MATHEMATICAL MODEL OF THYRISTOR’S SYSTEM CONTROL OF DC MOTOR WITH INDEPENDENT EXCITATION

MODEL MATEMATYCZNY TYRYSTOROWEGO SYSTEMU STEROWANIA SILNIKIEM PRĄDU STAŁEGO Z NIEZALEŻNYM WZBUDZANIEM

Abstract

The result of the research in this article is a mathematical model of thyristor’s system control of DC motor with independent excitation, which includes containing controlled three-phase rectifier. The nonlinear characteristics of transformer’s magnetization was being taken into account. Tyristors’ work in shown by means of a scheme of an ideal key.

Keywords: motor of direct-current, controlled three-phase rectifier, equation of dynamics, ideal key, logic variables

Streszczenie

W artykule przedstawiono model matematyczny tyrystorowego systemu sterowania silnikiem prądu stałego z niezależnym wzbudzaniem, który zawiera sterowany prostownik trójfazowy. Uwzględniono nieliniowe charakterystyki magnesowania transformatora. Działanie tyrystorów podano zgodnie z charakterystyką klucza idealnego.

Słowa kluczowe: silnik prądu stałego, sterowany prostownik trójfazowy, równania dynamiki, klucz idealny, zmienne logiczne

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1. Introduction

To compare other motors with DC motors, the last have considerable starting torque, that’s why they have proliferated in various actuators such as drives for electric, hoisting machines, and others. These motors are fed and followed by DC voltage, which is applied to the stator windings and armature motor. If there is no source of constant voltage it can be attained by transforming alternating voltage into constant. In such rectification, associated with the presence of pulsations, even when filters are used, the pulsation will appear, and as a result, we will have a deterioration of the dynamic characteristics of the system as a whole. Because of reducing such fluctuations, rectification of three-phase voltage should be applied. In this paper a mathematical model, which provides the use of three-phase half-wave rectifier with thyristors is considered. DC motor connected to the rectifier through capacitive filter, which provides filtration of output voltage ripple of rectifier.

Depending on combination of the motor windings, it is divided into engines with independent, parallel and consequent excitations. We consider the first type of combinations, when winding of excitation is fed using independent source of direct voltage. Armature’s winding is fed by output voltage of rectifier in which thyristors are managed by external circuit. Changing the angle of thyristors opens this scheme and changes the output voltage of rectifier, and therefore angular velocity of motor’s armature is also changed.

![Fig. 1. Structural scheme of system’s control of DC motor](image)

Anchor of motor is connected to tachogenerators, its output voltage is compared with the input voltage of control system. The achieved difference is transformed by microcontroller into angle of thyristors rectifier open. A block diagram of the system is shown in Fig. 1. The following notation are adopted: MC – microcontroller; TPR – three-phase rectifier; DCM – DC motor; TG – tachogenerators; $u_1$ – input system signal; $\Delta u$ – system’s error; $\phi$ – the angle of thyristors opening; $u_2$ – output voltage of rectifier; $\omega$ – speed of motor’s armature; $u_3$ – output tachogenerators’s voltage.

The development of a mathematical model of such a system is associated with certain difficulties, which are conditioned by nonlinear characteristics of its elements. It is the following features: magnetization cores of transformer, current-voltage characteristics of semiconductor valves and magnetic characteristics of the DC motor. The consideration of these types of nonlinearities makes the task of mathematical modeling extremely difficult. Therefore, to resolve it means to adopt certain assumptions which would take into account the most of elements’ characteristics of the system and, at the same time, it must simplify algorithm for analyzing its dynamics, which is suitable for numerical implementation on a computer. The presence of such a model enables us to investigate the influence of parameters on the system, and therefore, using appropriate methods to perform its optimization with an aim of output improvement of the mechanical properties of the system “controlled three-phase half-wave rectifier – DC motor with independent excitation”.

