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DYNAMIC ANALYSIS OF THE PEDESTRIAN AND CYCLIST FOOTBRIDGE BETWEEN KAZIMIERZ AND LUDWINOW QUARTER IN KRAKOW

ANALIZA DYNAMICZNA KŁADKI PIESZO-ROWEROWEJ KAZIMIERZ–LUDWINÓW W KRAKOWIE

Abstract

The paper is dedicated to issues relating to the dynamics of footbridges. It includes the basic information about the dynamic characteristics of pedestrian bridges – the frequencies and modes of vibrations, stiffness, and damping. An essential part of this work is the dynamic analysis of the Kazimierz–Ludwinow footbridge in Krakow with a span of 127.20 m. The analysis was completed on the basis of a numerical model in the Autodesk Algor Simulation 2011 (Student Version) package.

Keywords: footbridge, dynamics, footbridge vibrations

Streszczenie

W artykule omówione zostały podstawowe zagadnienia związane z dynamiką mostów dla pieszych. Praca zawiera aspekty teoretyczne tego szerokiego zagadnienia oraz ich zastosowanie praktyczne w inżynierii lądowej. Zostały omówione podstawowe pojęcia z teorii drgań, które bezpośrednio przekładają się na analizę dynamiczną kładek. W dalszej części pracy została zaprezentowana analiza dynamiczna kładki pieszo-rowerowej Kazimierz–Ludwinów w Krakowie obciążonej dynamicznym oddziaływaniem użytkowników. Postacie drgań własnych zostały wyznaczone przy użyciu programu Autodesk Algor Simulation 2011 (Student Version).

Słowa kluczowe: kładki dla pieszych, dynamika kładek dla pieszych, komfort wibracyjny

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1. Introduction

1.1. The footbridges – basic requirements

The pedestrian bridges commonly called footbridges are the structures whose primary task is to carry pedestrians over a physical obstacle [12]. Currently, in addition to the safe conduct of traffic on the other side of the obstacle, pedestrian bridges are for their users, a kind of opportunity to enjoy the advantages of the environment with which they are associated.

Pedestrian bridges should be designed according to recommendations of the standards which require an analysis of the superstructure in the two limit states: ultimate limit state (ULS) and serviceability limit state (SLS).

The requirements of the ultimate limit state include: exhaustion of the bearing capacity for critical cross-sections of the structure; loss of stability of elements or the whole structure; material fatigue in elements or connections. The requirements of the serviceability limit state include checking of the span deflection and vibration of the structure [2]. In addition, objects in public spaces such as footbridges, should meet the requirements of the Regulations of the Minister of Transport and Maritime Economy [13, 14].

1.2. The dynamics of footbridges – basic issues

Footbridges significantly differ from the conventional bridges in how they influence the users. The footbridges users are located directly on the object deck, staying there longer than during traveling by car across the bridge and directly feels its behavior. Users are exposed to a greater feeling of structural behavior. At the design stage, the footbridges require more attention to ensure the proper functional features and comfortable use.

In addition to static analysis, a dynamic analysis is also required. The primary objective of the dynamic analysis is to determine the effect of dynamic loads on the structure’s dynamic response and its influence on users. Making dynamic analysis, it should be verified that the proposed design provides a sufficiently high level of comfort. Dynamic calculations of civil engineering structures are carried out usually on a discrete model of the real system (model with lumped masses). The equation of motion has the form [6]:

$$M\ddot{q}(t) + C\dot{q}(t) + Kq(t) = P(t) \tag{1}$$

in which:

M, C, K – successively: mass matrix, damping matrix and stiffness matrix,
$q(t)$ – displacement vector,
$P(t)$ – dynamic load vector (external force).

An important stage in the dynamic analysis of the structure is modal analysis [7] that is, calculation of the mode shapes and natural vibration frequency of the structure. In modal analysis, the system of equations for the free non-damped oscillatory motion should be solved [6]:

$$M\ddot{q}(t) + Kq(t) = 0 \tag{2}$$

Next, based on the results of the preliminary analysis, it is necessary to analyze the possibility of the occurrence of resonance vibrations.

