MECHANIKA
CZASOPISMO TECHNICZNE
TECHNICAL TRANSACTIONS
MECHANICS
WYDAWNICTWO POLITECHNIKI KRAKOWSKIEJ
6-M/2012
ZESZYT 11
ROK 109
ISSUE 11
YEAR 109

DAWID TALER*, JAROSŁAW TOKARCZYK**

MODELING OF PIPELINE HEATING

MODELOWANIE NAGRZEWANIA RUROCIĄGÓW

A b s t r a c t

Heating of the pipeline with thermally insulated outer surface was modelled using the explicit finite difference method. The time variation of the fluid temperature at the pipeline inlet has the ramp form. An exact analytical solution was found using the method of superposition. The differences between the analytically and numerically predicted fluid and tube wall temperatures are small. The same method will be used in future for modeling, a heat storage unit, that is used in a combined electric-water heating system.

Keywords: *pipeline heating*, *finite difference method*, *analytical solution*

S t r e s z c z e n i e

Nagrzewanie rurociągu z izolowaną cieplnie powierzchnią zewnętrzną zostało zamodelowane z zastosowaniem metody różnic skończonych. Czasowy przebieg temperatury czynnika na wlocie do rurociągu ma kształt rampy. Ścisłe rozwiązanie analityczne zostało wyznaczone za pomocą metody superpozycji. Różnice między temperaturą cieczy wyznaczoną analitycznie i numerycznie są nieznaczne. Ta sama metoda różnic skończonych zastosowana zostanie w przyszłości do modelowanie akumulatora ciepła w hybrydowym elektryczno-wodnym układzie ogrzewania.

Słowa kluczowe: *nagrzewanie rurociągu*, *metoda różnic skończonych*, *rozwiązanie analityczne*

* Dr hab. inż. Dawid Taler, prof. PK, Instytut Inżynierii Cieplnej i Ochrony Powietrza, Wydział Inżynierii Środowiska, Politechnika Krakowska.

** Mgr inż. Jarosław Tokarczyk, Katedra Maszyn i Urządzeń Energetycznych, Wydział Mechaniczny, Politechnika Krakowska.

1. Introduction

Power units start-ups involve heating processes of steam pipelines. Steam temperature variation rate is of great importance not only because of pipe wall thermal stresses, but also because of stresses in fittings installed on the pipeline, often very expensive [1, 2]. Optimum transient fluid temperature changes at the inlet of the pipeline were determined from the solution of the Voltera integral equation of the first kind. The Duhamel's integral was solved numerically using the rectangle method. A review of existing literature suggests, however, that there is no paper devoted to the modeling of pipelines with varying in time medium temperature at the inlet to the pipeline.

Usually, the temperature and pressure of the flowing steam do not change considerably, so the steam can be assumed to be incompressible. A pipeline heating will be modeled using an explicit finite difference method. The accuracy of finite difference method, will be assessed by comparing the results with those obtained from the exact analytical solutions. The flow of an incompressible fluid in the thin walled pipeline will be analyzed. The fluid temperature at the inlet varies first linearly, then remains constant (Fig. 1). For such a case, frequently occurring in practice, the exact analytical solution can be found. It should be noted that the pipeline is a special case of the heat exchanger in which the heat transfer coeficient on the outer tube surface is equal to zero. Temperature changes of the fluid inside the pipeline are described by the energy conservation equation

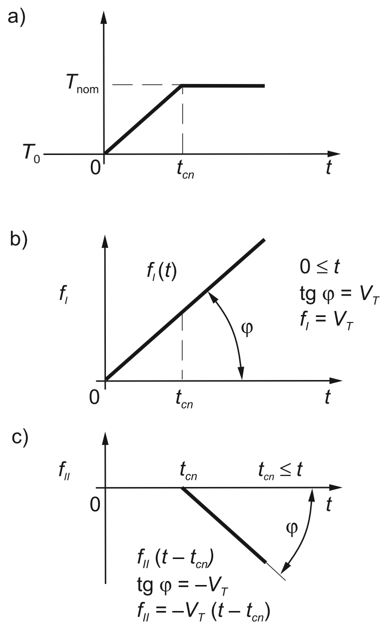

Fig. 1. Decomposition of fluid temperature changes at the inlet of the pipeline (a) prescribed in the boundary condition (6) into two parts: $f^{\mathrm{I}}(t)$ (b) and $f^{\mathrm{II}}(t - t_{cn})$ (c)

