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THE PILE STIFFNESS IN A PILED-RAFT FOUNDATION

SZTYWNOŚĆ PALA W FUNDAMENCIE PŁYTOWO-PALOWYM

S t r e s z c z e n i e

Artykuł przedstawia próbę zastosowania teorii sprężystości do oceny sztywności pala w fundamencie zespolonym płytowo-palowym w celu oszacowania osiadań konstrukcji. Zaprezentowano dwie metody obliczenia: uproszczony model komputerowy oraz rozwiązanie analityczne. Wyniki skonfrontowano z rezultatami badań polowych.

Słowa kluczowe: fundament płytowo-palowy

A b s t r a c t

The paper presents the trial of the theory of elasticity application to estimate pile stiffness in a combined piled-raft foundation. Two methods were applied: a simplified numerical model and closed-form analytical solution. Results were compared with the field tests.

Keywords: combined piled-raft foundation

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Symbols

K_r – raft stiffness [MN/m]
K_p – pile group stiffness [MN/m]
K_{FPP} – combined stiffness of a piled raft [MN/m]

1. Introduction

Because of difficult ground conditions some areas in the city centres stayed undeveloped, despite their convenient location. This is the case of Warsaw's Żoliborz glacial tunnel valley. Lack of land in the city centre and development of foundation technologies resulted in increasing interest of developers in sites located in this area. One of such investments was the inspiration and basis for this article.

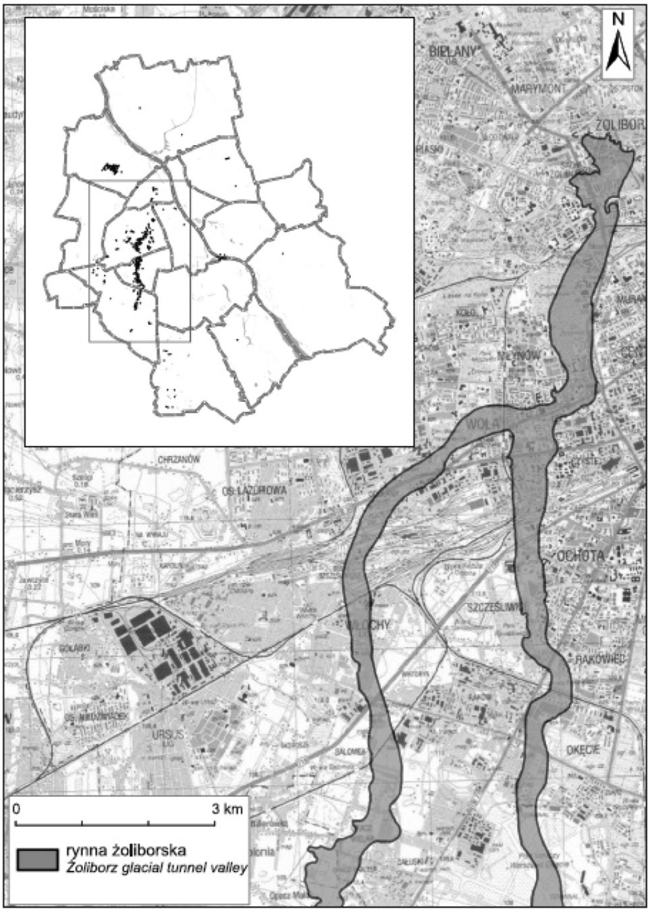

Fig. 1. Extents of Zoliborz glacial tunnel valley in Warsaw [1]

Development of tall buildings on weak soil requires taking into account both load bearing capacity (Ultimate Limit State) and settlement values (Serviceability Limit State). Raft foundation provides sufficient bearing capacity, but does not resolve the problem of strains exceeding maximum allowable values, or uneven settlement.