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**Rys. 1. Schemat blokowy układu sterowania silnikiem prądu stałego**

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2. The review of literature

It is obvious that the closer the mathematical model of control system to real physical processes are, the more accurate the results of computer simulation we will achieve. Our problem will be not too cumbersome if we are restricted by problem with lumped parameters. It means that a mathematical model will be confined to the record system of nonlinear differential equations of state. Otherwise, the problem should be considered from the point of view of the theory of electromagnetic field. The target system can be considered as a certain nonlinear electromagnetic range, which contains a transformer, semiconductor valves, DC motors, tachogenerator and microcontroller.

An executive mechanism in robotics is one of the most widespread uses of motors [4]. To reach the maximum capacity and minimal losses, the phenomenon of superconductivity in electrical winding in low temperature can be used [5]. Linear mathematical models of DC motors are described in the works [1, 7]. Here the windings’ inductive was not dependent on magnetic stream. The question of thyristor’s modeling of voltage transformers is not examined in this work. The publications [2, 3, 6, 9], offer the following information: [2] – the regulator which was based on neuron network, [3] – illegible logic, [6, 9] – parametric optimization which is executed by genetic algorithm. In [13] an experimental approach of DC motor’s parameter identification is observed. This question is especially important if the analysis of its work’s dynamics is used. But for counting of periodical processes it will be better to use the extrapolation e-algorithm, because the method of Poincarego-Lindstedta [14] has discrepancy.

In [17] an adaptive observer estimating all parameters and load torque is proposed for DC servo motors. The observer uses no direct feedback but the adaptation schemes’ use and speed measurements are actually valid. Both the observer and adaptations are simple to implement for real-time applications. Simulation results are satisfactory for the full adaptive observer. If the observer works in parallel with only load torque and armature resistance adaptations, the results are very good even if very low-quality sensors are used. In this simulation, only a single hall sensor is used as a rotational transducer, which produces a single pulse per revolution, and very high level noise and disturbance are added in order to provide a more realistic simulation.

However, we are more interested in electromagnetic devices which contain semiconductor valves. A large amount of work has been devoted to RLC-valve model [10–12]. Its main drawback lies in generating additional differential equations. Moreover, these equations are stiff, which makes the algorithm of solving more difficult.

Modern mathematical model of electromagnetic circles as an example of various electromagnetic devices can be met in papers [15, 16]. In these works the equation of dynamics is recorded concerning state’s variables, which include working magnetic flux, current and voltage windings on the capacitors. In this method the equation of dynamics is recorded in the normal form of Cauchy, which is very handy when explicit numerical integration methods are used. It must be remarked that to make the record more compact the equation of dynamics must be written in matrix form. The example is shown in work [8].

In [18] the mathematical models of thyristor converters, thyristors are modeled on the first proposed scheme of perfect key. Here, there is no disadvantage, which is inherent in the models where switching functions, namely duplication of equations of state are used. The introduction of a new concept of additional logical variables, which can take the values 0, ±1,
makes it possible. These variables are governed by the terms of opening and closing valves. It gives an opportunity to describe the whole – a set of states of open and closed valves of electromagnetic transducers as a single system of equations.

3. The equation of dynamics system of thyristor DC motor’s control with independent excitation

First of all, we consider the dynamic equation dynamic three-phase’s half-wave rectifier with capacitor in load. Parameters of the secondary winding must be presumed as leading to the use of the number of winding to the original. In fact, the equation of three-phase’s half-wave rectifier is reduced to a three-phase transformer with zero conductors, though the load of current is interfluent. For equations of state writing it is necessary to determine the total number of combinations of open and closed thyristors. To write an equation for each combination and with additional logical variables, to compile them into one system of algebraic-differential, equations are also necessary.