In the case of pedestrian bridges, the critical natural frequencies are within the ranges: 1.3 Hz ≤ f_i ≤ 2.3 Hz (in case of vertical vibration); 0.5 Hz ≤ f_i ≤ 1.2 Hz (in case of horizontal vibration).

1.3. Dynamic loading of footbridges

Pedestrian users impact on the construction of the bridge in a dynamic way. The user generates a load in three directions: vertical; horizontal; longitudinal [1]. An essential element of the dynamic analysis of a footbridge is to identify the frequency range of pedestrian impact.

The frequency of pedestrian impact for running, walking or jumping is within the range 1.4–3.4 Hz [1]. The basic assumption relating to the dynamic force generated by humans is their periodic character. The dynamic load vector can be represented as a Fourier series [1]:

$$F_z(t) = G + \sum\nolimits_{i=1}^{n} G\alpha_i \sin\left(2\pi i f_k t - \varphi_i\right) \tag{3}$$

where:

G – the pedestrian's weight (700–800 N).
φ_i – the Fourier's coefficient of the *i*-th harmonic.
f_k – the frequency of pedestrian activity (walking, running or jumping).
$P(t)$ – dynamic load vector (external force).
i – the number of the harmonic.
n – the total number of harmonic components.
φ_i – the phase shift of the 2nd or 3rd harmonic respectively, with respect to the 1st harmonic.

1.4. Approximate methods for estimation of impact of the dynamic load on footbridges

The approximate method for determining the impact of a single pedestrian whose stepping frequency is equal to the basic natural frequency of the vertical vibration of the structure is given in the standards BS 5400 78 (UK) and ONT 83 (Canadian).

For the footbridge which number of spans does not exceed three, the maximum vibration acceleration $a_{\max}$ is:

$$a_{\max} = 4\pi^2 f_1^2 z_{st} K \Psi \tag{4}$$

where:

z_{st} – the deflection (caused by static load, 700 N) in the middle of the span.
K – the configuration coefficient.
Ψ – the dynamic response coefficient.

In order to determine the dynamic influence of more than one pedestrian on the dynamic behavior of the footbridge, the results obtained for a single user $a_{1\max}$ should be multiplied by an increasing factor of M. The value of M factor depends on the type of vibration excitation and size of the pedestrians group N_g:

- synchronized group of footbridge users walking, running or jumping: $M = N_g$,
- random impact of a free stream of pedestrians: $M = \sqrt{N_g}$.

Intentional and malicious vibration excitations are not taken into account in the design of footbridges. It should be noted that there is a real risk of its occurrence on footbridges. It is advisable to take into account these loads during footbridge superstructure designing and stress analysis [8].

2. The Kazimierz–Ludwinow footbridge in Krakow – dynamic analysis

2.1. Description of the structure

The Kazimierz–Ludwinów pedestrian and bicycle footbridge (Fig. 1) was designed between Inflancki Boulevard (Kazimierz side) and the Wolynski Boulevard (Ludwinow side) near the closed Sofitel (Forum) Hotel in Krakow. The architectural project of the footbridge was realized by the Lewicki&Latak Design Office from Krakow. The structural project was prepared by Research and Design Team Mosty–Wroclaw. The project is very interesting both in terms of its architecture and structure.

Fig. 1. Visualization of the footbridge Kazimierz–Ludwinow in Krakow

The footbridge was designed to live load according to PN-85/S-10030 standard and a vehicle of 3.5 tons total weight as an exceptional load condition [4].

The structural system includes: main span (127.20 m) consisting of two external box girders and internal "sine"-shaped box girder, abutment from the Ludwinow side; abutment and ramps from the Kazimierz side. In the plan view, the ramps are designed parallel to the bank of the Vistula River.

External box girders are load-carrying superstructure. They are designed of structural steel S355J2G3 and S460NL. The rigidity of the girders is variable along the length of the span. In accordance with the brief foredesign, the girders are fixed in abutments. Both girders have the same cross-section and are located symmetrically in relation to the longitudinal axis of the footbridge. The deck width on the external box girders is 4.0 m and is constant over the entire length of the footbridge. The height of the girders changes smoothly to 15.79 m.

