manufacture becomes more popular in building structures. Fig. 10 illustrates how stainless steel consumption changes in Poland for the last 10 years. Maximum and minimum increase of stainless steel consumption in relation to previous year was recorded in 2004 (+40%) and 2012 (+4%), respectively. In 2009, due to global financial crisis, stainless steel consumption in Poland decreased about 7% but as can be seen in Fig. 10 trend line for stainless steel consumption in Poland in the last ten years is ascending.

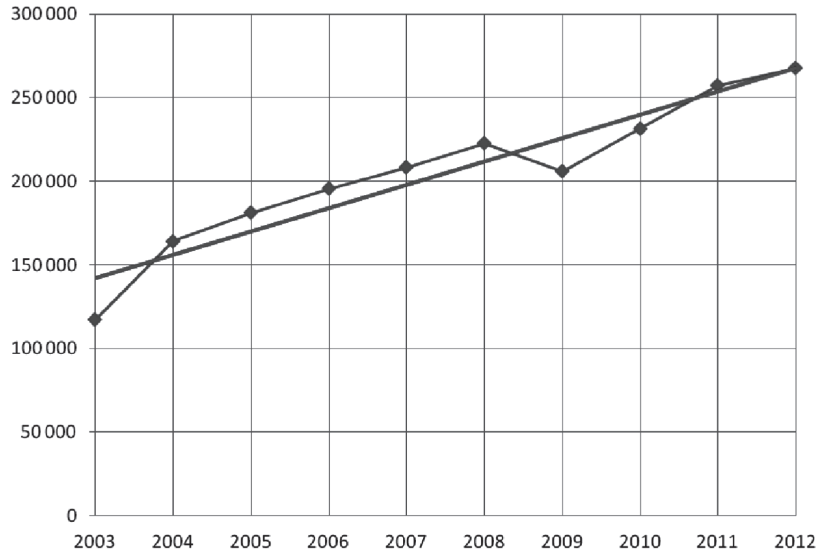

Fig. 10. Stainless steel consumption [t] in Poland in 2003–2012 [28]

The largest stainless steel distributor in Poland, Nova Trading S.A., achieved the mean annual growth of stainless steel sales for last 5 years at the level of 8%. In respect to stainless steel structural grades specified in Table 2.1 EN 1993–1–4 [8], the most popular among buyers was stainless steels: austenitic grade 1.4301 and ferrictic grade 1.4016.

In Poland the dominant was sale on stainless steel sheets and plates (hot– and cold rolled). Statistical data [28] indicates that 74% of 267 000 tones of sold stainless steel were sheets and plates, while consumption of tubes was equal about 17%, bars and profiles – about 2% and 4%, respectively.

One of the most important obstacle in common use of stainless steel structural elements is high price of stainless steel, currently about 4 times higher than price of carbon steel (compare Fig. 11 and Fig. 12).

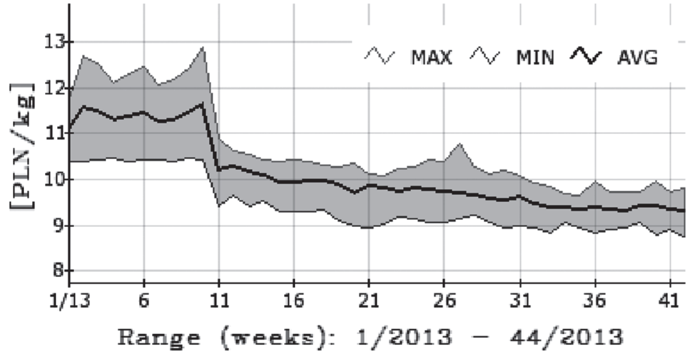

Fig. 11. Prices of 2 mm thick cold-rolled plate made of stainless steel grade 1.4301 with surface finish 2B [PLN/kg] according to The Stainless Steel Association – Stowarzyszenie Stal Nierdzewna (SSN) (source: www.stalenierdzewne.pl)

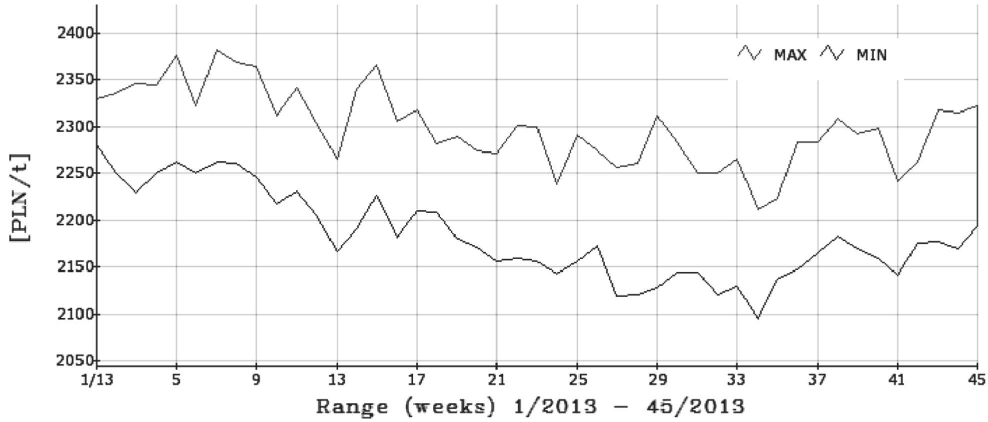

Fig. 12. Prices of 4 mm thick hot-rolled plate made of carbon steel grade S235JR2 (St3S) [PLN/t] according to The Polish Association of Steel Stockholders – Polska Unia Dystrybutorow Stali (PUDS) [www.puds.pl]

Stainless steel mill price is comprised of two parts:

– **the base production cost** that is set by the steel producer,
– **the Alloy Adjustment Factor** (AAF) that relates to the current price of alloy components.

Prices of stainless steel grade 1.4301 with division into base price and alloy surcharge on the German market in 2001–2010 are shown in Fig. 13.

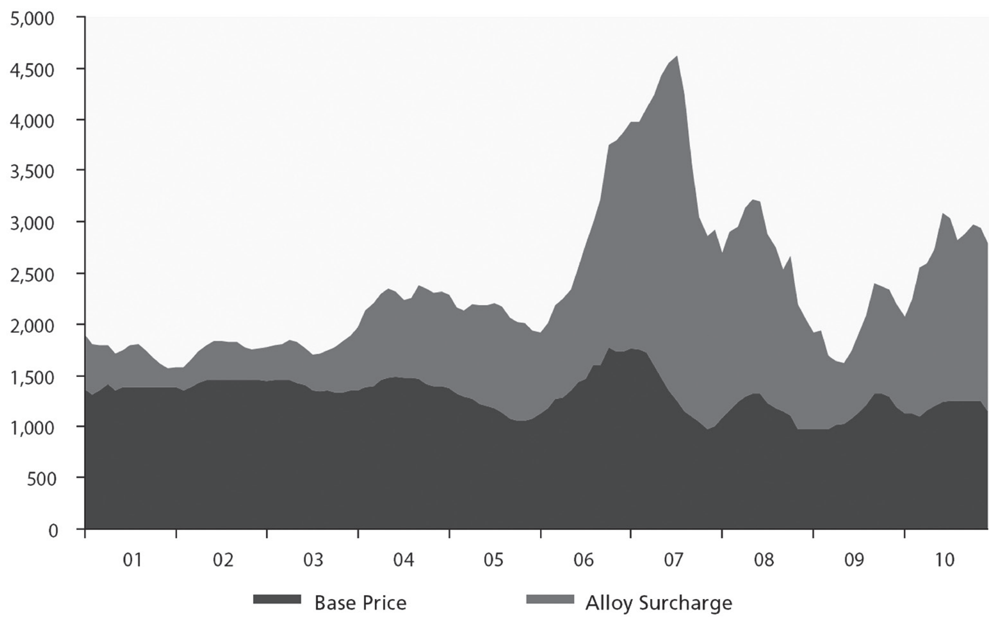

Fig. 13. Prices [x10³ €/t] of stainless steel grade 1.4301 divided into base price and alloy surcharge on the German market in 2001–2010 [27]

The price of the stainless steel product is mostly determined by the price of alloying components (particularly nickel) that is not directly controlled by the producer or distributor. The costs of molybdenum purchase also influence AAF although not as much as the nickel price. Variation of nickel price for last 12 months and 5 years are shown in Fig. 14. It should be noted that during last 5 years change of nickel price reached more than 250%.

Fig. 14. Variation of nickel price [$/LB] on London Metal Exchange (LME) (source: www.metalprices.com)

Nickel price adversely affect development of stainless steel applications in structural buildings, firstly by causing high costs of stainless steel production, secondly by causing lack of stability of stainless steel price due to frequent changes of nickel price causes, what may lead to difficulties of execution an investment project that in building industry is often planned for several years.

The most influenced by nickel price are austenitic and austenitic-ferritic (duplex) stainless steels because they have the highest content of this alloying component from all structural stainless steels listed in Table 2.1 EN 1993-1-4 [8]. However duplex steels have even 1.5 times higher yield strength than other stainless steels what can results in lower material consumption, so global costs of investment will be less sensitive to nickel price fluctuations. Currently European Standard EN 1993-1-4 [8] specifies in Table 2.1 mechanical properties for only two grades of duplex structural stainless steels: 1.4362 and 1.4462.

9. Conclusions

Stainless steel due to its good mechanical properties and usable values becomes more and more often used for building industry, including load bearing elements and pipelines.
Without any doubt one of the factors that slows down development of stainless steel in structural buildings is high and unstable price of the material. Stainless steels high price may be compensated by reduction of material consumption by using steels with yield strength higher than for carbon steels, e.g. duplex stainless steels.

Increase of structural use of stainless steel is connected with growing consciousness about advantages of stainless steel use, e.g. no necessity of anti-corrosion coatings application and renewing, high durability of material, long life cycle of a structure which decrease real costs of investment. Apart from that, producers introduce new grades of stainless steel with high yield strength and low content of expensive nickel what creates conditions conducive to designing structural elements from stainless steels.

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THE RISK OF FIRE OCCURRING IN BUILDING COMPARTMENT – WHAT ARE THE CONSEQUENCES IF IT IS ASSUMED TO BE TIME-INDEPENDENT

RYZYKO ZAISTNIENIA POŻARU W STREFIE POŻAROWEJ – JAKIE SĄ KONSEKWENCJE, JEŚLI PRZYJĄĆ, ŻE JEGO WARTOŚĆ NIE ZALEŻY OD CZASU

Abstract

The acceptance in advanced fire safety analysis of the formal model according to which the probability of fire occurrence is assumed as time-independent leads to the conclusion that random fire episodes in considered building compartment can be described by the formalism of a Poisson process. In the presented paper some consequences of such adoption are presented and widely discussed as well as the foundations are determined of the equivalence between this probability and a conditional failure rate, which is interpreted as a process intensity parameter.