Typical solution is to design additional pile supports to transfer load directly to deeper, stronger soil layers. In conventional design procedures it is assumed that total load has to be transferred by the piles – contribution of raft to foundation bearing capacity is ignored. The design of piled-raft foundation takes into consideration contribution of both piles and raft to the total foundation stiffness, leading to more economical design than traditional approach.

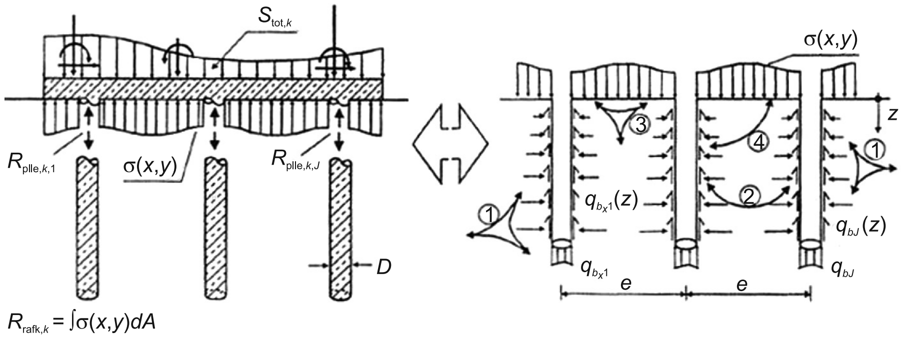

Fig. 2. Soil-structure interaction effects for a piled-raft foundation [2]

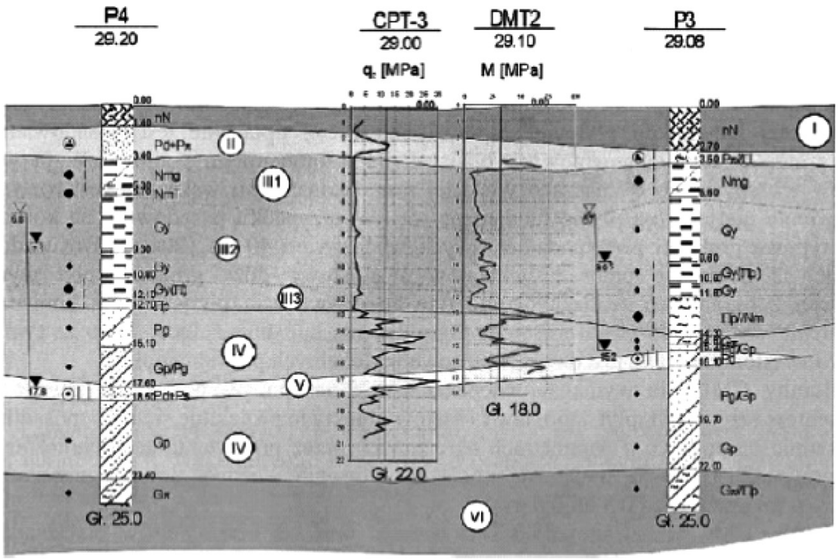

Fig. 3. Geotechnical cross-section presenting subsoil layers [2]

Unfortunately, neither former Polish standard PN-83/B-02482, nor European standard EN 1997 provides designers with guidelines concerning estimation of settlement of piled-raft foundation. Taking into account interaction of piles, raft and soil is a complicated task, indeed, as it has not been fully investigated, despite thorough research.

Of course it is possible to build a complete model of soil and piled-raft structure to be analysed with Finite Element Method. Theoretically, calculated displacements and stresses are more precise than computed by other methods. However, it should be noted that numerical errors and difficulties connected with modelling of soil behaviour may significantly influence reliability of the results, while complexity of a model extends time of modelling itself as well as the following calculation.

The paper presents alternative methods: analytical estimation of piled-raft stiffness and simplified model consisting of a plate on elastic supports, analysed then in a popular computer program ABC Płyta. Both methods require estimation of raft stiffness and single pile stiffness. Raft stiffness is quite easy to calculate; the major problem is how to estimate pile's behaviour.