The internal girder, so-called "sine", has a cross-sectional trapezoidal shape and is designed from the same steel as exterior girders. In the area of the abutment P1 (Ludwinow side), the internal girder is connected with abutment flexibly by means of the articulated nondisplaceable joint (pined joint). In the area of the abutment P2 (Kazimierz side), the girder is fixed in the abutment and the “sine”-shaped deck is lengthened by stairs descending to Kurlandzki Boulevard at the level of the bank of the Vistula River. Cooperation of external and internal girders is ensured by steel hangers, pillars and transverse beams.

The abutments were designed with architectural reinforced concrete and founded on large diameter reinforced concrete piles (1.50 m). Abutments are also integrated with the ground by means of ground anchors with multiple injected anchor bulb Anchors and piles are inclined at a ratio of 5:1.

2.2. Dynamic analysis of the Kazimierz–Ludwinow footbridge

In order to perform the dynamic analysis of the footbridge, the 3D beam FEM model using Autodesk Algor Simulation 2011 (Student Version) has been prepared. The model included the basic elements of the real system – two symmetrical exterior girders, sinusoidal internal girder pillars, hangers and transverse beams connecting and stiffening the girders.

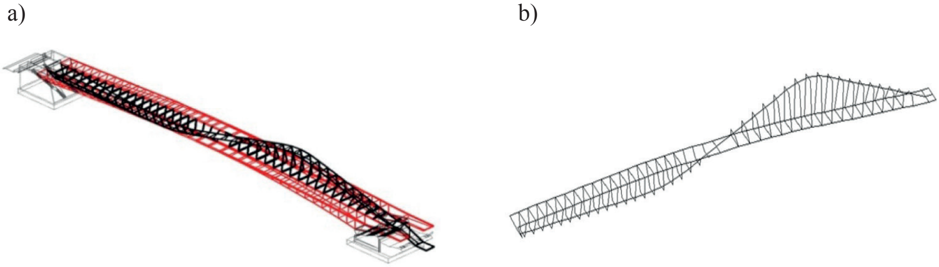

Fig. 2. Footbridge Kazimierz–Ludwinow a) axonometric view of the real structure [10], b) axonometric view of the beam computational model

All parts of the main span were modeled as beam elements. The symmetrical exterior girders and “sine” – shaped girder were modeled as beam elements, lying on the line running through the center of gravity of the girders cross-section. Pillars, hangers and transverse beams were modeled by beam elements placed along their axes in their center of gravity. The model consists of 215 nodes and 326 beam elements.

The footbridge girders were divided into 42 sections. The geometrical characteristics of the sections were calculated in accordance with the dimensions of the girder cross-section assumed in the brief foredesign [4]. In analysis, a linear elastic material model was adopted. The steel parameters were adopted in accordance with the parameters of the structural steel S355J2G3 and S4G2A.

As a result of the modal analysis, the natural frequencies and mode shapes of the footbridge were obtained. In Table 1 (below) basic frequencies less than 3.4 Hz (frequency in the range of the users dynamic action) are presented.

In order to estimate the vertical dynamic response of the structure due to pedestrian traffic, the third and fourth mode shape were selected for further analysis (these forms of vibrations are the most dangerous for the analyzed vertical vibration of the footbridge deck). Other vertical mode shapes were outside the critical range of considered pedestrian dynamic action or their form was complex and resonant vibrations have small probability of occurrence or in the event of occurrence the vibration were quickly damped by users walking with different frequencies on the further part of the footbridge deck). It should be mentioned that the further analysis should be performed for the second horizontal mode shape with frequency $f_2 = 1.14$ Hz within the range of frequency of the horizontal action of walking people. These calculations were beyond the scope of the analysis performed for the purposes of this article. In Fig. 3 analyzed third and fourth mode shapes of the footbridge are presented.

