MECHANIKA
CZASOPISMO TECHNICZNE
TECHNICAL TRANSACTIONS
MECHANICS
WYDAWNICTWO
POLITECHNIKI KRAKOWSKIEJ
5-M/2012
ZESZYT 10
ROK 109
ISSUE 10
YEAR 109

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ESTIMATION OF GEOMETRICAL PARAMETERS OF AN ELASTOKINEMATIC MODEL IN A CAR McPHERSON SUSPENSION

ESTYMACJA PARAMETRÓW GEOMETRYCZNYCH ELASTOKINEMATYCZNEGO MODELU ZAWIESZENIA McPHERSONA

A b s t r a c t

This paper deals with modelling of a McPherson car suspension including bushings. The position and orientation of each link and joint in the assembled and loaded suspension are measured by using a portable coordinate measuring manipulator. Based on experimental results elastic deflections are taken into account at the static equilibrium of the vehicle and used to estimate the real geometric parameters of the considered model.

Keywords: car suspension, elastokinematic model, estimation of parameters

S t r e s z c z e n i e

Artykuł dotyczy modelowania zawieszenia McPhersona kół samochodu przy uwzględnieniu odkształceń pod obciążeniem. Pozycje i orientacje członów i przegubów zawieszenia zmontowanego i obciążonego mierzono za pomocą przenośnego ramienia pomiarowego. Na podstawie wyników pomiarów uwzględniono odkształcenia w warunkach quasi-statycznych i estymowano rzeczywiste wartości parametrów geometrycznych rozpatrywanego modelu.

Sowa kluczowe: zawieszenie, model elastokinematyczny, estymacja parametrów

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Notations

Input variables: p – displacement of the rack; s – length of the spring-damper module. Output variables: δ – toe angle; γ – camber angle

$\boldsymbol{R}_{kp}$– orientation matrix of the reference system k with respect to p

$\boldsymbol{R}_{k}$– orientation matrix of reference system k with respect to the base

$\boldsymbol{R}_{p}$ – orientation matrix of reference system p with respect to the base

A_i– centre of the joint connected the link i to the frame of car body

B_i – centre of the joint connected to the wheel knuckle

$\boldsymbol{a}_i = \begin{bmatrix} a_{ix} & a_{iy} & a_{iz} \end{bmatrix}^T$ – position vector of point A_i described in the body reference frame

$\boldsymbol{b}_i = \begin{bmatrix} b_{ix} & b_{iy} & b_{iz} \end{bmatrix}^T$ – position vector of point B_i described in the body reference frame

$\boldsymbol{p}_i = \begin{bmatrix} p_{ix} & p_{iy} & p_{iz} \end{bmatrix}^T$ – position vector of measured point P_i described in the body reference frame

$\boldsymbol{a}_{ij}, \boldsymbol{b}_{ij}, \boldsymbol{d}_{ij}$ – position vectors: of point A_i with respect to A_i, point B_i with respect to B_i, and point B_i with respect to point A_i

$\boldsymbol{a}_{ij}^0, \boldsymbol{b}_{ij}^0, \boldsymbol{d}_{ij}^0, \boldsymbol{e}_{ij}^0$ – unit vectors of the position vectors

$\boldsymbol{x}_k^0, \boldsymbol{y}_k^0, \boldsymbol{z}_k^0$ – unit vectors of coordinate axes of the reference system k

$\boldsymbol{x}_p^0, \boldsymbol{y}_p^0, \boldsymbol{z}_p^0$ – unit vectors of coordinate axes of reference system p

d_{ii} – i-th link length (i =1, 3)

Mechanism positions – abbreviation codes:

–1	Suspension mechanism in rebound position (car unloaded)
–1; +38	– wheel steer angle maximum to the left (δ = +38°)
–1; +20	– wheel steer angle middle position to the left (δ = +20°)
–1; 0	– steering system is in straight ahead position (δ = 0°)
–1; –20	– wheel steer angle middle position to the right (δ = –20°)
–1; –38	– wheel steer angle maximum to the right (δ = –38°)
0	Suspension mechanism in the design position
0; +38	– wheel steer angle maximum to the left (δ = +38°)
0; +20	– wheel steer angle middle position to the left (δ = +20°)
0; 0	– steering system is in straight ahead position (δ = 0°)
0; –20	– wheel steer angle middle position to the right (δ = –20°)
0; –38	– wheel steer angle maximum to the right (δ = –38°)
+1	Suspension mechanism in bump position (car loaded)
+1; +38	– wheel steer angle maximum to the left (δ = +38°)
+1; +20	– wheel steer angle middle position to the left (δ = +20°)
+1; 0	– steering system is in straight ahead position (δ = 0°)
+1; –20	– wheel steer angle middle position to the right (δ = –20°)
+1; –38	– wheel steer angle maximum to the right (δ = –38°)

1. Introduction

Multi-body simulation model for vehicle behaviour analysis can be used to reduce the time needed to develop new suspension. However, before using a model for design studies, it is important to achieve a good correlation between the model simulations and the experimental results in order to ensure that modelling assumptions and parameters are valid. Several causes of discrepancy between model and reality can be given. Some parts, such as car chassis, McPherson strut or joint rubber bushings, considered as perfectly rigid in kinematic model, may show significant flexibility [1, 2]. These difficulties increase when no parametric data from the model of the suspension are known and when each parameter has to be determined in a limited number of measurements operations [7].

Former estimation methods addressing this problem are based on the analysis of he suspension behaviour on a kinematic and compliance (K&C) test bench [5, 6]. The first step is to use statistical analysis of experiments [1] in order to determine which parameters influence behaviour most. Then, the application of optimization routines for these parameters can be used to fit the model behaviour to experimental results. The estimation is then similar to a problem of optimal design.

