TECHNICAL TRANSACTIONS | CZASOPISMO TECHNICZNE
CIVIL ENGINEERING | BUDOWNICTWO

3-B/2013

MARIUSZ MAŚLAK*

THE RISK OF FIRE OCCURRING IN BUILDING COMPARTMENT – WHAT ARE THE CONSEQUENCES IF IT IS ASSUMED TO BE TIME-INDEPENDENT

RYZYKO ZAISTNIENIA POŻARU W STREFIE POŻAROWEJ – JAKIE SĄ KONSEKWENCJE, JEŚLI PRZYJĄĆ, ŻE JEGO WARTOŚĆ NIE ZALEŻY OD CZASU

Abstract

The acceptance in advanced fire safety analysis of the formal model according to which the probability of fire occurrence is assumed as time-independent leads to the conclusion that random fire episodes in considered building compartment can be described by the formalism of a Poisson process. In the presented paper some consequences of such adoption are presented and widely discussed as well as the foundations are determined of the equivalence between this probability and a conditional failure rate, which is interpreted as a process intensity parameter.

Keywords: fire occurrence, probability, Poisson process, risk, hazard rate, process intensity.

Streszczenie

Akceptacja w zaawansowanej analizie bezpieczeństwa pożarowego modelu formalnego, zgodnie z którym prawdopodobieństwo zaistnienia pożaru jest przyjmowane jako niezależne od czasu, prowadzi do wniosku, że losowe epizody pożaru w rozpatrywanej strefie pożarowej mogą być opisywane przy użyciu formalizmu procesu Poissona. W prezentowanym artykule przedstawiono i przedyskutowano niektóre konsekwencje wynikające z takiego przyjęcia, a także podano podstawy uznania równoważności pomiędzy rozpatrywanym prawdopodobieństwem a warunkową częstością zawodów, która jest interpretowana jako parametr intensywności procesu.

Słowa kluczowe: zaistnienie pożaru, prawdopodobieństwo, proces Poissona, ryzyko, stopień zagrożenia, intensywność procesu.

The Author is responsible for the language.

* Ph.D. D.Sc. Eng. Eng. Mariusz Maślak, prof. CUT, Institute of Building Materials and Structures, Faculty of Civil Engineering, Cracow University of Technology.

Symbols

$f_{fi}(t)$ – compartment random durability probability density function (*pdf*),
$F_{fi}(t)$ – compartment random durability cumulative distribution function (*cdf*),
$h(t) = h$ – hazard function (conditional failure rate) that fire will occur in a compartment at time $t + dt$ (specified per one year of building service and one square meter of the fire compartment area)
$m_{fi} = MTTF$ – mean time to fire in a compartment,
p_f – probability of structural failure due to fire, provided that fire ignition has occurred,
p_{ff} – probability of fire induced structural failure, related to the fire which can take place; but has not yet occurred,
$p_{f,ult}$ – ultimate acceptable value of the structural failure probability,
p_t – probability of fire occurring in a compartment,
$R(t)$ – reliability function specified for a compartment in the context of potential fire occurrence,
t – time the assessment of the hazard rate is made for,
t_0 – time related to the beginning of the building service,
t_{fi} – time related to the first fire ignition in a compartment,
T_{fi} – random durability of a compartment (time to the first fire),
$\lambda(t) = \lambda$ – the *Poisson* process intensity of fire occurring in a compartment.

1. Introduction

To protect the considered load-bearing structure against potential fire exposure in a more rational and better justified way the risk of fire occurrence should be adequately estimated in advance, in relation to the compartment inside of which such structure is located. The reliable knowledge of its value allows to differentiate the suitable safety requirements by assuming the acceptable risk level adjusted to real threat conditions. In consequence, in many cases less restrictive fire protection measures can be legally applied, being significantly cheaper and easier to use than those designed according to the traditional approach.

In fact, the failure probability p_f is usually adopted as a conclusive and objective valuation measure when the classical safety analysis is performed, for the case of unexpected event potentiality, such as threat to people or a building structure. Its application explicitly determines the understanding of limit state requirements. Such limit state is not reached exactly at the point-in-time when the considered event really takes place, but earlier, when the probability of its occurrence becomes too high and may no longer be tolerated. Conclusively, the safety condition is in general formulated as follows:

$$p_f \le p_{f,ult} \tag{1}$$

where the maximum acceptable values of failure probability, i.e. $p_{f,ult}$, are assigned arbitrarily, to be adequate for the assumed reliability class [1].