Fig. 2. Thyristor’s system control of DC motor with independent excitation

Rys. 2. Tvrystorowy układ sterowania silnikiem prądu stałego z niezależnym wzbudzeniem

In this scheme a large number of such combinations is available (Fig. 2). You must examine all options when there is one or two gates will be open. So, there are three combinations when one valve remains open, and three combinations when two gates are open, namely:

1) \( T_A \) open, \( T_B, T_C \) closed,
2) \( T_B \) open, \( T_A, T_C \) closed,
3) \( T_C \) open, \( T_A, T_B \) closed,
4) \( T_A, T_C \) open, \( T_B \) closed,
5) \( T_A, T_C \) open, \( T_B \) closed,
6) \( T_A, T_B \) open, \( T_C \) closed.
The combinations of two are also possible, when all valves are closed and all open, but such regimes almost never occur.

Closing of any valve will result in disabling the corresponding phase. When writing equations for three-phase transformer the switch off of phases can be modeled, equating inverse scattering inductance corresponding phase to zero, or multiplying it on additional logic variable, which takes the values of 0, 1. So there is no need for the transducer to record the equation of all combinations of open and closed valves. Just write the equation for the case when all the valves are open.

Equation circuit of primary side of the transformer can be written in a matrix form:

\[
\frac{d\Psi_1}{dt} = U_1 - R_1 I_1
\]

where:
- \(\Psi_1, U_1, I_1\) – matrix columns of complete linkages, voltages and currents
- \(h(h = \Psi_1, U_1, I_1) = (h_A, h_B, h_C)^T\),
- \(R_1 = \text{diag}(r_{1A}, r_{1B}, r_{1C})\) – diagonal matrix of resistance.

Subscripts \(A, B, C\) indicates the involvement in the relevant phases, 1 indicates the involvement in the primary side of the transformer.

Equation of secondary side can be written in a matrix form:

\[
\frac{d\Psi_2}{dt} = -H_C u_C - R_2 I_2
\]

where:
- \(\Psi_2, I_2\) – column matrix of complete linkages and currents
- \(h(h = \Psi_2, I_2) = (h_A, h_B, h_C)^T\),
- \(R_2 = \text{diag}(r_{2A}, r_{2B}, r_{2C})\) – diagonal matrix of resistance,
- \(H_C = (1, 1, 1)^T\) – structural matrix.

Subscript 2 indicates the involvement in the secondary side of the transformer.

Equations of currents are written in differential form:

\[
\frac{dI_1}{dt} = \alpha_1 \left( \frac{d\Psi_1}{dt} - \frac{d\psi}{dt} \right)
\]

\[
\frac{dI_2}{dt} = K \alpha_2 \left( \frac{d\Psi_2}{dt} - \frac{d\psi}{dt} \right)
\]

where:
- \(\psi = (\psi_A, \psi_B, \psi_C)^T\) – column matrix of basic linkages’ phases,
- \(\alpha_1 = \text{diag}(\alpha_{1A}, \alpha_{1B}, \alpha_{1C})\), \(\alpha_2 = \text{diag}(\alpha_{2A}, \alpha_{2B}, \alpha_{2C})\) – diagonal matrix of inverse leakage of inductances’ winding of the primary and secondary sides of the transformer,
- \(K = \text{diag}(k_A, k_B, k_C)\) – diagonal matrix of additional logical variables.

It takes the values of 0, 1.
Magnetic circuits are described by the equations:

\[ I_1 + I_2 = \Phi(\psi) + \alpha_0 E_1 \psi \]  
(5)

where:

\[ \Phi(\psi) = (\varphi_A(\psi_A), (\varphi_B(\psi_B), (\varphi_C(\psi_C))^T \] – column matrix of magnetic voltages in transformer’s core. It is defined by its magnetization curves,

\[ \varphi_0 \] – inverse leakage’s inductance of zero sequence,

\[ E_1 \] – matrix of dimension 3 × 3, in which all elements are equated to 1.

The equations (5) are differentiated by time (5):

\[ \frac{dI_1}{dt} + \frac{dI_2}{dt} = P \frac{d\psi}{dt} + \alpha_0 E_1 \frac{d\psi}{dt} \]  
(6)

where:

\[ P = \text{diag}(\alpha_A^*, \alpha_B^*, \alpha_C^*) \] – a diagonal matrix of inverse incremental inductances.

