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JAN TALAGA[1], IVAN FOŘT[2]

MODELLING OF THE VELOCITY PROFILES IN THE OUTLET STREAM FROM A RUSHTON TURBINE

MODELOWANIE PROFILI PRĘDKOŚCI CIECZY W STRUMIENIU WYLOTOWYM MIESZADŁA RUSHTONA

Abstract

This paper reports on the investigations of the mean velocity profiles of the radial and tangential components in the impeller discharge stream of a Rushton turbine in a pilot plant mixing vessel. The velocity of the liquid was determined on the basis of instantaneous velocity measurements made using a two-component Laser Doppler Anemometer (LDA). The influence of the radial distance and the blade height on the velocity profiles was processed in the form of correlation equations. The experimental results and the statistical correlations were also compared with a phenomenological model describing the velocity profiles of the radial component of the mean velocity in the impeller discharge stream.

Keywords: agitated vessel, Rushton turbine, impeller outlet stream, velocity profile, LDA

Streszczenie

W pracy przedstawiono wyniki badań profili składowej promieniowej i obwodowej średniej prędkości cieczy w strumieniu wylotowym z mieszadła turbinowego tarczowego (turbiny Rushtona). Pomiarów prędkości dokonano w mieszalniku modelowym za pomocą dwukanałowego anemometru laserowego (LDA). Wyniki pomiarów ujęto w postaci równań korelacyjnych przedstawiających wpływ odległości od mieszadła i wysokości łopatki mieszadła na profile prędkości w strumieniu cieczy tłoczonym przez mieszadło. Otrzymane wyniki w postaci modelu eksperymentalnego porównano z modelem fenomenologicznym opisującym profil składowej promieniowej prędkości w strumieniu wylotowym cieczy z mieszadła.

Słowa kluczowe: mieszalnik, mieszadło turbinowe, strumień wylotowy z mieszadła, profil prędkości, LDA

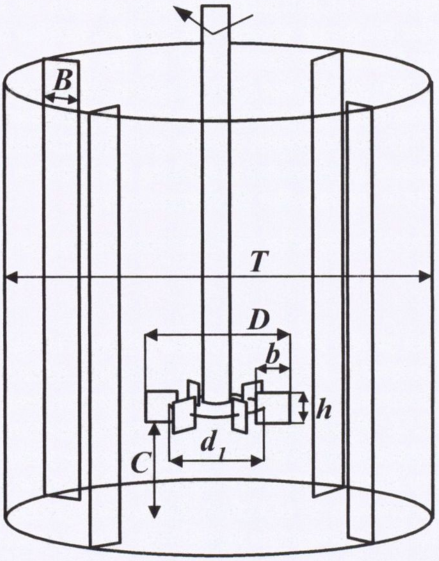
[1] Dr inż. Jan Talaga, Politechnika Krakowska.
[2] Doc. Ing. Ivan Fořt, DrSc., Czech Technical University in Prague.

1. Introduction

In mixing processes mechanical energy is introduced into the cylindrical vessel by means of a rotating impeller and converting this energy into hydrodynamics motion. The hydrodynamics of the flow in stirred tank, in particular knowledge of the velocity profiles, are essential for the correct design of mixing tanks. The most important thing is the velocity profile in the impeller stream that determines the global circulation pattern in stirred vessel and consequently e.g. power input, the mixing and circulation times, the heat and mass transfer rates. The velocity profiles and flow patterns obviously depend on the type and geometry of the impeller.

2. Experimental investigations

Experimental studies were carried out in a pilot plant agitated vessel (Fig. 1) with internal diameter $T = 286$ mm with a flat bottom and equipped with four standard radial baffles. The impeller was a typical Rushton turbine with diameter of $D = 1/3T$ and a standard geometry. The velocity of the liquid in the discharge stream from the impeller was determined on the basis of instantaneous velocity measurements made using a Laser Doppler Anemometer (LDA) [1].



