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# SIMULATION OF CAVITATION IN JET PUMPS

# SYMULACJE ZJAWISKA KAWITACJI W POMPACH STRUMIENICOWYCH

Abstract

This paper presents simulation of cavitation phenomena at jet pumps with use of CFD methods. There was also presented theoretical consideration of cavitation formation and models applied at the CFD codes. On example of liquid-jet liquid pump, which working medium is water was presented simulation of cavitation process which was conducted in Ansys CFX code. This paper also includes some selected results of simulation for fluid flow at the jet pump with consideration of cavitation process.

*Keywords*: *Cavitation*, *CFD*, *jet pumps*

Streszczenie

W artykule przedstawiono podstawy matematycznego modelowania zjawiska kawitacji, w szczególności przedstawiono modele kawitacji wykorzystywane w systemach CFD. Na przykładzie pompy strumieniowej przedstawiono symulację pracy pompy strumienicowej typu ciecz–ciecz z uwzględnieniem zjawiska kawitacji. Symulacje numeryczne przeprowadzono w systemie Ansys CFX.

*Słowa kluczowe*: *kawitacja*, *symulacja CFD*, *pompa strumienicowa*

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## Notations

$R_b$ – bubble radius
$p_v$ – vapor pressure
$p$ – pressure of liquid surrounding the bubble
$\rho$ – liquid density
$\rho_g$ – vapor density
$\sigma$ – surface tension
$U$ – liquid velocity
$\alpha$ – vapor volume fraction
$n$ – bubble number density
$R$ – phase change rate
$fv$ – vapor mass fraction
$f_g$ – noncondesable gases
$\Gamma$ – diffusion coefficient

## 1. Intruduction

Jet pumps particularly liquid-liquid type operation is mostly limited by cavitation process. The principle of jet pump operation is conversion of velocity energy into a pressure energy and vice versa. This may lead to that pressure drops below the level below saturation pressure of working liquid and in the consequences vapor bubbles. Such phenomena is recognized as cavitation and it is one of the parameters that limits usage of jet pumps. Cavitation is common met problems not only in jet pumps but also in a lot of engineering application. Their nature does not only disturb a flow by sudden phase changes but also is a cause of sudden wear and noise. That reasons caused the this phenomena has been a subject of studies from many years and still is. Cavitation is a complex problem on which a lot of factors may have influence. The studies presented in books of Frenkel and Skripov [1] deal with a fundamental physics of nucleation, however they were conducted on pure liquids, which engineering liquids are not. And what later studies shown the contamination as well as a aeration of working liquids have influence on cavitation process significantly. The nature of cavitation at liquid flow is a complex phenomenon and makes difficulties in modeling and simulation. There are, however, some formulas than allows to evaluate flow conditions at which cavitation may appear in jet pumps but this is only a rough estimation. Therefore, in this paper an attempt of simulation of liquid flow at Liquid Jet Liquid pump with the use of CFD method was undertaken.

## 2. Mathematical models of cavitation

An attempts of preparing mathematical description of cavitation has been conducted from many years. The studies presented in book [1] shows a mathematical description of cavitation and bubble dynamics. This work presents selected models of cavitation which are applied in CFD codes. Generally, tendency a flow to cavity may be defined as the cavitation number:

$$c_a = \frac{p - p_v}{0.5\rho U^2} \tag{1}$$

One of the common approach to describe bubble dynamics is The Rayleigh Plesset equation [3]:

$$R_b \frac{d^2 R_b}{dt^2} + \frac{3}{2}\left(\frac{dR_b}{dt}\right)^2 + \frac{2\sigma}{\rho R_b} = \frac{p_v - p}{\rho} \tag{2}$$

After deriving this equation, and neglecting second order terms and surface tension the equation is reduced to the following:

$$\frac{dR_b}{dt} = \sqrt{\frac{2}{3}\left(\frac{p_v - p}{\rho}\right)} \tag{3}$$

The rate of bubble volume changes is as follow:

$$\frac{dV_b}{dt} = 4\pi R_b^2 \sqrt{\frac{2}{3}\left(\frac{p_v - p}{\rho}\right)} \tag{4}$$