Geometric and stiffness parameters can be obtained using optimization routines on an elastokinematic model, but this approach needs a high number of simulations. This paper presents an estimation method that is based on the measurements using a portable coordinate manipulator (Sigma arm made by Romer) [3], without disassembly the suspension mechanism. The measured point coordinates of the knuckle at selected positions are used to compute the position of joints and orientation of axes specific to considered suspension.

2. Procedure for suspension measurements

In order to measure knuckle displacement on the assembled suspension, four reference points, also called “marks” (Fig. 2) are made as small conic holes located on the surface to ensure an easy coordinate measurement on the assembled suspension in different position and orientation. The minimum number of points required to compute the position and orientation of a solid part in space is three. No precise location is required for the mark, provided they are sufficiently spaced from each other and accessible for measurement [7].

The measurement operations are done for 15 selected positions of the suspension and steering mechanism. The design position is describe with fixed steering wheel in straight ahead position and standard load case of the vehicle (two person of 75 kg at the front places and full tank). The car body coordinate system (body reference frame) is defined with the x axis in the forward oriented longitudinal direction of the vehicle, and the origin centred between two measured points (A_{1r}, A_{1l}) marked on the outside faces of rear bushings of the left of right wishbones (Fig. 1).

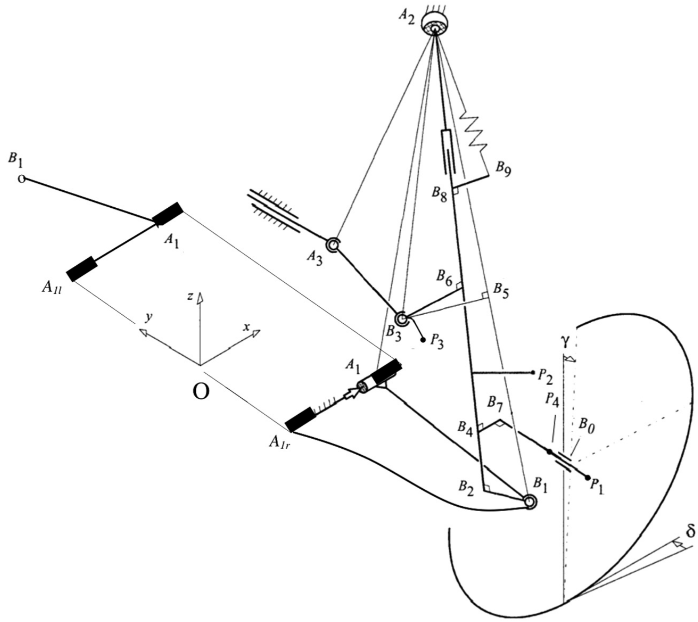

Fig. 1. Kinematical model of a McPherson suspension. Notations: A_1 – centre point of the revolute joint connecting the lower arm to the body frame; A_{1r}, A_{1l} – measured points marked on the outside faces of rear bushings of the left of right wishbones; A_2 – spherical joint between the strut and the chassis; A_3 – spherical joint between the tie rod and the steering rack; B_i – characteristic points on the wheel knuckle: B_1 – spherical joint between the knuckle and the lower arm; B_3 – spherical joint between the knuckle and the steering rod; P_i (i = 1, 2, 3, 4) – measured points marked on the wheel knuckle

Rys. 1. Kinematyczny model zawieszenia McPhersona. Oznaczenia: A_1 – centralny punkt obrotowego złącza łączącego dolne ramię z ramą nadwozia; A_{1r}, A_{1l} – mierzone punkty zaznaczone na zewnętrznych powierzchniach tylnych tulejek lewej i prawej dźwigni; A_2 – złacze kuliste między rozpórką a podwoziem; A_3 – złącze kuliste między zwrotnicą i dolnym ramieniem; B_i – punkty charakterystyczne na zwrotnicy koła: B_1 – złącze kuliste między zwrotnicą i dolnym ramieniem; B_3 – kuliste złącze między zwrotnicą a drążkiem kierowniczym; P_i (i = 1, 2, 3, 4) – mierzone punkty zaznaczone na zwrotnicy koła

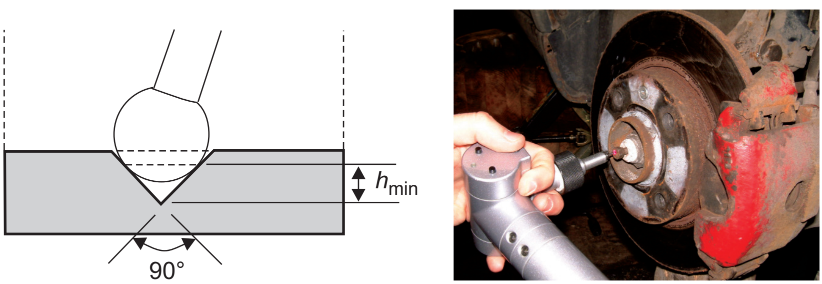

Fig. 2. Cross section and view of one mark used for coordinate measurement

Rys. 2. Przekrój poprzeczny i widok jednego znaku używanego do pomiaru współrzędnych