$T = 286$ mm

$D = 1/3\ T$

$d_1 = 0.66\ D$

$C = D$

$b = 0.25\ D$

$h = 0.2\ D$

$B = 1/10\ T$

Fig. 1. Geometry of the stirred vessel equipped with a Rushton turbine

Rys. 1. Parametry geometryczne mieszalnika z mieszadłem turbinowym Rushtona

On the basis of the results of measurements of the radial velocity component, its distribution along the height of the impeller blade, the velocity profile can be described by the following exponential function [2]:

$$W_r^* = \exp(a_1 + b_1 \cdot z^* + c_1 \cdot z^{*2}) \tag{1}$$

where W_r^* is the dimensionless radial component of the mean velocity normalised with the impeller tip velocity

$$W_r^* = \frac{W_r}{\pi \cdot D \cdot n} \tag{2}$$

and z^* is the dimensionless axial coordinate normalised with the half-height of the impeller blade

$$z^* = \frac{2 \cdot z}{h}. \tag{3}$$

Appearing in equation (1) the coefficients a_1, b_1 and c_1 include the dependence on the distance from the tip of the turbine blades. The coefficients are a function of the dimensionless radius of the tank:

$$r^* = \frac{2 \cdot r}{D} \tag{4}$$

and they are defined using the following quantitative relations:

$$a_1 = 3.36 - 5.56 \cdot r^* + 3.57 \cdot r^{*2,5} - 1.68 \cdot r^{*3} \tag{5}$$

$$b_1 = -1.41 + 6.27 \cdot r^{*2} - 7.14 \cdot r^{*2,5} + 2.11 \cdot r^{*3} \tag{6}$$

$$c_1 = (0.02 - 0.30 \cdot r^{*3})^{-1} \tag{7}$$

The graph reproduced in Fig. 2 shows the radial component of the mean velocity distributions along the height of the impeller blade – expressed on the basis of equation (1) – for various distances r^* from the tip of the blades. The radial velocity distribution in the flow of liquid near the agitator blades is practically symmetrical along the entire height of the blade (the plane of the impeller separating disk), and the maximum speeds take place in the middle of its height, while further away from the blade this maximum is shifted toward the upper half of the blade, because of the asymmetrical position of the impeller in the vessel.

During the experimental investigations the velocity profile of the tangential component was also determined and it can be described by equation of the same form as for the radial component this is:

$$W_{tg}* = \exp(a_2 + b_2 \cdot z * + c_2 \cdot z *^2) \tag{8}$$

where the coefficients a_2, b_2 and c_2 are defined using the following quantitative relations:

$$a_2 = 2.30 - 2.54 \cdot r * + 0.11 \cdot r *^3 \tag{9}$$

$$b_2 = 0.332 - 0.012 \cdot r *^3 - 0.010 \cdot (\ln r*)^{-1} \tag{10}$$

$$c_2 = -10.66 - 3.80 \cdot r * + 12.71 \cdot r *^{0.5} \tag{11}$$

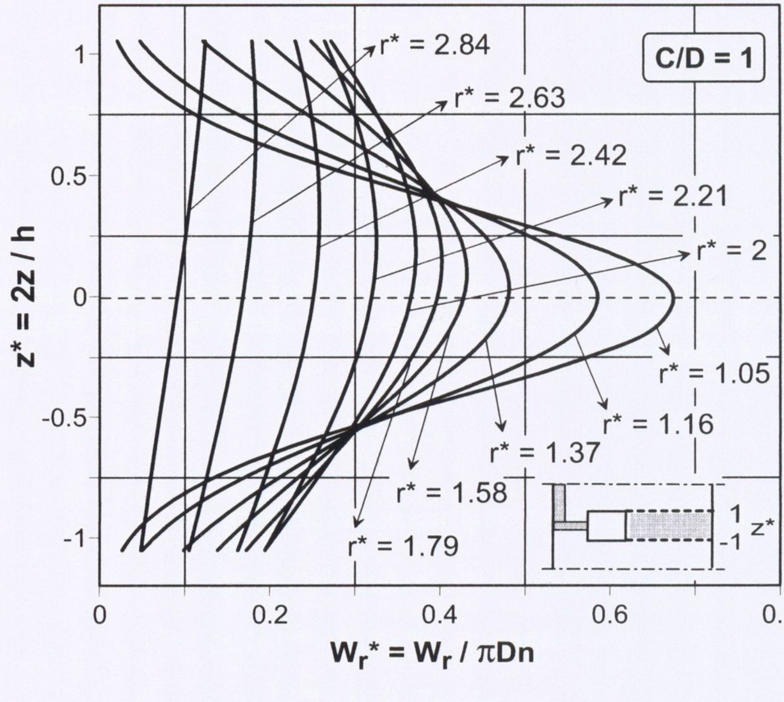


Fig. 2. Impeller discharge profiles of the dimensionless radial component of the mean velocity, according to Eq. (1)