As results of field tests are available [3], it is possible to compare results of theoretical modeling with real values. Geometry, load value and soil parameters applied in analysis comply with physical model and results of geotechnical research.

2. Raft stiffness

Raft stiffness may be defined as a ratio of imposed stress to resulting settlement. Provided modulus of compressibility of the soil layers are known, consolidation under predicted load may be easily estimated.

Modulus of compressibility of the layer of gyttja was estimated in CPT and dilatometer tests to be approximately 15 MPa. The total estimated settlement of a 10-meters thick layer of compressible organic soils under 120 kPa load equals 40 mm. The total raft stiffness is:

$$K_r = \frac{120\,\mathrm{kPa} \cdot 25\,\mathrm{m}^2}{40\,\mathrm{mm}} = 75\,\mathrm{MN/m}$$

It can also be represented as stiffness of the soil below the foundation:

$$K_r = \frac{120\,\mathrm{kPa}}{40\,\mathrm{mm}} = 3\,\mathrm{MPa/m}$$

3. Pile stiffness

Determining stiffness of a pile requires deriving mathematical relation between imposed load and settlement. Traditional approach bases on calculating maximum bearing capacity and resulting settlement, but leads to linear stiffness of a pile support, which does not comply with load-settlement curve observed in field loading tests. Furthermore, the estimated stiffness is lower than real. In usual range of loads full bearing capacity will never be mobilized and the pile stiffness will be greater. According to Gwizdała [9], pile reaches its limit load at settlement of 0.10 D (D is a pile diameter), while in fact the range of loads imposed is usually much lower than maximum values.

Even though it is well-known that soils do not behave as elastic materials, the theory of elasticity is a useful tool to calculate stresses and displacement in soil masses. The key requirement is to define elastic "constants" for an appropriate range of stress in analysed soil mass [6]. According to tests, elastic "constant" for ground have no constant values – modulus G and E depend on current state of stress and strain in soil. It can be observed in triaxial compression test, when one of the results is $E(\varepsilon)$ relation.

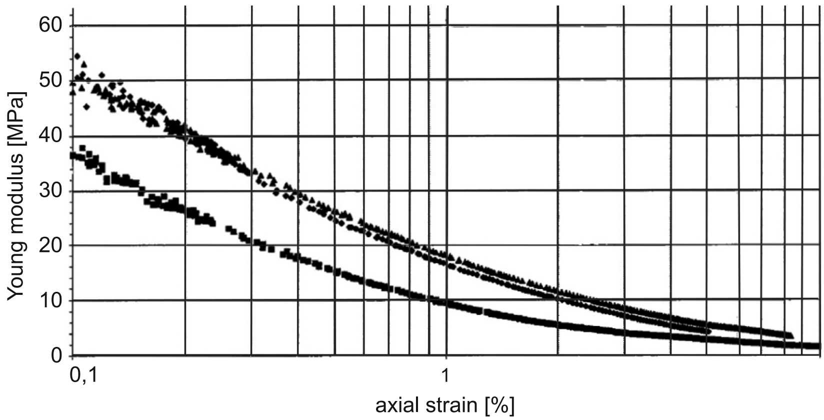

Fig. 4. $E(\varepsilon)$ relation – laboratory triaxial compression tests result

Possible solution of pile head settlement was derived by Randolph and Wroth, presented in [6]. The solution applies to a single pile in a homogenous soil, with a shear modulus increasing linearly with depth.