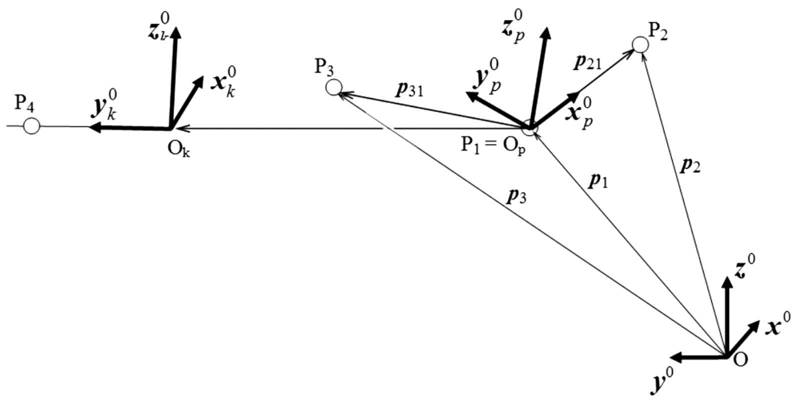

Fig. 3. The coordinate frames (*p* and *k*) and their correlation with measured points

Rys. 3. Ramki współrzędnych (*p* i *k*) i ich korelacja z punktami mierzonymi

The reference frame p is fixed to the wheel knuckle at point P_1 axis with the unit vector $\mathbf{x}_p^0$ pointing at P_2. (P_1, P_2 and P_3 are measured points on the wheel knuckle). The reference frame k is fixed to the point O_k belonged to the wheel axis with the unit vector $\mathbf{y}_p^0$ oriented into the direction of the wheel axis. $\boldsymbol{p}_1$, $\boldsymbol{p}_2$ and $\boldsymbol{p}_3$ are position vectors of the mark points on the wheel knuckle, where $\boldsymbol{p}_1$ is a position vector of a point P_1 on the wheel axis.

$$\boldsymbol{p}_{21} = \boldsymbol{p}_2 - \boldsymbol{p}_1 \tag{1}$$

$$\boldsymbol{p}_{31} = \boldsymbol{p}_3 - \boldsymbol{p}_1 \tag{2}$$

The unit vectors of the coordinate frame p can be obtained as follows:

$$\boldsymbol{x}_p^0 = \frac{\boldsymbol{p}_{21}}{|\boldsymbol{p}_{21}|} \tag{3}$$

$$\boldsymbol{z}_p^0 = \frac{\boldsymbol{p}_{21} \times \boldsymbol{p}_{31}}{|\boldsymbol{p}_{21} \times \boldsymbol{p}_{31}|} \tag{4}$$

$$\boldsymbol{y}_p^0 = \boldsymbol{z}_p^0 \times \boldsymbol{x}_p^0 \tag{5}$$

The orientation of the frame k with respect to the frame p can be described by using the known elements of their orientation matrices with respect to the base frame (such as toe and camber angle):

$$\boldsymbol{R}_{kp} = \boldsymbol{R}_p^T \boldsymbol{R}_k \tag{6}$$

or

$$\begin{bmatrix} \boldsymbol{x}_k^0 \cdot \boldsymbol{x}_p^0 & \boldsymbol{y}_k^0 \cdot \boldsymbol{x}_p^0 & \boldsymbol{z}_k^0 \cdot \boldsymbol{x}_p^0 \\ \boldsymbol{x}_k^0 \cdot \boldsymbol{y}_p^0 & \boldsymbol{y}_k^0 \cdot \boldsymbol{y}_p^0 & \boldsymbol{z}_k^0 \cdot \boldsymbol{y}_p^0 \\ \boldsymbol{x}_k^0 \cdot \boldsymbol{z}_p^0 & \boldsymbol{y}_k^0 \cdot \boldsymbol{z}_p^0 & \boldsymbol{z}_k^0 \cdot \boldsymbol{z}_p^0 \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_p^0 \cdot \boldsymbol{x}^0 & \boldsymbol{x}_p^0 \cdot \boldsymbol{y}^0 & \boldsymbol{x}_p^0 \cdot \boldsymbol{z}^0 \\ \boldsymbol{y}_p^0 \cdot \boldsymbol{x}^0 & \boldsymbol{y}_p^0 \cdot \boldsymbol{y}^0 & \boldsymbol{y}_p^0 \cdot \boldsymbol{z}^0 \\ \boldsymbol{z}_p^0 \cdot \boldsymbol{x}^0 & \boldsymbol{z}_p^0 \cdot \boldsymbol{y}^0 & \boldsymbol{z}_p^0 \cdot \boldsymbol{z}^0 \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{x}_k^0 \cdot \boldsymbol{x}^0 & \boldsymbol{y}_k^0 \cdot \boldsymbol{x}^0 & \boldsymbol{z}_k^0 \cdot \boldsymbol{x}^0 \\ \boldsymbol{x}_k^0 \cdot \boldsymbol{y}^0 & \boldsymbol{y}_k^0 \cdot \boldsymbol{y}^0 & \boldsymbol{z}_k^0 \cdot \boldsymbol{y}^0 \\ \boldsymbol{x}_k^0 \cdot \boldsymbol{z}^0 & \boldsymbol{y}_k^0 \cdot \boldsymbol{z}^0 & \boldsymbol{z}_k^0 \cdot \boldsymbol{z}^0 \end{bmatrix} \tag{7}$$

In order to simplify the following operations the elements of matrix $\mathbf{R}_k$ are written as:

$$R_k = \begin{bmatrix} \boldsymbol{x}_k^0 \cdot \boldsymbol{x}^0 & \boldsymbol{y}_k^0 \cdot \boldsymbol{x}^0 & \boldsymbol{z}_k^0 \cdot \boldsymbol{x}^0 \\ \boldsymbol{x}_k^0 \cdot \boldsymbol{y}^0 & \boldsymbol{y}_k^0 \cdot \boldsymbol{y}^0 & \boldsymbol{z}_k^0 \cdot \boldsymbol{y}^0 \\ \boldsymbol{x}_k^0 \cdot \boldsymbol{z}^0 & \boldsymbol{y}_k^0 \cdot \boldsymbol{z}^0 & \boldsymbol{z}_k^0 \cdot \boldsymbol{z}^0 \end{bmatrix} = \begin{bmatrix} l_x & m_x & n_x \\ l_y & m_y & n_y \\ l_z & m_z & n_z \end{bmatrix} \tag{8}$$

