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ВАДИМ МИЗОНОВ, АНДРЕЙ МИТРОФАНОВ, ВАЛЕРИЙ МАГНИЦКИЙ,
АНТОН ВАЛЕРЬЕВИЧ ОГУРЦОВ[1]

# MODELLING OF FLUIDIZED BED BY MEANS OF THE THEORY OF MARKOV CHAINS

# MODELOWANIE ZŁOŻA FLUIDALNEGO Z ZASTOSOWANIEM TEORII ŁAŃCUCHÓW MARKOVA

Abstract

A mathematical model of particulate solids fluidization based on the theory of Markov chains is proposed. The model deals with one-dimensional cell presentation of the process at which the matrix of transition probabilities controls travel particles over the cells. It allows describing the bed expansion and particle content distribution over the bed height as well as particle residence time distribution for continuous fluidization.

*Keywords: fluidized bed, state vector, matrix of transition probabilities, particle content distribution, hold-up, residence time distribution*

Streszczenie

Zaproponowano model matematyczny fluidyzacji cząstek ciała stałego opartej na teorii łańcuchów Markowa. W zaproponowanym modelu proces fluidyzacji przedstawiono w postaci jednowymiarowej, gdzie macierz prawdopodobieństw przejścia kontroluje drogę cząstki przez komórkę. Pozwoliło to na opis ekspansji złoża i rozkładu zawartości cząstek wzdłuż wysokości złoża oraz wyznaczenie rozkładu czasu przebywania dla ciągłej fluidyzacji.

*Słowa kluczowe: złoże fluidalne, wektor stanu, macierz prawdopodobieństw przejścia, rozkład zawartości cząstek, zatrzymanie, rozkład czasu przebywania*

[1] Д.т.н., профессор Вадим Евгеньевич Мизонов, к.т.н., доцент Андрей Васильевич Митрофанов, к.т.н., доцент Валерий Александрович Магницкий, к.т.н., доцент Антон Валерьевич Огурцов, Ивановский государственный энергетический университет.

## 1. Introduction

There are two extreme cases in fluidized bed modeling: a model can be too simplified to be used for reliable prediction of bed behavior (for example, [1]), or too complex to be used in engineering practice [2, 3, etc]. Sometimes the simple models can provide more or less good prediction but only if they use a lot of empirical correction coefficients that are valid only for already existing reactors. The objective of the study is to develop a simple but informative model that deals with particle concentration distribution over the height of a bed in contrast to classical simple models that consider a bed as a single perfectly mixed cell. An appropriate mathematical tool for this purpose is the theory of Markov chains. It was successfully used to describe some of similar processes [4, 5].

## 2. A cell model of a batch fluidized bed reactor

Let us begin with a cylindrical reactor and separate its operating volume into n perfectly mixed cells like it is shown in Fig. 1a. At no gas flow action the particulate solids occupies several cells at the bottom. Then let us begin to supply the bed with air with the velocity $W$ related to the empty cross section of the reactor. The concentration of particles is high at this moment, and the local velocity $W_i$ of flow around particle is much higher than $W$. If this velocity is higher than the particle settling velocity, particles begin moving up with the velocity $V_i = W_i - V_s$ and occupying upper cells. Their concentration is getting smaller that leads to decreasing of $W_i$ and $V_i$ respectively. Finally at a certain level (in the $h$-th cell) $V_i$ becomes equal to zero, and we get the upper level of the bed (if the equilibrium between $W_i$ and $V_s$ is not reached in the cell $n$, the bed will be blown up and the stable bed cannot exist at this regime). The transitions caused by interaction between gas and particles can be called the convection transitions. If the equilibrium is reached in the cell $h$, we can "lock" the bed on this level.

Let us describe the evolution of particles content in the cells due to the convection transitions only. The particles content distribution over the chain can be presented as a column state vector $\mathbf{S} = \{S_i\}$, $i = 1, 2, \ldots h$. Its evolution can be described by the non-linear recurrent matrix equality:

$$\mathbf{S}^{k+1} = \mathbf{P}(\mathbf{S}^k)\mathbf{S}^k \tag{1}$$

where $\mathbf{P}$ is the matrix of transition probabilities that depends itself on the current state vector, $k$ is the number of time transition of duration $\Delta t$. In order to describe the transition probabilities let us use the following presentation of $W_i$:

$$W_i = \frac{W}{\left(1 - \varepsilon \cdot \frac{\sum_{e=1}^{m} S_{ei}}{S_{\max}}\right)} \tag{2}$$