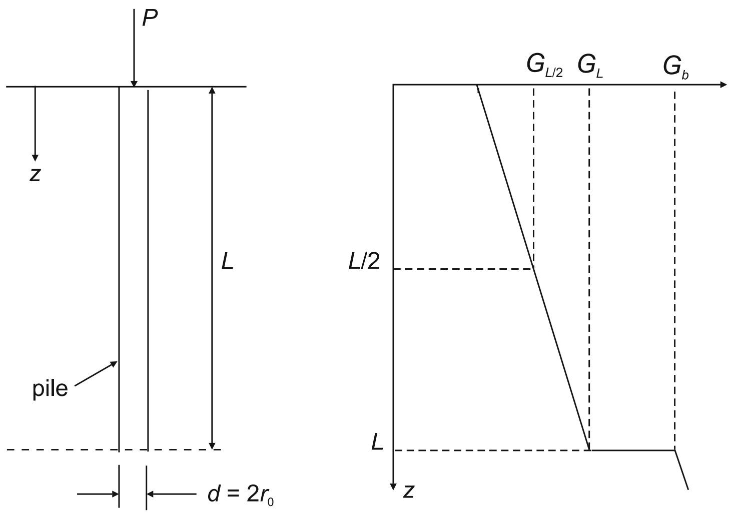

Fig. 5. Single axially loaded end bearing pile [6]

The pile head settlement d is given by a following formula:

$$\frac{P}{\delta r_0 G_L} = \frac{\dfrac{4\eta_r}{(1-\nu)} + \dfrac{2\pi\rho}{\zeta}\dfrac{\tanh(\mu L)}{\mu L}\dfrac{L}{r_0}}{1+\dfrac{1}{\pi\lambda}\dfrac{4}{(1-\nu)}\dfrac{\eta_r}{\xi}\dfrac{\tanh(\mu L)}{\mu L}\dfrac{L}{r_0}} \tag{1}$$

where:

- P – applied load,
- r_0 – radius of pile shaft,
- r_b – radius of pile base,
- G_L – shear modulus of soil at depth L,
- G_b – shear modulus of soil at pile base (in end bearing piles),
- v_v – Poisson' ratio of soil,
- $\eta_r = r_b/r_0$,
- $\xi = G_L/G_b$,
- $\rho = G_{L/2}/G_L$,
- $\lambda = E_p/G_L$.
- $\mu L = \frac{L}{r_0} \cdot \sqrt{\frac{2}{\zeta\lambda}}$,
- $\zeta = \ln\left(\left[0,25 + \left(2,5\rho(1-\nu) - 0,25\right)\xi\right] L / r_0\right)$.

Solution is derived from a differential equation of pile compression [10]:

$$(EA)_p \frac{d^2 w(z)}{dz^2} - k(z)w(z) = 0 \tag{2}$$

where:

$$k(z) = \frac{\pi B q_s}{w(z)} \tag{3}$$

is a Winkler constant for depth z. It should be noted that equation (2) is a simplified one; it ignores influence of soil compression around the pile shaft.

In a homogenous soil, a modulus k is constant along the pile shaft.

Poisson's ratio was assumed according to [7] to be 0.5 for undrained and 0.3 for drained conditions. It is claimed that laboratory tests do not give reliable values of Young's modulus to be used in case of pile analysis – it should be back-calculated on the basis of pile load tests. However, for the need of theoretical analysis, results of triaxial tests were implemented.

Unfortunately, analysis of Young's modulus was limited by range of triaxial test results, as the solution is provided only for strain exceeding 0.1%. Evaluation of smaller strains is possible using geophysical techniques and the results of such tests were not available yet. Relation between E and ε was therefore determined as an exponential function. For the full range of strain hyperboloid functions are more appropriate [9]. Iterative calculations based on equation (1) resulted in a non-linear curve of pile head displacement plotted against imposed axial load. Lacking part of the plot is connected with displacements too small to be measured with triaxial testing machine equipment.