Rys. 2. Profile bezwymiarowej składowej promieniowej średniej prędkości cieczy w strumieniu wylotowym z mieszadła zgodnie z równaniem (1)

In Fig. 3 is shown the profile of the tangential component of the mean velocity in the discharge stream from the Rushton turbine along the height of the impeller blade – expressed on the basis of equation (8) – for various distances $r*$ from the tip of the blades.

Comparison of the measurement values of tangential velocity components and the values calculated based on the obtained Eq. (8) has been shown in the graphs contained in Fig. 4.

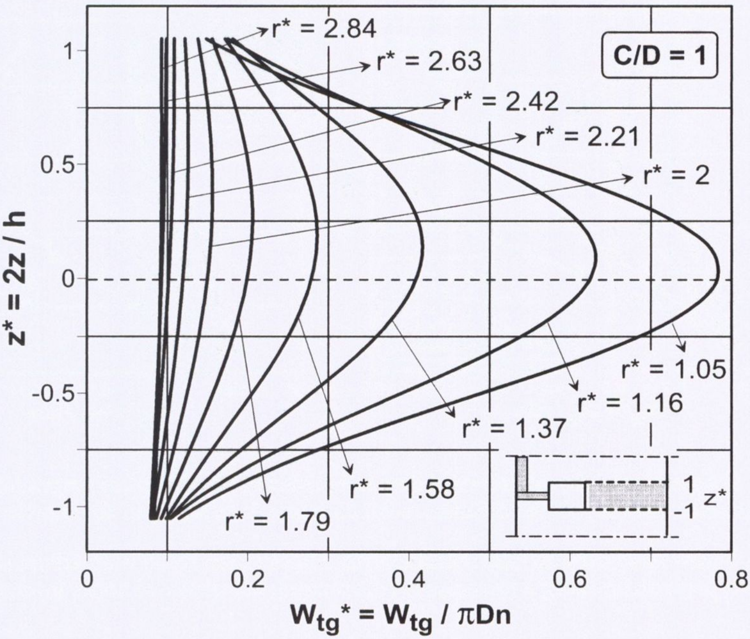


Fig. 3. Impeller discharge profiles of the dimensionless tangential component of the mean velocity, according to Eq. (8)

Rys. 3. Profile bezwymiarowej składowej obwodowej średniej prędkości cieczy w strumieniu wylotowym z mieszadła zgodnie z równaniem (8)

3. Phenomenological model of the velocity profiles in a discharge stream

The proposed phenomenological model for the description of the discharge flow from the impeller is based on a tangential jet model [3] and gives results for the profile of the radial velocity component. It is assumed that the impeller as a source of motion may be replaced by an axially symmetric cylindrical slot – the tangential cylindrical jet (Fig. 5). The radius of the cylindrical tangential jet was calculated as the arithmetic mean of values:

$$a = r \cdot \sin \alpha \tag{12}$$

determined from the measured values of the radial and tangential mean velocities and angle alpha calculated from the relation:

$$\alpha = \mathrm{arctg}\,(W_{tg} / W_r) \tag{13}$$

for various radial positions given by the radial coordinate expressed in dimensionless form $r^* = 2 \cdot r / D$.