By using the measured coordinates of two points P_1 and P_4 (in an arbitrary position) marked on the wheel axis it is possible to calculate the elements of the matrix $\mathbf{R}_k$

$$m_x = \frac{\boldsymbol{p}_{4x} - \boldsymbol{p}_{1x}}{|\boldsymbol{p}_{4x} - \boldsymbol{p}_{1x}|} \tag{9}$$

$$m_y = \frac{\boldsymbol{p}_{4y} - \boldsymbol{p}_{1y}}{|\boldsymbol{p}_{4y} - \boldsymbol{p}_{1y}|} \tag{10}$$

$$m_z = \frac{\boldsymbol{p}_{4z} - \boldsymbol{p}_{1z}}{|\boldsymbol{p}_{4z} - \boldsymbol{p}_{1z}|} \tag{11}$$

Now δ – steer angle and γ – camber angle can be calculated by using formulae [2, 4]:

$$\delta = \arctan\left(\frac{m_x}{m_y}\right) \tag{12}$$

$$\gamma = \operatorname{arctg}\frac{-m_z}{\sqrt{1-m_z^2}} \tag{13}$$

The value of the angle δ obtained by using formulae (9)–(12) can be used to calculate:

$$l_x = \cos(\delta) \tag{14}$$

The remaining elements of the orthogonal matrix $\boldsymbol{R}_k$ can be obtained by using respective formulae selected from 6 independent orthogonal conditions for the unit vectors of the frame axes. With the correctly calculated orientation matrix $\boldsymbol{R}_{kp}$ according to an initial position it is now possible to compute the orientation matrix $\boldsymbol{R}_{ki}$ for i position (I = 1, 2, ..):

$$\boldsymbol{R}_{ki} = \boldsymbol{R}_{pi}\boldsymbol{R}_{kp} \tag{15}$$

3. Numerical example

The coordinates of the points P_i (i = 1, 2, 3) with respect to the body reference frame:

$$\boldsymbol{p}_1 = [281.74;\quad -780.03;\quad 47.38]^T;\ \boldsymbol{p}_2 = [145.95;\quad -664.92;\quad 0.87]^T;$$

$$\boldsymbol{p}_3 = [273.97;\quad -618.61;\quad 197.21]^T;\ \boldsymbol{p}_4 = [281.86; -746.79; 47.90]^T$$

The following vectors are calculated by using formulas (1) and (2):

$$\boldsymbol{p}_{21} = [-135.79;\quad 115.11;\quad -46.51]^T;\ \boldsymbol{p}_{31} = [-7.77;\quad 161.42;\quad 149.83]^T$$

The axis unit vectors of the frame p are obtained by using:

$$\boldsymbol{x}_p^0 = [-0.7380;\quad 0.6256;\quad -0.2528]^T;\ \boldsymbol{y}_p^0 = [0.2056;\quad 0.5653;\quad 0.7988]^T$$

$$\boldsymbol{z}_p^0 = [0.6427;\quad 0.5376;\quad -0.5458]^T$$

which gives the orientation matrix of the frame p with respect to the base frame:

$$\boldsymbol{R}_p^T = \begin{bmatrix} -0.7380 & 0.6256 & -0.2528 \\ 0.2056 & 0.5653 & 0.7988 \\ 0.6427 & 0.5376 & -0.5458 \end{bmatrix}$$

Knowing δ and γ in a selected position some elements of matrix $\boldsymbol{R}_k$ can be calculated:

$$m_x = 0.0132;\ m_y = -0.9999;\ m_z = -0.0096$$

To fulfil the criterion of an orthogonal matrix is calculated now:

$$\boldsymbol{R}_k = \begin{bmatrix} 0.9999 & 0.0132 & 0.0000 \\ 0.0132 & -0.9999 & 0.0096 \\ 0.0000 & -0.0096 & -0.9999 \end{bmatrix}$$

By using formula (6) the orientation matrix is computed:

$$\boldsymbol{R}_{kp} = \begin{bmatrix} -0.7297 & -0.6329 & 0.2589 \\ 0.2131 & -0.5702 & -0.7934 \\ 0.6497 & -0.5238 & 0.5509 \end{bmatrix}$$

By using formula (15):

$$\boldsymbol{R}_{k9} = \begin{bmatrix} 0.8265 & 0.5503 & -0.1189 \\ 0.5445 & -0.8350 & -0.0798 \\ -0.1432 & 0.0012 & -0.9897 \end{bmatrix}$$

$\delta_9 = -33.39°$; $\gamma_9 = -0.08°$

As result, by using MATLAB's spline interpolation, the following figures can be obtained:

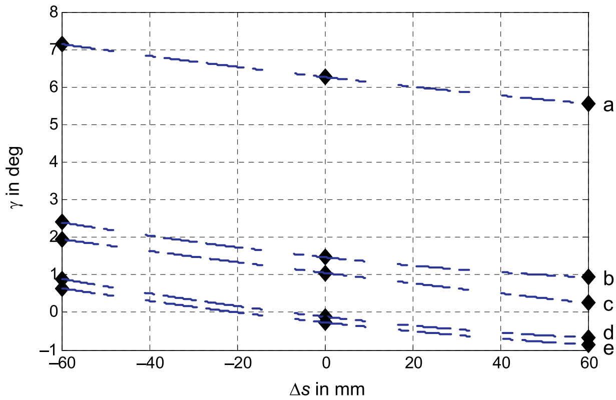

Fig. 4. Camber angle versus vertical displacement of the wheel axis at the steering wheel position: a) in the maximum on the left; b) in the middle left; c) in the middle; d) in the middle right; e) in the maximum right steering wheel position