The rate of change bubble mass is:

$$\frac{dm_g}{dt} = 4\pi R_b^2 \rho_g \sqrt{\frac{2}{3}\left(\frac{p_v - p}{\rho}\right)} \tag{5}$$

The $N_b$ bubbles per unit volume the volume fraction $r_g$ is expressed by:

$$r_g = \frac{4}{3}\pi R_b^2 N_b \tag{6}$$

The total interphase mass transfer per unit volume is:

$$\dot{m}_{fg} = 3\frac{r_g \rho_g}{R_b}\sqrt{\frac{2}{3}\frac{p_v - p}{p_f}} \tag{7}$$

When including condensation this expression is as follows:

$$\dot{m}_{fg} = 3F\frac{r_g \rho_g}{R_b}\sqrt{\frac{2}{3}\frac{|p_v - p|}{p_f}\operatorname{sgn}(p_v - p)} \tag{8}$$

Vapor transport equation has the following form:

$$\frac{\partial}{\partial t}(\alpha\rho_v) + \nabla\cdot(\alpha\rho_v\vec{V}_v) = R_e - R_c \tag{9}$$

The other cavitation model, which base on full cavitation model was developed by Singhal et al. [3]. Two phase continuity equations may be presented as below. Liquid phase is described as:

$$\frac{\partial}{\partial t}[(1-\alpha)\rho] + \nabla\cdot[(1-\alpha)\rho\vec{V}] = -R \tag{10}$$

while vapor phase:

$$\frac{\partial}{\partial t}(\alpha\rho_v)+\nabla\cdot(\alpha\rho_v\vec{V})=R \tag{11}$$

and finally mixture:

$$\frac{\partial}{\partial t}(\rho_m)+\nabla\cdot(\rho_m\vec{V})=0 \tag{12}$$

where:

subscript $m$ – is a mixture phase,
$v$ – vapor phase.

Mixture density is defined as:

$$\rho_m=\alpha\rho_v+(1-\alpha)\rho \tag{13}$$

Combining above equation gives relation between mixture density and vapor volume fraction $\alpha$

$$\frac{D\rho_m}{Dt}=\frac{D\alpha}{Dt}(\rho_v-\rho) \tag{14}$$

Vapor volume fraction can be related to the bubble number density ($n$) and the bubble radius $R_b$

$$\alpha=n\left(\frac{4}{3}\pi R_b^3\right) \tag{15}$$

Using above equations leads to the evaporation rate $R$

$$R=2\alpha(4\pi n)^{\frac{1}{3}}\frac{\rho_v\rho}{\rho_m}\sqrt{\frac{2}{3}\left(\frac{p_B-p}{\rho}\right)} \tag{16}$$

All terms are known except „$n$" wich is constant or dependent variable. The phase change expression migth be rewritten as a function of bubble radius $R_b$

$$R=\frac{3\alpha\rho_v\rho}{R_B\rho_m}\sqrt{\frac{2}{3}\left(\frac{p_B-p}{\rho}\right)} \tag{17}$$

Vapor mass fraction is the dependent variable in vapor transport equation:

$$\frac{\partial}{\partial t}(f_v\,\rho)+\nabla\cdot(f_v\,\rho\vec{V}_v)=\nabla\cdot(\Gamma\nabla\vec{V}_v)+R_e-R_c \tag{18}$$

Rates of mass exchange are given by:

for $p\le p_v$

$$R_e=F_{vap}\frac{\max(1.0,\sqrt{k})(1-f_v-f_g)}{\sigma}\rho\rho_v\sqrt{\frac{2}{3}\left(\frac{p_v-p}{\rho_{ell}}\right)} \tag{19}$$

for $p>p_v$

$$R_e = F_{\text{cond}} \frac{\max(1.0, \sqrt{k}) f_v}{\sigma} \rho\rho_v \sqrt{\frac{2}{3}\left(\frac{p_v - p}{\rho}\right)} \tag{20}$$

The saturation pressure is as follows:

$$p_v = p_{\text{sat}} + 0.5(0.39k) \tag{21}$$

where $F_{vap}$, $F_{\text{cond}}$ are constatns.