Keywords: fire occurrence, probability, Poisson process, risk, hazard rate, process intensity.

Streszczenie

Akceptacja w zaawansowanej analizie bezpieczeństwa pożarowego modelu formalnego, zgodnie z którym prawdopodobieństwo zaistnienia pożaru jest przyjmowane jako niezależne od czasu, prowadzi do wniosku, że losowe epizody pożaru w rozpatrywanej strefie pożarowej mogą być opisywane przy użyciu formalizmu procesu Poissona. W prezentowanym artykule przedstawiono i przedyskutowano niektóre konsekwencje wynikające z takiego przyjęcia, a także podano podstawy uznania równoważności pomiędzy rozpatrywanym prawdopodobieństwem a warunkową częstością zawodów, która jest interpretowana jako parametr intensywności procesu.

Słowa kluczowe: zaistnienie pożaru, prawdopodobieństwo, proces Poissona, ryzyko, stopień zagrożenia, intensywność procesu.

The Author is responsible for the language.

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Symbols

$f_{fi}(t)$ – compartment random durability probability density function (*pdf*),
$F_{fi}(t)$ – compartment random durability cumulative distribution function (*cdf*),
$h(t) = h$ – hazard function (conditional failure rate) that fire will occur in a compartment at time $t + dt$ (specified per one year of building service and one square meter of the fire compartment area)
$m_{fi} = MTTF$ – mean time to fire in a compartment,
p_f – probability of structural failure due to fire, provided that fire ignition has occurred,
p_{ff} – probability of fire induced structural failure, related to the fire which can take place; but has not yet occurred,
$p_{f,ult}$ – ultimate acceptable value of the structural failure probability,
p_t – probability of fire occurring in a compartment,
$R(t)$ – reliability function specified for a compartment in the context of potential fire occurrence,
t – time the assessment of the hazard rate is made for,
t_0 – time related to the beginning of the building service,
t_{fi} – time related to the first fire ignition in a compartment,
T_{fi} – random durability of a compartment (time to the first fire),
$\lambda(t) = \lambda$ – the *Poisson* process intensity of fire occurring in a compartment.

1. Introduction

To protect the considered load-bearing structure against potential fire exposure in a more rational and better justified way the risk of fire occurrence should be adequately estimated in advance, in relation to the compartment inside of which such structure is located. The reliable knowledge of its value allows to differentiate the suitable safety requirements by assuming the acceptable risk level adjusted to real threat conditions. In consequence, in many cases less restrictive fire protection measures can be legally applied, being significantly cheaper and easier to use than those designed according to the traditional approach.

In fact, the failure probability p_f is usually adopted as a conclusive and objective valuation measure when the classical safety analysis is performed, for the case of unexpected event potentiality, such as threat to people or a building structure. Its application explicitly determines the understanding of limit state requirements. Such limit state is not reached exactly at the point-in-time when the considered event really takes place, but earlier, when the probability of its occurrence becomes too high and may no longer be tolerated. Conclusively, the safety condition is in general formulated as follows:

$$p_f \le p_{f,ult} \tag{1}$$

where the maximum acceptable values of failure probability, i.e. $p_{f,ult}$, are assigned arbitrarily, to be adequate for the assumed reliability class [1].

However, if the fire safety is to be examined in detail, one should precisely define what kind of such failure probability is considered, as at least two interpretations are possible, differing not only from quantitative but also from qualitative point of view. These are as follows [2, 3]:

- probability of fire caused failure, if it is known that fire ignition occurred and; moreover, this fire reached the flashover point (and thus may be described as a fully developed fire) – in further analysis such probability is generally denoted by p_f,
- probability of fire caused failure, due to fire which did not occur so far (so the designer has no information on its ignition and flashover) – let us assign a symbol p_{ff} to designate it.

According to *Lie* [4] p_f and p_{ff} are related by:

$$p_{ff} = p_t p_f \tag{2}$$

where p_t denotes the probability of fire occurrence (not only of fire ignition but also reaching the flashover point). As one can see, probability p_f is interpreted here as a conditional probability with a condition that fire has already occurred and the exhaust gas temperature in the whole compartment is uniform (the fire is fully developed). Not only a qualitative but also a quantitative distinction between probabilities p_f and p_{ff} seems to be very significant. Even if conditional probability p_f is high, probability p_{ff} is usually quite low, because in reality the value of probability p_t is also low.

Let us underline, that the value of the probability p_{ff}, related to the potential fire which did not occur so far, seems to be essential both for building occupants who will be able to occupy the considered compartment if fire ignition and flashover takes place and also for firemen taking part in a future firefighting action. To responsibly assess such value according to formula (2), the appropriate value of a probability of fire occurrence p_t has to be accepted beforehand by the safety expert for considered compartment, depending on the way such compartment is used as well as on the fire loads accumulated in its volume and on the possible ventilation conditions. The probability p_t is usually assumed to be constant during the whole lifetime of the building. The main advantage of such design approach lies in its simplicity; however, it leads to both quantitative and qualitative consequences important for the safety level really ensured for inhabitants. The detailed analysis of those repercussions, resulting directly from the adopted analytical model, is the main objective of the presented paper.

2. The risk, the hazard function or the failure probability – which is the most accurate interpretation of p_t

One may find many quantitative evaluations of the probability p_t in professional literature. Some of those are listed in Table 1. Let us notice that in [5] the author proposed to rename this quantity and interpret it as the hazard function *h*. The basic reason was that if a function has a dimension, then it cannot be treated as a typical probability, in spite of the fact that it is defined in this way by many authors.

It is noteworthy that all proposed values of the considered hazard function are assumed to be constant-in-time. This also means that they do not depend on the point-in-time when they are assessed. Thus, in consequence, the value of a probability p_t is quantitatively the same both at the beginning of the building life and after many years of its service.

Table 1

Recommended values of fire ignition hazard function *h* $\left[\left(m^2 \cdot year\right)^{-1}\right] \times 10^{-6}$

Building type	*Kersken – Bradley* [6]	BSI DD240 [7]	DIN 18230 [8]	JCSS [9]	*Schleich, Cajot* et al. [10]
Apartments	0.1÷0.5	2.0	0.2	0.5÷0.4	30.0
Schools	0.1÷1.0	–	0.5	0.5÷0.4	–
Hotels	0.1÷1.0	–	1.0	–	–
Shops	0.5÷5.0	–	1.0	1.0	–
Offices	0.1÷1.0	1.0	0.5	1.0	10.0
Industrial buildings	1.0÷5.0	2.0	2.0	2.0÷10.0	10.0

In such a formal model the quantity p_t should not be interpreted as the classical risk value because in a common understanding the risk is the probability of something happening multiplied by the resulting cost or benefit of its occurrence.

In the approach proposed by the author the quantity $p_t = h$ is interpreted as a conditional probability that potential fire will occur between the t and $t + dt$ points-in-time if it is known (including the condition) that it did not occur in time-period $[t_0,t)$ – t_0 denotes commissioning of the building. To make such definition more illustrative let us study in detail the following explanation:

- Let the event A mean that considered fire occurred between t and $t + dt$; whereas, the event B, that this fire did not occur in $[t_0,t)$.
- Next, let us calculate the value of a conditional probability $P\left(A|B\right) = \dfrac{P\left(A \cap B\right)}{P\left(B\right)}$. In this formula $P\left(A \cap B\right) = P\left(A\right)$ because if the fire occurs between t and $t + dt$ it could not occur prior to time t.
- Let the point-in-time t_0 denote the commissioning of the building, while the point-in-time t_{fi} the moment of the first fire ignition in considered compartment. Then a difference $T_{fi} = t_{fi} - t_0$ is the measure of a compartment random durability in context of building fire safety analysis. It is assumed that the considered compartment is reliable at a random point-in-time t if no fire occurred prior to this point-in-time.
- Concluding, the reliability function $R(t)$ may be formulated as follows:

$$R\left(t\right) = P\left(T_{fi} \geq t\right) = 1 - P\left(T_{fi} < t\right) = 1 - F_{fi}\left(t\right) \tag{3}$$

  where $F_{fi}(t)$ is a compartment durability *cdf* (cumulative distribution) function. The corresponding *pdf* (probability density) function $f_{fi}(t)$ is then equal to $f_{fi}\left(t\right) = dF_{fi}\left(t\right)/dt$. Consequently, the probability that fire occurs in time period $[t,t + dt)$ is equal to $P\left(A\right) = f_{fi}(t)dt$, while it is also true that $P\left(B\right) = 1 - F_{fi}\left(t\right)$.
- Finally, the value of the examined hazard function can be estimated by the formula:

$$h(t) = P(A|B) = \frac{P(A)}{P(B)} = \frac{f_{fi}(t)dt}{1 - F_{fi}(t)} = \lim_{dt \to 0} \frac{P(t \le T_{fi} < t + dt | T_{fi} \ge t)}{dt} \tag{4}$$

As one can see, transition of considered compartment to the failed state (failure is understood here as the occurrence of fire) is characterized by the conditional failure rate. This can be interpreted as a measure of the rate at which failures occur, taking into account the size of the population with the potential to fail, i.e. those compartments of the examined type, which did function without fire until time *t*.

3. The hazard rate *h*(t) constant-in-time – basic consequence

As it is shown in Table 1, the hazard rate *h*(*t*), constituting conditional probability measure of fire occurrence, is usually assumed to be time independent. This means that its value remains constant during the whole building lifetime. In this chapter the basic consequence of such assumption is demonstrated, dealing with the predicted reliability of considered fire compartment. In order to study the resultant trend characterizing the adequate reliability function *R*(*t*) – see Eq. (3) – the formula (4) has to be integrated. In fact:

$$\int_0^t h(t)dt = \int_0^t \frac{f_{fi}(t)}{1 - F_{fi}(t)} dt = -ln\left[1 - F_{fi}(t)\right] \tag{5}$$

yielding:

$$F(t) = 1 - exp\left(-\int_0^t h(t)dt\right) \tag{6}$$

Assumption that *h*(*t*) = const (*t*) leads to the simple equation:

$$F(t) = 1 - e^{-ht} \tag{7}$$

and also, regarding the reliability:

$$R(t) = e^{-ht} \tag{8}$$

It is a common knowledge that such relation is associated with the formalism of *Poisson* process, according to which hazard rate $h(t) = h$ is equivalent to process intensity $\lambda(t) = \lambda$ [4]. This is not a surprise because the fire episodes, treated by designers as accidental events, should occur very rarely. Moreover, despite the fact that fire duration is always marked as a sectioning line along the time axis, it is very short in relation to the human lifetime or to the whole building service time. Probability p_t is then understood as the probability that fire ignition occurred at least once prior to the considered point-in-time *t* (most frequently the time period $[t_0,t)$ is the whole building lifetime, but if only one year of service life is to be considered the point-in-time t_0 denotes beginning of such year). Consequently, the probability that fire will occur *x* times prior to time *t* is given as follows:

$$p_x(x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}, \quad \mathrm{x} = 1, 2, \ldots, \infty \tag{9}$$

where parameter λ is called process intensity. Application of this formula leads to the following formulae for probabilities p_x and p_t:

- probability that fire will not occur at all prior to time *t*:

$$p_x(x = 0) = e^{-\lambda t} \tag{10}$$

  One can easily see that such equation is equivalent to formula (8). In fact, the examined compartment remains to be reliable only if no fires occur prior to time *t*.

