TECHNICAL TRANSACTIONS | **CZASOPISMO TECHNICZNE**
CIVIL ENGINEERING | BUDOWNICTWO

2-B/2013

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SHEAR CAPACITY OF PRESTRESSED HOLLOW CORE SLABS ON FLEXIBLE SUPPORTS

NOŚNOŚĆ NA ŚCINANIE SPRĘŻONYCH PŁYT KANAŁOWYCH NA PODPORACH PODATNYCH

Abstract

It is widely believed that reduction of unfavorable effects of shear stress in the HC slab's web can be achieved by filling the cores with concrete or through the arrangement of the monolithic layer or concrete topping. The *fib* guidelines are practically the only document which allows for determining the shear capacity of HC slabs, including the influence of concrete topping or core filling – a description of this design model and the calculation analysis are presented in the paper. In order to determine the effect of concrete topping on the behaviour of prestressed HC slabs on flexible supports, a number of calculation analyses were performed. The results of the calculations and the conclusions resulting from it are presented in the paper.

Keywords: Hollow Core, flexible support, Slim Floor, concrete topping, precast slabs

Streszczenie

Powszechnie uważa się, że zmniejszenie niekorzystnego wpływu stycznych naprężeń ścinających w żeberku płyty można osiągnąć poprzez wypełnienie betonem kanałów w skrajnych fragmentach płyty HC lub ułożenie monolitycznej warstwy nadbetonu. Wytyczne fib są praktycznie jedynym dokumentem pozwalającym określić nośność stropów SF z uwzględnieniem wpływu nadbetonu lub wypełnienia kanałów – w artykule opisano ten model obliczeniowy. W celu określenia wpływu nadbetonu na pracę sprężonych płyt HC na podporach podatnych, wykonano szereg analiz obliczeniowych, których wyniki, wraz z wnioskami z nich wynikającymi, przedstawiono w niniejszym artykule.

Słowa kluczowe: Hollow Core, podpora podatna, Slim Floor, nadbeton, płyty prefabrykowane

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1. Slim Floor Structures

Slim Floors (SF) are structures made of Hollow Core (HC) slabs supported on slender beams where the beam height is usually slightly greater than the height of the precast slab element.

When designing these structures, the fact that deformation of the slabs occurs with the increase of deflection of supports should be taken into account. As a consequence, a complex stress state arises in the slabs – additional transverse normal and shear stress appears in the precast unit (Fig. 1). HC slabs tend to move alongside the beams. On the other hand, the adhesive forces and friction between the slab ends and the beam tend to prevent this displacement which generates incidental stress (Fig. 2). This leads to cracking of the joint between the filling concrete and the beam or between the filling concrete and the slab end. Opening of these cracks reduces the stiffness of the connection and eventually, shear flow between the slab end and the beam is transmitted mainly by the interface between the beam and the soffit of the slab [1].

If the HC slabs are supported so low on the beam that bending of the beam gives rise to transverse tensile stress at the bottom of the slab, the soffit of the slab tends to crack longitudinally. Longitudinal cracks reduce the transverse bending stiffness of the slabs.

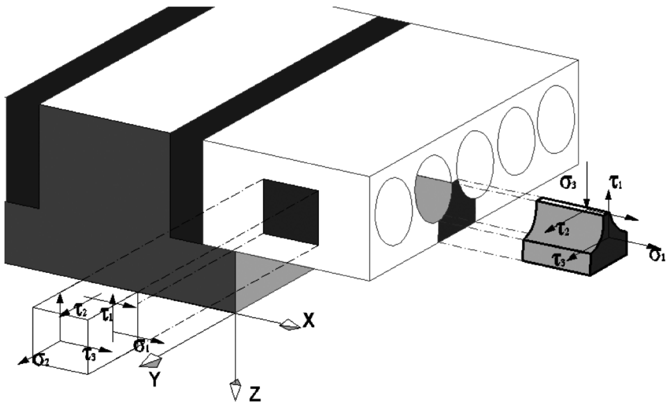

Fig. 1. Stress components in HC web

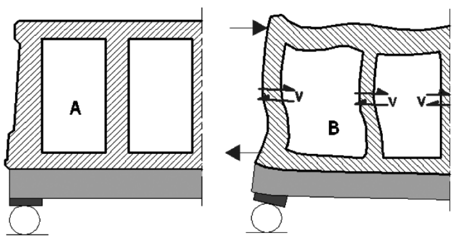

Fig. 2. Deformation of the HC cross-section as a result of deflection of the beam: A – before deflection, B – frame effect