Another approach, which was the assumption that bubbles have the same size was proposed by Zwarat-Gerber-Belamri [2]. They assumed that interphase mass transfer per unit volume ($R$) is calculated used bubble density number ($n$) and the mass change rate of a single bubble

$$R = n\left(4\pi R_B^2 \rho_v \frac{DR_B}{Dt}\right) \tag{22}$$

After including Eq 15. we obtain:

$$R = \frac{3\alpha\rho_v}{R_B} \sqrt{\frac{2}{3}\left(\frac{p_B - p}{\rho}\right)} \tag{23}$$

$$R_e = F \frac{3\alpha\rho_v}{R_B} \sqrt{\frac{2}{3} \frac{|p_B - p|}{\rho}} \operatorname{sign}(p_B - p) \tag{24}$$

where $F$ is empirical coefficient.

In this model is proposed of replacing $\alpha_v$ with $\alpha_{\text{nuc}}(1 - \alpha)$. And the final model of cavitation is as follows:

for $p \le p_v$

$$R_e = F_{vap} \frac{3\alpha_{\text{nuc}} \rho_v (1 - \alpha_v)}{R_B} \sqrt{\frac{2}{3}\left(\frac{p_v - p}{\rho}\right)} \tag{25}$$

for $p > p_v$

$$R_{ec} = F_{\text{cond}} \frac{3\alpha_v \rho_v}{R_B} \sqrt{\frac{2}{3}\left(\frac{p - p_v}{\rho}\right)} \tag{26}$$

where:

$\alpha_{\text{nuc}}$ – is nucleation site volume fraction,
$F_{vap}$ – evaporation coefficient,
$F_{\text{cond}}$ – is condensation coefficient.

Another mathematical model of cavitation has been presented by Schnerr and Sauer [2]. The vapor fraction equation has the following general form:

$$\frac{\partial}{\partial t}(\alpha\rho_v) + \nabla \cdot (\alpha\rho_v \vec{V}) = \frac{\rho_v \rho}{\rho_m} \tag{27}$$

The mass source term is:

$$R = \frac{\rho_v \rho}{\rho_m} \frac{d\alpha}{dt} \tag{28}$$

Unlike mentioned previously models, the relation between vapor fraction and number of bubbles has the following form:

$$\alpha = \frac{4}{3}\pi \frac{n_b R_B^3}{1 + \frac{4}{3}\pi n_b R_B^3} \tag{29}$$

And finally the mass transfer rate is:

$$R = \frac{\rho_v \rho}{\rho_m} \alpha(1-\alpha) \frac{3}{R_B} \sqrt{\frac{2}{3}\left(\frac{p_v - p}{\rho}\right)} \tag{30}$$

while bubble radius is:

$$R_B = \left(\frac{3\alpha}{4\pi n(1-\alpha)}\right)^{\frac{1}{3}} \tag{31}$$

The final form of this model is:

When $p_v \geq p$

$$R_e = \frac{\rho_v \rho}{\rho_m} \alpha(1-\alpha) \frac{3}{R_B} \sqrt{\frac{2}{3}\left(\frac{p_v - p}{\rho}\right)} \tag{32}$$

When $p_v < p$

$$R_c = \frac{\rho_v \rho}{\rho_m} \alpha(1-\alpha) \frac{3}{R_B} \sqrt{\frac{2}{3}\left(\frac{p - p_v}{\rho}\right)} \tag{33}$$

## 3. CFD simulation of flow LJL pump

The object of simulation was two stage liquid jet liquid pump, which working medium is water. This pump diameter of throat is approx. 200 mm and is equipped with two annular motive nozzle. Due to the symmetry only a half of geometrical model was used. The grid which is presented in Fig. 1, it consists of tetrahedral prism layers cells and was prepared in Ansys Workbench Mesh module. Despite the pump is two stage pump for the simulation of cavitation formation only one stage was used. CFD simulation was conducted in Ansys CFX code in two stages, the first, without considering cavitation to check if there is possibility for cavitation formation at pump working conditions. The second stage was full cavitation simulation using Rayleigh Plesset model, which was presented in chapter 2. Some selected results of simulation for both stages are presented in below figures. There are presented pressure distributions, path lines as well as a distribution of volume fractions in case of cavitation simulations.