Rys. 1. Dekompozycja zmian temperatury czynnika na wlocie do rurociągu (a) zadanej w warunku brzegowym (6) na dwie składowe: $f^{\mathrm{I}}(t)$ (b) i $f^{\mathrm{II}}(t - t_{cn})$ (c)

$$\tau_1 \frac{\partial T_1}{\partial t} + \frac{1}{N_1} \frac{\partial T_1}{\partial x^+} = -(T_1 - T_w) \tag{1}$$

where:

T_1 = fluid temperature,
t – time,
$x^+ = x/L_x$ – dimensionless coordinate,
L_x – pipeline length,
T_w – tube wall temperature.

The number of transfer units N_1 and the time constant τ_1 are defined as:

$$N_1 = \frac{\alpha_1 A_w}{\dot{m}_1 c_{p1}} = \frac{\alpha_1 A_w}{\rho_1 w_1 A_1 c_{p1}}, \qquad \tau_1 = \frac{m_1 c_{p1}}{\alpha_1 A_w} = \frac{\rho_1 A_1 L_x c_{p1}}{\alpha_1 A_w} = \frac{\rho_1 d_w c_{p1}}{4\alpha_1} \tag{2}$$

where:

α_1 – heat transfer coefficient at the inner pipeline surface,
$A_w = \pi d_w L_x$ – area of the pipeline inner surface,
$\dot{m}_1$ – fluid mass flow rate,
c_{p1} – fluid specific heat at constant pressure,
ρ_1 – fluid density,
w_1 – fluid velocity,
$A_1 = \pi d_w^2 / 4$ – cross-section area of the pipeline,
d_w – tube inner diameter.

The energy balance equation for the pipeline wall is:

$$\tau_w \frac{\partial T_w}{\partial t} = T_1 - T_w \tag{3}$$

where the symbol τ_w denotes the time constant of the tube wall, defined as:

$$\tau_w = \frac{m_w c_w}{\alpha_1 A_w} = \frac{\left(d_z^2 - d_w^2\right)\rho_w c_w}{4 d_w \alpha_1} \tag{4}$$

where:

$m_w = \pi\left(d_z^2 - d_w^2\right) L_x \rho_w / 4$ – pipeline mass,
c_w – specific heat of the pipeline material,
ρ_w – density of the pipeline material.

Equation (1) is subject to the boundary condition at the pipeline inlet:

$$T_1\big|_{x=0} = T_0 + f(t) \tag{5}$$

where $f(t)$ is the known function of time, describing the changes in the excess fluid temperature at the inlet to the pipeline. The excess temperature over the initial temperature T_0 is defined as the difference between the real and the initial fluid temperature. The initial temperature T_0 of the fluid and pipeline is assumed to be constant, i.e. the initial conditions have the form:

$$T_1\big|_{t=0} = T_0 \tag{6}$$

$$T_w\big|_{t=0} = T_0 \tag{7}$$

The initial-boundary value problem (1)–(7) is solved using the finite difference method and analytically for a time variation in the inlet temperature of the fluid in the form of a ramp (Fig. 1). The function $f(t)$ in the boundary condition (5) is given by the following expressions:

$$\begin{aligned} f(t) &= v_T t, \qquad 0 \le t \le t_{cn} \\ f(t) &= T_{nom}, \qquad t_{cn} \le t. \end{aligned} \tag{8}$$

where the fluid temperature rate is given by: $v_{\mathrm{T}} = (T_{\mathrm{nom}} - T_0)/t_{\mathrm{cn}}$.