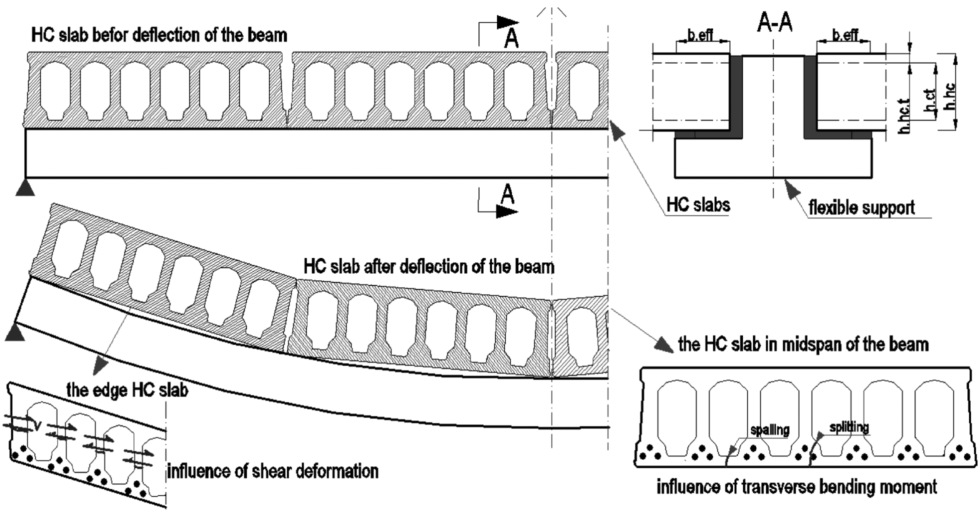

Fig. 3. Behavior of HC slabs on the flexible support

Sometimes the cracks under grow along the strands. It reduces the bond between the strands and the concrete – this has a negative effect on the shear capacity of the slab.

As a result of the deflection of the beam, the difference in work schema of the HC elements, depending on its position along the beam, appears (Fig. 3). The element in the midspan of the beam is subjected mainly to transverse bending moments. While the deflection of the beam occurs, the HC slabs are supported only in the area of their outermost webs [2]. The edge slabs are subjected to shear deformation, which is the main reason for reducing the shear capacity. On the basis of a series of experimentally full-scale tests [3], it has been shown that the failure of the floor always begins from the outermost slabs. The results of the transverse loads state in the SF structures are cracking and shear failure of the HC slabs in their weakest area – non-reinforced edge webs and reduction of the shear capacity [6].

Despite the frequent implementation of these types of structures, the existing standard EN 1168 [7] gives only a brief record of the need to take into account the reduction in design shear capacity, but does not give any calculation procedure [5] may be useful for the design of these types of structures. These recommendations are based on a series of studies carried out in Finland [4] and were created on the basis of the calculation model by Leskelä and Pajari [8]. It is a model of the composite beam and HC slab. This model is a development of the concept of HC elements based on rigid supports and is based on a classical Euler-Bernoulli beam theory. The model for the rigid supports is based on the assumption that the HC elements failure mechanism is due to shear [2, 9].

2. Concrete topping in composite beam model

Reduction of unfavorable effects of shear stress τ_2 in the HC slab's web can be achieved by the applied technological treatments e.g., filling the cores of the outermost slabs with concrete or arrangement of the monolithic layer or concrete topping.

