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CONTROL OF ELECTRIC DRIVE BY MEANS OF INVERSE DYNAMICS

STEROWANIE NAPĘDU ELEKTRYCZNEGO METODĄ DYNAMIKI ODWROTNEJ

Abstract

This paper presents a method for positioning an electric drive with an elastic mechanical part by applying the inverse problem of dynamics. The presented assumptions take into account technological requirements and limitations of dynamic variables. The desired trajectory of the mechanical part of the electromechanical system has also been determined. On this basis, an algorithm for determining the control voltage waveform is proposed.

Keywords: *electromechanical systems, control, inverse dynamics*

Streszczenie

W artykule przedstawiono metodę pozycjonowania napędu elektrycznego z elementem elastycznym z zastosowaniem zagadnienia odwrotnego dynamiki. Sformułowano założenia i uwzględniając wymagania technologiczne oraz ograniczenia wielkości dynamicznych, określono przebieg pożądanej trajektorii układu elektromechanicznego. Na tej podstawie opracowano algorytm obliczeniowy w celu określenia przebiegu napięcia sterującego.

Słowa kluczowe: *układy elektromechaniczne, sterowanie, odwrotne zagadnienie dynamiki*

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Nomenclature

x – position of the winding engine mass [m],
$x^{(1)}$ – velocity of the winding engine mass [m/s],
$x^{(2)}$ – acceleration of the winding engine mass [m/s^2],
x_m – position of the point on the circumference of the drive wheel [m],
u_s – control signal calculated by the controller [V],
ε – correction signal (output of the correction block) [V],
ω – angular velocity of the motor [rad/s].

1. Introduction

Fast dynamic algorithms are a prerequisite in the positioning of real-time control systems of flexible manipulators and winding machines [2]. Analysis of elastic vibrations in such systems is usually carried out by investigating the drive transmission system. In general, the drive transmission system consists of a set of masses with elastic and damping connections.

In a multi-mass system, the most significant oscillation frequencies should be selected. If a system has only one significant natural frequency, its dynamic properties may be reproduced with a two-mass system with a fair degree of accuracy. Flexibility in mechanical joints results in a dynamic torque variable component which makes it difficult to obtain the desired dynamic characteristics [1].

As a result, the system can only imperfectly reproduce a predetermined trajectory in the state space. These imperfections can be partly compensated for by classical methods, i.e. by adaptation of the gain factor feedback. [13]. An example of applying nonlinear feedback methods to reducing overshoots in the positioning system when the actuator reaches the desired position is shown in [9]. Another approach is presented in [8], where the authors discuss the inverse dynamics velocity control of a direct drive manipulator based on the sliding mode compensation technique. The compensated inverse dynamic velocity control scheme is robust against the detrimental effects of model uncertainties and exhibits robust tracking performance of the desired velocity trajectories. In [7], a discrete-time sliding mode control strategy combined with an inverse dynamics approach to motion control of robot manipulators is proposed.

When positioning such systems, three key issues must be addressed – determination of the control system structure, calculation of parameter values and determination of control variable waveforms ensuring the desired trajectory.

Electrical drives with linear or angular position control require the use of specialized digital systems. Such systems often implement complex control algorithms [3]. The aim of this paper is to present a simple example of a drive system loaded with elastic and damping elements. The drive control system works by solving the inverse dynamics problem [4], which requires relatively little computational effort.

The determination of input values (such as control voltages and electromagnetic moments) from the given kinematic elements of motion or from the given properties of motion is one of the main problems associated with the dynamics of electromechanical systems [13]. The inverse dynamics problem has attracted the attention of engineers due to its wide scope of potential applications and general solvability [14]. Computer-aided modeling of various systems by applying the of inverse problems of dynamics has led to a considerable broadening of the concept of the inverse problem itself [11]. If the given properties of motion of the electromechanical system can be represented analytically as first integrals of the appropriate equations of motion, then, in general, the solution of the inverse problem of dynamics is reduced to the construction of an adequate system of differential equations. The coefficients of these differential equations can be computed on the basis of technological requirements, limitations imposed on state variables and other properties known. Solving these differential equations yields information regarding the forces and moments which act upon the system [15].

2. Structure of the control system

It is assumed that the general structure of the control system comprises the following elements:

– a controller which implements the control task
– an object, i.e. a DC motor drive with a permanent magnet in a closed-loop control system with speed and current controllers, containing a transistor-based amplifier,
– mechanical load, involving a flexible damping element and the mass of the mechanism itself,
– a correction block which limits the impact of changes in system conditions and disturbances.

It is assumed that the structure of the motor speed control system is a cascade consisting of two PI regulators whose settings have been optimized. The speed controller acts as the outer loop controller which controls velocity, while the current controller acts as the inner loop controller which treats the output of the outer loop controller as its setpoint and controls the current of the motor.

The structure of this electromechanical drive system is depicted in Fig. 1 with two additional labels: *u* – corrected control signal; *z* – disturbance signal.

