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GENETIC ALGORITHM IN OPTIMIZATION OF CYCLOID PUMP

ALGORYTM GENETYCZNY W OPTYMALIZACJI POMPY CYKLOIDALNEJ

A b s t r a c t

The paper presents a method of optimizing the geometry of a cycloid positive displacement pump using genetic algorithm. This allowed for increasing its delivery and efficiency and reduce pulsation.

Keywords: *optimization positive displacement pump genetic algorithms*

S t r e s z c z e n i e

W artykule przedstawiono metodę optymalizacji geometrii pompy wyporowej cykloidalnej z wykorzystaniem algorytmu genetycznego. Pozwoliło to na zwiększenie jej wydajności i sprawności oraz obniżenie pulsacji.

Słowa kluczowe: *optymalizacja pompa wyporowa algorytm genetyczny*

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1. Introduction

The paper presents the task of optimizing the displacement gear pump with three teeth. Due to the small number of teeth is obtained discharge and suction chamber with large volume. This allows on the one hand, to obtain a high delivery in a unitary pump with small dimensions, on the other hand a low sensitivity of the pump parameters for the pumped medium and the contamination contained in pump. Selection of geometric parameters is essential for pump parameters. To help select solution optimization task can be formulated. The paper presents own software that using a genetic algorithm [1,2,6,7] which by API interface is connected to the CAD system [8,9], and can automatically generate 3D models and perform multi-criteria optimization for gear pump [5].

2. Cycloid gear wheel with three teeth

Gear wheels of cycloidal profile are designed using whole cycloid curves arcs and have the form as on figure 1.

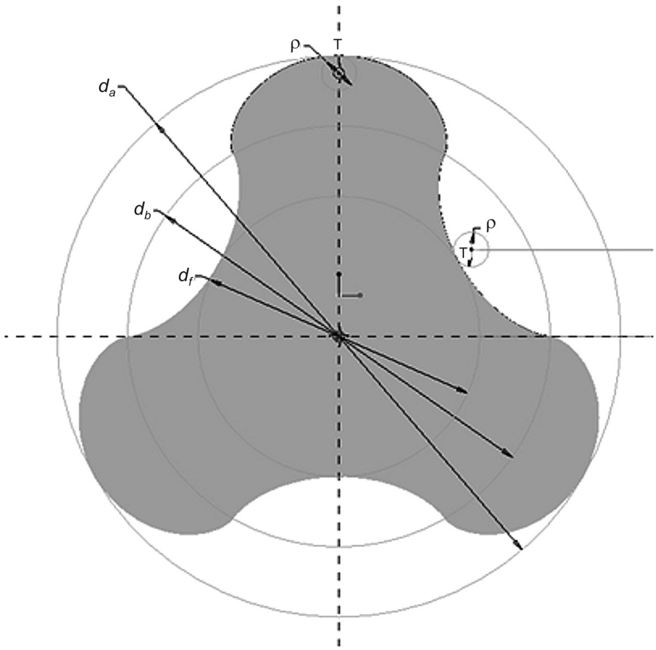

Fig. 1. Gear wheel with cycloidal profile

Rys. 1. Koło zębate o profilu cykloidalnym

The tooth profile is created by rolling in a circle with a diameter d_b of a circle of radius ρ_h, on the inner side, resulting in that a point *M* which lies on that circle creates an arc of ordinary hypocycloid. From the outside circle of radius ρ_e rolls creating an arc of ordinary Hypocycloid. However, at the point P, which is located on the diameter d_b of the two arcs [3, 10]. Ratio of base circle d_b and radius ρ rolling circle must be:

$$\frac{d_b}{\rho} = 4z \tag{1}$$

Length of the pitch circle is:

$$m = 4\rho \tag{2}$$

Pitch circle divides tooth on the head with a height h_a and foot with a height h_f, according to the relationship:

$$h_a = h_f = 2\rho \tag{3}$$

Formulas for determining the diameter of addendum circle d_a and the diameter of root circle d_f have form:

$$d_a = d + 2h_a = 4\rho z + 4\rho = 4\rho\ (z + 1) \tag{4}$$

$$d_f = d - 2h_f = 4\rho z - 4\rho = 4\rho\ (z - 1) \tag{5}$$

3. Optimization of pump parameters

Optimization of pumps can be carried out taking into account the different objective functions. In the case of positive displacement pumps it may be getting the biggest pump delivery or the total efficiency, minimalize pulsation or cost of manufacture product, getting higher operating pressure, etc. The most common approach known in the literature rely on formulating one objective with a fixed range of decision variables [4, 11], which facilitates the solution of the problem. However, it appears that higher effects are obtainable using a multiobjective optimization [7, 11]. As the objectives of optimizing for the analyzed pump in this article were selected: maximum unitary delivery, maximum pump delivery, minimum of pulsation and the total efficiency. Mathematical relations for these functions can be determined from the following relations [3]:

Geometric dependencies of the positive displacement pump

Geometrical relations are as the following formulas:

– Unitary delivery:

$$Q_u = \frac{bm^2}{8}[2(z_1+1)^2 - 2z^2 - 2] \tag{6}$$

where:

Q_u – unitary delivery,
b – gearwheel width,
m – module,
z – number of teeth.

– Theoretical delivery[3]:

$$Q_t = \frac{\pi b m^2}{4}[2(z+1)^2 - 2z^2 - 1] \tag{7}$$

where:

Q_t – theoretical delivery.

– Pulsation:

$$\delta = \frac{2\frac{z^2}{R^2}\sin 2\frac{\pi R}{2z}}{2\left(\frac{z}{2}+1\right)^2 - \frac{z^2}{2} - \frac{z^2}{R^2}\left(1+\frac{z}{\pi R}\sin\frac{\pi R}{z}\right)} \tag{8}$$

where:

δ – pulsation,

R – tooth shape factor

The efficiency of the pump

Parameters related to the operation of the pump are: working pressure, fluid parameters and above all, the total pump efficiency, which is defined as the product of the volumetric, mechanical and hydraulic efficiency [3, 4]. Total efficiency η_c is defined as the quotient of output power P_{out} to the power input P_{in} or can be calculated as the product of the volumetric efficiency η_v and hydraulic-mechanical efficiency η_{hm}.

$$\eta_c = \frac{P_{out}}{P_{in}} = \eta_v \eta_{hm} \tag{9}$$

Volumetric efficiency is defined as the quotient of real delivery Q_r to a theoretical:

$$\eta_v = \frac{Q_r}{Q_t} = \frac{Q_t - \Delta Q}{Q_t} = 1 - \frac{\Delta Q}{Q_t} \tag{10}$$

where:

ΔQ – volumetric losses.

$$\Delta Q = Q_\mu + Q_\varsigma \tag{11}$$

where:

Q_u – losses associated with viscosity of the liquid,

Q_ς – losses associated with the density of the liquid.

$$Q_u = c_\mu \frac{p}{2\pi\mu_d} q \tag{12}$$

where:

μ_d – dynamic viscosity of liquid,

c_μ – loss coefficient associated with the viscosity, depending on the unitary delivery.

$$Q_\varsigma = c_r \sqrt{\frac{2p}{\varsigma}} \sqrt[3]{q^2} \tag{13}$$

where:

ς – density of the liquid,

c_r – loss coefficient associated with the density, depending on the type and size of the slots, and the unitary delivery.

Substituting relations (11), (12), (13) for (10) receives a formula which defines the volumetric efficiency:

$$\eta_v = 1 - c_\mu \frac{p}{2\pi\mu_d n} - c_r \frac{1}{n}\sqrt{\frac{2p}{\varsigma}}\sqrt[3]{q-1} \tag{14}$$

Hydraulic-mechanical efficiency is defined as the quotient of the theoretical moment M_t on the pump shaft to the actual moment M_r:

$$\eta_{hm} = \frac{M_t}{M_r} = \frac{M_t}{M_t + \Delta M} \tag{15}$$

where:

ΔM – a moment of loss.

$$M = M_v + M_\varsigma + M_\mathrm{p} \tag{16}$$

where:

M_v – moment of losses associated with the resistances caused by the friction of viscous liquids,

M_ς – moment of losses associated with the density of the working medium,

M_p – moment of losses associated with the mechanical losses dependent on the working pressure of the pump.

These moments can be calculated from the following formulas [3]:

$$M_v = c_v \mu_d nq \tag{17}$$

$$M_\varsigma = c_\varsigma \frac{n^2}{4\pi}\sqrt[3]{q^5} \tag{18}$$

$$M_p = c_p \frac{pq}{2\pi} \tag{19}$$

where:

c_v – proportionality coefficient,

c_ς – ensity coefficient,

c_p – pressure coefficient.