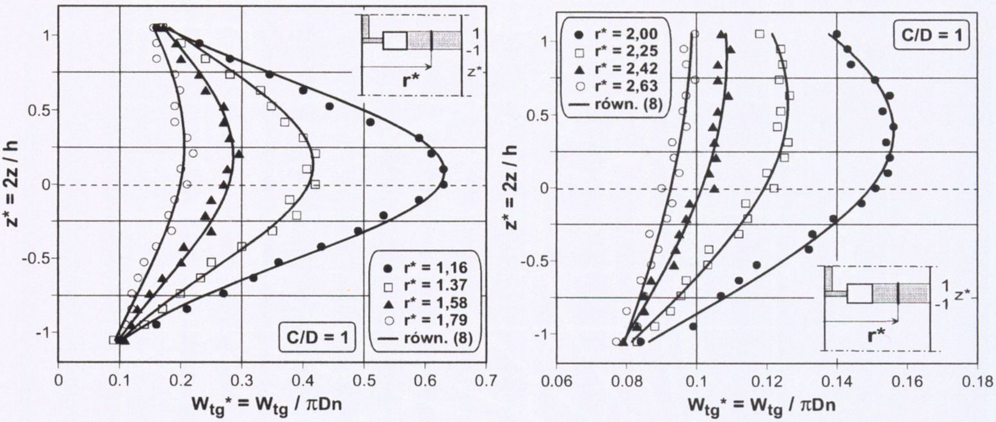


Fig. 4. Comparison of the experimental data and predicted dimensionless tangential velocity components from Eq. (8) for different radial distances from the tip of the impeller blades

Rys. 4. Porównanie wartości eksperymentalnych z bezwymiarowymi wartościami składowej obwodowej prędkości średniej z równania (8) dla różnych odległości od końca łopatek mieszadła

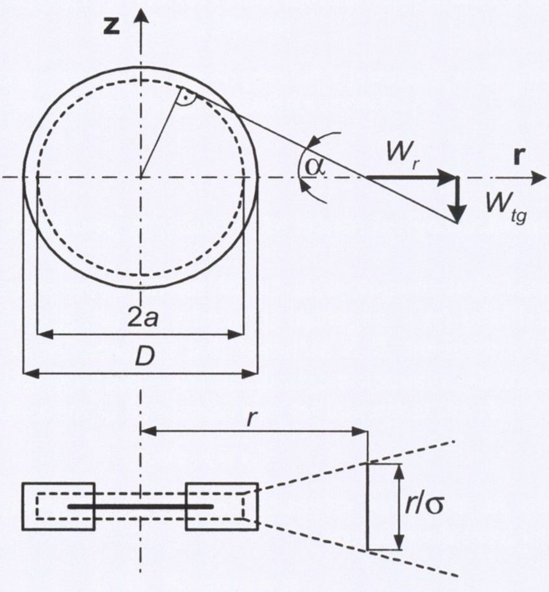


Fig. 5. Cylindrical tangential jet

Rys. 5. Styczny strumień cieczy wypływającej z mieszadła cylindrycznego

The dependence of the radial component of mean velocity W_r on the axial coordinate z is expressed in dimensionless form as [4,5]:

$$w_r{}^* = A_1 \cdot \left\{1 - \mathrm{tg}\,h^2\left[A_2(z^* - A_3)\right]\right\}, \qquad r^* = \text{const} \tag{14}$$

Parameter A_1 in Eq. (14) represents the maximum dimensionless radial velocity component of the liquid on a given profile along the blade height and for the given radial coordinate. This parameter is determined by the relation:

$$A_1 = \frac{A}{2 \cdot \pi \cdot D \cdot n} \cdot \left(\frac{\sigma}{r^3}\right)^{1/2} \cdot (r^2 - a^2)^{1/4} \tag{15}$$

Parameter A_2 represents the reciprocal width of the stream on a given position, $\sigma/2r$ (Fig. 5), normalised by the half-height of the blade $h/2$:

$$A_2 = \frac{\sigma \cdot h}{4 \cdot r} \tag{16}$$

Parameter A_3 in Eq. (14) expresses the dimensionless axial coordinate of the maximum on the velocity profile, *i.e.* its shift off the horizontal plane of coordinate $z = 0$.

Parameters A_1, A_2, A_3 were found by nonlinear regression and fit the experimental data of the velocities profiles $w_r{}^* = f(z^*)$ for various radial coordinates r^*.