- probability that fire will occur exactly once prior to time *t*:

$$p_x(x = 1) = \lambda t e^{-\lambda t} \tag{11}$$

- probability that fire will occur at least once prior to time *t* (i.e. once or more than once):

$$p_x(x \geq 1) = 1 - p_x(x = 0) = 1 - e^{-\lambda t} = p_t \tag{12}$$

4. Conditional failure rate as a process intensity parameter

The conditional failure rate *h*, generally adopted as the measure of the fire occurrence risk, is most frequently specified per one year of building lifetime and per one square meter of the considered fire compartment area.

The method to evaluate the intensity parameter λ, for buildings having only one type of fire compartments, was given by *Lie* [4]:

$$\lambda = hA \tag{13}$$

where $h[m^{-2}]$ denotes the fire occurrence risk (calculated per $1m^2$ of fire compartment); whereas, $A[m^2]$ is the area of the considered fire compartment. If several fire compartment types, with various sizes assigned to each of them, are located in the examined building, then the generalized formula proposed by *Burros* [11] may be applied in safety analysis:

$$\lambda = h\overline{A} = h\frac{A_F}{N} \tag{14}$$

according to which A_F is the total area of all compartments in the whole building; whereas N is the number of such compartments.

Furthermore, because in relation to one year of the building service (and even to the whole service life) $x << 1$ holds, the following simplification of Eq. (12) is acceptable and commonly used:

$$p_x(x \geq 1) = 1 - e^{-\lambda t} = 1 - e^{-h\overline{A}t} \approx h\overline{A}t = p_t \tag{15}$$

If the formalism characterizing the *Poisson* process is accepted as describing the probability of consecutive fire episodes, it may be used to evaluate the sought failure probability p_{ff} (see Eq. (2)), thus making possible to the rearrangement of Eq. (3) to the following form, identical with Eq. (8):

$$R(t) = p_x(x=0) = P(T_{fi} \geq t) = 1 - P(T_{fi} < t) = 1 - F_{fi}(t) = e^{-\lambda t} \tag{16}$$

Consequently:

$$F_{fi}(t) = 1 - e^{-\lambda t} \tag{17}$$

and also:

$$f_{fi}(t) = \frac{dF_{fi}(t)}{dt} = \lambda e^{-\lambda t} \tag{18}$$

This means that the random time to the first fire occurrence exhibits exponential probability distribution. As a result of such substitutions the hazard rate $h(t) = h$ may be calculated as follows:

$$h(t)dt = \frac{f_{fi}(t)dt}{1 - F_{fi}(t)} = \frac{(\lambda e^{-\lambda t})dt}{e^{-\lambda t}} = \lambda dt \tag{19}$$

This proves that it is really an equivalent of the process intensity parameter.

5. The mean time to fire occurrence

In previous chapter of this paper it was shown that the random time to the first fire occurrence is characterized by exponential probability density function (see Eq. 18). Hence the value of the mean time to fire occurrence m_{fi} (the so called mean time to failure – *MTTF* – in this example interpreted as the mean time to fire ignition) may be calculated as follows:

$$m_{fi} = E(t) = \int_0^\infty t f_{fi}(t)dt = \int_0^\infty t \lambda e^{-\lambda t} dt \tag{20}$$

Substituting $u = \lambda t$ yields:

$$m_{fi} = (1/\lambda)\int_0^\infty u e^{-u} du = (1/\lambda)\left[e^{-u}(-u-1)\right]\Big|_0^\infty = 1/\lambda \tag{21}$$

This result seems to be important. It means that, if the constant value of the hazard rate h is adopted for fire safety analysis, the mean time to fire is simply its reciprocal. In the same way one may calculate the variance of the time to fire σ^2_{fi}, and also its coefficient of variation (*cov*) v_{fi}. As a result, one obtains:

$$\sigma^2_{fi} = 1/\lambda^2 \text{ and } v_{fi} = 1 \tag{22}$$

6. Time to *k*-th fire episode

Application of a *Poisson* process model allows to examine not only the random time to the first fire episode in the considered building compartment but also the time to the *k*-th fire incident occurring at the same location. Let $i = 1,2,\ldots,k$, and $t_1,t_2,\ldots,t_k$ denote the points-in-time connected with succeeding fire episodes, then the sought time is equal to the sum $(t_k - t_0) = (t_1 - t_0) + (t_2 - t_1) + \ldots + (t_k - t_{k-1})$. Time periods $(t_i - t_{i-1})$ between successive fires are statistically independent and exhibit also the exponential probability distribution, characterized by a common intensity parameter λ. As a consequence of such assumptions, the probability density function (*pdf*) $f_{fi,k}(t)$ has the following form:

$$f_{fi,k}(t) = \frac{\lambda(\lambda t)^{k-1}\, e^{-\lambda t}}{(k-1)\,!} \tag{23}$$

As one can see, the time $t = t_k$ to the *k*-th fire incident exhibits the gamma probability distribution characterized by the parameters k and λ. Because the first of those parameters, k, is the natural number, this gamma distribution is a special one, commonly called the *Erlang* probability distribution. Moreover, it is possible to show that [12]:

$$m_{fi,k} = E(t) = k/\lambda \quad \text{and} \quad \sigma^2_{fi,k} = k/\lambda^2 \quad \text{and} \quad v_{fi,k} = 1/\sqrt{k} \tag{24}$$

7. Concluding remarks

In engineering practice the value of the hazard rate h, constituting a risk measure of fire occurring in examined building compartment and for considered point-in-time if it is known that no fire occurred previously, is statistically estimated to reliably evaluate the real threat conditions in case of a fire. It is usually assumed to be constant during the whole building service period. This means that such a risk does not depend on the time when the fire safety assessment is made. This hazard function, discussed in presented paper, has clear and univocal interpretation, especially if the formal model of a *Poisson* process is taken into account in the analysis. Such model is fully adequate for the fire cases. This conclusion results from the fact that the real fires can be treated as rare random events, very short in relation to the whole building service time. Application of such simplified mathematical formalism allows to estimate in an easy way the probability of fire ignition (and flashover) p_t. The hazard function h may be interpreted as a parameter of the process intensity λ, in accordance with the suggestion given by *Lie* (see Eq. 13). The inverse of such parameter is quantitatively equal to the mean time to fire m_{fi}. It is obvious that such mean time seems to be very large in relation to the typical building service period, especially if the hazard rate h is small. In fact, fire does not occur at all in a great majority of buildings so the time to fire estimated for the whole population of the examined objects tends to infinity. The random time to the first fire is the measure of a compartment durability in context of a fire safety analysis. In the presented article it is shown that such durability is described by means of exponential probability distribution. The same kind of probability density function characterizes random time-periods between succeeding fires in the examined building compartment. Thus, as a result of such conclusions, the random time to the *k*-th fire episode conforms to the *Erlang* probability distribution.

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ANALIZA TECHNICZNO-EKONOMICZNA WYBRANYCH ROZWIĄZAŃ OCIEPLANIA ŚCIAN ZEWNETRZYCH W POLSCE

TECHNICAL AND ECONOMIC ANALYSIS OF CERTAIN EXTERNAL WALL INSULATION SOLUTIONS IN POLAND

Abstract

This paper presents the characteristics of four types of thermal insulation, with the application of: polystyrene panels, mineral wool panels, polyurethane foam panels, and phenolic foam panels. The technical parameters, prices, and amount of labour related to wall construction technology with the application of the thermal insulation materials listed above were discussed. The goals of this article are: to compare technical parameters of certain types of thermal insulation, to provide an economic analysis of the chosen wall insulation solutions, and to choose the type of thermal insulation that is most beneficial in terms of the relationship of price to quality.

Keywords: wall insulation, thermal insulation material, technical and economic analysis

Streszczenie

W artykule przedstawiono charakterystyki czterech rodzajów termoizolacji: z zastosowaniem płyt styropianowych, płyt z wełny mineralnej, płyt z pianki poliuretanowej oraz płyt z pianki fenolowej. Omówiono parametry techniczne, ceny i nakłady pracy związane z technologią wykonania ścian z zastosowaniem wymienionych materiałów termoizolacyjnych. Celami artykułu są: porównanie parametrów technicznych wybranych rodzajów termoizolacji, analiza ekonomiczna wybranych rozwiązań ocieplania ścian ograniczona do wykonanycha kosztorysów i wybór rodzaju termoizolacji, który jest najkorzystniejszy ze względu na stosunek jakości do ceny.

Słowa kluczowe: ocieplanie ścian, materiał termoizolacyjny, analiza techniczno-ekonomiczna

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1. Introduction

External walls are the most important structural element of a building. They support ceilings and the roof and protect the interior of a building against external factors. At a time of rising energy prices, walls should, above all, protect a house from the escape of heat. Appropriate thermal insulation can prevent heat from escaping. A wide assortment of insulation materials in various forms is available on the market. Every manufacturer praises its product, and if it decides to compare it with a different type of thermal insulation, it does so only on the basis of those parameters which compare favourably. In addition, the thermal insulation product sector is dominated by manufacturers of conventional materials. Currently, despite increasing knowledge on the subject of profit related to energy savings, related in turn to well-made thermal insulation, the choice of thermal insulation material is mainly dependent on price. Thus, when deciding to insulate a building, an investor often chooses the product about which it has the most information and that is generally available in large quantities at a low price. However, it is possible that the selection of a more expensive material with better parameters is more advantageous due to lower costs of building use. The subject matter of thermal insulation of walls has been presented in many papers [1, 4, 5, 7, 11, 12]; however, there are very few publications that list and compare thermal insulation materials according to a series of criteria that can help e.g. an investor to make a decision.