However, if the fire safety is to be examined in detail, one should precisely define what kind of such failure probability is considered, as at least two interpretations are possible, differing not only from quantitative but also from qualitative point of view. These are as follows [2, 3]:

- probability of fire caused failure, if it is known that fire ignition occurred and; moreover, this fire reached the flashover point (and thus may be described as a fully developed fire) – in further analysis such probability is generally denoted by p_f,
- probability of fire caused failure, due to fire which did not occur so far (so the designer has no information on its ignition and flashover) – let us assign a symbol p_{ff} to designate it.

According to *Lie* [4] p_f and p_{ff} are related by:

$$p_{ff} = p_t p_f \tag{2}$$

where p_t denotes the probability of fire occurrence (not only of fire ignition but also reaching the flashover point). As one can see, probability p_f is interpreted here as a conditional probability with a condition that fire has already occurred and the exhaust gas temperature in the whole compartment is uniform (the fire is fully developed). Not only a qualitative but also a quantitative distinction between probabilities p_f and p_{ff} seems to be very significant. Even if conditional probability p_f is high, probability p_{ff} is usually quite low, because in reality the value of probability p_t is also low.

Let us underline, that the value of the probability p_{ff}, related to the potential fire which did not occur so far, seems to be essential both for building occupants who will be able to occupy the considered compartment if fire ignition and flashover takes place and also for firemen taking part in a future firefighting action. To responsibly assess such value according to formula (2), the appropriate value of a probability of fire occurrence p_t has to be accepted beforehand by the safety expert for considered compartment, depending on the way such compartment is used as well as on the fire loads accumulated in its volume and on the possible ventilation conditions. The probability p_t is usually assumed to be constant during the whole lifetime of the building. The main advantage of such design approach lies in its simplicity; however, it leads to both quantitative and qualitative consequences important for the safety level really ensured for inhabitants. The detailed analysis of those repercussions, resulting directly from the adopted analytical model, is the main objective of the presented paper.

2. The risk, the hazard function or the failure probability – which is the most accurate interpretation of p_t

One may find many quantitative evaluations of the probability p_t in professional literature. Some of those are listed in Table 1. Let us notice that in [5] the author proposed to rename this quantity and interpret it as the hazard function h**.** The basic reason was that if a function has a dimension, then it cannot be treated as a typical probability, in spite of the fact that it is defined in this way by many authors.

It is noteworthy that all proposed values of the considered hazard function are assumed to be constant-in-time. This also means that they do not depend on the point-in-time when they are assessed. Thus, in consequence, the value of a probability p_t is quantitatively the same both at the beginning of the building life and after many years of its service.

Table 1

Recommended values of fire ignition hazard function *h* $\left[\left(m^2 \cdot year\right)^{-1}\right] \times 10^{-6}$

Building type	*Kersken – Bradley* [6]	BSI DD240 [7]	DIN 18230 [8]	JCSS [9]	*Schleich, Cajot* et al. [10]
Apartments	0.1÷0.5	2.0	0.2	0.5÷0.4	30.0
Schools	0.1÷1.0	–	0.5	0.5÷0.4	–
Hotels	0.1÷1.0	–	1.0	–	–
Shops	0.5÷5.0	–	1.0	1.0	–
Offices	0.1÷1.0	1.0	0.5	1.0	10.0
Industrial buildings	1.0÷5.0	2.0	2.0	2.0÷10.0	10.0

In such a formal model the quantity p_t should not be interpreted as the classical risk value because in a common understanding the risk is the probability of something happening multiplied by the resulting cost or benefit of its occurrence.

In the approach proposed by the author the quantity $p_t = h$ is interpreted as a conditional probability that potential fire will occur between the t and $t + dt$ points-in-time if it is known (including the condition) that it did not occur in time-period $[t_0,t)$ – t_0 denotes commissioning of the building. To make such definition more illustrative let us study in detail the following explanation:

- Let the event A mean that considered fire occurred between t and $t + dt$; whereas, the event B, that this fire did not occur in $[t_0,t)$.
- Next, let us calculate the value of a conditional probability $P\left(A|B\right) = \dfrac{P\left(A \cap B\right)}{P\left(B\right)}$. In this formula $P\left(A \cap B\right) = P\left(A\right)$ because if the fire occurs between t and $t + dt$ it could not occur prior to time t.
- Let the point-in-time t_0 denote the commissioning of the building, while the point-in-time t_{fi} the moment of the first fire ignition in considered compartment. Then a difference $T_{fi} = t_{fi} - t_0$ is the measure of a compartment random durability in context of building fire safety analysis. It is assumed that the considered compartment is reliable at a random point-in-time t if no fire occurred prior to this point-in-time.
- Concluding, the reliability function $R(t)$ may be formulated as follows:

$$R\left(t\right) = P\left(T_{fi} \geq t\right) = 1 - P\left(T_{fi} < t\right) = 1 - F_{fi}\left(t\right) \tag{3}$$

  where $F_{fi}(t)$ is a compartment durability *cdf* (cumulative distribution) function. The corresponding *pdf* (probability density) function $f_{fi}(t)$ is then equal to $f_{fi}\left(t\right) = dF_{fi}\left(t\right)/dt$. Consequently, the probability that fire occurs in time period $[t,t + dt)$ is equal to $P\left(A\right) = f_{fi}(t)dt$, while it is also true that $P\left(B\right) = 1 - F_{fi}\left(t\right)$.
- Finally, the value of the examined hazard function can be estimated by the formula:

$$h(t) = P(A|B) = \frac{P(A)}{P(B)} = \frac{f_{fi}(t)dt}{1-F_{fi}(t)} = \lim_{dt \to 0} \frac{P\left(t \le T_{fi} < t+dt \middle| T_{fi} \ge t\right)}{dt} \tag{4}$$

As one can see, transition of considered compartment to the failed state (failure is understood here as the occurrence of fire) is characterized by the conditional failure rate. This can be interpreted as a measure of the rate at which failures occur, taking into account the size of the population with the potential to fail, i.e. those compartments of the examined type, which did function without fire until time *t*.

3. The hazard rate *h*(t) constant-in-time – basic consequence

As it is shown in Table 1, the hazard rate *h*(*t*), constituting conditional probability measure of fire occurrence, is usually assumed to be time independent. This means that its value remains constant during the whole building lifetime. In this chapter the basic consequence of such assumption is demonstrated, dealing with the predicted reliability of considered fire compartment. In order to study the resultant trend characterizing the adequate reliability function *R*(*t*) – see Eq. (3) – the formula (4) has to be integrated. In fact:

$$\int_0^t h(t)dt = \int_0^t \frac{f_{fi}(t)}{1-F_{fi}(t)}dt = -ln\left[1-F_{fi}(t)\right] \tag{5}$$

yielding:

$$F(t) = 1 - exp\left(-\int_0^t h(t)dt\right) \tag{6}$$

Assumption that *h*(*t*) = const (*t*) leads to the simple equation:

$$F(t) = 1 - e^{-ht} \tag{7}$$

and also, regarding the reliability:

$$R(t) = e^{-ht} \tag{8}$$

It is a common knowledge that such relation is associated with the formalism of *Poisson* process, according to which hazard rate $h(t) = h$ is equivalent to process intensity $\lambda(t) = \lambda$ [4]. This is not a surprise because the fire episodes, treated by designers as accidental events, should occur very rarely. Moreover, despite the fact that fire duration is always marked as a sectioning line along the time axis, it is very short in relation to the human lifetime or to the whole building service time. Probability p_t is then understood as the probability that fire ignition occurred at least once prior to the considered point-in-time *t* (most frequently the time period $[t_0,t)$ is the whole building lifetime, but if only one year of service life is to be considered the point-in-time t_0 denotes beginning of such year). Consequently, the probability that fire will occur *x* times prior to time *t* is given as follows:

$$p_x(x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}, \quad \mathrm{x} = 1, 2, \ldots, \infty \tag{9}$$

where parameter λ is called process intensity. Application of this formula leads to the following formulae for probabilities p_x and p_t:

- probability that fire will not occur at all prior to time *t*:

$$p_x(x = 0) = e^{-\lambda t} \tag{10}$$

  One can easily see that such equation is equivalent to formula (8). In fact, the examined compartment remains to be reliable only if no fires occur prior to time *t*.
- probability that fire will occur exactly once prior to time *t*:

$$p_x(x = 1) = \lambda t e^{-\lambda t} \tag{11}$$

- probability that fire will occur at least once prior to time *t* (i.e. once or more than once):

$$p_x(x \geq 1) = 1 - p_x(x = 0) = 1 - e^{-\lambda t} = p_t \tag{12}$$

4. Conditional failure rate as a process intensity parameter

The conditional failure rate *h*, generally adopted as the measure of the fire occurrence risk, is most frequently specified per one year of building lifetime and per one square meter of the considered fire compartment area.