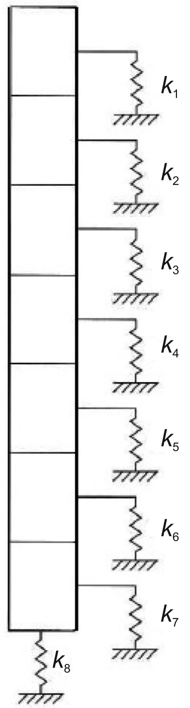

Fig. 6. Discretization of the soil along the pile shaft by Winkler springs [10]

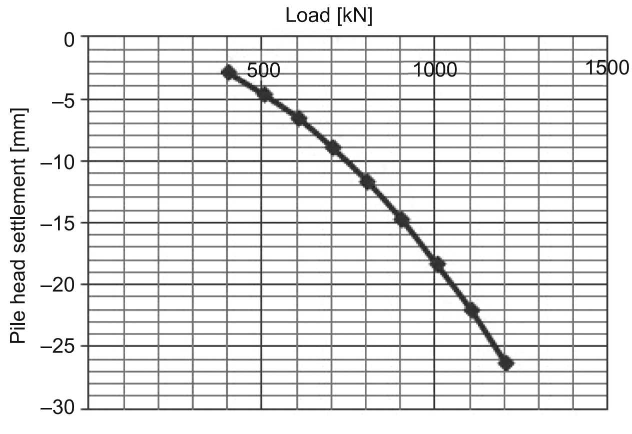

Fig. 7. Load-settlement curve for a single pile

In a piled-raft foundation loads are transferred to the soil by both piles and raft. Therefore pile head settlement equals raft settlement, a result of compression of soil layers below. Majority of settlement results from weak organic soil compression – underlying silt has insignificant effect in total settlement value. ε value was determined as a ratio of pile head settlement to total thickness of gyttja layers.

As presented in the plot (Fig. 7), stiffness of a pile has no constant value. To reflect behaviour of soil, a computer model should consider non-linearity of Young's modulus or compressibility modulus of the elastic solid. However, as stated above, the range of allowable strains is limited to 0.10 D, while in engineering practice the value is limited to c.a. 0.05 D. Taking 40 cm piles and 10 m layer of gyttja, calculated maximum strain equals merely 2 cm/10 m = 0.002 = 0.2%.

4. Modelling – ABC Płyta program

Two created models reflected geometry of the physical models constructed at site. Raft dimensions were 5.0 × 5.0 × 0.3 m – one of the rafts was supported by nine CMC columns (diameter: 0.4 m) arranged in a grid of 2.0 × 2.0 m.

Piles were introduced as single springs (assumed displacement was around 2–3 cm), and soil was characterized by Winkler coefficient 3 MPa/m.

Results of modeling are presented in Table 1, along with field tests' results. Figure 8 presents reaction forces in all of the piles.

Table 1

Settlement values in field tests and computer modelling in ABC Płyta

	Physical model		Computer model (ABC Płyta)	
	raft	piled raft	raft	piled raft
settlement [mm]	25 (16–35)	5.5 (3.9–8.6)	40	3

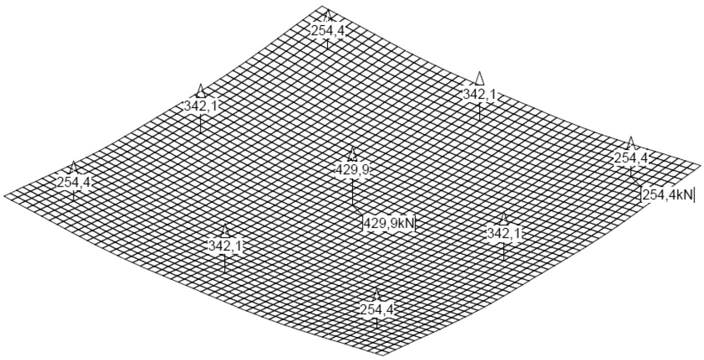

Fig. 8. Reaction forces in piles under 120 kPa load

Settlement calculated proves good correspondence with real values. Results of field tests exceeded displacements estimated in the program – this may be assigned to ignoring block deformation of the foundation in a computer model. In case of a pile group, soil between the piles moves together with the whole piled-raft structure. Therefore effective friction along pile shafts is reduced, especially in the middle piles. Load bearing capacity of the columns is limited and contact pressure below raft increased, leading to increased settlement values. Trial of estimation of the influence of group work on a pile capacity was presented by Katzenbach in [8].