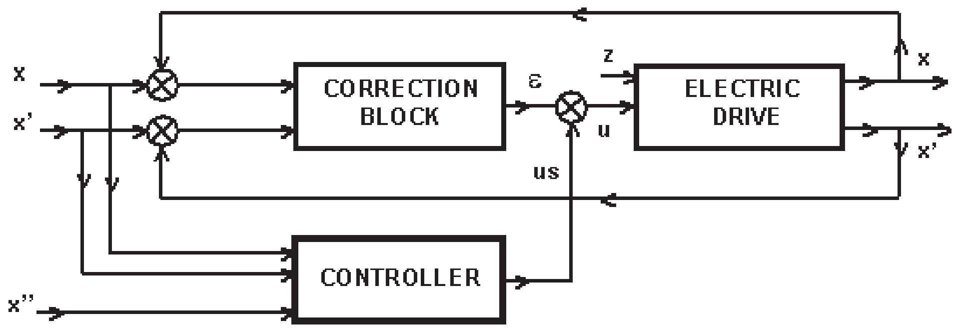

Fig. 1. The control system structure of a DC drive control of an elastic element

3. Electromechanical model and its motion control

The purpose of control is to carry the winding engine mass from the initial state (x_0, $x_0^{(1)} x_0^{(2)}$) to the final state (x_k, $x_k^{(1)} x_k^{(2)}$) in minimum time t_k while satisfying limitations imposed on selected state variables such as $x^{(1)}$, $x^{(2)}$.

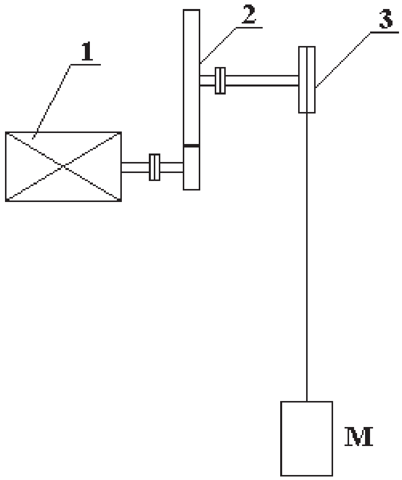

Fig. 2. The general scheme of the electromechanical system, where: 1 – motor, 2 – gear, 3 – drive wheel, M – winding engine mass on elastic rope

The paper presents the results of simulating this process in an electromechanical system. The corresponding block diagram is shown in Fig. 2.

The problem can be reduced to computing the optimal waveform of the engine control variable, which limits transient oscillations of the winding engine mass. The desired trajectory of the winding engine mass can be calculated on the basis of limits imposed on state variables describing the dynamics of the motor and the winding engine mass while minimizing the duration of the process.

The dynamics of the presented electromechanical system is described by the following relationship:

$$x(t)^{(1)} = f\,[x(t), u(t), t] \tag{1}$$

where:

$x(t) = \{x_1(t), ..., x_n(t)\}^T$	– vector of state variables,
$u(t) = \{u_1(t), ..., u_m(t)\}^T$	– vector of control variables,
m, n	– dimensions of each vectors,
T	– transpose of a vector,
$f\,[x(t), u(t), t]$	– predetermined vector function (generally nonlinear),
$x(t)^{(1)}$	– first derivative of the state vector.

In this paper, the following assumptions were made:

– the desired trajectory of the winding engine mass in the state space was selected in such a way as to satisfy the following technological limitations:

$$x_z(t) = \xi(t); \quad t_0 < t < t_0 + t_k; \quad 0 < t_k < \infty \tag{2}$$

where:

t_0 – initial time,
t_k – duration of the transient state;

– the right side of the equation (1) can be expressed as the following sum:

$$x(t)^{(1)} = g[x(t), t] + h[u(t), t] \tag{3}$$

where the input vector function may be represented as:

$$h[u(t), t] = \xi(t)^{(1)} - g[\xi(t), t] \tag{4}$$

4. Specification of the control task

The control task can be expressed as follows: we need to find a vector control function $u(t)$ for which the trajectory of the winding engine mass corresponds with the desired trajectory $\xi(t)$ with the assumed accuracy [5].

In practice, the j-th component of the vector function can typically be expressed as:

$$h_j[u(t), t] = \sum_{i=1}^{m} \alpha_{ji}(t)\, u_i(t) \qquad \text{for} \quad j = 1, ..., n \tag{5}$$

where the $\alpha_{ji}(t)$ coefficients follow from time-dependent parameters of the system and are not difficult to calculate.

An approximation of the difference $\xi(t)^{(1)} - g[\xi(t), t]$ can be computed using linear combinations $\sum_{i=1}^{m} \alpha_{ji}(t)\, u_i(t)$ for $j = 1, ..., n$.