The effects of these treatments are included in the model given in *fib* recommendations [5] by multiplying the basic shear stress component τ_2 by the reduction factor β_f (for filling cores) and β_{top} (for concrete topping). The failure criterion is achieving the tensile strength value of concrete f_{ctd} by the main stress σ_1 ($\sigma_1 < 0$).

$$\sigma_{ps} = \frac{\sigma_1}{2} + \sqrt{\left(\frac{\sigma_1}{2}\right)^2 + \tau_1^2 + \left[\beta_f\left(\tau_{2,top} + \beta_{top} \cdot \tau_{2,imp}\right)\right]^2} \tag{1}$$

where:

- $\tau_{2,top}$ – shear stress component of the weight of topping concrete,
- $\tau_{2,imp}$ – shear stress component of the additional loads.

Normal compression stress σ_1 and transverse shear stress τ_1 (vertical) are given by the formulae described in (2) and (3):

$$\sigma_1 = -\frac{P_t}{A} + \frac{-P_t \cdot e_p + M_t}{I_y} \cdot z \tag{2}$$

$$\tau_1 = \frac{V_z \cdot S_y}{I_y \cdot b_w} \tag{3}$$

A more difficult issue is the identification of transverse shear τ_2 (horizontal) in the HC web, emerging from the longitudinal compressive transverse shear flows ν in the composite cross-section, consisting of the beam, the concrete grouting of the joint and the upper flange of HC slab, working at the length b_{eff} (4).

$$v = \frac{e_{sl.top} \cdot (\mathrm{EA})_{sl.top}}{(\mathrm{EI})_{com}} \cdot V_b \tag{4}$$

where:

- $(\mathrm{EA})_{sl.top}$ – axial stiffness of the whole upper flange,
- $e_{sl.top}$ – centroidal distance of the top flange of the HC slab from the centroid axis of the composite cross-section,
- EI_{com} – bending stiffness of the composite cross-section,
- V_b – shear force of the beam due to imposed load.

On the basis of the compressive transverse shear flows, the value of the transverse shear stress τ_2 can be determined. The model acknowledges τ_2 at the length $x = b_{cr}$, as the average value of the stress for the h_{sl} distance.

$$\tau_2 = \frac{3}{2} \cdot \frac{v \cdot b_{sl}}{2 \cdot b_w \cdot x} = \frac{3}{4} \cdot \frac{v \cdot b_{sl}}{b_w \cdot b_{cr}} \tag{5}$$

where:

- b_w, b_{sl} – sum of webs' width in the HC slab, width of the HC slab,
- $v \cdot b_{sl}$ – transverse shear force from the slabs supported on both sides of the beam,
- $2 \cdot b_w \cdot x$ – cross-section of HC webs supported on both sides of the beam.

The influence of concrete topping is taken into account by a reduction factor given by the formula:

$$\beta_{top} = \frac{\upsilon_{web}}{\upsilon_{top} + \upsilon_{web}} = \frac{\frac{0.5 \cdot F_{web}}{\Delta x}}{\frac{0.5 \cdot F_{top}}{\Delta x} + \frac{0.5 \cdot F_{web}}{\Delta x}} = \frac{F_{web}}{F_{top} + F_{web}} \leq 1.0 \quad (6)$$

where υ_{top} and υ_{web} are horizontal flow shear strength, carried by the reinforced concrete topping and slab' webs. The sum of the impacts of these streams reduces the total shear stress, given by the formula $\upsilon = 2\Delta Nx/\Delta x$ (Fig. 4a) [11].

Value of the shear flows υ_{web} and υ_{top}, could be calculated by the formula:

$$\upsilon_{web} = \frac{4}{3} \cdot \frac{b_{w.sl}}{b_{sl}} \cdot b_{cr} \cdot \tau_2 \quad (7)$$

$$\upsilon_{top} = \min\left[\left(\frac{A_{sv} \cdot f_{yk}}{s} \cdot \mu\right), \left(0.2 \cdot f_{ck} \cdot h_{top}\right)\right] \quad (8)$$