This article presents the characteristics of four types of thermal insulation: polystyrene panels, mineral wool panels, polyurethane foam panels, and phenolic foam panels. The technical parameters, prices, amount of labour, and the technologies of wall construction required for the application of each will be discussed. These parameters are compiled in a table and compared. Economic analysis is limited to the preparation of cost estimates in which the amount of labour is determined [6, 9]. The prices of building a wall insulated with a given insulating material are determined based on cost estimates of the construction of these partitions for a single-family house that serves as an example in this paper.

The goals of this article are:

- to compare the technical parameters of certain types of thermal insulation,
- to provide economic analysis of certain wall insulation solutions,
- to select the type of thermal insulation that is most advantageous in terms of its cost-to--quality ratio.

This article can serve as a source of data and information that will be helpful when deciding on a type of external wall insulation.

Cost estimates will be made using the example of the single-family house design ‘Dom pod jarząbem 4 (G2)’ [17] in the Zuzia cost estimate program. The prices contained therein will be accepted as average prices according to the SEKOCENBUD price list for the fourth quarter of 2012 and data made available by manufacturers and contractors.

2. Comparison of the technical parameters of external wall thermal insulation solutions

2. 1. Thermal insulation parameters

Panels for insulating external walls made from 4 alternative materials were selected for analysis. The parameters of the four analysed thermal insulation materials are compiled in Table 1. Technical data concerning individual parameters was taken from standards [13–16] as well as from manufacturers. Some values are given quantitatively, while others can only be presented descriptively. In addition, numerical parameters are presented in charts (except porosity, due to very similar values, and tensile strength perpendicular to surfaces, due to a lack of data for polyurethane foam) Fig. 1–4.

The parameters which can be given only in a descriptive (qualitative) way have been determined on the basis of the adopted definitions, presented below.

Fire resistance – resistance to the destructive impact of fire during its spontaneous and uncontrolled spread over the material, in the form of changes to e.g. its structure, shape or mechanical durability [8].

Acoustic insulation – the insulation of a building partition from airborne sounds or/and impact noise, expressed as a difference between the sound in front of and behind the partition [18].

Durability – this parameter determines the impact of atmospheric factors, such as temperature, light, air, rain, ultraviolet radiation, on the properties of the material [8]

Material storage – this parameter determines the rules of material storage and the related difficulties.

Transport – this parameter determines the correct method of material protection during its transport.

Assembly – this parameter determines the degree of difficulty of work with the material.

Resistance to biological factors – the resistance of the material to the destructive activity of microorganisms, bacteria, fungi and certain insect species [8].

Resistance to chemical factors – this parameter determines the resistance of the material to various chemical substances which may cause its destruction during their contact with it.

Impact on human health – this parameter determines the impact (harmfulness) of the material on the human organism.

Ecology – this parameter determines the impact (harmfulness) of the material on the natural environment.

Method of destruction – this parameter determines the possibilities of material utilisation.

Table 1

Tabular compilation of the parameters of certain types of thermal insulation (authors' table)

Parameter	Unit	Polystyrene	Mineral Wool	Polyurethane Foam	Phenolic Foam
λ coefficient	$\frac{W}{m \cdot K}$	0.038–0.045	0.036–0.042	0.023–0.028	0.021–0.024
Bulk density	kg/m^3	15	150–180	30–35	35
Porosity	%	98	98	90–96	98
Absorbability	%	0.65–1.6	4–10	1–3	N/A

Table 1 con.

Fire resistance	–	E	A1	E, up to 300°C	B-s1,d0
Acoustic insulation	–	low	high	medium	no data
Durability	–	high, but not resistant to UV radiation	high, but loses properties when damp	high	very high
Material storage	–	dry, covered rooms without access to flame	dry, covered rooms	dry, covered rooms without access to flame	for short-term storage – no requirements, for long-term storage – in covered rooms or with polyethylene foil covering
Transport	–	any mode of transport with safeguards	covered means of transport with safeguards	any mode of transport with safeguards	any mode of transport with safeguards
Assembly	–	easy	requires the observance of special health and safety measures, heavy panels	very easy	easy
Compression stress at 10% deformation	MPa	0.05–0.07	0.03–0.04	0.10–0.15	0.1
Tensile strength perpendicular to surfaces	MPa	0.1	0.01–0.08	not specified	0.08
Resistance to biological factors	–	resistant	resistant	resistant	resistant
Resistance to chemical factors	–	not resistant to petroleum derivatives or organic solvents	resistant	not resistant to acids with high concentrations	not resistant to acids with high concentrations
Impact on human health	–	harmless	emits dust and stings during assembly, harmless to the users of insulated rooms	emits harmful substances during a fire	harmless
Ecology	–	harmless	harmless	harmless	harmless
Method of destruction	–	burning or recycling	recycling	burning or recycling	burning or recycling

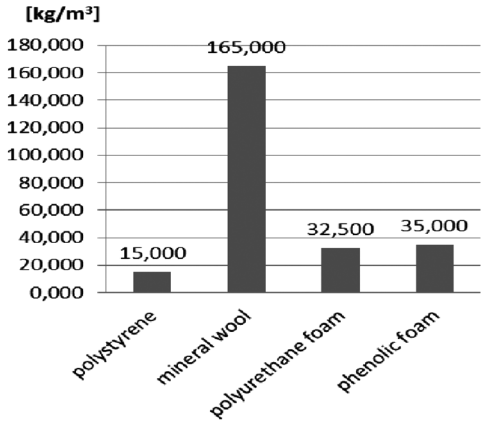

Fig. 1. Comparison of average bulk density values for 4 chosen thermal insulation materials (authors' chart)

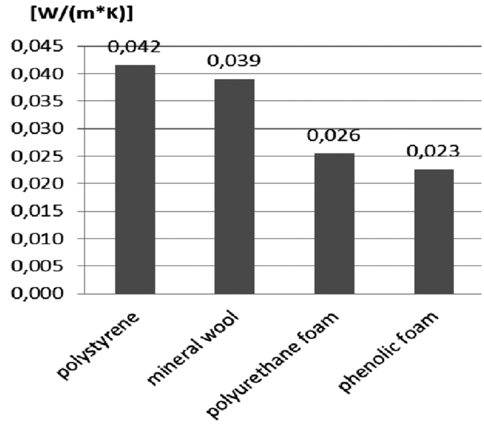

Fig. 2. Comparison of average λ coefficient values for 4 chosen materials (authors' chart)

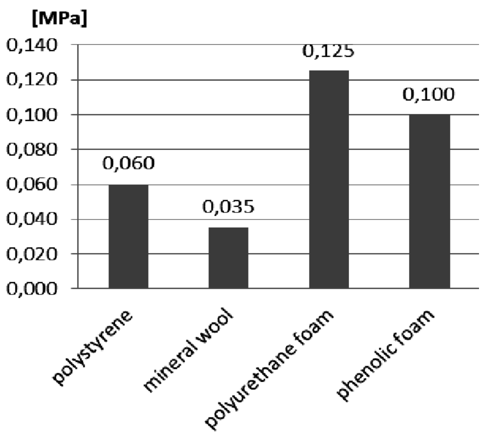

Fig. 3. Comparison of average compressive stress values at 10% deformation for 4 chosen thermal insulation materials (authors' chart)

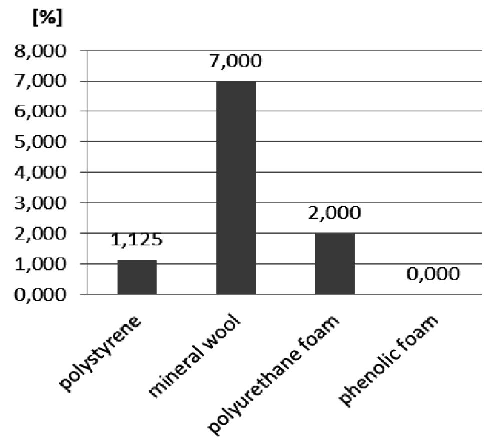

Fig. 4. Comparison of average absorbability values of 4 chosen thermal insulation materials (authors' chart)

The thermal conductivity coefficient λ for mineral wool and polystyrene is comparable, and the situation is similar for phenolic and polyurethane foam; however, the value of these foams is half that of the first two materials. Thus, when foam insulation is applied, the thickness of the material can be reduced by a(neral wool has the best resistance to fire. It is non-flammable and fully protects a building against fire. Phenolic foam has slightly inferior fire parameters: it is slow-burning. Polystyrene and polyurethane foam are classified in category *E* (self-extinguishing); however, polyurethane foam resists higher temperatures than polystyrene.

Mineral wool decidedly affords the best acoustic insulation. It is also the only material of the four that is resistant to all chemical substances. The other materials are characterised by high mechanical strength. Polystyrene has the best tensile strength perpendicular to surfaces; however, foams have better resistance to compressive stresses at 10% deformation.

In terms of absorbability, mineral wool is the most absorbent of all of the materials. Its durability, transport, and storage is related to this quality. When damp, it loses its properties, and so, in order to prevent this, it must be transported and stored appropriately. In contrast, phenolic foam, with an absorbability of zero, does not require special means of transport; it can be stored outside for a short time, and it lasts for a lifetime as long as the panel is not damaged.

Mineral wool is the heaviest material, and, because of this, the most difficult to install. Its mass may be up to 12 times greater than the mass of polystyrene. Dust and stinging from the material make mineral wool panels more difficult to install.

Porosity, impact on the natural environment, resistance to biological factors, and the method of destruction are comparable or the same for each of these insulating materials.

Table 2

Tabular compilation of the point score of parameters of certain types of thermal insulation (authors' table)

Parameter	Polystyrene	Mineral Wool	Polyurethane Foam	Phenolic Foam
λ coefficient	5	4	2	1
Bulk density	1	5	2	3
Porosity	1	1	2	1
Absorbability	2	5	3	1
Fire resistance	5	1	4	2
Acoustic insulation	4	1	2	3
Durability	4	4	2	1
Material storage	4	3	4	2
Transport	3	4	3	3
Assembly	2	3	1	2
Compression stress at 10% deformation	4	5	1	2
Tensile strength perpendicular to surfaces	1	2	3	3
Resistance to biological factors	1	1	1	1
Resistance to chemical factors	5	1	3	3
Impact on human health	1	3	3	1
Ecology	1	1	1	1
Method of destruction	1	1	1	1
TOTAL	45	45	38	31

2.2. Point scoring of thermal insulation parameters

The assessment of variants of the solutions in decision-making problems and taking into consideration multiple criteria makes use of the methods of multi-criteria analysis, e.g. TOPSIS, ELEKTRE, AHP, DEMATEL, BIPOLAR and many others [2,3,10]. Comparing measurable and non-measurable factors is usually done as non-measurable assessment by means of various methods of the so-called measure encoding, e.g. using the Peter method or standardisation [2]. The present study applies a much more simplified, practical approach to this issue. Point scoring system was used, which assigns each parameter of each material an appropriate number of points on a scale of 1–5, where: 1 – best, 2 – good, 3 – average, 4 – bad, 5 – worst.