Equations of currents are substituted (3), (4) in the equation of state’s magnetic circuits (6):

\[ \alpha_1 \left( \frac{d\Psi_1}{dt} - \frac{d\psi}{dt} \right) + K \alpha_2 \left( \frac{d\Psi_2}{dt} - \frac{d\psi}{dt} \right) = P \frac{d\psi}{dt} + \alpha_0 E_1 \frac{d\psi}{dt} \]  
(7)

or:

\[ (P + \alpha_0 E_1 + \alpha_1 + K \alpha_2) \frac{d\psi}{dt} = \alpha_1 \frac{d\Psi_1}{dt} + K \alpha_2 \frac{d\Psi_2}{dt} \]  
(8)

whence it appears:

\[ \frac{d\psi}{dt} = D_1 \frac{d\Psi_1}{dt} + D_2 \frac{d\Psi_2}{dt} \]  
(9)

\[ D_1 = G \alpha_1, \quad D_2 = G K \alpha_2, \quad G = (P + \alpha_0 E_1 + \alpha_1 + K \alpha_2)^{-1}. \]

Equations (1), (2) are written as a single expression:

\[ \frac{d\Psi}{dt} = U - RI \]  
(10)

where:

\[ \Psi = (\Psi_1, \Psi_2)^T \] – column matrix of complete linkages,

\[ I = (I_1, I_2)^T \] – column matrix of currents,

\[ U = (U, -H \mu_c)^T \] – column matrix of voltages,

\[ R = \text{diag}(R_1, R_2) \] – diagonal matrix of active resistances.

Taking into consideration (10) the equation (9) can be written as:

\[ \frac{d\psi}{dt} = D(U - RI), \quad D = (D_1, D_2) \]  
(11)
The differential equations (9) are substituted in equations of currents (3), (4):

\[
\frac{dI_1}{dt} = \alpha_1 \left( \frac{d\Psi_1}{dt} - D_1 \frac{d\Psi_1}{dt} - D_2 \frac{d\Psi_2}{dt} \right)
\]

(12)

\[
\frac{dI_2}{dt} = K \alpha_2 \left( \frac{d\Psi_2}{dt} - D_1 \frac{d\Psi_1}{dt} - D_2 \frac{d\Psi_2}{dt} \right)
\]

(13)

Taking into consideration the notation (10) the system of equations (12), (13) can be written as:

\[
\frac{dI_1}{dt} = A_1 (U - RI), \quad A_1 = (\alpha_1 (E - D_1), -\alpha_1 D_2)
\]

(14)

\[
\frac{dI_2}{dt} = A_2 (U - RI), \quad A_2 = (-K \alpha_2 D_1, K \alpha_2 (E - D_2))
\]

(15)

Equation (15) will be added by capacitor equation:

\[
\frac{du_C}{dt} = C^{-1} \left( H^T_C I_2 - i_o \right)
\]

(16)

Equation (11), (15) and (16) can be written as single expression:

\[
\frac{dX}{dt} = B(\psi) Z(t), \quad Z(t) = (U - RI, H^T_C I_2 - i_o)^T
\]

(17)

where:

\[
X = (\psi, I_x, u_C)^T \quad \text{– vector of state variables,}
\]

\[
B = \text{diag}(M, C^{-1}) \quad \text{– matrix of coefficients; } M = (D, A_2)^T.
\]

Primary currents are determined in accordance with (5):

\[
I_1 = \Phi(\psi) + \alpha_0 E_i \psi - I_2
\]

(18)

These equations must be supplemented by the conditions of closing and opening thyristors. If the valve of the j phase \((k = 0)\) is closed the dependence \(\Psi_j = \psi_j, j = A, B, C\) will be correct. That’s why its opening condition will be determined by attaching positive voltage onto it:

\[
-\frac{d\psi_j}{dt} - u_C > 0
\]

(19)