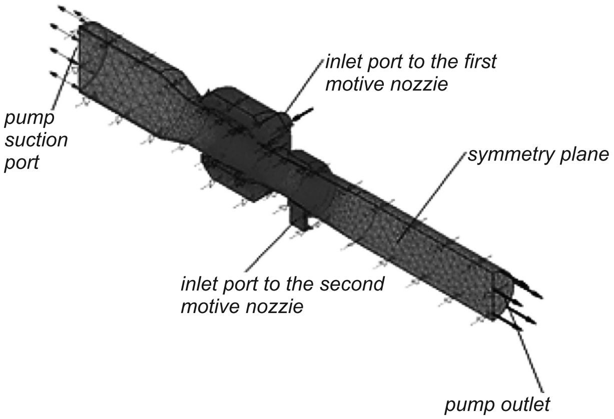

Fig. 1. CFD model of LJL pump

Rys. 1. Model CFD pompy strumienicowej

Fig. 2. Path lines of motive liquid

Rys. 2. Linie prądu wygenerowane dla strumienia napędowego

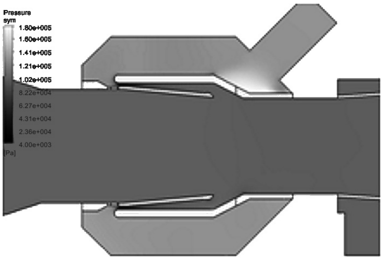

Fig. 3. Pressure distribution

Rys. 3. Rozkład ciśnienia statycznego [Pa]

Presented results for initial simulation in Figure 2 and 3 allowed to investigate the potential area where cavitation may appear. Water flowing to the motive nozzle gather velocity in the nozzle what leads to pressure drop. What was found is the biggest pressure drop was not in

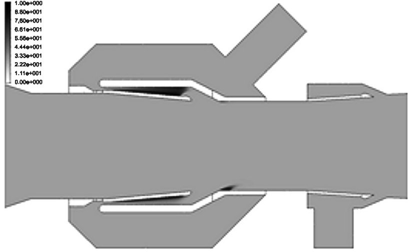

Fig. 4. Distribution of vapor fraction

Rys. 4. Rozkład fazy gazowej podczas przepływu wody the pump throat but in the entrance to the motive nozzle. Therefore the simulation which includes cavitation models were conducted. As may be found in Figures 4 and 5 process of bubble formation begins in the area where pressure drop was observed *t* initial simulation. Figure 4 presents a distribution of vapor phase during pump operation in the symmetry plane, while Figure 5 presents the surface with vapor fraction only.

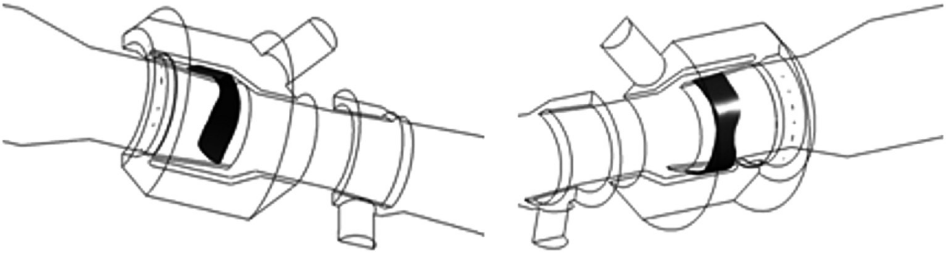

Fig. 5. Surface of vapor fraction

Rys. 5. Kształt powierzchni fazy gazowej

## 4. Conclusions

This paper presents a mathematical models of cavitation models which are used in CFD codes. One of such model was used in simulation of flow with cavitation at liquid jet liquid pump. There was presented selected simulation results which shown the way the cavitation may appear during water flow in the pump.

## References

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