By specifying for each state variable ($j = 1, ..., n$), the error of the approximation as:

$$\varepsilon_j(t) = g_j[\xi(t), t] - \xi_j^{(1)}(t) + \sum_{i=1}^{m} \alpha_{ji}(t)\, u_i(t) \tag{6}$$

and minimizing the functional:

$$I = \int_{t_0}^{t_0+t_k} \varepsilon(t)^T \varepsilon(t)\, dt \tag{7}$$

in each step of the procedure we can determine the coefficients $\alpha_{ji}(t)$ and the control functions $u_i(t)$.

Taking into account the technological limitations of the components of the state vector (e.g. regulator output signals, armature current, speed and acceleration of the winding engine mass), additional conditions may be obtained:

$$x_j(t)_{\min} < x_j(t) < x_j(t)_{\max} \qquad j = 1, ..., n \tag{8}$$

where:

$x_j(t)_{\min}$, $x_j(t)_{\max}$ – the lowest and highest admissible values of the j-th component of the state vector.

The limitations imposed on vector control signal components (e.g. speed setpoint signal voltage) can be expressed as follows:

$$u_j(t)_{\min} < u_j(t) < u_j(t)_{\max} \qquad j = 1, ..., m \tag{9}$$

where:

$u_j(t)_{\min}$, $u_j(t)_{\max}$ – the lowest and highest admissible values of the j-th component of the vector control signal.

Taking into account the above mentioned conditions, the components of the input vector function $h[u(t), t]$ may be defined as:

$$\begin{aligned} h_j\ [u(t), t] &= \sum_{i=1}^{m} \alpha_{ji}(t)\ u_i(t) = \xi_j(t)^{(1)} - g_j\ [\xi(t), t] = \\ &= h_j^*[t, t_0, t_k, u_{\max}, u_{\min}, x_{\min}, x_{\max}, \alpha(t)] \end{aligned} \tag{10}$$

where:

h^* – in general, a nonlinear vector function of parameters (e.g. limiting values of state variables such as start and end time), which defines a desired trajectory of the winding engine mass,

$\alpha(t)$ – a time-dependent parameter matrix.

The control task leads to the solution of equation (10) with respect to vector control components.

5. Simulation studies

Simulation studies were carried out for a DC motor drive with permanent magnets, manufactured by Wamel, series 5680, type DPM 56-DF4 K-7707, designed to feed the drives of numerically controlled machines [5]. Its basic parameters and the limiting values of the control system are as follows: armature resistance 1.75 Ω; armature inductance 0.037 H, motor torque constant, 0.85 Nm/A; rated torque with the rotor stopped 7.4 Nm, peak torque with the rotor stopped 62.0 Nm, rated speed 1200 rpm, rated current 9.0 A; peak current 71 A; maximum supply voltage 107 V; reduced moment of inertia of the rotating masses 0.015 kgm^2; velocity measurement constant k_{tg} = 0.301 Vs; current feedback gain 0.33 V/A; maximum control voltage $u_{\max}$ = 8.5 V; gain rectifier built on power transistors 50 V/V. The signal delay introduced by the rectifier was seen as negligible.

The study used a cascade armature current control system with speed and current PI controllers. Settings have been optimized on the basis of the polynomial Ellert criterion, limiting the output signals of current and speed controllers to between –10.0 V and +10 V; gain of the speed controller: 4.084 V/V; time constant of the speed controller: 0.08 s; gain of the current controller: 0.698 V/V; time constant of the current controller: 0.021 s.

The parameters of the mechanical part of the system and limitations placed upon the state variables associated with it were as follows: mass of the winding engine $M = 20$ kg; elasticity modulus of the rope $k = 100$ N/m; substitute damping factor of the rope $c = 20$ Ns/m, radius of the drive wheel $r = 0.2$ m; speed limit of the winding engine mass $x^{(1)}_{\max} = 5.648$ m/s; mass acceleration limits: $-1.25\ \mathrm{m/s^2} \le x^{(2)}_{\max} \le 1.25\ \mathrm{m/s^2}$; derivative of mass acceleration limits: $-5.0\ \mathrm{m/s^3} \le x^{(3)}_{\max} \le 5.0\ \mathrm{m/s^3}$; second derivative of mass acceleration limits: $-25.0\ \mathrm{m/s^4} \le x^{(4)}_{\max} \le 25.0\ \mathrm{m/s^4}$.

The aim of the process control was to position the winding machine mass. This was achieved by appropriately shaping the engine speed waveform. The whole dynamic process lasted 20 s and consisted of three stages: startup (0–5 s); movement with maximum established speed (5–15 s); and braking (15–20 s).

To determine the trajectory of the winding engine mass $x(t)$, iit is necessary to analyze the technological limitations and find the relationships between them. The maximum values of the speed and acceleration of the mass correspond to linear velocity and linear acceleration of a point on the circumference of the drive wheel. Peak values of the linear velocity $x^{(1)}_{m\ \max}$ and linear acceleration of a point on the circumference of the drive wheel $x^{(2)}_{m\ \max}$ can be expressed as follows:

$$x^{(1)}_{m\ \max} = \frac{u_{\max}\ r}{k_{tg}}; \qquad x^{(2)}_{m\ \max} = \frac{i_{\max}\ k_m\ r}{J + M\ r^2} \tag{11}$$

where:

- r – radius of the drive wheel,
- k_m – motor torque constant,
- k_{tg} – velocity-feedback gain factor,
- $u_{\max}$ – control voltage limit,
- $i_{\max}$ – larmature current limit,
- $x^{(1)}_{m\ \max}$ – peak value of the linear position derivative of a point on the circumference of the drive wheel,
- J – moment of inertia of the rotating masses (reduced to the motor shaft).