If condition (19) is executed, the counting of angle of opening thyristors will begin. For the opening angle of thyristors must be \(\pi/6\). The period of the supply voltage is \(\tau = 0.02\) c, which corresponds to a frequency of \(f = 50\) Hz. This period corresponds to the meaning of \(2\pi\) rad. Therefore angle \(\pi/6\) are formed by 1/12 period, it means \(\Delta\tau = 0.01/6\) s. From the moment of executing condition (9) the time \(\Delta\tau = 0.01/6\) was counted. The signal of permitting for thyristor’s opening \(T_j\) is submitted and additional logical variable value \(k_j = 1\) are assigned. Thyristors are closed by their currents:
\[ i_{2j} = 0, \quad \frac{di_{2j}}{dt} < 0 \quad (20) \]

If condition (20) is executed the valve \( T_j \) will be closed and additional logical variable value will be assigned \( k_j = 0 \).

Now the equation of DC motor with independent excitation is being considered. Differential equations of motor’s winding currents take this form:

\[ \frac{di_A}{dt} = S_A u_C + T_A u_F + E_A, \quad \frac{di_F}{dt} = T_F u_C + S_F u_F + E_F \quad (21) \]

where:

\[ S_A = \frac{1}{(L_A + L_{AF} L_{FA} / L_{FF})}, \quad T_A = -S_A L_{AF} / L_{FF}, \quad T_F = -S_A L_{FA} / L_{FF} \]

\[ E_F = -(L_{FA} E_A + r_F i_F) / L_{FF}, \quad E_A = S_A (L_{AF} r_F i_F / L_{FF} - c \omega \Phi - \Delta u - r_A i_A) \]

\[ S_F = \left(1 - L_{FA} T_A \right) / L_{FF} \]

\( L_A \) – total inductance of consecutive circuit’s armature,
\( L_{FF} \) – inductance of exciting winding,
\( L_{AF}, L_{FA} \) – mutual inductances of circuit’s armature and circuit of excitation,
\( r_A, r_F \) – active resistances of circuit’s armature and circuit of excitation,
\( \omega \) – angular velocity of motor armature,
\( \Phi \) – magnetic flux of motor,
\( c \) – constitutive constant of motor armature,
\( \Delta u \) – voltage drop in the brush contact.

In compensated motors the account of magnetic saturation can be performed approximately using the magnetization curve \( \Phi = \Phi(i_F), L_{FF} = L_{FF}(i_F) \). In the unsaturated motor it can be performed by \( \Phi = ki_F, L_{FF} = \text{const} \).

The equations of motion can be written by d’Alembert equation:

\[ \frac{d\omega}{dt} = (c \Phi i_A - M_O) / J \quad (22) \]

where:

\( J \) – a moment of inertia of the rotor motor,
\( M_O \) – a moment of resistance.

The motor is connected with tachogenerator, which can be considered as a linear object. It is described by the equation of the first order:

\[ \frac{du_g}{dt} = \left( k_g \omega - u_g \right) / T_g \quad (23) \]

where:

\( u_g \) – input voltage of tachogenerator,
\( k_g \) – a coefficient of tachogenerator’s transmission,
\( T_g \) – constant of tachogenerator’s.
The equation of mechanic elements can be written in matrix form:

\[
\frac{dX_M}{dt} = B_M Z_M(t)
\]

(24)

where:
- \(X_M = (i_A, i_F, \omega, u_g)^T\) – a vector of mechanical state variable,
- \(Z_M(t) = (u_C, u_F, 1, c\Phi_i A - M_G, k_g \omega - u_g)^T\) – a vector of time functions,
- \(B_M\) – a matrix of coefficient.

\[
B_M = \begin{bmatrix}
S_A & T_A & E_A & 0 & 0 \\
T_F & S_F & E_F & 0 & 0 \\
0 & 0 & 0 & J^{-1} & 0 \\
0 & 0 & 0 & 0 & T_g^{-1}
\end{bmatrix}
\]

(25)

The error of control system is formed as the difference between control voltage \(u_1\) and the output voltage of the generator \(u_g\):

\[
u_2 = u_1 - u_g
\]

(26)

The microcontroller performs the role of the regulator of the system. It reconstructs a linear dependence of the transistor’s opening angle on system’s error. It played with the linear dependence of the angle of opening on thyristors system error:

\[
\Phi_O = \begin{cases}
\Phi_m (1 - u_2 / u_m), & \text{if } u_2 \leq u_m \\
0, & \text{if } u_2 > u_m
\end{cases}
\]

(27)

where:
- \(u_m\) – voltage at which the opening angle of thyristors becomes equal to zero,
- \(\Phi_m\) – the meaning of opening angle of thyristors, when the error of system is equal to 0.