The method to evaluate the intensity parameter λ, for buildings having only one type of fire compartments, was given by *Lie* [4]:

$$\lambda = hA \tag{13}$$

where $h[m^{-2}]$ denotes the fire occurrence risk (calculated per $1m^2$ of fire compartment); whereas, $A[m^2]$ is the area of the considered fire compartment. If several fire compartment types, with various sizes assigned to each of them, are located in the examined building, then the generalized formula proposed by *Burros* [11] may be applied in safety analysis:

$$\lambda = h\overline{A} = h\frac{A_F}{N} \tag{14}$$

according to which A_F is the total area of all compartments in the whole building; whereas *N* is the number of such compartments.

Furthermore, because in relation to one year of the building service (and even to the whole service life) $x << 1$ holds, the following simplification of Eq. (12) is acceptable and commonly used:

$$p_x(x \geq 1) = 1 - e^{-\lambda t} = 1 - e^{-h\overline{A}t} \approx h\overline{A}t = p_t \tag{15}$$

If the formalism characterizing the *Poisson* process is accepted as describing the probability of consecutive fire episodes, it may be used to evaluate the sought failure probability p_{ff} (see Eq. (2)), thus making possible to the rearrangement of Eq. (3) to the following form, identical with Eq. (8):

$$R(t) = p_x(x=0) = P(T_{fi} \geq t) = 1 - P(T_{fi} < t) = 1 - F_{fi}(t) = e^{-\lambda t} \tag{16}$$

Consequently:

$$F_{fi}(t) = 1 - e^{-\lambda t} \tag{17}$$

and also:

$$f_{fi}(t) = \frac{dF_{fi}(t)}{dt} = \lambda e^{-\lambda t} \tag{18}$$

This means that the random time to the first fire occurrence exhibits exponential probability distribution. As a result of such substitutions the hazard rate $h(t) = h$ may be calculated as follows:

$$h(t)dt = \frac{f_{fi}(t)dt}{1 - F_{fi}(t)} = \frac{(\lambda e^{-\lambda t})dt}{e^{-\lambda t}} = \lambda dt \tag{19}$$

This proves that it is really an equivalent of the process intensity parameter.

5. The mean time to fire occurrence

In previous chapter of this paper it was shown that the random time to the first fire occurrence is characterized by exponential probability density function (see Eq. 18). Hence the value of the mean time to fire occurrence m_{fi} (the so called mean time to failure – *MTTF* – in this example interpreted as the mean time to fire ignition) may be calculated as follows:

$$m_{fi} = E(t) = \int_0^{\infty} t f_{fi}(t)dt = \int_0^{\infty} t\lambda e^{-\lambda t}dt \tag{20}$$

Substituting $u = \lambda t$ yields:

$$m_{fi} = (1/\lambda)\int_0^{\infty} u e^{-u} du = (1/\lambda)\left[e^{-u}(-u-1)\right]\Big|_0^{\infty} = 1/\lambda \tag{21}$$

This result seems to be important. It means that, if the constant value of the hazard rate h is adopted for fire safety analysis, the mean time to fire is simply its reciprocal. In the same way one may calculate the variance of the time to fire σ^2_{fi}, and also its coefficient of variation (*cov*) v_{fi}. As a result, one obtains:

$$\sigma^2_{fi} = 1/\lambda^2 \text{ and } v_{fi} = 1 \tag{22}$$

6. Time to *k*-th fire episode

Application of a *Poisson* process model allows to examine not only the random time to the first fire episode in the considered building compartment but also the time to the *k*-th fire incident occurring at the same location. Let $i = 1,2,\ldots,k$, and $t_1,t_2,\ldots,t_k$ denote the points-in-time connected with succeeding fire episodes, then the sought time is equal to the sum $\left(t_k - t_0\right) = \left(t_1 - t_0\right) + \left(t_2 - t_1\right) + \ldots + \left(t_k - t_{k-1}\right)$. Time periods $(t_i - t_{i-1})$ between successive fires are statistically independent and exhibit also the exponential probability distribution, characterized by a common intensity parameter λ. As a consequence of such assumptions, the probability density function (*pdf*) $f_{fi,k}(t)$ has the following form:

$$f_{fi,k}\left(t\right) = \frac{\lambda\left(\lambda t\right)^{k-1} e^{-\lambda t}}{\left(k-1\right)!} \tag{23}$$