The armature current limit in a transient state is indirectly related to the limit of the second derivative of the linear position of a point on the circumference of the drive wheel. The differential equation of the drive transmission system can be expressed as:

$$M\, x^{(2)}(t) + c\, x^{(1)}(t) + k\, x(t) = c\, x^{(1)}_m(t) + x_m(t) \tag{12}$$

where:

- $x^{(1)}(t), x^{(2)}(t)$ – first and second derivatives (respectively) of coordinates specifying the position of the winding engine mass.

The architecture of the electromechanical system described above is depicted in Fig. 3.

The top branch of the electromechanical control system comprises a trajectory controller which is used to solve the inverse problem of dynamics, as well as a DC drive system with PI

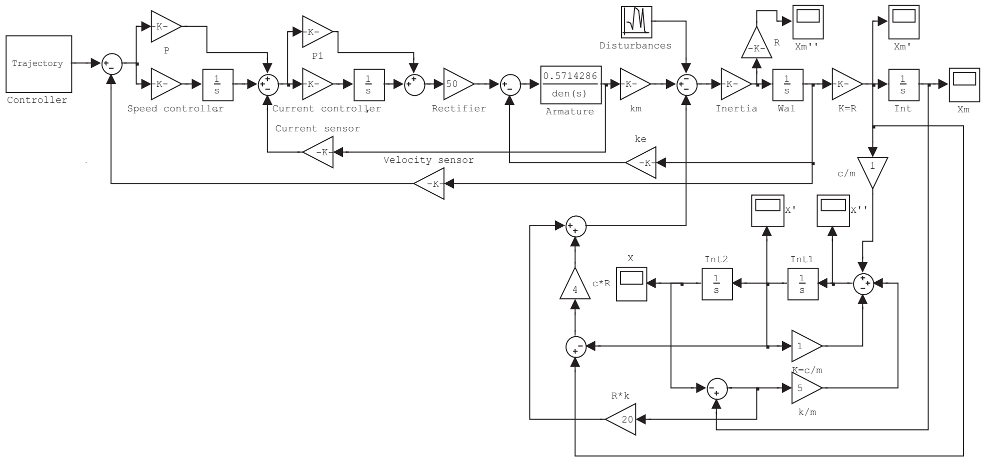

Fig. 3. The general structure of the model tested electromechanical system

speed and current controllers. The bottom part of the system includes a model elastic damping element of the winding engine with mass M. It should be noted that the natural frequency of the mechanical part of the system is approximately 2.25 Hz, whereas the time constant of the armature is 0.0211 s. The significant differences between the own frequencies of the motor and the driven mechanical system, and the excess power of the DC motor, make it possible to reduce the control problem to control over rotating masses connected to the rotor, linear motion of the winding engine mass and a point on the circumference of the drive wheel. As can be seen, fast transients in the motor control system are sufficiently quickly suppressed and do not have a significant influence on the dynamics of the winding engine. Taking this into account, and based on the limit condition (11) which should be achieved as soon as possible, we can build a computational algorithm which determines the desired trajectory of the winding engine mass. Subsequently, we can apply equation (10) and differential equation (12) to construct an algorithm for the inverse dynamic controller.

The velocity controller in the DC drive control system limits the rate of rise in armature current. Equation (12) imposes a limit on the second derivative of the winding engine mass acceleration. The technological limitations (11) placed upon regulator output signals, armature current, speed and acceleration of the winding engine mass result in additional conditions.

To satisfy the technological limits imposed on the armature current and voltage control (11), the second derivative of acceleration is set to the maximum value $x_{\max}^{(4)}$ in the interval $T_1 = \frac{x_{\max}^{(3)}}{x_{\max}^{(4)}}$ = 0.2 s. The duration of the stage of linear acceleration changes is consequently calculated as $T_2 = \frac{x_{\max}^{(2)}}{x_{\max}^{(3)}} - \frac{x_{\max}^{(3)}}{x_{\max}^{(4)}}$ = 0.05 s. In the startup stage, the need to minimize transient state duration results in the maximum speed being achieved in $T_3 = \frac{x_{\max}^{(1)}}{x_{\max}^{(2)}} - \frac{x_{\max}^{(2)}}{x_{\max}^{(3)}} - \frac{x_{\max}^{(3)}}{x_{\max}^{(4)}}$ = 4.0684 s. This determines the duration of the startup and braking stages when the DC motor produces the maximum permissible acceleration.