Thus, the microcontroller must:
- a) fix the moment of time \(t_1\) of condition’s (19) executor,
- b) according to (26) determine the opening angle of thyristors,
- c) transfer this angle in time \(t_O = \Phi_O \pi/(2\pi)\),
- d) when the passage time will exceed the moment \(t_1\) on value \(t_O\) the signal of opening the thyristor will be given.

The last point is an identity of the condition \(t \geq t_1 + t_O\). Mathematically, it boils down to assigning additional logical variable value the meaning of \(k_j = 1\). In the program code, this delay can be realized by usual transfer time \(t_O\) in the number of discrete steps, namely:

\[
n = t_O / \Delta t
\]

(28)

where:
- \(\Delta t\) – an integration step.
4. Results of computer simulations

The Program was developed and computer simulation of the dynamic processes of controlling the speed of rotation of DC motor with separate excitation was carried out. Here the managing element is a three-phase half-wave rectifier which is built on three thyristors.

Voltage feed is set by expressions:

\[ u_{1A} = U_m \sin(\omega_0 t + \gamma), \quad u_{1B} = U_m \sin(\omega_0 t + \gamma - \gamma_0), \quad u_{1C} = U_m \sin(\omega_0 t + \gamma + \gamma_0) \]

\[ U_m = 311 \text{ V}, \quad \omega_0 = 314.1593 \text{ rad/s}, \quad \gamma = 1.8 \text{ rad}, \quad \gamma_0 = 2\pi/3 \text{ rad}. \]

Using these parameters of rectifier calculations was executed:

\[ r_{1A} = r_{1B} = r_{1C} = 2.0 \text{ Ohm}; \quad r_{2A} = r_{2B} = r_{2C} = 1.0 \text{ Ohm}; \quad \alpha_{1A} = \alpha_{1B} = \alpha_{1C} = 172 \text{ H}^{-1}; \quad \alpha_{2A} = \alpha_{2B} = \alpha_{2C} = 200 \text{ H}^{-1}; \quad \alpha_0 = 1.2 \text{ H}^{-1}; \quad C = 9.0 \text{ mF}. \]

Magnetized curve was approximated by the equation of formula’s choosing:

\[ \varphi(\psi) = \begin{cases} a_1 \psi, & |\psi| > \psi_1 \\ S_3(\psi), & \psi_1 \leq |\psi| \leq \psi_2 \\ a_2 \psi - a_0, & |\psi| > \psi_2 \end{cases} \quad (28) \]

where:

\[ a_1 = 0.25 \text{ H}^{-1}; \quad a_2 = 3.0 \text{ H}^{-1}; \quad a_0 = 1.8 \text{ A}; \]

\[ \psi_1 = 0.2 \text{ Wb}; \quad \psi_2 = 0.9 \text{ Wb}; \]

\[ \varphi(\psi_1) = 0.05 \text{ A}; \quad \varphi(\psi_2) = 0.9 \text{ A}; \quad S_3(\psi) - \text{a cube spline}; \]

\[ \alpha'(\psi_1) = a_1, \quad \alpha''(\psi_2) = a_2 \quad \text{must be taken into account}. \]