It was assumed that every parameter has the same weight.

Based on the point scoring system, it can be stated that phenolic foam has the best technical parameters, and polystyrene and mineral wool, tied with the same number of points, have inferior properties.

3. Price and amount of labour of external wall insulation for a sample design

3.1. Design characteristics

The construction design on the basis of which cost estimates of external walls and the amount of labour will be made is 'Dom pod jarząbem 4 (G2)'. This design was developed by the ARCHON design company in Myślenice [17]. This design is a free-standing, ground-floor, single-family residential building without a basement. The house consists of 4 rooms, 1 kitchen, 1 bathroom, a pantry, boiler room, utility room, and garage. The usable area is 118.7 m^2 + garage 32.6 m^2. The walls were designed with Porotherm 30 P+W hollow bricks.
Data:

- perimeter of external walls – 68.66 m,
- wall height – 2.62 m,
- area of external walls – 179.89 m^2,
- area of openings – 41.95 m^2.

For calculations, the area reduced by the surface of openings was assumed to be 137.94 m^2.

3.2. Cost estimates, amount of labour, price

The prepared cost estimates of individual external wall insulation solutions are intended to indicate both the cheapest and the most expensive solutions. For every analysed solution, the structural wall is made of Porotherm 30 P+W hollow bricks, cleaned before insulation is laid down, and installed on a cove base. Polystyrene, mineral wool, and phenolic foam panels were glued to the walls with adhesive mortar, covered with a glass fibre mesh, and covered with a thin layer of plaster. Mineral wool and phenolic foam panels were additionally reinforced with metal connectors with a galvanised pin. Polyurethane foam panels were fastened to the walls with metal connectors with a galvanised pin and covered with a facade wall of full brick construction. The thickness of thermal insulation was calculated for a partition with a *U* coefficient of at least 0.3 W/m^{2}K.

The amount of labour determines the time necessary for the performance of all work and the composition of brigades employed at the construction site. For masonry work, a brigade composed of 2 masons, 1 carpenter, and 2 workers was assumed; for installation of thermal insulation glued to the wall, 4 plasterers and 1 worker; and for fastening, 1 plasterer and 1 worker. The following brigade was assumed for installation of thermal insulation fastened with metal connectors: 2 assembly men and 1 worker. For plaster work, 2 plasterers and 1 worker were assumed.

The price of construction of an external wall with insulation is dependent, above all, on the price of material. The cheapest among the four analysed materials is polystyrene, so it is cheapest to insulate walls with it. The mineral wool solution is not much more expensive (approximately 5,500 PLN more). In the case of foams, the situation is different. Installation of polyurethane foam panels on walls is over 4,000 PLN cheaper than installation of phenolic foam panels; however, the fact that polyurethane foam panels can only cover the facade wall makes the construction of walls insulated with polyurethane foam over 19,500 PLN more expensive than with phenolic foam. The difference between the cheapest solution (polystyrene) and the most expensive (polyurethane foam) amounts to over 27,500 PLN. Thus, if polystyrene is chosen, a building with an external wall surface of nearly twice the area can be insulated in comparison to polyurethane foam.

The time to perform the work for all solutions is very similar. Differences between solutions range from 1–2 days. The time required for raising and insulating walls with phenolic and polyurethane foams is the same. The analysed house design can be insulated the most quickly by using polystyrene panels, while mineral wool takes the longest to install.

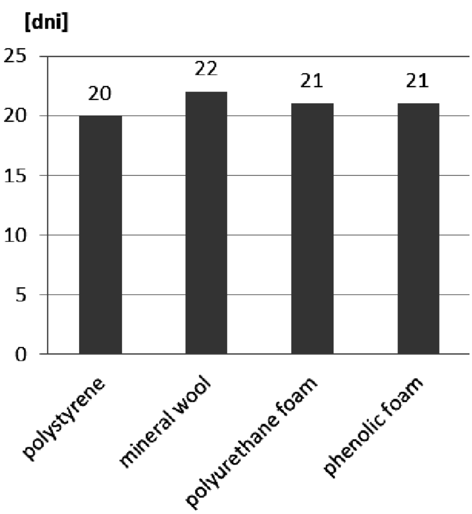

Fig. 5. Number of days required for construction of a wall insulated with one of the four chosen thermal insulation materials (authors' chart)

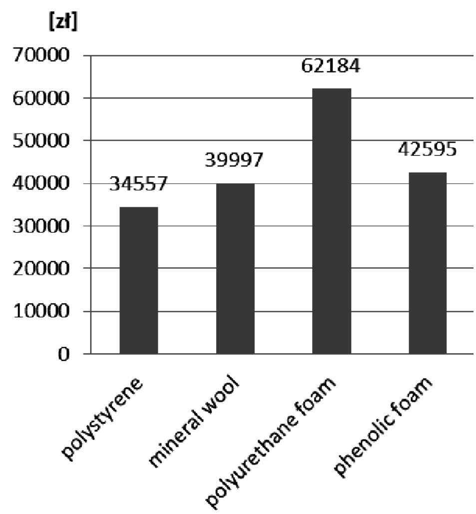

Fig. 6. Price of construction of an external wall insulated with one of the 4 chosen thermal insulation materials (authors' chart)

3.3. Point score of price and time of insulation installation

Because the price and time of insulation installation are specified using different units, it is difficult to evaluate them. In relation to this, a point scoring system was used, which assigns each wall insulation solution with the application of a given material an appropriate amount of points on a scale of 1–5 according to the price and time of installation, where 1 – best, 2 – good, 3 – average, 4 – bad, 5 – worst.

Table 3

Tabular comparison of the point scores of external wall insulation solutions according to time and price of installation (authors' table)

Parameter	Polystyrene	Mineral Wool	Polyurethane Foam	Phenolic Foam
Price	1	2	5	3
Time	1	3	2	2
TOTAL	2	5	7	5

Based on the point score, the best solution is the one with polystyrene panels and the worst with polyurethane foam, as evaluated by time and price of installation.

4. Summary of the technical and economic analysis

Based on the point scoring system, each material received an appropriate number of points in the technical and economic analysis, respectively. To compare these analyses, points were assigned on a scale of 1–4, where: 1 – first place, 2 – second place, 3 – third place, 4 – fourth place. Based on the sum of the points from the two analyses, materials were assigned places that determine which type of thermal insulation has the most advantageous price-to-quality ratio.

Table 4

Comparison of the results of technical and economic analysis and specification of the thermal insulation material with the most advantageous price-to-quality ratio (authors' table)

Analysis	Polystyrene	Mineral Wool	Polyurethane Foam	Phenolic Foam
Technical	45	45	38	31
Score	3	3	2	1
Economic	2	5	7	5
Score	1	2	3	2
Sum of points	4	5	5	3
PLACE	2	3	3	1

Phenolic foam panels took first place among the four types of thermal insulation. They mainly owe this to possession of the best technical parameters. Polystyrene panels placed second due, above all, to the lowest cost of insulation and the shortest time of installation. Mineral wool and polyurethane foam tied for third place.

5. Final conclusions

The decided majority of investors look at the price of material when selecting an insulation system, and only later consider the parameters of the system. According to the authors of this paper, this is a poor approach. The type of thermal insulation is often chosen with less precision than the house's furniture or finishing elements, even though the cost of replacement of the latter is much cheaper and less troublesome than thermal modernisation of the entire building. The analysis shows that the price of material alone does not always mean that a given solution is the cheapest, as can be seen from the example of polyurethane foam panels. In addition, the lowest cost of building insulation does not go hand in hand with low maintenance costs. Technical parameters have an impact on the future use of the building, and the better the technical parameters, the less the use-related costs. That is why it is worth paying attention to the parameters, not only the price, of a given material when choosing thermal insulation.

This paper describes and compares the technical parameters, cost, and time of construction of walls insulated with polystyrene, mineral wool, polyurethane foam, and phenolic foam. A point-scoring system was used to evaluate each analysis and to determine which thermal insulation material is the best and which is the worst among the four analysed materials. Next, both analyses were combined in order to determine which material has the best price-to-quality ratio.

It is impossible to change the mentality of people, and the materials most often used for external wall insulation will continue to be conventional materials like polystyrene and mineral wool. Investors oriented towards insulating a building at the lowest cost choose polystyrene, and, according to the conducted analysis, their decision is justified. However, those who decide to insulate a building with mineral wool should think about choosing the material that placed first according to the analysis: phenolic foam panels. The cost of insulating a building with phenolic foam panels is only 6% greater than the cost of mineral wool panels, while its technical parameters are significantly better and will enable the user to enjoy a durable, very thin, and impact-resistant facade for years.

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TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
CIVIL ENGINEERING | BUDOWNICTWO

3-B/2013

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OPTIMAL MODELLING OF PRESTRESSED GIRDERS USING THE GRADIENT-ITERATIVE METHOD

OPTYMALNE KSZTAŁTOWANIE DŹWIGARÓW SPRĘŻONYCH METODĄ GRADIENTOWO-ITERACYJNĄ

A b s t r a c t

The paper describes the gradient-iterative optimization method, outlines the method's basic assumptions and illustrates its general use. The method's implementation was illustrated with the help of a prestressed beam. Calculations were made to illustrate a girder prestress and height optimization. The method makes it possible to quickly obtain optimal results using universally-available programming. In addition, the method makes it possible to find optimal solutions without the use of complicated mathematical formulas. The article solved the problem of a statically indeterminate prestressed beam with three decision variables, thus proving that the gradient-iterative method is both an efficient and quick optimization method. To illustrate the effectiveness of the optimization method calculations were performed for double- and three-span beam.

Keywords: gradient-iterative method, structure optimization, prestressed beams

S t r e s z c z e n i e

W artykule przedstawiono gradientowo-iteracyjną metodę optymalizacji. Zostały opisane podstawowe założenia metody oraz pokazano jej ogólny sposób stosowania. Na przykładzie dźwigara sprężonego pokazano zastosowanie prezentowanej metody. Opisany przykład obliczeniowy dotyczy optymalnego doboru sprężenia oraz wysokości belki. Metoda umożliwia szybkie uzyskanie rozwiązania optymalnego przy wykorzystaniu ogólnodostępnego oprogramowania. Dodatkowo metoda pozwala na znalezienie rozwiązania optymalnego bez konieczności stosowania skomplikowanego opisu matematycznego. Rozwiązany w pracy problem statycznie niewyznaczalnej belki sprężonej przy założeniu trzech zmiennych decyzyjnych dowodzi skuteczności i szybkości metody gradientowo-iteracyjnej obliczeń optymalizacyjnych. W celu pokazania skuteczności opisanej metody obliczeń optymalizacyjnych, przeprowadzono obliczenia dla belki dwu i trójprzęsłowej.