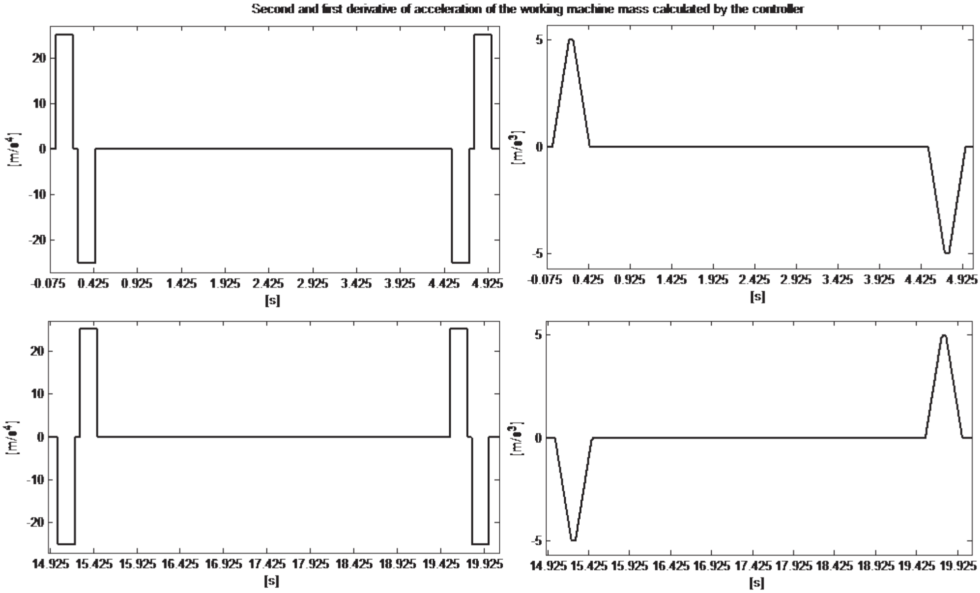

Fig. 4. The second and first derivative of the winding engine mass in the engine startup and braking stages, as calculated by the controller

Fig. 4 shows the first and second derivatives of the winding engine mass in the engine startup and braking stages. Calculations were performed using the trajectory controller, which implements the principle of inverse dynamics. In this way, we can obtain the desired waveforms of variables describing the motion of the winding engine, which define the components of the right side of differential equation (10) and allow us to proceed with its numerical integration. The waveform of the winding engine mass acceleration in the real system may deviate from the typical trapezoidal shape (dotted line) – the controller should therefore provide for a limitation of its second derivative (solid line), as can be seen in Fig. 5.

In order to calculate acceleration waveforms for the electromechanical transmission system, the controller implements the algorithm resulting from equation (12). The starting point for the calculation is provided by the computed acceleration of the winding engine mass and the initial condition imposed on the position of a point on the circumference of the drive wheel. The procedure consists of iterative numerical integration of equation (13), each time placing the integral calculated by the previous iteration on the right-hand side of the formula

$$x_m^{(1)}(t) = \frac{M}{c}\, x^{(2)}(t) + x^{(1)}(t) + \frac{k}{c}\, x(t) - \frac{k}{c} x_m(t) \qquad (13)$$

Fig. 5 shows the waveform of the control voltage signal, which was calculated using the inverse problem of dynamics. On the basis of the mathematical model of the electromechanical transmission system, the controller calculates the linear velocity waveform of a point on the circumference of the drive wheel. Subsequently, by applying the mathematical model

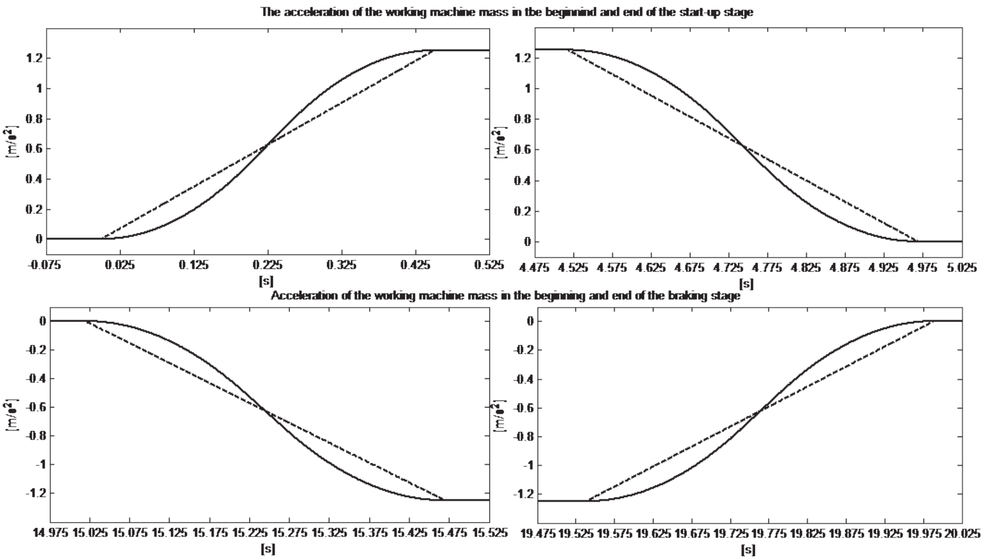

Fig. 5. Programmed waveform of the acceleration of the winding engine mass

of the DC motor control system, it calculates the control voltage signal. As can be shown, the controller modifies the motor speed waveform in stages corresponding to the assumed maximum allowed value of the second derivative of the winding engine acceleration coefficient:

– in the startup stage: 0.0–0.45 s and 4.55–5.0 s,
– in the braking stage: 15.0–15.45 s and 19.55–20.00 s.