Substituting dependeces (17), (18), (19) and (16) for (15)receives a formula defining the hydraulic-mechanical efficiency:

$$\eta_{hm} = \frac{1}{1 + c_v 2\pi \frac{\mu_d n}{p} + c_p \frac{\varsigma n^2}{2\pi}\sqrt[3]{q^2} + c_p} \tag{20}$$

Substituting the formula for volumetric efficiency (14) and model of the hydraulic--mechanical efficiency (20) we get the following formula on the total efficiency:

$$\eta_c = \frac{1 - c_\mu \dfrac{p}{2\pi\mu_d n} - c_r \dfrac{1}{n}\sqrt{\dfrac{2p}{\varsigma}}\sqrt[3]{q-1}}{1 + c_v 2\pi \dfrac{\mu_d n}{p} + c_\varsigma \dfrac{\varsigma n^2}{2p}\sqrt[3]{q^2} + c_p} \tag{21}$$

The objective function

On basis of formulas (6), (7), (8), (21) constituting mathematical model of the pump was built four objective functions . The following notation were prepared: f_1 – function of the unitary delivery (22), f_2 – function of the theoretical delivery (23), f_3 – function of the pulsation, for it is used inverse function (24) (for compatibility with other functions, i.e. maximum f_3 means the minimum pulsation), f_4 – function of the total efficiency (25).

$$f_1 = f_1\,(q\,(b, r_w, r_t)) \tag{22}$$

$$f_2 = f_2\,(Q_t\,(x, y, z, b)) \tag{23}$$

$$f_3 = f_3\,(\delta\,(\mathrm{z}, y, z, b)) \tag{24}$$

$$f_4 = f_4\,(\eta_c\,(Q_{ew}, p, \mu_d, \varsigma)) \tag{25}$$

For the defined objective function multi-criteria optimization problem can be written as [11]:

$$f = \max\,(w_1 \cdot f_1 + w_2 \cdot f_2 + w_3\,(-1)\,f_3 + w_4 f_4) \tag{26}$$

4. The simulation studies

Calculations were carried out for various variants of the pump design.

To carry out the calculations a hydraulic oil with the following specifications was selected: dynamic viscosity of liquid u_k=41 mm²/s, liquid density ς = 840 kg/m³, and the number of teeth equal to 3.

Decision variables x_i:

- height tooth coefficient y $\quad x_{1\min} < x_1 < x_{1\max}$
- gearwheel width b $\quad x_{2\min} < x_2 < x_{2\max}$
- the pumping pressure p $\quad x_{3\min} < x_3 < x_{3\max}$

The range of values of the decision variables are shown in Table 1.

Table 1

The Range of the decision variables

x_i	x_i min	x_i max	units
×1	0.6	1.2	–
×2	30	50	mm
×3	1.2	2.5	MPa

For optimization using a genetic algorithm the following parameters were selected:
– The type of representation: floating point
– Crossover: 2-point, with a probability: 0.9
– The probability of mutation: 0.1
– The size of population: 20
– The number of algorithm iterations: 2000

The greatest effect of the conducted optimization was reduction of the pump pulsation by 47.2%, total efficiency improved by 9.2%, unitary delivery improved by 2.9% and theoretical delivery improved by 2.8%. This effect is presented in Figure 2.

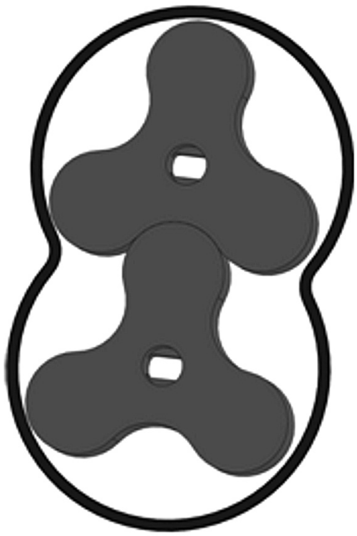

Fig. 2. Gear wheels after optimization

Rys. 2. Koła zębate po optymalizacji

5. Conclusions

This paper presents the problem of multi-criteria optimization displacement pump design using a genetic algorithm. To optimize pump was used known in the literature mathematical models describing the basic properties of the pump, such as unitary delivery, pump delivery, pulsation and the total efficiency. As the object of the investigations the standard model of the positive displacement pump used in industry was selected. On pump was conducted simulation studies looking for better solutions to reduce pulsation and increase unitary delivery, pump delivery and total efficiency. A software Creo (Pro/Engineer) was used for the purpose of construction of the gear geometry. Own software with the use genetic algorithm was developed for the optimization. The developed software proved to be an effective tool in the search for better solutions, which improved pump parameters in particular significantly reduce the pulsation.

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