In analogy, design formulas for the F_{top} and F_{web} forces can be written, which are the longitudinal shear capacity per unit length beam in the vertical plane of the composite action between the beam and slab elements, divided respectively for concrete topping and for the slab's webs:

$$F_{web} = \frac{4}{3} \cdot L_e \frac{b_{w.sl}}{b_{sl}} \cdot \sqrt{2} \cdot \frac{f_{ctm}}{\beta_f} \quad (9)$$

$$F_{top} = \min\left[\left(2 \cdot L_e \frac{A_{sv} \cdot f_{yk}}{s} \cdot \varpi\right), \left(0.2 \cdot f_{ck} \cdot h_{top} \cdot L_e\right)\right] \quad (10)$$

In the above formulae, L_e represents the span of the considered element in the direction of the longitudinal axis of the girder, $b_{w.sl}$ is a total width of webs, and b is the width of the precast element. A_{sv} and f_{yk} are respectively the cross section area and yield strength of reinforcing bars in concrete topping, spaced along the beam with a distant of s. Not only reinforcement in concrete topping layer, but also its thickness, surface roughness and cracked concrete strength affect the value of the coefficient β_{top}. The *fib* guidelines assumed the constant coefficient of friction with a value equal to $\mu = 2.0$ [5].

β_{top} – reduction factor in concrete topping – reduces the shear stress τ_2 when the minimum reinforcement in concrete topping are bars with a diameter of minimum 5 mm spaced at 150 mm and the reinforcement is so anchored to take the tensile force that appears after cross scratches [3]. Based on the FEM analysis [6], it was found that the typical reinforcement in concrete topping (grid 15 × 15 mm, ø = 6 mm) increases the value of the horizontal shear force in the webs by 1–3%, and a strong reinforcement in concrete topping (10 × 10 mm mesh, diameter = 10 mm) with 5–12%.

The continuous reinforcement in the area of the support in concrete topping introduces a level of unintended fixing on the support plates which can lead to cracking (Fig. 4b). Type A cracks are not dangerous, but crack types B and C reduce the load plate shear HC. If the model of the composite beam with a layer of unreinforced concrete topping is used with regard to the same effective width of plates without concrete topping as the HC b_{eff}, additional concrete layer can negatively affect shear capacity on the plates. It seems probable that the reinforcement in the concrete topping layer, parallel to the axis of the plate HC, can reduce the deformation of elements in the direction of the girder, thereby positively affecting the shear capacity [6]. In the light of these studies, for slabs with longer spans, a concrete topping layer is a better option to increase the shear capacity of the HC element than filling the cores. Although topping generates additional load, it increases the stiffness of the slab in the SLS [8]. Positioning of the concrete topping layer is particularly often used for HC slabs with large spans.

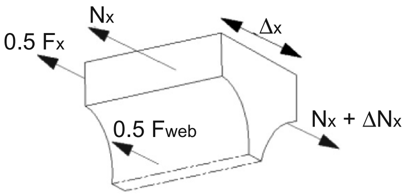

Fig. 4a. Shear strengths balanced the influence of the shear stresses

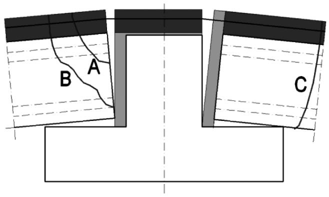

Fig. 4b. Cracking in the HC slab due to unintentional clamping moment in the support

By providing an adequate adhesion of the top surface of the concrete slabs to the concrete topping, one can achieve a positive impact on the reduction of transverse shear stress in the web and an increase in the capacity of the shear elements. However, the presence of a concrete topping can also have negative consequences, i.e. by increasing the surface of the compressed flange, increasing the total flow of the longitudinal tangential stresses [8].

3. Parametric design analysis

In order to determine the impact of the various parameters of the design model described above, a large number of calculations were performed.

The subject of the first analysis was to determine the effort HC slab (expressed by the ratio of principal stresses σ_{ps} to the design value of the tensile strength of concrete f_{ctd}) for a variety of cooperation between the HC element and the concrete topping: HC without any topping, full composite action between the HC and the reinforced concrete topping, full composite action between the HC and the unreinforced concrete topping and the topping layer treated only as a ballast. For example, the results obtained for the slab HC 320 are shown in Fig. 5.