Słowa kluczowe: metoda gradientowo-iteracyjna, optymalizacja konstrukcji, belki sprężone

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Symbols

$\sigma_{d.s}$, $\sigma_{g.s}$ – accordingly: set of normal bend values for each calculation combination for: bottom and top edge of cross-section

a – set vertical displacement
a_{dop} – maximum admissible vertical displacement
b – beam width
$f_{e.i}$ – nodal forces vector for i-th finite element
g – dead external load
h – beam hight
h_p – distance of prestressed cable's path
n – number of finite elements
q – linear load
q_2, q_3, q_1 – live load

$A(x)$ – the cross-section field for the element length function
B_i – Boolean matrix for i-th finite element
$F_{I \div VI}$ – load phase
G_1 – restriction of maximum compression stress on top edge of cross-section
G_2 – restriction of maximum tensile stress on top edge of cross-section
G_3 – restriction of maximum compression stress on bottom edge of cross-section
G_4 – restriction of maximum tensile stress on bottom edge of cross-section
G_5 – restriction of maximum beam bend
K_{wb} – stiffness matrix incorporating boundary conditions
L – element's overall length
L_1 , L_2 , L_3 – span length
L_{ES} – length of finite element
$K_{e.i}$ – stiffness matrix calculated in accordance with equation (3) for i-th finite element
N_p – prestress force
Q – nodal displacement vector
R – reaction vector
S_{wb} – nodal load vector with boundary conditions
$Z_{e.i}$ – vector of alternatives for linear load calculated according with equation (8) for every i-th finite element.

1. Introduction

The use of prestress technology allows for the design of elements with great spans which significantly spread out load. Due to considerable prestress costs, the design of prestressed elements should encompass a search for optimal economic solutions.

Designed elements must fulfill all capacity and utility specifications and must simultaneously meet optimal economic demands. In most cases an optimally-modeled element will weigh as little as possible as this minimizes consumption of materials, thus also minimizing production costs.

The article outlines optimal modeling of prestressed girders. The gradient-iterative method was utilized to pinpoint optimal solutions.

The article is an example of a new use of the gradient-iterative method for optimization calculations; it allows rapid selection of optimal solutions.

2. Calculation method and its groundwork

Literature pertaining to construction optimization contains solutions which utilize mathematical methods of optimal control. Methods based on refined control theory make it possible to find optimal solutions or set boundary conditions and restrictions. Optimization based on the maximum principle gives good results but has two significant, design-related, restrictions:

- The necessity to translate the complexity of optimization into mathematical terms. Such a description does not correlate with the manner in which bearing capacity is verified in practice,
- Lack of efficient or intuitive programming used to find solutions to complex problems formulated in categories of complete control.

The optimization method presented in this article makes it possible to quickly find an optimal solution without the necessity of utilizing difficult to obtain numeric programs which calculate multipoint boundary value problems.

The method combines the gradient descent method and an iterative solution to the formulated optimization problem. The method can be expressed in six points:
(1) A mathematical formula of functions describing the assessed optimization task.
(2) Determining the objective function and decision variables.
(3) Determining optimization restrictions.
(4) Determining the optimization starting point and the direction of the sought solution.
(5) A description of the increment function.
(6) Iteratively obtaining a solution which fulfills optimization criteria.

The described method can be used to solve various optimization problems with the help of universally-available software. Furthermore, it is considerably quicker than other optimization methods.

In conjunction with the finite-element method, the gradient-iterative method can be used to find very quick optimization solutions for statically undetermined constructions.

Simple functions which thoroughly describe the optimization problem are all that is required to formulate a task for the gradient-iterative method. As a result of these simple mathematical formulas, numerical calculations can be carried out quickly.

When describing the problem, it is extremely important to correctly determine the increment function. Incorrect input can lengthen calculations considerably, and in the worst case can even make it impossible to receive a reliable result.

3. A general description of the problem

The task involves optimal modeling of prestressed beams with a rectangular cross-section and linear path for prestressed cables. Fig. 1–2 below show a static diagram and assumed load phases.

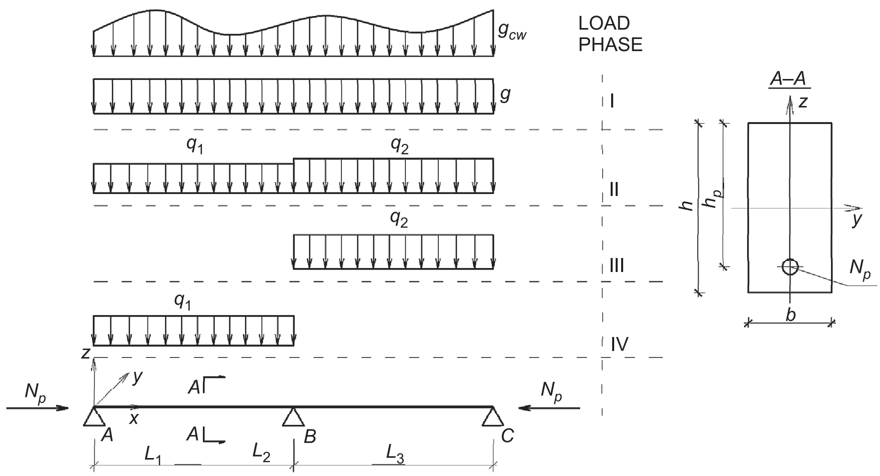

Fig. 1. Static diagram, cross-section and configuration of external forces for a double-span beam

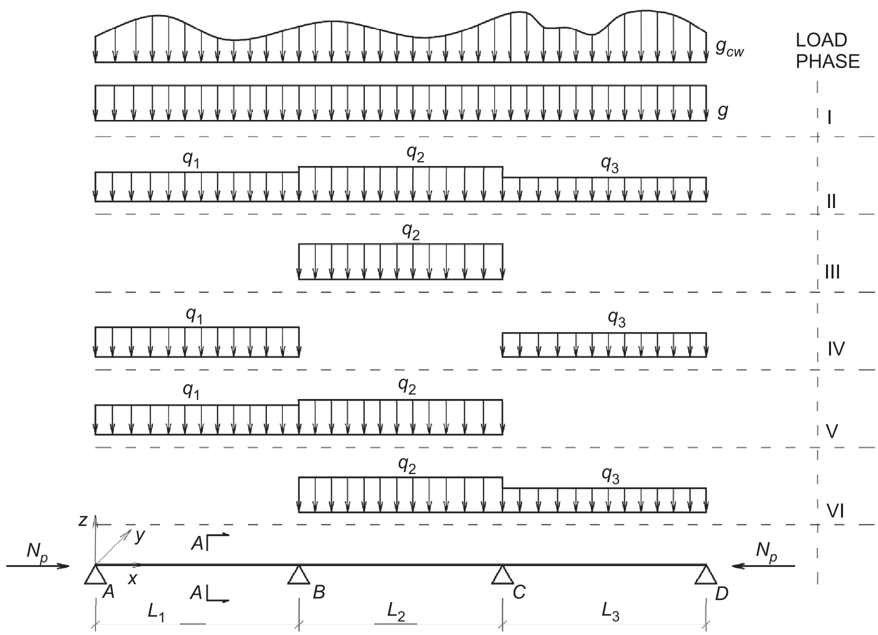

Fig. 2. Static diagram and configuration of external forces for a three-span beam

Load phase I incorporates the element's own weight g_{cw} and dead load g. Phases II-VI illustrate live load in various calculated cases.

The analyzed example incorporates 4 (in the case of double-span beams) and 6 (in the case of three-span beams) calculation combinations of each load phase:

$$\begin{aligned} C1 &= F_I \\ C2 &= F_I + F_{II} \\ C3 &= F_I + F_{III} \\ C4 &= F_I + F_{IV} \\ C5 &= F_I + F_V \\ C6 &= F_I + F_{VI} \end{aligned} \tag{1}$$

where:

$F_{\mathrm{I \div VI}}$ – load phase

4. Calculation procedure

It is necessary to establish optimal cross-section dimensions which will minimize the assumed objective function (the element's volume).

$$V = \int_0^L A(x)dx \tag{2}$$

where:

$A(x)$ – the cross-section field for the element length function

L – element's overall length

Optimization pertains to the selection of three decision variables:

- Fixed prestress force along the length of the element (prestress force after considering all loses)
- Fixed resultant distance h_p of prestressed cable's path from upper edge of cross-section (linear path)
- Cross-section's height variable along the girder's axis

The optimal solution minimizes the objective function (2) and fulfills all assumed optimization restrictions.

4.1. Finite-element method in the analysed example

The beam was discretized into finite elements with constant stiffness *EI(h)* and fixed length L_{ES} (Fig. 2). When discretizing a beam with the help of multiple finite elements, the assumption that one finite element has a fixed moment of inertia does not lead to significant calculation errors.

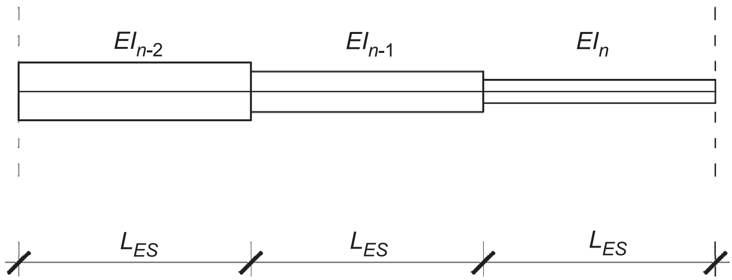

Fig. 3. Beam discretization

The stiffness matrix was defined (3) and a Boolean matrix was constructed for n finite elements (4).

$$k(EI,L)=\begin{pmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{-12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & \frac{-6EI}{L^2} & \frac{2EI}{L} \\ \frac{-12EI}{L^3} & \frac{-6EI}{L^2} & \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & \frac{-6EI}{L^2} & \frac{4EI}{L} \end{pmatrix} \tag{3}$$

$$B_i = \left| \begin{array}{l} B_{(1,2\cdot(top^i-1)+1)} = 1 \\ B_{(2,2\cdot(top^i-1)+2)} = 1 \\ B_{(3,2\cdot(top^i-1)+3)} = 1 \\ B_{(4,2\cdot(top^i-1)+4)} = 1 \end{array} \right. \tag{4}$$

where:

top^i – fragment of topology matrix corresponding with i-th finite element.