The appointed values of A_1 and A_2 were used to calculate – for the known values of the parameter a (Eq. (12)) and the radial coordinate r^* – the quantity σ (Eq. (16)) and A in dimensionless form as $A/(2{\cdot}\pi{\cdot}D^2{\cdot}n)$ (Eq. (15)). The values of determined parameters σ and $A/(2{\cdot}\pi{\cdot}D^2{\cdot}n)$, and also parameter a (Eq. (12 and 13)), are presented in Table 1. The Table 1 also presents the results of experiments and calculations made by other authors.

Table 1

Determined parameters of the velocity field in the discharge stream from a Rushton turbine

Author	D/T	C/T	D [m]	a [m]	σ	$A/2{\cdot}\pi{\cdot}D^2{\cdot}n$
This work	**1/3**	**1/3**	**0.286**	**0.0429 ± 0.0049**	**14.88 ± 3.51**	**0.102 ± 0.012**
Fořt et al, [4]	1/3	1/2	0.25	– [1]	11.86	0.113
	1/3	1/2	0.3	– [1]	14.81	0.104
	1/3	1/2	0.5	– [1]	13.3	0.111
Obeid et al, [5]	1/3	1/3	0.4	0.0493	15.3	0.12
	1/3	1/2	0.4	0.0521	17.9	0.122
Drbohlav [6]	1/3	1/3	1.0	– [1]	15.35	0.140
	1/4	1/3	1.0	– [1]	13.6	0.092
Bertrand [7]	1/3	1/2	0.4	– [1]	17.87	0.122
Möckel [8]	1/2	1/2	0.5	– [1]	14.60	0.106
	1/3	1/2	0.5	– [1]	17.15	0.093

[1] The values of the parameter a in each case depends on the conditions of mixing according to the relation: $2{\cdot}a/D = 2.087{\cdot}Re_M{}^{-0.106}$ [5].

The value of parameter a is dependent on the impeller Reynolds number [5], so it depends in particular on the rotational speed of the impeller. On the basis of the results of experimental investigations and calculated results for impeller rotation speed n in the range from 4.17 to 7.50 s^{-1}, the following relationship has been found between the dimensionless parameter $2 \cdot a/D$ and the impeller Reynolds number:

$$\frac{2 \cdot a}{D} = 1.811 \cdot {Re_M}^{-0.07} \tag{17}$$

where the impeller Reynolds number is defined as follows:

$$Re_M = \frac{n \cdot D^2}{\nu} \tag{18}$$

where ν is the kinematic viscosity of an agitated liquid.

The values of parameters σ and $A/(2\pi \cdot D^2 \cdot n)$ correspond fairly well to the same parameters determined by Fořt at al. [4] and Obeid et al. [5]. The exponent in Eq. (17) is slightly lower than the value published by the same authors, probably because in the cited articles the diameter of the vessel $T = 1$ m, while in this study $T = 0.286$ m.

It follows from the results of experimental studies for the case of $D/T = 1/3$ and $C/D = 1$, that parameter A_3 in Eq. (11) is not equal to zero, because there is an axial displacement of the velocity profile maximum relative to the horizontal plane $z = 0$. The value of this displacement is a function of the radial coordinate r in accordance with the found relationship:

$$A_3 = 0.0069 \cdot \left(\frac{2 \cdot r}{D}\right)^{4,95} \tag{19}$$

The quantities a, σ, $A/(2 \cdot \pi \cdot D^2 \cdot n)$ and on this basis the calculated parameters A_1, A_2, A_3 were used to calculate the axial profiles (along the impeller blade height) of the dimensionless radial component of the mean velocity for various radial coordinates, *i.e.* various positions in the discharge stream from the Rushton turbine. The results of these calculations are presented in Figure 6.

The results for the radial component of the mean velocity obtained on the basis of the phenomenological model (Eq. (14)) were compared with the results from the statistical model described by Eq. (1). A comparison of the radial velocity profiles in the discharge stream of stirrer set on the basis of equations (1) and (14), and the corresponding experimental results are presented in Figures 7 for the cases of four different distances r^* from the tip of impeller blades.

4. Discussion and conclusions

The two velocity profiles (Equations 1 and 14) describe with sufficient accuracy the radial velocity distribution in the range of the dimensionless radius r^* of 1 to 2.00. The velocity distributions along the impeller blades are comparable, and are consistent with the experimental data. The average relative error of Eq. (14) in relation to the measurement data is 19.87%, while average relative error of Eq. (1) is 8.22%. The differences between the accuracies in determining these profiles corresponds to the form of expression of the governing equations for the models.