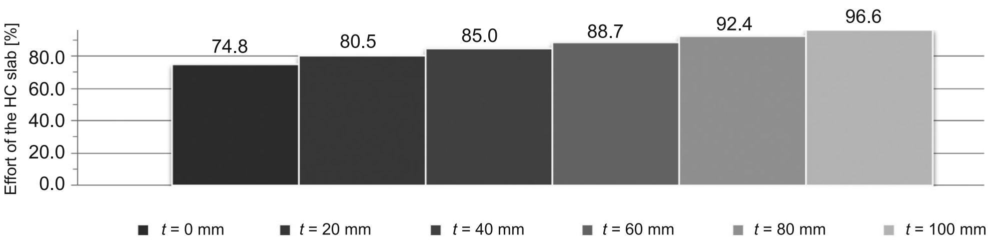

Fig. 5. Effort of the HC slab (320 mm) according to the thickness of concrete topping

Moreover, in the next calculations, the influence of the thickness of the concrete topping (variable from 0 mm to 100 mm) on the effort of the composite slab, as well as the effect of the value of the coefficient of friction between the upper edge of the HC element and the topping, were considered – see Fig. 6. The impact of the order of concreting the vertical joints and laying the concrete topping has been also analyzed. To estimate the influence of the topping reinforcement ratio on the effort of the HC slab's web, other calculations were made. The results are presented on Fig. 7.

The summary of the results for different types of HC elements, working in various conditions is given on Fig. 8–11.

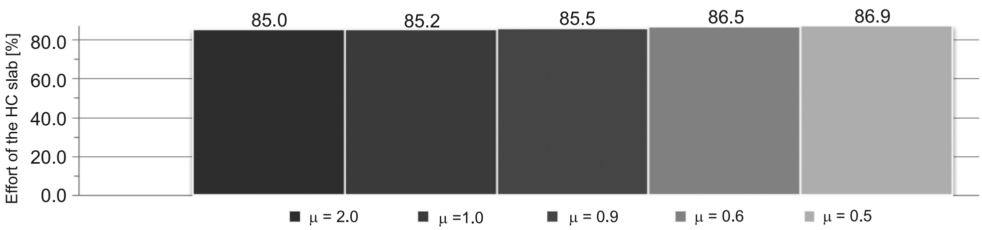

Fig. 6. Effort of the HC slab (320 mm) according to the value of the friction coefficient between the concrete topping and the precast unit

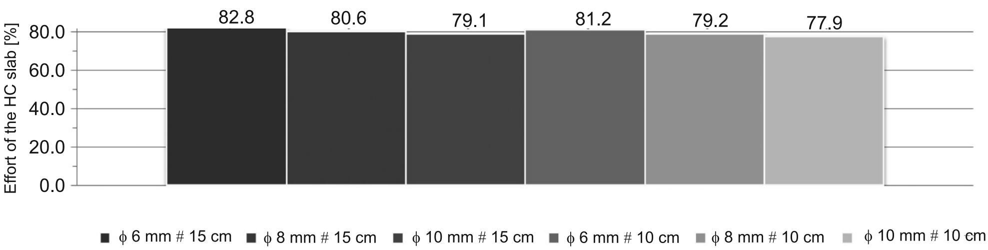

Fig. 7. Effort of the HC slab (320 mm) according to amount of supplementary reinforcement in the concrete topping