Rys. 4. Kąt pochylenia koła w funkcji pionowego przemieszczenia osi koła przy pozycji koła kierownicy: a) maksymalnie w lewo; b) położenie środkowe w lewo; c) położenie środkowe w prawo; d) położenie środkowe w prawo; e) maksymalnie w prawo

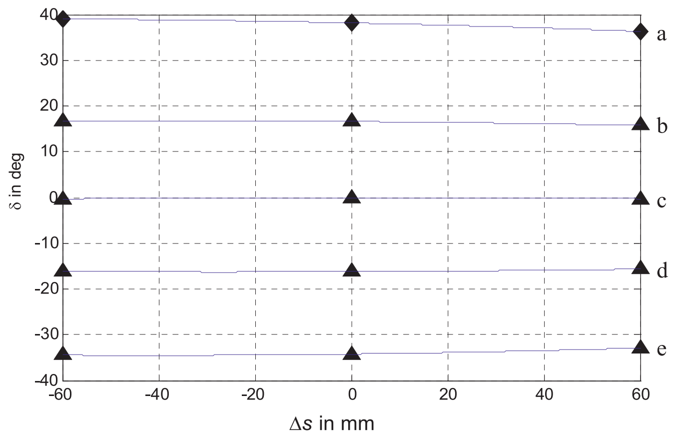

Fig. 5. Toe angle versus vertical displacement of the wheel axis when the steering wheel position is: a) in the maximum on the left; b) in the middle left; c) in the middle; d) in the middle right; e) in the maximum right steering wheel position

Rys. 5. Kąt zbieżności kół w funkcji pionowego przemieszczenia osi koła przy pozycji koła kierownicy: a) maksymalnie w lewo; b) położenie środkowe w lewo; c) położenie środkowe; d) położenie środkowe w prawo; e) maksymalnie w prawo

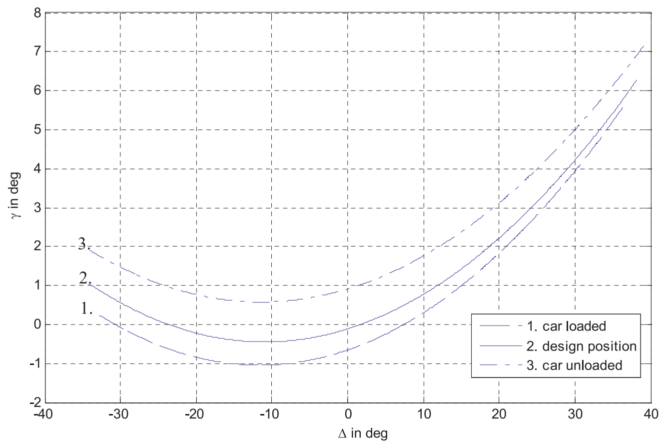

Fig. 6. Steer angle δ vs. camber angle γ calculated by using formulae (9)–(15) and taking the coordinates of points P_i measured in 15 positions of the suspension mechanism (see Table 1)

Rys. 6. Kat skrętu w funkcji koła pochylenia obliczony za pomocą wzorów (9)–(15) i współrzędnych punktów P_i zmierzonych w 15 pozycjach mechanizmu zawieszenia (p. Tabela 1)

Table 1

Coordinates of the measured points (P_i) of the wheel knuckle in the body reference frame

Measured coordinates \ Mechanism position	0; –38	0; –20	0	0; +20	0; +38
*p*1*x*	209,69	247,85	280,20	313,18	347,34
*p*1*y*	–754,54	–783	–794,74	–786,05	–767,75
*p*1*z*	86,28	91,3	95,56	94,04	92,48
*p*2*x*	184,72	155,47	145,27	148,43	165,78
*p*2*y*	– 581,08	–633,79	–679,99	–714,19	–751,35
*p*2*z*	28,09	35,56	46,35	54,11	65,55
*p*3*x*	279,93	276,12	272,36	270,31	268,81
*p*3*y*	–617,74	–624,29	–630,53	–627,34	–625,85
*p*3*z*	244,20	241,52	242,63	241,10	242,43

Measured coordinates \ Mechanism position	–1; –38	–1; –20	–1	–1; +20	–1; +38
*p*1*x*	212,77	252,14	281,74	315,53	349,32
*p*1*y*	–736,21	–764,02	–780,03	–770,59	–753,54
*p*1*z*	37,72	43,72	47,38	47,62	46,13
*p*2*x*	189,63	159,30	145,95	150,60	168,81
*p*2*y*	–561,30	–613,74	–664,92	–698,02	–737,60
*p*2*z*	–16,51	–8,36	0,87	9,20	20,23
*p*3*x*	286,29	281,77	273,97	272,83	270,66
*p*3*y*	–603,67	–607,99	–618,61	–614,25	–613,65
*p*3*z*	197,87	196,69	197,21	196,78	198,49
*P*4*x*			281,86		
*P*4*y*			–746,79		
*P*4*z*			47,90		

Measured coordinates \ Mechanism position	+1; –38	+1; –20	+1	+1; +20	+1; +38
*p*1*x*	212,51	249,97	281,64	313,67	347,08
*p*1*y*	–753,29	–777,88	–788,11	–779,87	–762,30
*p*1*z*	151,90	158,70	163,35	162,91	160,83
*p*2*x*	182,58	155,73	146,49	149,87	166,08
*p*2*y*	–581,53	–630,67	–674,21	–706,74	–741,93
*p*2*z*	90,92	100,72	111,92	121,26	132,73
*p*3*x*	277,34	247,87	272,21	271,47	271,13
*p*3*y*	–611,97	–617,01	–622,65	–619,33	–616,92
*p*3*z*	308,15	307,35	308,83	308,56	309,06

In order to validate the method presented the iterative estimation procedure has been written using Matlab programming software based on formulas (16)–(34). Part dimensions are given by portable measuring arm. The implementation of the estimation procedure is represented in Fig. 7.