The parameters of motor and generator are \( L_A = 9.67 \text{ H}; \quad L_{FF} = 110.8 \text{ H}; \quad L_{AF} = L_{FA} = 0 \text{ H}; \quad r_A = 33.2 \text{ Ohm}; \quad r_F = 173 \text{ Ohm}; \quad J = 0.09 \text{ N} \cdot \text{m} \cdot \text{s}^2/\text{rad}; \quad M = 4.0 \text{ N} \cdot \text{m}; \quad c = 70.8 \text{ N} \cdot \text{m}/(\text{Wb} \cdot \text{A}); \quad u_f = 300.0 \text{ V}; \quad T_g = 0.04 \text{ s}; \quad k_g = 0.1 \text{ V} \cdot \text{s}/\text{rad}; \quad u_m = 10.0 \text{ V}; \quad \varphi_m = 5.934 \text{ rad}. \]

In Fig. 3 the time dependence of the rate’s rotation of the DC motor for different values of the input voltage of the system \( u_1 \) is shown. Thus, the curve 1 corresponds to the meaning 5 V, the curve 2 corresponds to the meaning 10 V, curve 3 corresponds to the meaning 15 V, curve 4 corresponds to the meaning 20 V. We see that the time of transition process of control system increases with input voltage. When the voltage is maximum (curve 4) the transition process will have exponential character without control.

The dependence of constant meaning of DC motors winding speed on input voltage \( u_1 \) is shown in Fig. 4. This dependence has practically a lineal character. When \( u_1 \leq 3.5 \text{ V} \) the motor does not wind because his electromagnetic moment is less than the moment of resistance.

Constant meaning of transformer’s secondary phase current for two meanings of input voltage \( u_i \) is shown in Fig. 5. For meaning \( u_i = 10 \text{ V} \) (Fig. 5a) the opening angle of thyristors was approximately equal to 3 rad. Instead of it, when \( u_i \) was equal to 20 V this angle would become equal to 0.7 rad. Looking at Fig. 4, we can simply say that after 20 V the speed of motor winding remains constant. Comparing Fig. 5b with Fig. 5a, the form of current phase is narrower and has greater amplitude in Fig. 5a.
Constant meaning of capacitor for two meanings of input voltage is shown on Fig. 6. For meaning \( u_1 = 10 \text{ V} \) (Fig. 6a) amplitude of capacitor voltage is equal to 64 V. This voltage has constant and variable component. The constant component dominates and it is approximately equal to 163,5 V. Amplitude of variable component is not overstated 0,5 V. For meaning \( u_1 = 20 \text{ V} \) (Fig. 6b) the constant component of capacitor’s voltage approximately grew up to 101 V, but variable component practically did not change.

![Graph](image)

**Fig. 3.** Time dependence of the speed’s rotation of the motor for different values of the input voltage \( u_1 \): 1–5 V, 2–10 V, 3–15 V, 4–20 V

**Rys. 3.** Zależności czasowe prędkości obrotowej silnika dla różnych wartości napięcia wejściowego \( u_1 \): 1–5 V, 2–10 V, 3–15 V, 4–20 V

![Graph](image)

**Fig. 4.** The dependence of speed’s rotation of the motor input voltage

**Rys. 4.** Zależność prędkości obrotowej silnika od napięcia wejściowego

![Graph](image)

**Fig. 5.** Well-established values of currents phase transformer secondary for two values of the input voltage \( u_1 \): a) 10 V, b) 20 V

**Rys. 5.** Przebiegi ustalone prądów faz na wyjściu transformatora dla dwu wartości napięcia wejściowego a) 10 V, b) 20 V
5. Conclusion

This article proposed a mathematical model of thyristor control system of winding speed of DC motor’s control with independent excitation. Tyristors’ work is shown by scheme of an ideal key, which gives an opportunity to minimize number of system’s differential equation. The influence of tyristor’s opening angle on winding motor’s speed was investigated. Microcontroller controls tyristors. Using linear law, it counts the opening angle which depends on system’s error meaning. It was clarified that the speed of armature motor depends on input voltage in the strip from 4 V to 18 V. This dependence has a linear character. When system’s input voltage grows up, the time of transitional process will grow up also. Thus, the developed mathematical model of the system’s work regimes is distinguished by universality and essence of algorithmization.

References