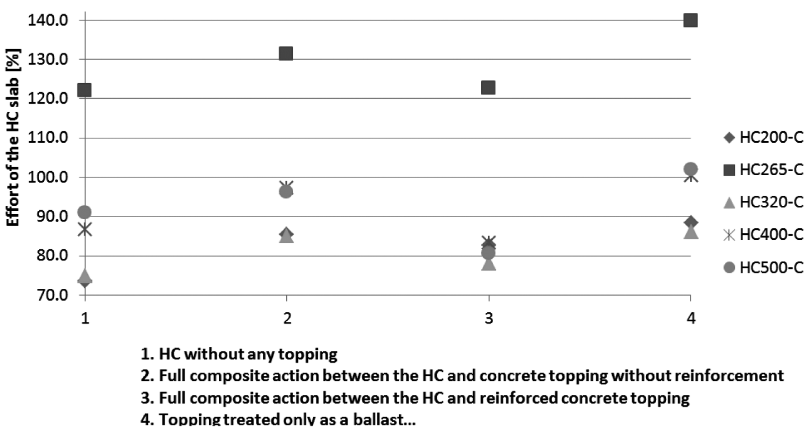

Fig. 8. Effort of the different types of HC slabs with a small number of the webs according to different behaviour of the topping and precast unit

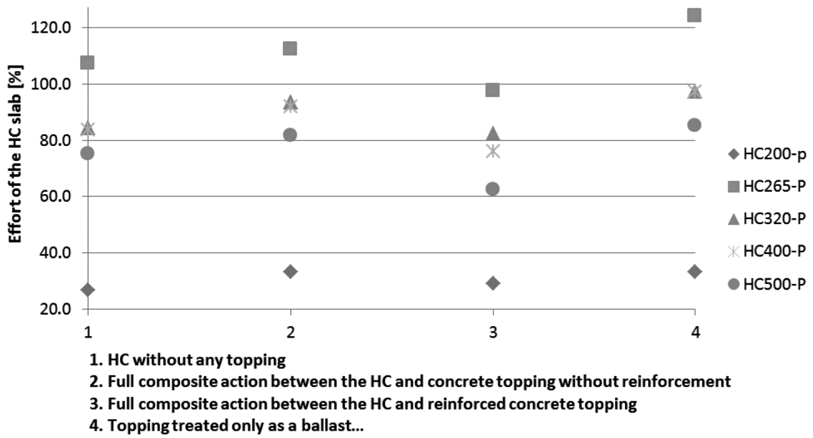

Fig. 9. Effort of the different types of HC slabs with a large number of webs according to different behaviours of the topping and precast unit

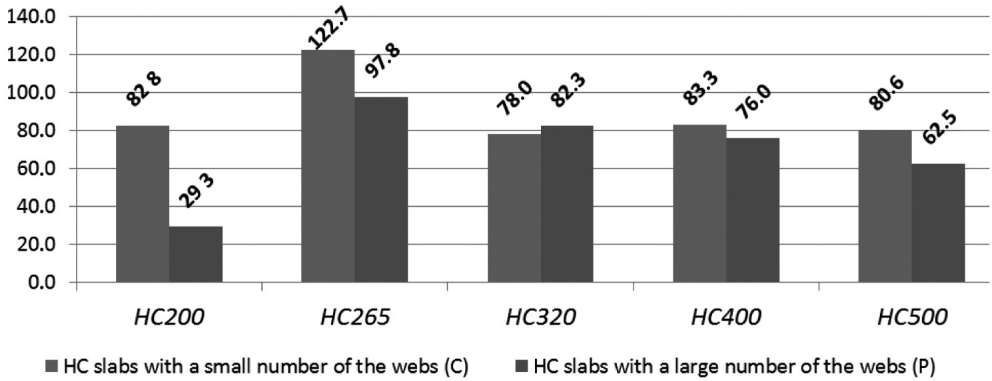

Fig. 10. Effort of the different types of HC slabs with small and large numbers of webs with topping layer

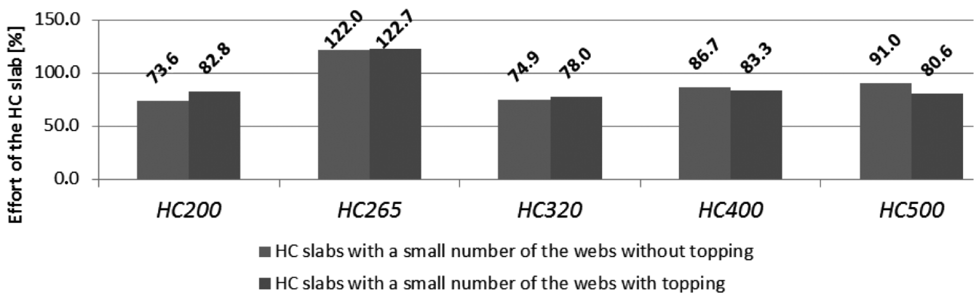

Fig. 11. Effort of the different types of HC slabs with a small number of webs with and without topping layer