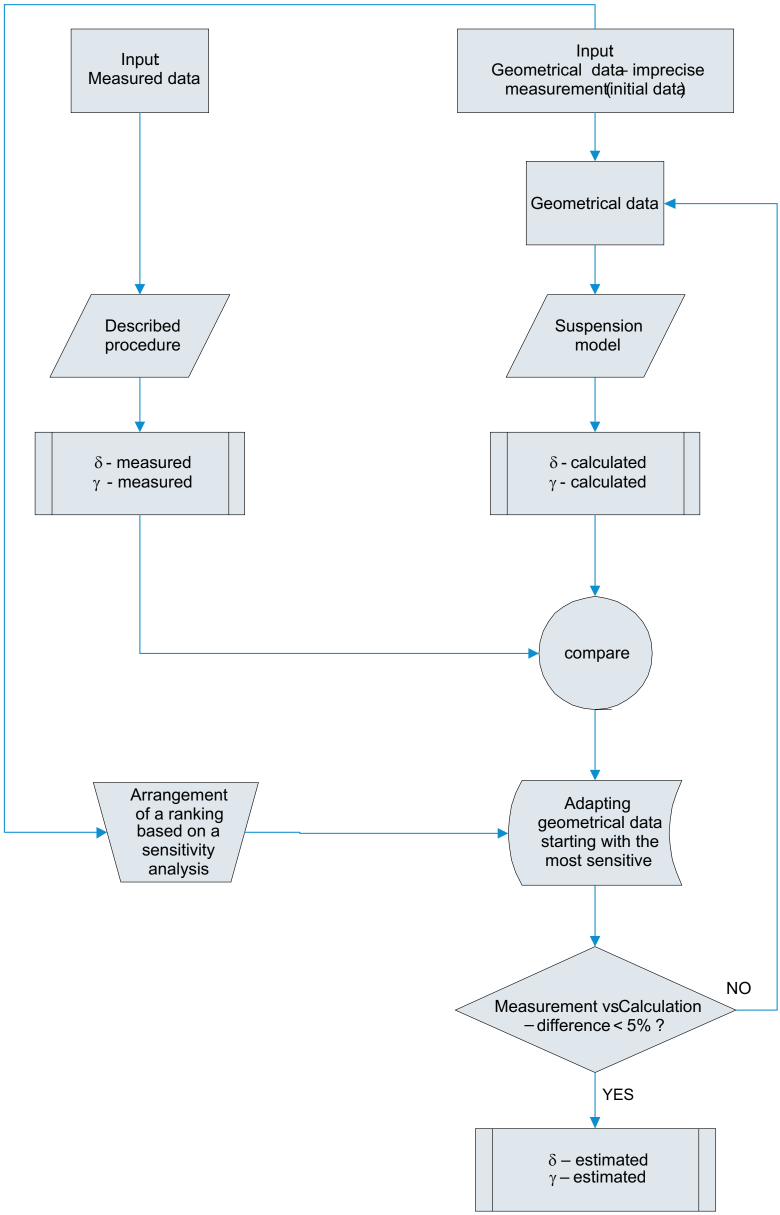

Fig. 7. Flowchart of the estimation procedure and experimental verification

Rys. 7. Schemat procedury estymacji i eksperymentalnej weryfikacji

4. Sensitivity analysis and parameter estimation

The aim of a sensitivity analysis is to arrange a ranking which gives the information of how selected tolerances of geometrical parameters have influence on the toe angle and camber angle. To observe how the angles will change, one geometrical parameter after the other is modified in a range of ± 5%. The chosen objective is the approximation of the simulated camber angle and steer angle to the measured with the assumption that the toe (and steer) angle has a ten times higher importance than the camber angle. The decision to converge like this is due to the fact that δ is the decisive angle during the vehicle motion and has mainly influence on the safety and comfort. The highest importance of a parameter describes a value of 1 down to the minor importance – value 13.

This comparison shows slightly different results between a +5% and a –5% value modification but one fact is clearly recognizable. There are parameters like a_{2y}, a_{1y}, a_{3y}, d_{11}, d_{33} which are the most important. This does mean that the accuracy of these parameters has to be the biggest within the synthesis of a new suspension system.

Table 2

Comparison of the order of influence of both analyses

	a_{2x}	a_{2y}	a_{2z}	b_0	a_{3x}	a_{3y}	a_{3z}	a_{1x}	a_{1y}	a_{1z}	d_{11}	d_{33}	b_{26}
Order of influence +5%	9	5	6	13	12	3	11	7	4	10	2	1	8
Order of influence –5%	10	1	7	13	12	5	9	6	2	11	3	4	8

Table 3

Geometrical data – Starting values vs. estimated values

		$\boldsymbol{a}_1$	$\boldsymbol{a}_2$	$\boldsymbol{a}_3$	d_{11}	b_{63}	b_{26}	d_{33}	b_{12}	b_{13}
Initial data	a_{ix}	239.9	246.9	140.0	369.1	146.0	48.7	434.0	94.5	129.9
	a_{iy}	–309.0	–519.5	–233.6						
	a_{iz}	38.0	634.7	75.0						
Estimated data	a_{ix}	245.7	271.9	147.0	395.8	126.0	51.5	435.7	57.3	124.2
	a_{iy}	–308.0	–504.5	–239.9						
	a_{iz}	22.0	616.7	51.0						