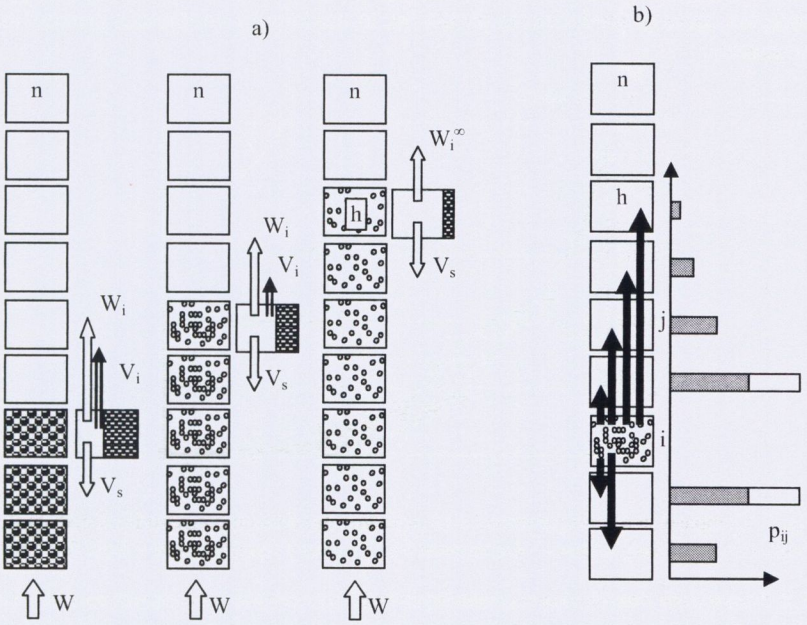


Fig. 1. To calculation of convection (a) and diffusion (b) transition probabilities

Rys. 1. Schemat prawdopodobieństwa przejścia dla konwekcji (a) i dyfuzji (b)

where $S_{ei}$ is the content of particles of the $e$-th fraction in the $i$-th cell, $S_{max}$ is the maximum particulate phase content in the cell, ε is the porosity of particles packing. Eq. (2) allows calculating $V_i$, and then the transition probability $v_i = V_i \cdot \Delta t / \Delta x$. The matrix **P** contains the values of $v_i$ under the main diagonal if $v_i > 0$, and above it if $v_i < 0$. The main diagonal consists of the values $1 - v_i$; all the rest entries of the matrix are equal to zero.

The matrix **P** written only for convection transitions can describe the bed expansion but not mixing of particles inside the bed. The diffusion transitions are responsible for particles mixing, and they are also to be included in the matrix **P**. In contrast to usual Markov chain where transitions are allowed to neighboring cell only, it was proposed to allow transitions to any cell from the given one with transition probabilities decreasing with the increase of the distance between the cells. This presentation is shown schematically in Fig. 1b. It was also supposed that the transition probabilities and distances between cells are subjected to the normal distribution law. The rule of introducing the probabilities into the matrix **P** can be found in [4, 5]. One of examples of modeling is shown in Fig. 2.

The model is the only calibrating parameter: the variance of the normal distribution of diffusion transition probabilities. The experimental data were obtained at a laboratory set of fluidized bed for the binary mixture of two fractions of the same size but different density. Good correlation between the both can be seen. This is not the only example of the model validation. It was checked at different laboratory sets with different regimes of fluidization.

In all these cases good correlation was obtained too. This allows developing the approach to more complex cases of modeling, in particular, to modeling a continuous process in a fluidized bed reactor.

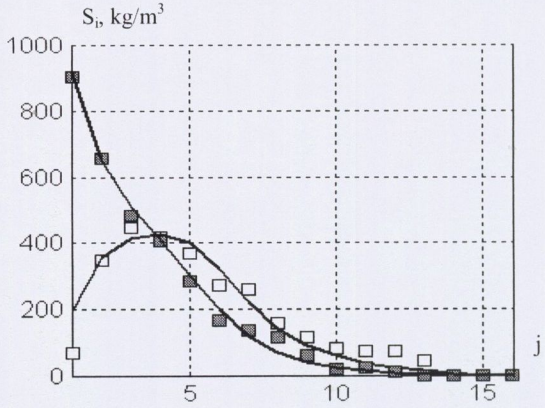


Fig. 2. Comparison of computational and experimental data on concentration distribution: lines – computation, squares – experiment (dark – heavy fraction, light – light fraction)

Rys. 2. Porównanie danych eksperymentalnych i obliczeniowych dotyczących rozkładu stężeń: linie – obliczenia, punkty – eksperyment (ciemny – ciężka frakcja, jasny – lekka frakcja)