Fig. 6 presents a comparison between the linear velocity waveforms of the winding engine mass and a point on the circumference of the drive wheel. The aim of the control process was to position the winding engine mass. This was achieved by appropriately shaping the engine speed waveform.

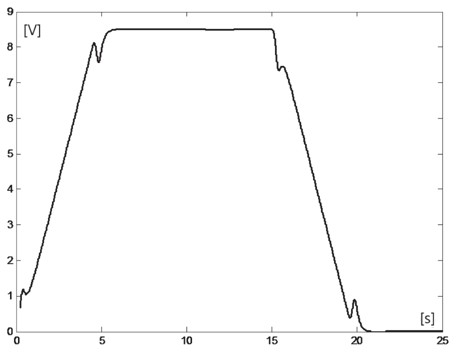

Fig. 6. Control voltage waveform calculated using the proposed algorithm

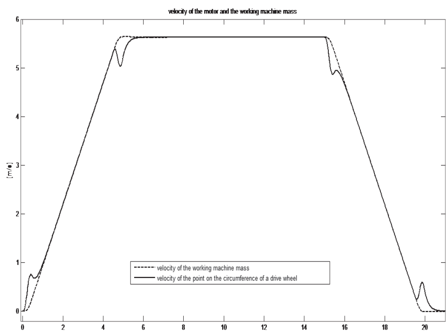

Fig. 7. Linear velocities of the winding engine mass and a point on the circumference of the drive wheel

Fig. 8 and Fig. 9 show the waveforms extending in the initial part of the startup stage (0.2–1.2 s) and in the final part of the startup stage (4.5–6.0 s). Fig. 10 and Fig. 11 show the waveforms extending in the early braking stage (14.8–16.2 s) and the late braking stage (15.9–20.7 s).

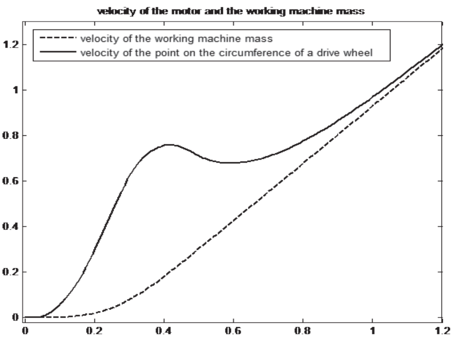

Fig. 8. Linear velocities of the winding engine mass and a point on the circumference of the drive wheel during the early startup stage

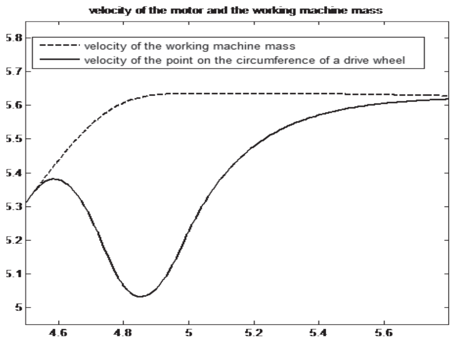

Fig. 9. Linear velocities of the winding engine mass and a point on the circumference of the drive wheel during the late startup stage

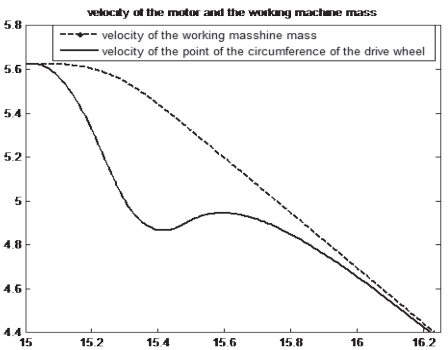

Fig. 10. Linear velocities in the early braking stage

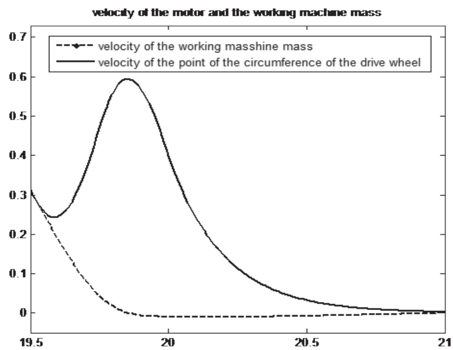

Fig. 11. Linear velocities in the late braking stage

Fig. 12 and Fig. 13 present a comparison between the linear acceleration of the winding engine mass and the linear acceleration of a point on the circumference of the drive wheel during the final stages of startup and braking. It can be shown that the values of the second derivative of the linear acceleration of the winding engine mass satisfy the limit conditions $-25.0\ \mathrm{m/s^4} \leq x^{(4)}_{M\ \max} \leq 25.0\ \mathrm{m/s^4}$.