Three parameters of Eq. (1) were calculated by the method of the least squares of the best fit correlation (Eqs. 5, 6 and 7), but parameters A_1 and A_2 of Eq. (14) depend on the radial coordinate r according to the model (Eqs. 15 and 16). However, the phenomenological turbulent flow model in the discharge stream from the turbine impeller enables some turbulent characteristics (*e. g.* the eddy viscosity) to be determined, while the empirical model is based on the best fit of the chosen formula to the experimentally determined velocity profile $W_r = W(r)$.

On the basis of the experimental results, it can be concluded that, in the case of impeller off bottom clearance $C/T = 1/3$, parameter A_3 in the phenomenological model (Eq. 14) is different from zero and must be taken into account. However, for impeller off bottom clearance $C/T = 1/2$ its value amounts zero [9]. It follows from our experimental data that the dependence of parameter A_3 on the dimensionless radial coordinate is expressed by the power form – Eq. 19.

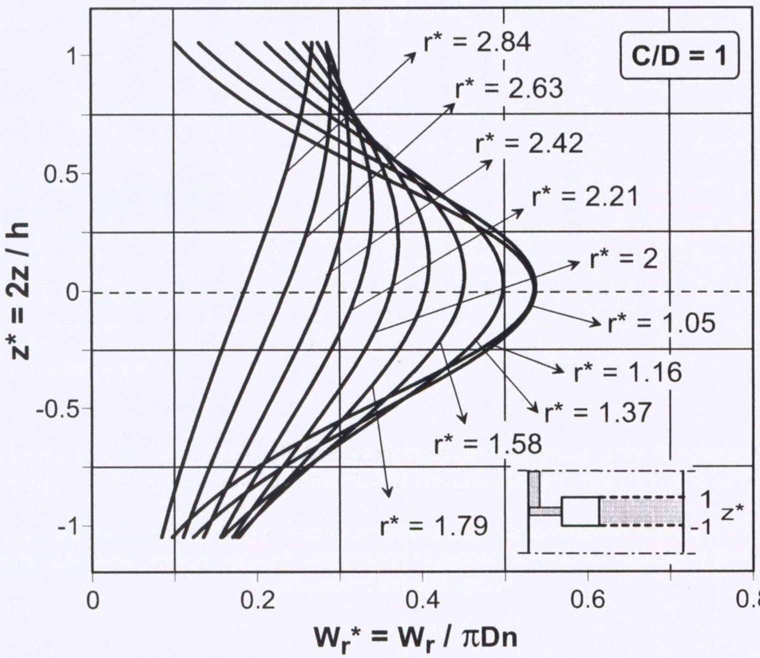


Fig. 6. Impeller discharge profiles of the dimensionless radial component of the mean velocity, according to Eq. (14)

Rys. 6. Profile bezwymiarowej składowej promieniowej średniej prędkości cieczy w strumieniu wylotowym z mieszadła zgodnie z równaniem (14)