For finite elements with variable inertia, the stiffness matrix will obviously take on a different form.

Function (5) describes incorporated matrix topology for n finite elements.

$$top = \begin{array}{l} for\; i \in 1 \ldots n \\ \left| \begin{array}{l} top_{i,1} = i \\ top_{i,2} = i+1 \end{array} \right. \end{array} \tag{5}$$

The above matrices (4) define the Boolean matrix for every i-th finite element, making it possible to later perform automatic calculations for any number of elements. The overall size of the Boolean matrix for the discussed task is $4 \times (2n + 2)$. Function (4) shows only non-zero elements of the matrix. Below is a Boolean matrix for every i-th finite element.

$$B_i = \begin{pmatrix} \underbrace{\begin{matrix} 0 & \dots & 0 \\ 0 & \dots & 0 \\ 0 & \dots & 0 \\ 0 & \dots & 0 \end{matrix}}_{2i-2} & \underbrace{\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}}_{4} & \underbrace{\begin{matrix} 0 & \dots & 0 \\ 0 & \dots & 0 \\ 0 & \dots & 0 \\ 0 & \dots & 0 \end{matrix}}_{(2n+2)-(2i-6)} \end{pmatrix} \tag{6}$$

Stiffness matrix aggregation:

$$K = \sum_{i=1}^{n} B_i^T K_{e.i} B_i \tag{7}$$

where:

$K_{e.i}$ – stiffness matrix calculated in accordance with equation (3) for i-th finite element
B_i – Boolean matrix for i-th finite element

Definition of alternative vectors for linear load:

$$Z_c\left(q, L_{ES}\right) = \begin{pmatrix} \dfrac{qL_{ES}}{2} \\ \dfrac{qL_{ES}^{\ 2}}{12} \\ \dfrac{qL_{ES}}{2} \\ \dfrac{-qL_{ES}^{\ 2}}{12} \end{pmatrix} \tag{8}$$

where:

q – linear load
L_{ES} – length of finite element

Aggregation of alternative vectors:

$$Z = \sum_{i=1}^{n} B_i^T Z_{e.i} \tag{9}$$

where:

$Z_{e.i}$ – vector of alternatives for linear load calculated according with equation (8) for every i-th finite element.

Dead load was approximated to fixed load (to linear load within one finite element). Approximation of dead load with discontinuous linear load is exact enough in the case of a minimum of 5 finite elements within one span section.

Boundary conditions for a three-span beam were described by vector (10):

$$w = \begin{vmatrix} w_1 = 1 \\ w_2 = 0 \\ for\, i \in 2...(n+1) \\ \qquad w_{(2i-1)} = \begin{vmatrix} 1\ if\ \left((i-1)L_{ES} = L_1\right) \vee \left((i-1)L_{ES} = L_1 + L_2\right) \vee \\ \qquad \left((i-1)L_{ES} = L_1 + L_2 + L_3\right) \\ 0\ otherwise \end{vmatrix} \\ w_{2n+2} = 0 \end{vmatrix} \tag{10}$$

The boundary conditions vector was established analogically for double-span beams.

The solution to the set of equations and calculation of displacement vectors for finite elements:

$$Q = K_{wb}^{-1} S_{wb} \tag{11}$$

$$R = K_{wb} Q - S_{wb} \tag{12}$$

where:

Q – nodal displacement vector
R – reaction vector
K_{wb} – stiffness matrix incorporating boundary conditions
S_{wb} – nodal load vector with boundary conditions

Calculation of node forces in elements:

$$f_{e.i} = K_{wb.i} B_i Q - Z_{e.i} \tag{13}$$

where:

$f_{e.i}$ – nodal forces vector for *i*-th finite element

Equations (3-13) were used to formulate function *MES(EI,x)*, which makes it possible to determine the value of internal forces as well as horizontal and angular displacement.

For statically indeterminate girders exposed to prestress forces, it is necessary to additionally take under consideration the effect of inducted internal forces. The impact of prestress force on the value of internal forces was taken under consideration by stressing the configuration with forces causing an equally great bearing reaction. The calculations did not incorporate the effect of rheological phenomena.

A computer program was written based on the above assumptions. It enables finding an optimal solution to the discussed problem. The algorithm was written in the Mathcad environment. The calculation program makes it possible to save a legible record of the calculation procedure and quickly solve matrix equations.

4.2. Optimization restrictions

The following restrictions were set:

G_1 – restriction of maximum compression stress on top edge of cross-section
G_2 – restriction of maximum tensile stress on top edge of cross-section
G_3 – restriction of maximum compression stress on bottom edge of cross-section
G_4 – restriction of maximum tensile stress on bottom edge of cross-section

$$G_1 = R - \max(|\sigma_{g.s}|), \quad s = 1 \div 4 \tag{14}$$

$$G_2 = R - \min(|\sigma_{g.s}|), \quad s = 1 \div 4 \tag{15}$$

$$G_3 = R - \max(|\sigma_{d.s}|), \quad s = 1 \div 4 \tag{16}$$

$$G_4 = R - \min(|\sigma_{d.s}|), \quad s = 1 \div 4 \tag{17}$$

where:

$\sigma_{d.s}$, $\sigma_{g.s}$ – accordingly: set of normal bend values for each calculation combination for: bottom and top edge of cross-section
G_5 – restriction of maximum beam bend

$$G_5 = a_{dop} - \max(a) \tag{18}$$

where:

a_{dop} – maximum admissible vertical displacement,
a – set vertical displacement.

Restrictions G_2 and G_4 protect from exceeding concrete resistance to tensile. In practice the prestressed elements are usually designed with cracking as the ultimate limit state. There are categories of elements, however, for which – due to favorable environmental conditions – are permitted to cross the concrete's tensile limit state to tensile. When optimizing beams that may crack Young's modulus should be correctly implemented.

Additionally, restrictions were set for permissible area of decision variables.

4.3. Optimization restrictions

The starting point of the optimization process was obtaining minimal dimensions of the cross-section due to set geometric restrictions and minimal value of prestress force.

A stepwise increment of the decision variable was assumed within one calculation loop. The direction of the increment Δh depends on fulfilling bearing capacity conditions and is determined in relation to the result of the cross-section verification result. Additionally, the increment value Δh decreases with subsequent calculation phases.

The result of a calculation loop is the optimal height of the cross-section of one finite element. What follows, the total amount of calculation loops within one iteration is equal to n+1.
where:
n – number of finite elements.

Analogically, a stepwise increment in value of the remaining two decision variables was also assumed.

Calculations were carried out in the following steps:
The value of the initial decision variables was determined
(1) An optimal solution for the determined value and location of net prestress force were determined in accordance with procedure A
(2) stepwise increment of the prestress force and finding an optimal solution for a fixed location and value of net prestress force within one incrementation (procedure A iteratively repeated for various force values N_p, until expected convergence is reached)
(3) stepwise increment of the decision variable h_p and finding an optimal solution for value h_p within one iteration (point 3 repeated iteratively)
(4) choice of optimal result from set of results

Procedure A:
- Finding the optimal cross-section height for each finite element.
- Cross-section fulfills all determined bearing conditions and minimizes determined objective function (1). Calculations are conducted for internal forces determined via the finite element method for initial values.
- MES calculations and updating value of cross-section forces and linear displacement.
- Re-determining optimal cross-section height along girder's length.
- Verification of boundary nods displacement.
- Iterative calculations (MES calculations are carried out for each iteration, an optimal solution is determined for defined internal forces and boundary nods displacement is verified).
- Iterative calculations are stopped upon receiving an expected iterative convergence.

5. Calculation results

Below are examples of optimization results for the following data:
- concrete class: C35/45,
- dead external load: $g = 20 \cdot 10^3$ N/m
- live load: $q_1 = 25 \cdot 10^3$ N/m, $q_2 = q_3 = q_1$
- fixed geometric dimensions: $b = 0.4$ m
- length: $L_1 = 18$ m, $L_2 = L_3 = L_1$
- length of finite element: $L_{ES} = 0.2$ m

Calculations were conducted for three decision variables:
- Fixed prestress force along the length of the element (prestress force after considering all loses)
- Fixed resultant distance h_p of prestressed cable's path from upper edge of cross-section (linear path)
- The element's cross-section's height

The following area of variability was assumed for the above decision variables:
- height h: <0.4 ; 2.0> [m]
- distance h_p: <0.3 ; 2.0> [m]
- prestress force N_p: <1000 ; 7000> [10^3 N]
-

The illustrations below show: boundary of the bending moment and shearing force (Fig. 4, 5), boundary of nodal vertical displacement (Fig. 6) and optimal beam height (Fig. 7) for double-span beam.

Figure 6 shows the boundary of nods vertical displacement along with marked boundary values of admissible displacement. The graph shows that restriction G_5 is not active in any beam cross-section.

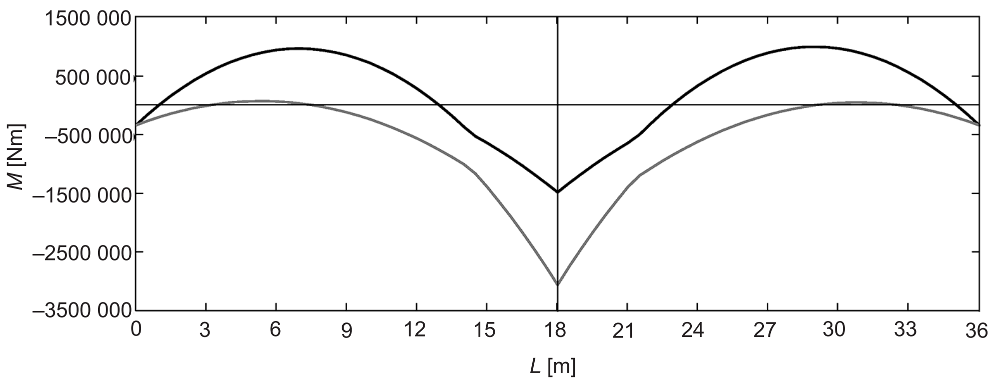

Fig. 4. Boundary of bending moments (double-span beam)