Fig. 14–Fig. 17 show a comparison between the waveforms of the electromagnetic moment of the motor and the dynamic load torque on the motor shaft in the working cycle.

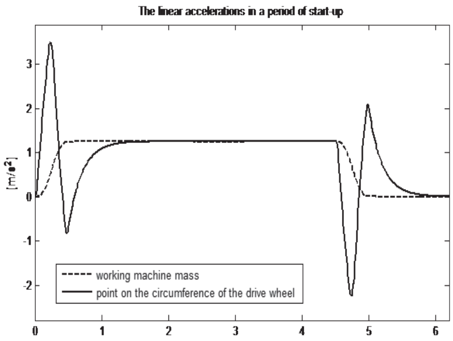

Fig. 12. Linear acceleration of the winding engine mass and a point on the circumference of the drive wheel during startup

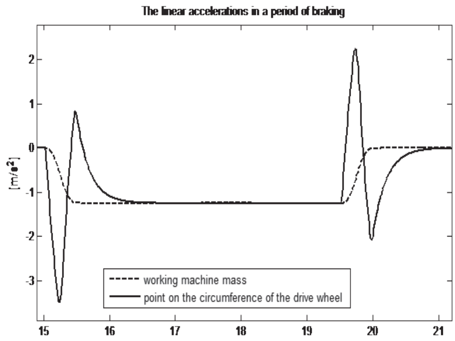

Fig. 13. Linear acceleration of the winding engine mass and a point on the circumference of the drive wheel during braking

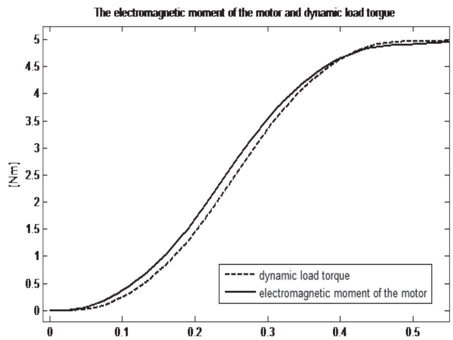

Fig. 14. Electromagnetic moment of the motor and dynamic load torque in the early startup stage

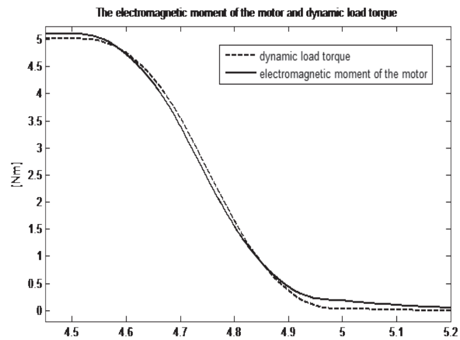

Fig. 15. Electromagnetic moment of the motor and dynamic load torque in the late startup stage

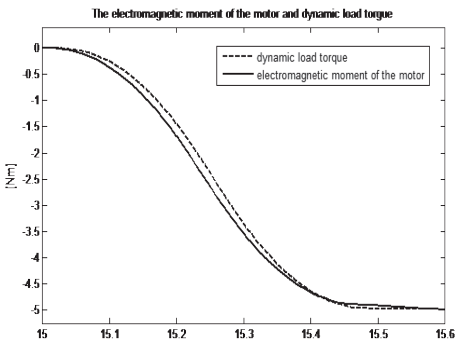

Fig. 16. Electromagnetic moment of the motor and dynamic load torque in the early braking stage

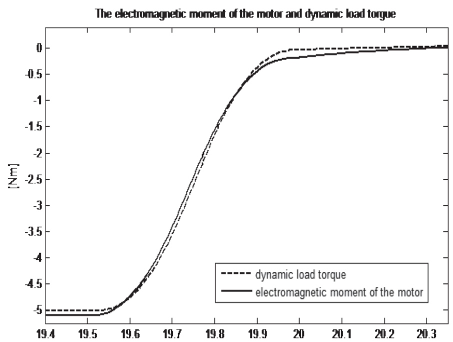

Fig. 17. Electromagnetic moment of the motor and dynamic load torque in the late braking stage

6. Conclusions

One of the major problems associated with the dynamics of electromechanical systems is deriving control variables from equations which describe the behavior of a physical system in terms of its motion as a function of time. The properties of motion of the electromechanical system can be computed in many different ways, one of which is by applying the inverse dynamics method [10, 11]. In this paper, the construction of the programmed motion of a winding engine is reduced to equations of motion under which the motion of the winding engine mass is characterized by minimum oscillations.