### 3. A cell model of a continuous fluidized bed reactor

The same strategy is used to model the continuous process. In this case Eq.(1) takes the following form:

$$\mathbf{S}^{k+1} = \mathbf{P}(\mathbf{S}^k)\mathbf{S}^k + \mathbf{S}_f - \mathbf{S}_d^k \tag{3}$$

where $\mathbf{S_f}$ is the feed vector, and $\mathbf{S_d}^{\mathbf{k}}$ is the discharge vector. The feed vector has the only non-zero element for the cell, which is supplied with the feed. If the reactor is fed to the very top of it, this is the cell $n$ and $S_f(n) = G_f \cdot \Delta t$ where $G_f$ is the feed rate. In order to define the discharge vector let us suppose that the fluidized particulate system behave exactly like liquid and the discharge velocity can be found from Bernoulli law. If the discharge opening is placed in the cell m, the $m$-th element of the discharge vector can be calculated as:

$$S_{dm}^k = G_d^k \cdot \Delta t = a \cdot S_m^k \cdot \sqrt{2 \cdot g \cdot (n_u^k - m) \cdot \Delta x} \cdot A \tag{4}$$

where $G_d$ is the discharge flow rate, A is the cross section area of outlet tube, $a$ is the experimental coefficient, $(n_u - m) \cdot \Delta x$ is the height of the bed above the discharge device. The value of $n_u$ varies from one time transition to another until the equality of the feed and discharge flow rate is reached.

At steady state regime the matrix of transition probabilities and particle content distribution become constant. If we inject the unit portion of a tracer into the feed, we can get the residence time distribution at the system outlet. The outflow of the tracer can be calculated as follows:

$$q(k) = a \cdot S_m^{\infty} \cdot \sqrt{2 \cdot g \cdot \left(n_u^{\infty} - m\right) \cdot \Delta x} \cdot A \tag{5}$$

where the symbol $^{\infty}$ is assigned to the steady state parameters. The mean residence time and the variance of residence time distribution can be calculated using Eq. (5):

$$t_m = \Delta t \cdot \sum_{k=1}^{\infty} k \cdot q(k) \tag{6}$$

$$\sigma^2 = \sum_{k=1}^{\infty} \left(k \cdot \Delta t - t_m\right)^2 \cdot q(k) \tag{7}$$

It should be noted that in the steady state the mean residence time $t_m$ must coincide with the mean flow time

$$t_f = \frac{\sum_{j=1}^{n} S_j^{\infty}}{G_f} \tag{8}$$

The steady state is calculated as the limit of transient computational recurrent process, at which different parameters limit to their steady state values (with a given accuracy) with different speed. This is why the steady state must be defined by the parameter that limits to the state with the smallest speed.

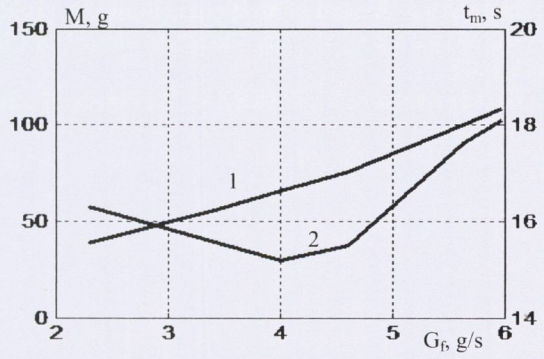


Fig. 3. Influence of the throughput on the hold-up and mean residence time:
1 – hold-up in steady-state regime, 2 – mean residence time ($m$ = 5)

Rys. 3. Wpływ przepustowości na zatrzymanie i średni czas przebywania:
1 – zatrzymanie w warunkach ustalonych, 2 – średni czas przebywania ($m$ = 5)

Fig. 3 illustrates an example of process modeling. Influence of the throughput on the

hold-up and mean residence time is shown. The outlet cell has the number 5 in this example. It is obvious that the hold-up grows with growing throughput. However, the dependence of mean residence time on throughput demonstrates more complex behavior. First, it is decreasing, then, after reaching the minimum, it is growing. The similar behavior was found in a continuous vibration mill [4].

## 4. Conclusions

Despite this was not the first attempt to apply the theory of Markov chains to model different aspects of fluidization, the proposed approach in contract to other ones is directed to modeling all aspects of the process on the basis of universal computational algorithm. It allows describing and predicting the particulate phase distribution over bed height in a batch process and particle residence time distribution and hold-up in a continuous one. These characteristics are the key characteristics to describe heat and mass transfer and chemical reactions in fluidized bed reactors they are built for. The model is open for further development to take into account another specific feature of the process on the basis of the same computational algorithm.

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