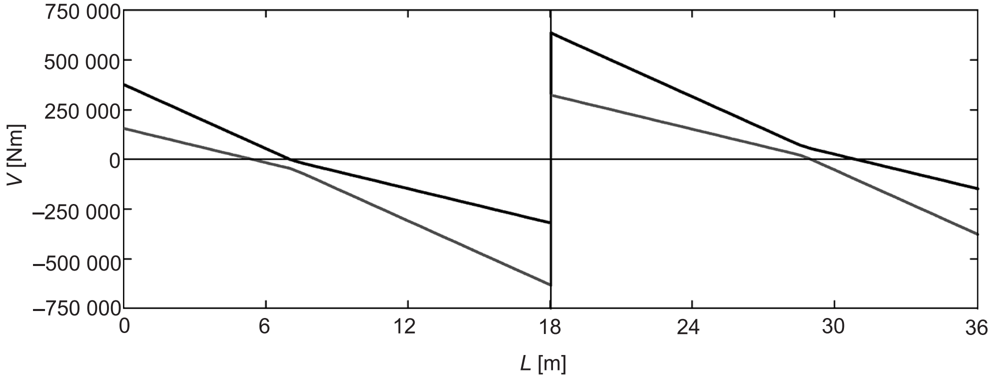

Fig. 5. Boundary of shearing force (double-span beam)

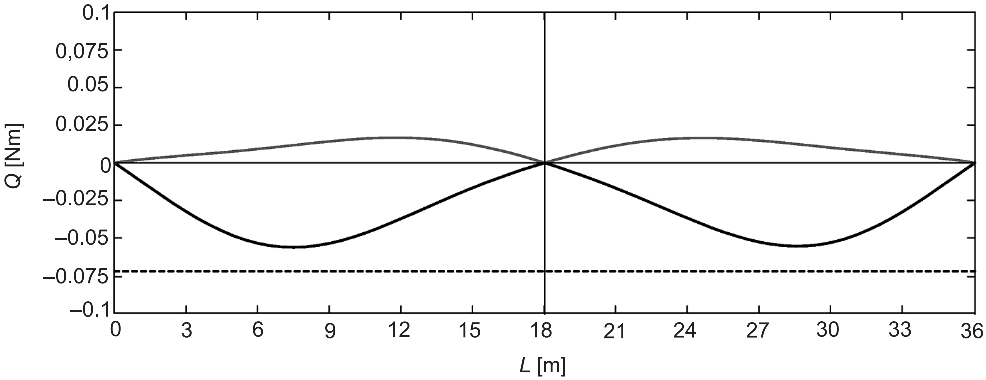

Fig. 6. Boundary of nodal vertical displacements (double-span beam)

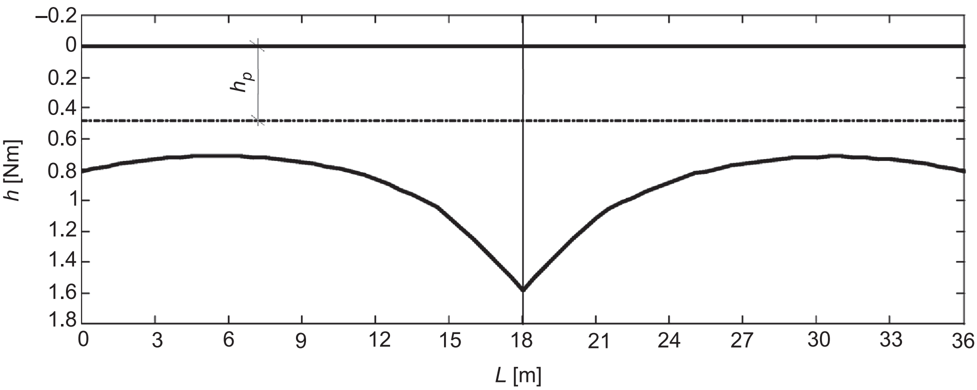

Fig. 7. Optimal solution (double-span beam)

Maximum vertical displacement was equal to 0.056 m. Bend limit for examined girder was equal to L/250 = 0.072 m.

Optimal values were set for:

- Prestress force N_p: 4507.6·10^3 N
- Distance h_p: 0.485 m

Objective function (2) value for the obtained result is equal to V = 12.831 m^3.

The figures below show: boundary of the bending moment and shearing force (Fig. 8, 9), boundary of nodal vertical displacement (Fig. 10) and optimal beam height (Fig. 11) for three-span beam.

Figure 10 shows the boundary of nodal vertical displacement along with marked boundary values of admissible displacement. The graph shows that restriction G_5 is not active in any beam cross-section.

Maximum vertical displacement was equal to 0.065 m. Bend limit for examined girder was equal to L/250 = 0.072 m.

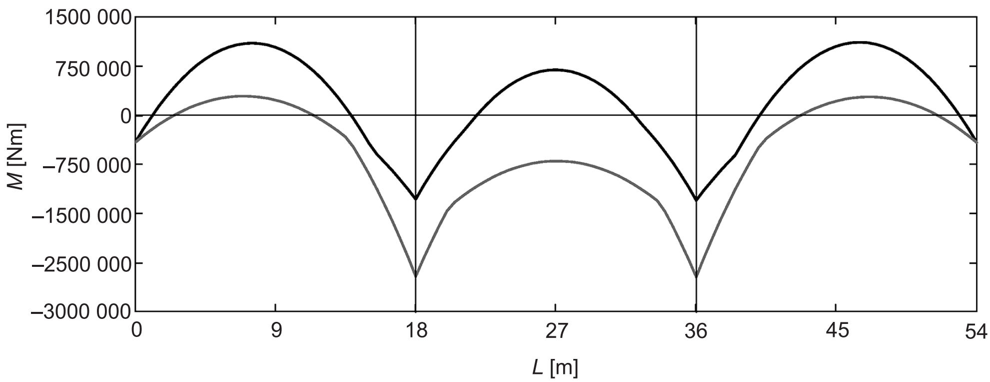

Fig. 8. Boundary of bending moments (three-span beam)

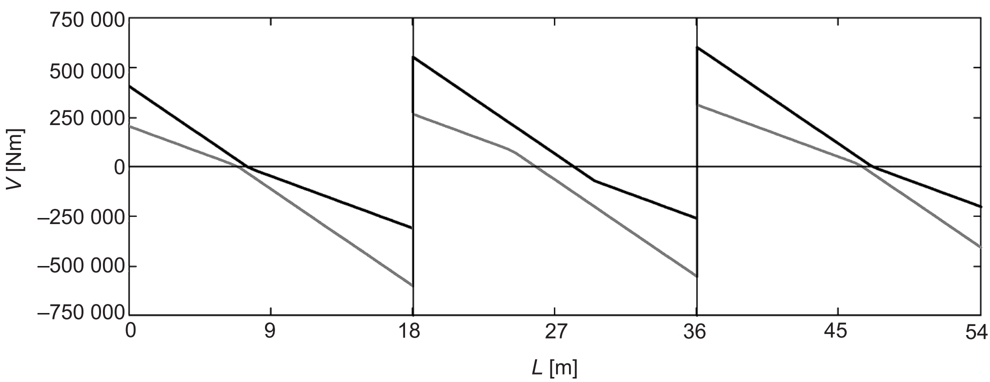

Fig. 9. Boundary of shearing force (three-span beam)

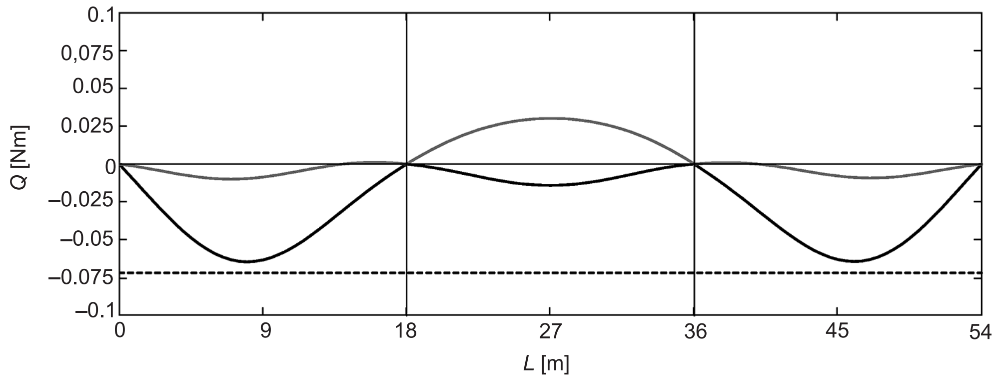

Fig. 10. Boundary of nodal vertical displacements (three-span beam)

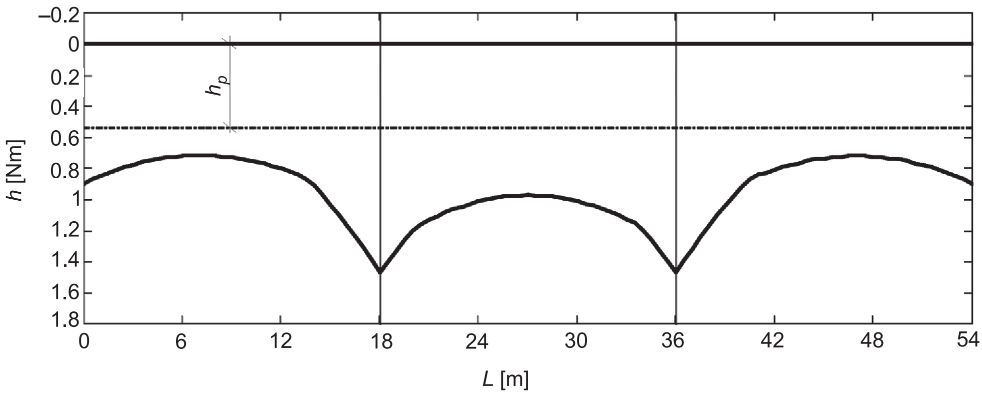

Fig. 11. Optimal solution (three-span beam)

Optimal values were set for:

- Prestress force N_p: $4652.9 \cdot 10^3$ N
- Distance h_p: 0.540 m

Objective function (2) value for the obtained result is equal to $V = 20.459$ m^3.

6. Conclusions

In the case of complex construction configurations, the necessity to find optimal solutions incorporating the maximum principle requires solving very intricate multi-point boundary value problems. Additionally, the optimal control method requires the use of numeric programs for solving formulated boundary value problems; these programs are difficult to obtain.

The gradient-iterative method makes it possible to find quick solutions even in the case of complex problems. By formulating the task with the help of simple functions and carrying out calculation loops, the solution set contains an optimal result which fulfills all predefined optimization criteria.

The gradient-iterative method in conjunction with the MES algorithm offers designers vast possibilities. As the method takes relatively little time it can be used in construction design studios to optimize various construction elements.

The optimization task can be formulated in any programming language or in popular calculation software (e.g. Mathcad, Mathlab).

Finding the optimal solution using the optimal control method for statically indeterminate prestressed beam with three decision variables requires an extreme effort and is time-consuming. The gradient-iterative method makes it possible to find the solution much quicker.

The described method can of course be used to solve any number of optimization problems that do not pertain to the optimal shape of structures

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