In order to solve the inverse problem of dynamics, the necessary and sufficient conditions for ensuring motion with specific properties were formulated. The motion properties of an electromechanical system whose state is defined by a vector of generalized coordinates and a vector of generalized velocities were given as a system of inequalities in compliance with technological requirements. The following interdependences were found in the scope of technological limitations. In a transient state, the motor velocity and winding engine mass limits depend on control voltage limits, while the motor and winding engine acceleration limits depend on armature current limits. It is assumed that the transient state should be as short as possible.

The trajectory of the programmed motion was calculated by assuming that the variables should reach their limits in the shortest time. From the equations of motion constructed on the basis of inverse dynamics, controlling variables and corresponding parameters were determined.

The inverse dynamics method is suitable for direct control over the motion of the winding engine mass in order to avoid negative dynamic effects. Nevertheless, disturbances in a real electromechanical system may result in the loss of control quality. If load torque disturbances are directly transmitted to the motor shaft, the presented engine control system compensates them well. When disturbances interact with the winding engine mass instead, and momentary values describing the dynamic state of this mass can be measured in real time, the correction block shown in Fig. 1 should be applied. Solving such problems is important in the context of hoisting machine control systems in the mining industry [6].

Similar results can be obtained by formulating the issue as a problem of finding a control law for a given system assuming a certain optimality criterion with a square indicator of quality. The task of the control system is then to find a control variable waveform $u_s(t)$ which will minimize energy losses during transients (e. g. startup or braking) at time t_s. The work performed by the electromagnetic torque of the motor during startup is equal to the sum of the final kinetic energy of the motor and the actuator, and the action of dissipative forces.

The concept of minimizing energy losses during startup may be replaced by minimizing losses associated with the work of dissipative forces. This leads to minimization of the following functional:

$$W_1 = \int_0^{t_s} c\left(x_m^{(1)} - x^{(1)}\right)^2 dt \tag{14}$$

where:

c – is damping factor,
T – is the startup duration,
$x_m^{(1)}$ – is the linear velocity of a point on the circumference of the drive wheel.

References

[1] Dwivedy S.K., Eberhard P., *Dynamic analysis of flexible manipulators, a literature review*, Mechanism and Machine Theory, Vol. 41, Issue 7, July, 2006, 749-777.

[2] Ider S.K., *Inverse dynamics of parallel manipulators in the presence of drive singularities*, Mechanism and Machine Theory, Vol. 40, Issue 2, January, 2005, 33-44.

[3] Jarzębowska E., *Model-Based Tracking Control of Nollinear Systems*, CRC Series: Modern Mechanics & Mathematics, CRC Press, Taylor & Francis Group, 2012.

[4] Galiullin A.S., *Inverse Problems of Dynamics*, Mir Publishers, Moscow 1984.

[5] Zając M., *Wybrane zagadnienia syntezy sterowania programowego układów elektromechanicznych*, Rozwój zautomatyzowanych układów napędu elektrycznego, Komitet Elektrotechniki PAN, PWN, 1992, 129-139.

[6] Zając M., *New Metod of Mine Winder Control in Deep Shafts*, Prace Instytutu Elektrotechniki, Vol. 151, 1988, 81-93.

[7] Calanca A., Capisani L.A., Ferrara A. et al., *An Inverse Dynamics-Based Discrete-Time Sliding Mode Controller for Robot Manipulators*, Robot Motion and Control, 2007, 137-146.

[8] Barai R.K., Dey A., *Sliding mode compensation of inverse dynamics velocity control*, 12th Int. Workshop on Variable Structure Systems (VSS), 2012, 155-160.

[9] Morales B., Carelli R., *Robot control with inverse dynamics and non-linear gains*, Latin American Applied Research, Vol. 33, No. 4, Oct./Dec. 2003, 393-397.

[10] Kuo A.D., *A Least-squares Estimation Approach to Improving the Precision of Inverse Dynamics Computations*, Transactions of the ASME, Journal of Biomechanical Engineering, Vol. 120, Feb. 1999, 148-159.

[11] Kato O., Sugiura I., *An interpretation of airplane general motion and contol as inverse problem*, Journal of Guidance, Control, and Dynamics, Vol. 9, No. 2, 1986, 198-204.

[12] Venture G., Kojima T., Tagawa Y., *Fast Motion Control of Robotics Systems Using Inverse Dynamics via 'Simulation of Feedback Control Systems' (IDCS)*, The 1'st Joint International Conference on Multibody System Dynamics, May 25–27, 2010, Lappeenranta, Finland.

[13] Wach P., *Dynamics and control of Electrical Drives*, Springer-Verlag, 2011.

[14] Крутько П.Д., Лакота Н.А., Попов Е.П., *Конструирование алгоритмов управления приводных систем по методу обратных задач динамики*, Доклады Академии Наук СССР, Кибернетика и теория регулирования, 1982, 1086-1089.

[15] Крутько П.Д., *Обратные задачи динамики управляемых систем. Линейные модели*, Наука 1987.