Table 4

Estimated coordinates of the knuckle points $\boldsymbol{B}_i$ described in the body reference frame

s		526.5			586.5 (Δs = 0)			646.5		
		$\boldsymbol{b}_1$	$\boldsymbol{b}_3$	$\boldsymbol{b}_2$	$\boldsymbol{b}_1$	$\boldsymbol{b}_3$	$\boldsymbol{b}_2$	$\boldsymbol{B}_1$	$\boldsymbol{b}_3$	$\boldsymbol{b}_2$
Estimated values (Δp = 0)	b_{ix}	299.0	184.2	306.9	296.4	182.0	304.3	292.5	178.5	299.9
	b_{iy}	–687.6	–660.4	–632.5	–698.6	–671.3	–643.4	–700.3	–674.3	–644.8
	b_{ix}	120.5	159.0	107.1	60.8	100.8	47.8	–1.8	40.2	–13.8

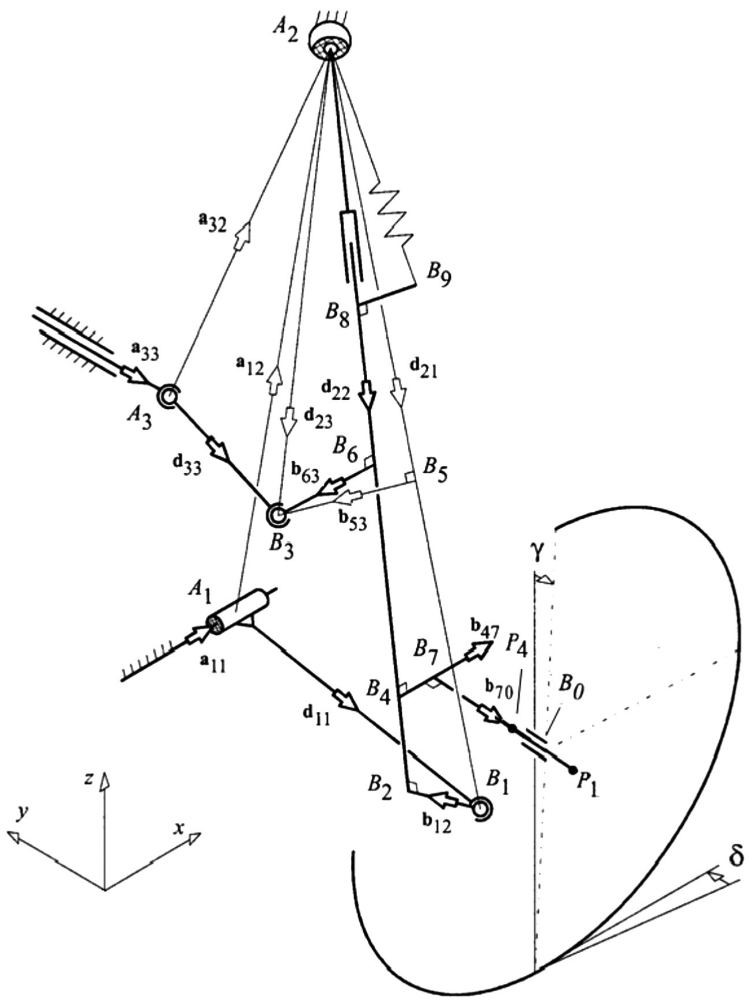

Fig. 8. Kinematical model of a McPherson suspension. Notations: $\boldsymbol{a}_{11}$ – unit vector of the revolute joint axis; $\boldsymbol{a}_{33}$ – unit vector of the rack axis; $\boldsymbol{d}_{11}$ – unit vector of the lower arm; $\boldsymbol{d}_{33}$ – unit vector of the steering rod; $\boldsymbol{d}_{21}$ – unit vector of the kingpin axis; $\boldsymbol{d}_{22}$ – unit vector of the shock absorber axis

Rys. 8. Kinematyczny model zawieszenia McPhersona. Oznaczenia: $\boldsymbol{a}_{11}$ – wektor jednostkowy osi złącza obrotowego; $\boldsymbol{a}_{33}$ – wektor jednostkowy osi zębatki; $\boldsymbol{d}_{11}$ – wektor jednostkowy dolnego ramienia; $\boldsymbol{d}_{33}$ – wektor jednostkowy drążka kierowniczego; $\boldsymbol{d}_{21}$ – wektor jednostkowy osi sworznia zwrotnicy; $\boldsymbol{d}_{22}$ – wektor jednostkowy osi amortyzatora

The following formulae [2] are used to calculate the position and orientation coordinates of the wheel knuckle as functions of the input variables: s – the strut displacement and p – the rack displacement.

$$\boldsymbol{d}_{11}^{0} = \boldsymbol{a}_{12}^{0} c_1 - \boldsymbol{a}_{11}^{0} c_{12} c_1 - (\boldsymbol{a}_{12}^{0} \times \boldsymbol{a}_{11}^{0})\sqrt{D_1} \,/\, (1 - c_{12}^{2}) \tag{16}$$

$$c_{12} = \boldsymbol{a}_{12}^{0} \cdot \boldsymbol{a}_{12}^{0} = \text{const.} \tag{17}$$

$$c_1 = d_{11}^2 + a_{12}^2 - b_{12}^2 - s^2 / (2d_{11}a_{11}) \tag{18}$$

$$D_1 = 1 - c_{12}^2 - c_1^2 \tag{19}$$

$$\boldsymbol{d}_{21}^{0} = \boldsymbol{d}_{11}^{0} d_{11} - \boldsymbol{a}_{12}^{0} a_{12} / \sqrt{s^2 + b_{12}^2} \tag{20}$$