Table 1

Natural vibration frequencies (in the range of the users dynamic action)

No. of mode shape	Frequency [Hz]	Period [s]	Type of mode shape
1	1.05	0.95	Vertical symmetric vibration of the side and the middle (sinusoidal) decks (one extreme)
2	1.14	0.88	Horizontal symmetric vibration of the side and the middle (sinusoidal) decks (one extreme)
3	2.06	0.49	Torsional asymmetric vibration of the middle (sinusoidal) deck (two extrema) and vertical symmetric vibration of the side decks in anti-phase (one extreme)
4	2.38	0.42	Vertical asymmetric viration of the side and the middle sinusoidal decks (two extrema)
5	2.94	0.34	Vertical symetric vibration of the side and the middle (sinusoidal) decks (three extrema)
6	2.98	0.34	Torsional asymmetric vibration of the middle (sinusoidal) deck (two extrema) and vertical asymmetric vibration of the side decks in anti-phase (two extrema)

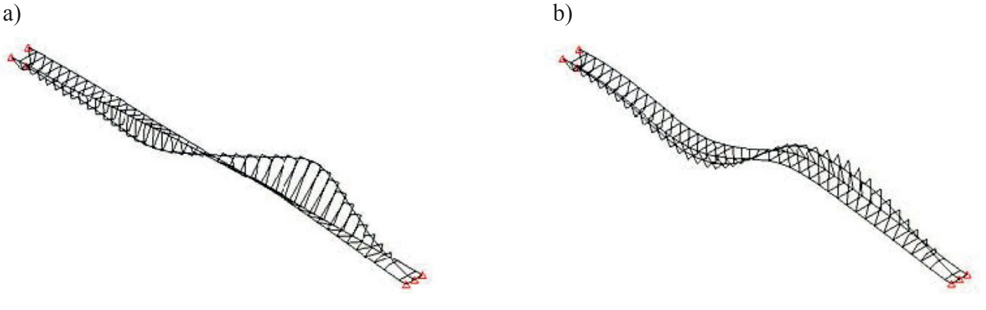

Fig. 3. Selected vertical mode shapes of the footbridge: a) third mode shape $f_3 = 2.06$ Hz, b) fourth mode shape $f_4 = 2.38$ Hz

The value of the vertical vibration acceleration was estimated according to equation (4). The deflection z_{st} caused by a static load of 700 N applied to the middle of the footbridge span is z_{st} = 0.077 mm. For K = 0.7, Ψ = 15 the vibration acceleration during a dynamic action of a single pedestrian equals:

$$a_3 = 4\pi^2 f_3^2 z_{st} K\Psi = 0.135\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right],\quad a_4 = 4\pi^2 f_4^2 z_{st} K\Psi = 0.169\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]$$

Below is an example of estimation of the value of vibration acceleration caused by more than one user the group of 25 users was adopted $N_g = 25$. The following cases were considered:

- unsynchronized pedestrians group:

$$a_{3.25un} = Ma_3 = \sqrt{25}a_3 = 0.677\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right],\ a_{4.25un} = Ma_4 = \sqrt{25}a_4 = 0.844\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]$$

- synchronized pedestrians group:

$$a_{3.25s} = Ma_3 = 25a_3 = 3.386\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right],\ a_{4.25s} = Ma_4 = 25a_4 = 4.221\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]$$

The results should be compared with the requirements of the vibrational comfort criteria.

Based on PN-EN 1990/A1, the limit value of vertical vibration acceleration of $0.7\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]$ was accepted. It should be noted that vertical vibrations with an acceleration of $0.7\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]$ can be felt by walking users (and very clearly by standing users) but in the case of very rare occurrences, can be accepted as permissible. Full comfort of the footbridge use will be ensured for vertical vibration acceleration less than $0.5\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]$. The results of comfort estimations are given below:

- in the case of resonant dynamic action of one pedestrian, for both analyzed natural frequency (3 and 4 mode shapes), the requirements of the vibrational comfort criteria for the vertical vibrations are fulfilled.
- in the case of dynamic action of a non-synchronized group of 25 pedestrians in case of third, torsional mode shapes f_3 = 2.06 Hz the requirements of the vibrational comfort criteria for the vertical vibrations are fulfilled. In the work the analysis of the horizontal vibration acceleration was not carried out, for the fourth natural frequency (f_4 = 2.3 Hz)
- the vertical vibration acceleration exceeds the accepted as permissible value of $0.7\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]$
- which indicates a high risk of disturbance of comfort of use of the footbridge, particularly on the “sine” – shaped deck. It should be noted that the vibration acceleration is obtained by extrapolation of the acceleration determined for case of a single pedestrian walking at a frequency of 2.3 Hz. This frequency corresponds to slow running (jogging). The occurrence of five synchronized running users (M = 5), although possible, is very rare (with low probability of occurrence) especially on the “sine” – shaped deck. Therefore, the comfort of use of the footbridge can be considered as fulfilled. A more likely case of two or three

runners does not lead to the accepted limit of vibrational comfort criteria for the vertical vibrations being exceeded. However, further analyses of the dynamic action of running users are required.

- in the case of dynamic action of a fully synchronized group of 25 persons, the requirements of the vibrational comfort criteria for the vertical vibrations were significantly exceeded (by about 6 times). However, the case of 25 synchronized pedestrians on the footbridge deck is a very rare with a very low probability of occurrence. It can be assumed that under everyday use, the comfort of using the footbridge will not be disturbed. It should be pointed out that this kind of pedestrian traffic should be forbidden on lightweight footbridges.

3. Conclusions

The results obtained during analysis were compared with the results of two analyses of the footbridge provided by the Lewicki and Latak Project Office presented in [4] and [9]. After comparison of the dynamic characteristics of the footbridge, all stages of modeling can be considered as having been performed correctly and effectively.

The natural frequencies obtained during the described analysis are within the range of the results obtained by other authors presented in Tab. 2. It should be noted that the first mode shape of vibration does not occur in the quoted results [4, 9]. It can be caused by the fact that compared dynamic analyses were carried out on various mechanical models – both with regard to the model shape, the number of the finite elements and the number of nodes. The results of modal analyses are affected by parameters related to the inertia and stiffness of the structure, which are directly changed by the simplifications adopted in the computational model (both in terms of the geometry and parameters of the material of the real system). Additional explanations for the differences in the compared results are the different mass distribution for translational and rotational degrees of freedom in both models and the different way of modeling an eccentric connections of the structural member. For a more accurate estimation of the vibration frequency and mode shapes of the footbridge, the 3D model of type e^2, p^3 (two-dimensional elements in three dimensional space) should be prepared. For the purpose of the preliminary estimation of the comfort of use of the footbridge, the accuracy of the results of the vertical vibration frequency obtained by using the described simplified computational model were accepted as sufficient.

The results of the analysis should be considered as the forecasted level of vibration accelerations encumbered with an error resulting from the estimation of the vibration damping parameter (fraction of critical damping) which also affects the coefficient Ψ in equation (4) (dynamic response coefficient).

In the analysis, the vibration damping level and the associated dynamic response coefficient from equation (4) was adopted as recommended by the literature for typical steel beam footbridges. The parameter of the vibration damping in the analyzed footbridge, due to a unique form of the construction, can be higher. In this case, the risk of instances of exceeding the acceptable level of vibration acceleration will be favorably reduced.

Further analyses of running persons and analyses of the horizontal vibrations occurring in horizontal and torsional mode shapes are required.

Table 2

The dynamic characteristics of the footbridge [9]

No. of mode shape	Frequency [Hz]	Period [s]	Description
1	1.15	0.87	Horizontal symmetric vibration of the side and the middle (sinusoidal) decks (one extreme)
2	1.20	0.83	Vertical symmetric vibration of the side and the middle (sinusoidal) decks (one extreme)
3	2.62	0.38	Vertical asymmetric viration of the side and the middle (sinusoidal) decks (two extrema)
4	2.67	0.37	Torsional asymmetric vibration of the middle (sinusoidal) deck (two extrema) and vertical symmetric vibration of the side decks in anti-phase (one extreme)
5	2.96	0.33	Vertical symetric vibration of the side and the middle (sinusoidal) deck (three extrema)
6	3.36	0.29	Torsional asymmetric vibration of the middle (sinusoidal) deck (two extrema) and vertical asymmetric vibration of the side decks in anti-phase (two extrema)

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