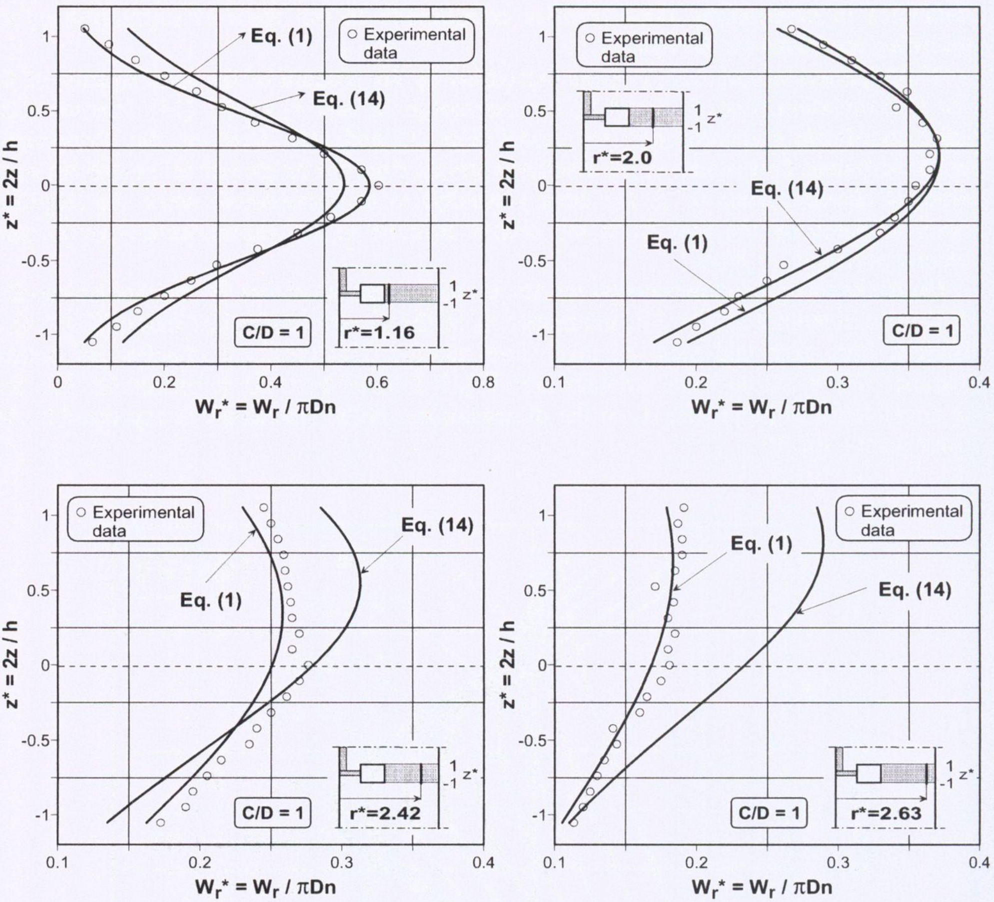


Fig. 7. Comparison of the experimental data and predicted dimensionless mean radial velocities from Eq. (1) and Eq. (14) for various dimensionless radial distances from the tip of the impeller blades r^*

Rys. 7. Porównanie wartości eksperymentalnych z bezwymiarowymi wartościami składowej promieniowej prędkości średniej z równań (1) i (14) dla różnych bezwymiarowych odległości od końca łopatek mieszadła r^*

For a dimensionless radius greater than 2.42, the compared velocity distributions show significant differences. In this case, the error of Eq. (14) is 55.23% and the error of Eq. (1) is 7.29%. Fluid flow in the range of $r^* > 2$ is already determined by the effects of baffles [10]. This is not consistent with the adopted model of free and uninterrupted flow in the impeller discharge stream.

Symbols

A	–	universal parameter of the axial profile of the radial velocity component	[m^2/s]
A_j	–	(j = 1, 2, 3) parameter of axial profile of radial mean velocity component – Eq. (14)	[–]
a_j, b_j, c_j	–	(j = 1, 2) coefficients in the correlation equations	[–]
a	–	radius of cylindrical tangential jet	[m]
B	–	width of baffle	[m]
b	–	width of impeller blade	[m]
C	–	impeller clearance	[m]
D	–	vessel diameter	[m]
d	–	impeller diameter	[m]
d_1		diameter of the impeller disk	[m]
h	–	height of impeller blade	[m]
n	–	impeller speed	[1/s]
r	–	radial coordinate	[m]
r*	–	dimensionless radial coordinate	[–]
Re_M		Reynolds number for mixing	[–]
T	–	vessel diameter	[m]
W_r	–	radial component of the mean velocity	[m/s]
W_{tg}	–	tangential component of the mean velocity	[m/s]
W_r*	–	dimensionless radial component of the mean velocity	[–]
W_{tg}*	–	dimensionless tangential component of the mean velocity	[–]
z	–	axial coordinate	[m]
z*	–	dimensionless axial coordinate	[–]
α	–	angle	[deg]
σ	–	universal parameter of axial profile of radial mean velocity component	[–]
ν	–	kinematic viscosity	[m^2/s]

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