4. Discussion of conducted studies

The fib recommendations are based on a series of studies, carried out in Finland by M. Pajari, which were accompanied by a quite gently worded conclusion: “If the composite beam model is applied using the same effective width as for slabs without topping, the topping has a negative effect on shear capacity. It is possible that a reinforcement parallel to the slab units in the concrete topping may have a positive effect on the shear capacity of HC slabs by reducing shear deformation of HC slab in the direction of the beam” [6].

Only one experimental floor specimen was tested out within the scope of the Finnish studies mentioned above for HC slabs with reinforced concrete topping (HC 265 with a topping height of 60 mm). A comparison of the test results obtained for HC slabs with and without concrete topping showed that a topping layer increased shear capacity of the HC slab ca. 50% [12].

Experimental research carried out in Finland and Germany (1990–2009) for the HC slabs without topping showed that these elements supported on a slender beams have a lower shear capacity than one on rigid supports and the difference of their effort reaches 30–60% [13].

On the basis of computational analyses which were carried out using calculation procedures set exactly in the *fib* recommendations, the following conclusions can be drawn:

- The effort of the HC webs under shear increases with the addition of a concrete topping layer and is aggravated by an increase in its thickness. Although the fact of the incidental loads from self-weight of the topping is indisputable, it seems that the increase of the cross-sectional dimensions of the composite beam should lead to a decrease in its effort. It should be noted that in his research, M. Pajari [6] drew attention to the negative consequences of laying the topping, such as increasing the compression zone resulting in an increase in the shear longitudinal stream in the web.
- Analysis of the influence of the coefficient of friction between the concrete topping and the upper surface of the precast element (varying from 0.5 to 2.0) showed the negligible effect of this parameter on the slab’s effort, such as the impact of the order of the vertical joints concreting and pouring of the topping layer.
- The concreting sequence of vertical joints between the slab and the beam is coincident with the expectation, but it does not substantially affect the slab’s effort.
- The modification (reduction) of the effective width of the topping on precast elements due to the different classes of concrete of precast slabs and the topping layer does not significantly affect the effort of the slab.

By modifying the reinforcement work described in F_{top} formula (10), by selecting the maximum and not the minimum value of the two given functions, the following conclusions can be drawn:

- The increase in the reinforcement ratio for the topping slightly improves the working conditions of the composite beam. The topping reinforcement may reduce the deformation slab in the direction of the beam, thereby positively affecting its shear capacity. The relationship between the height of the slab and the type of its cross-sectional shape and the increase in shear capacity due to the presence of reinforced topping can be observed. For slabs with a small number of webs and circular cores, the topping layer reduces the effort only for elements with a height higher than 400 mm. However, for slabs with a large number of webs and slender cores, a positive effect of topping is already observed in the elements with a height higher than 265 mm.
- Assuming a simplified cross-section geometry of the precast element, especially for slabs with circular cores, is not meaningless for the calculation result. Shear capacity decreases for the

bigger flange height, but at the same time the topping has a positive impact and slightly increases the shear capacity of the slab. This behavior is observed for all types of HC elements.

5. Conclusions

The results obtained from the analytical calculations, which took into account the presence of the concrete topping, do not fully correspond to the common belief about the positive impact of the topping, as well as to the results of experimental tests [6]. The conducted analysis showed also that the shape of the element's cross-section strongly influences its shear capacity. The increase in the reinforcement ratio for the topping slightly improves the working conditions of the whole slab.

According to the authors of this paper, there is a justifiable need for further experimental research in the field of Slim Floors working with topping and for attempts to modify the design model described for such structures.

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