$$\boldsymbol{d}_{23}^{0} = \boldsymbol{d}_{21}^{0}(c_3 - c_4 c_5) - \boldsymbol{a}_{32}^{0}(c_4 - c_3 c_5) + (\boldsymbol{d}_{21}^{0} \times \boldsymbol{a}_{32}^{0})\sqrt{D_2} / (1 - c_5^2) \tag{21}$$

$$c_3 = s^2 + b_{12}^2 + (s - b_{26})^2 + b_{63}^2 - b_{13}^2 / (2\sqrt{[(s - b_{26})^2 + b_{63}^2](s^2 + b_{12}^2)}) \tag{22}$$

$$c_4 = (s - b_{26})^2 + b_{63}^2 + a_{32}^2 - d_{33}^2 / (2a_{32}\sqrt{(s - b_{26})^2 + b_{63}^2}) \tag{23}$$

$$c_5 = (-\boldsymbol{d}_{21}^{0}) \cdot \boldsymbol{a}_{32}^{0} \tag{24}$$

$$D_2 = 1 - c_3^2 - c_4^2 - c_5^2 + 2c_3 c_4 c_5 \tag{25}$$

$$\boldsymbol{d}_{22}^{0} = \boldsymbol{d}_{21}^{0}(c_6 - c_7 c_8) - \boldsymbol{d}_{23}^{0}(c_7 - c_6 c_8) + (\boldsymbol{d}_{21}^{0} \times \boldsymbol{d}_{23}^{0})\sqrt{D_3} / (1 - c_8^2) \tag{26}$$

$$c_6 = s / \sqrt{s^2 + b_{12}^2} \tag{27}$$

$$c_7 = (s - b_{26}) / \sqrt{(s - b_{26})^2 + b_{63}^2} \tag{28}$$

$$b_8 = \boldsymbol{d}_{23}^{0} \cdot \boldsymbol{d}_{21}^{0} \tag{29}$$

$$D_3 = 1 - c_6^2 - c_7^2 - c_8^2 + 2c_6 c_7 c_8 \tag{30}$$

$$\boldsymbol{b}_{63}^{0} = (\boldsymbol{d}_{23}^{0}\sqrt{(s - b_{26})^2 + b_{63}^2} - \boldsymbol{d}_{22}^{0}(s - b_{26})) / b_{63} \tag{31}$$

$$\boldsymbol{b}_{70}^{0} = (-\boldsymbol{d}_{22}^{0})c_9 + \boldsymbol{b}_{63}^{0} c_{10} + ((-\boldsymbol{d}_{22}^{0}) \times \boldsymbol{b}_{63}^{0})\sqrt{D_4} \tag{32}$$

$$c_{10} = \boldsymbol{b}_{70}^{0} \cdot \boldsymbol{b}_{63}^{0} \tag{33}$$

$$D_4 = 1 - c_9^2 - c_{10}^2 \tag{34}$$

Steer angle δ and camber angle γ – measured and calculated (by using simulation model) versus displacement of the strut (Δs) for fixed steering wheel in straight ahead position ($\Delta p = 0$) are shown in Fig. 9. Similar results presented by Mantaras at all [5, 6] were obtained by using ADAMS/Car.

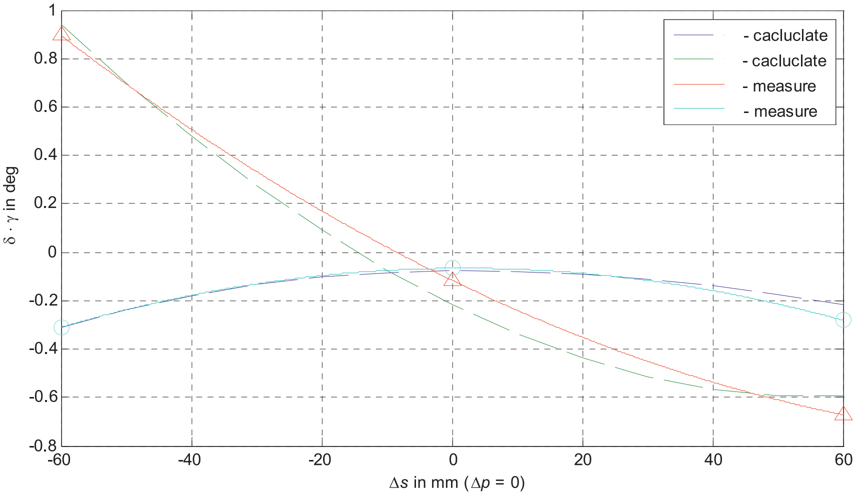

Fig. 9. Steer angle δ and camber angle γ – measured and calculated (by using simulation model) versus displacement of the strut (Δs) for fixed steering wheel in straight ahead position ($\Delta p = 0$)

Rys. 9. Kat skrętu δ oraz kat pochylenia γ – zmierzone i obliczone (za pomocą modelu symulacyjnego) w funkcji przemieszczenia rozpórki (Δs) dla ustalonej do jazdy na wprost pozycji koła kierownicy ($\Delta p = 0$)

5. Conclusions

The method presented in this paper can be used to associate measurements on assembled vehicle suspension in order to obtain geometric parameters according to assumed elastokinematic model. The implementation of this method is simple and the only required device is a portable measuring arm. Geometric parameters are estimated relatively simple on the basis of measurements results at the static equilibrium of the vehicle by using closed formulae and sensitive analysis.

The geometric estimation is a first step towards a fully representative model. The next step in improving the model behaviour is to achieve a better estimation of bushing stiffnesses. The measurement of successive positions of marked points could be used to perform stiffness parameter estimation.

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