As one can see, the time $t = t_k$ to the *k*-th fire incident exhibits the gamma probability distribution characterized by the parameters k and λ. Because the first of those parameters, k, is the natural number, this gamma distribution is a special one, commonly called the *Erlang* probability distribution. Moreover, it is possible to show that [12]:

$$m_{fi,k} = E\left(t\right) = k/\lambda \quad \text{and} \quad \sigma^2_{fi,k} = k/\lambda^2 \quad \text{and} \quad v_{fi,k} = 1/\sqrt{k} \tag{24}$$

7. Concluding remarks

In engineering practice the value of the hazard rate h, constituting a risk measure of fire occurring in examined building compartment and for considered point-in-time if it is known that no fire occurred previously, is statistically estimated to reliably evaluate the real threat conditions in case of a fire. It is usually assumed to be constant during the whole building service period. This means that such a risk does not depend on the time when the fire safety assessment is made. This hazard function, discussed in presented paper, has clear and univocal interpretation, especially if the formal model of a *Poisson* process is taken into account in the analysis. Such model is fully adequate for the fire cases. This conclusion results from the fact that the real fires can be treated as rare random events, very short in relation to the whole building service time. Application of such simplified mathematical formalism allows to estimate in an easy way the probability of fire ignition (and flashover) p_t. The hazard function h may be interpreted as a parameter of the process intensity λ, in accordance with the suggestion given by *Lie* (see Eq. 13). The inverse of such parameter is quantitatively equal to the mean time to fire m_{fi}. It is obvious that such mean time seems to be very large in relation to the typical building service period, especially if the hazard rate h is small. In fact, fire does not occur at all in a great majority of buildings so the time to fire estimated for the whole population of the examined objects tends to infinity. The random time to the first fire is the measure of a compartment durability in context of a fire safety analysis. In the presented article it is shown that such durability is described by means of exponential probability distribution. The same kind of probability density function characterizes random time-periods between succeeding fires in the examined building compartment. Thus, as a result of such conclusions, the random time to the *k*-th fire episode conforms to the *Erlang* probability distribution.

This paper was prepared with the financial support of the project granted by Polish Ministry of Science and Higher Education (N N506 243938).

References

[1] PN-EN 1990 Eurocode – Basis of Structural Design.
[2] Maślak M., *Prawdopodobieństwo katastrofy ustroju nośnego budowli w warunkach pożaru*, Czasopismo Techniczne, 12-B/2005, 67–80.
[3] Maślak M., *Selected aspects of failure probability assessment for fire situation*, Archives of Civil Engineering, LVII, 3, 2011, 297–311.
[4] Lie T.T., *Optimum Fire Resistance of Structures*, Journal of the Structural Division, 98, ST1, 1972, 215-232.
[5] Maślak M., *Measures of the risk of fire occurrence – interpretation and quantification*, Proceedings of IABSE Conference "Global Thinking in Structural Engineering: Recent Achievements", Sharm El Sheikh, Egypt, May 7-9, 2012, IABSE Report, vol. 98, 174-175 (abstract) + CD (6 pages).
[6] Kersken-Bradley M., *Basic Notes on Structural Fire Protection*, Proceedings of XI JCSS Meeting, Lisbon, Portugal, April 21-22, 1980.
[7] BSI DD240, *Fire Safety Engineering in Building*, Part 1: *Guide to the Application of Fire Safety Engineering Principles*, British Standard Institution, 1997.
[8] DIN 18230, *Baulicher Brandschutz im Industrienbau*, Teil 1: *Rechnerisch Erforderliche Feuerwiderstandsdauer*, Beuth Verlag Gmbh, Berlin, Germany, 1995.
[9] JCSS, *Probabilistic Model Code*, Part 2: *Load Models*, 2.20 *Fire*, Joint Committee on Structural Safety, Zurich, Switzerland, 2001.
[10] Schleich J.-B, Cajot L.B., et al., *Competitive Steel Buildings through Natural Fire Safety Concept*, ECSC Research 1210-SA/125, 126, 213, 214, 323, 423, 522, 623, 839, 937, 194-98, Final Report EUR 20360EN, European Coal and Steel Community, Esch/Alzette, Luxembourg 2002.
[11] Burros R.H., *Probability of Failure of Building from Fire*, Journal of the Structural Division, ST9, September 1975, 1947-1960.
[12] Benjamin J.R., Cornell C.A., *Rachunek prawdopodobieństwa, statystyka matematyczna i teoria decyzji dla inżynierów*, Wydawnictwa Naukowo-Techniczne, Warszawa 1977.