5. Analytical solution

Another method of estimating settlement of a piled-raft is based on calculating total stiffness of the foundation using simple functions of raft and pile stiffness defined separately. One of the solutions derived is the one by Randolph (Kacprzak [3]). Total foundation stiffness is defined as follows:

$$K_{FPP} = \frac{1-0.6\cdot(K_r / K_p)}{1-0.64\cdot(K_r / K_p)}\cdot K_p \tag{4}$$

where:

K_r – raft stiffness [MN/m],
K_p – pile group stiffness [MN/m],
K_{FPP} – combined stiffness of a piled raft [MN/m].

Pile group stiffness may be calculated as product of single pile stiffness and coefficient $n^{0.5}$, where n is number of piles in the group. Estimated raft stiffness is K_r = 75 MN/m and pile group stiffness $K_p = 143 \cdot 9^{0.5}$ MN/m. Therefore total piled-raft stiffness equals 432 MN/m and settlement associated with 3MN load – 7 mm.

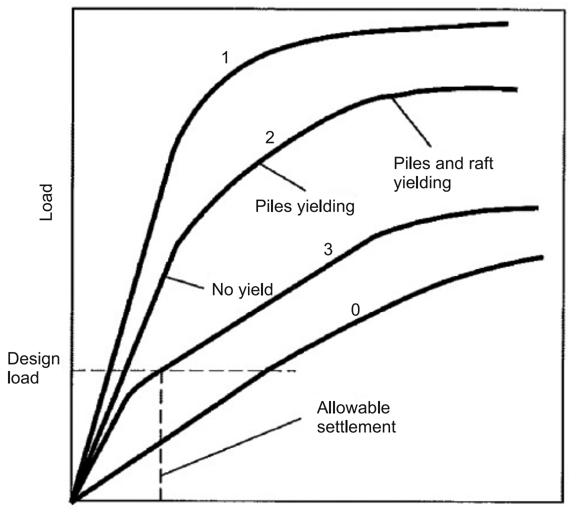

Fig. 9. Load-settlement curves depending on design approach: 0 – raft without piles, 1 – conventional piles, 2 – possible interaction of piles and raft in load transmission (safety factor for piles lowered to 1,5), 3 – full mobilization of piles' bearing capacity, full interaction [3]

Table 2

Settlement values – comparison

	Physical model	Computer model (ABC Płyta)	Analytical solution (Randolph)
settlement [mm]	5.5 (3.9–8.6)	3	7

Randolph has also provided a closed-form solution estimating load distribution between piles and raft:

$$\beta_p = \frac{1}{1+\alpha} \tag{5}$$

where:

$$\alpha = \frac{0,2}{1-0.8(K_r / K_p)} \cdot \frac{K_r}{K_p} \tag{6}$$

Results of modelling and calculation are compared with field tests in Table 3.

Table 3

Load distribution – % of load transmitted by piles

Physical model	Computer model (ABC Płyta)	Analytical solution (Randolph)
92–97%	93.9%	96.7%

6. Conclusions

Final comparison of the results of pile stiffness obtained in theoretical analysis with values measured in field load tests are presented in Tables 2 and 3. Both computer model and analytical calculation give results similar to field tests. However, it can be noticed that numerical solution results in lower value of settlement. This may be assigned to ignoring influence of the group work (introduced in Randolph's solution).

Methods presented above are easy to apply in practice and the results are close to real values measured in field test. Even though the solution provided is restricted to homogenous soil along the pile shaft, it is a common scheme for piled-raft foundation. As there is an assumption to design piles for full utilization bearing capacity (safety factor = 1), in most cases columns may reach only to the roof of the stronger layers below the single weak layers of similar low parameters. However, more field research on full-sized structures should be conducted, especially to analyse influence of superstructure's stiffness on soil-combined foundation interaction.

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