2. Finite difference method

The explicit finite difference method was used to solve the problem (1)–(8). The arrangement of nodes in the finite difference grid is depicted in Figure 2. Equations (1) and (3) are approximated by the finite difference equations:

$$\tau_1 \frac{T_{1,i+1}^{n+1} - T_{1,i+1}^{n}}{\Delta t} = -\frac{1}{N_1}\frac{T_{1,i+1}^{n} - T_{1,i}^{n}}{\Delta x^+} - \left(\frac{T_{1,i}^{n} + T_{1,i+1}^{n}}{2} - T_{w,i}^{n}\right), \quad i = 1,\ldots,N, \quad n = 0,1,\ldots \tag{9}$$

$$\tau_w \frac{T_{w,i}^{n+1} - T_{w,i}^{n}}{\Delta t} = \frac{T_{1,i}^{n} + T_{1,i+1}^{n}}{2} - T_{w,i}^{n} \tag{10}$$

The coordinates of the difference grid are:

$$x_i = (i-1)\cdot \Delta x, \quad i = 1,\ldots,N+1 \quad \text{for fluid,}$$

$$x_i = (i-1)\cdot \Delta x + \frac{1}{2}\Delta x, \quad i = 1,\ldots,N+1 \quad \text{for pipeline wall,}$$

$$t_n = n\cdot \Delta t, \quad n = 0,1,\ldots \quad \text{for fluid and pipeline wall.}$$

The dimensionless grid size is:

$$\Delta x^+ = \Delta x / L_x = 1/N.$$

Solving Equation (9) for $T_{1,i+1}^{n+1}$ and Equation (10) for $T_{w,i}^{n+1}$ gives, respectively:

$$T_{1,i+1}^{n+1} = T_{1,i+1}^{n} - \frac{\Delta t}{\tau_1}\left[\frac{1}{N_1}\frac{T_{1,i+1}^{n} - T_{1,i}^{n}}{\Delta x^+} + \left(\frac{T_{1,i}^{n} + T_{1,i+1}^{n}}{2} - T_{w,i}^{n}\right)\right], \quad i = 1,\ldots,N, \quad n = 0,1,\ldots \tag{11}$$

$$T_{w,i}^{n+1} = T_{w,i}^{n} + \frac{\Delta t}{\tau_w}\left(\frac{T_{1,i}^{n} + T_{1,i+1}^{n}}{2} - T_{w,i}^{n}\right) \tag{12}$$

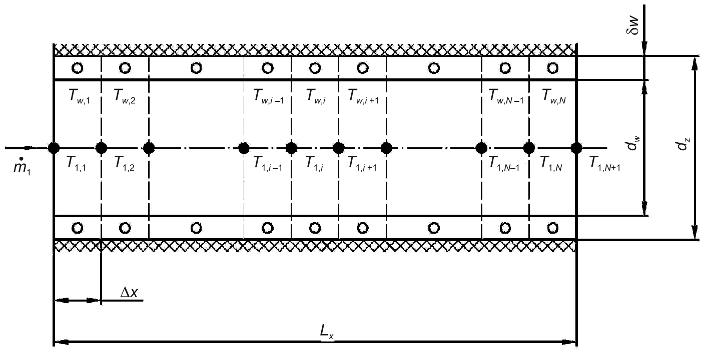

Fig. 2. Finite difference grid for fluid and pipe wall; • – fluid, o – pipeline wall

Rys. 2. Siatka różnicowa w obszarze cieczy i ścianki rury; • – płyn, o – ścianka rury

The boundary condition (8) can be written as:

$$T_{1,1}^n = T_0 + v_T\, t_n, \quad t \le t_{cn}, \quad n = 0,1,\ldots$$
$$T_{1,1}^n = T_{nom}, \quad t \ge t_{cn} \tag{13}$$

The initial conditions (6) and (7) take the following form:

$$T_{1,i}^0 = T_0, \quad i = 1,\ldots,N+1 \tag{14}$$

$$T_{w,i}^0 = T_0, \quad i = 1,\ldots,N \tag{15}$$

Formulas (11) and (12), after taking into account the boundary condition (11) and the initial conditions (14) and (15), allow to determine the temperature of the fluid and the wall as a function of position and time.

3. Exact analytical solution

To determine the temperature distribution in the fluid and the wall of the pipeline the method of superposition [4] will be used. Applying the principle of superposition, the boundary condition (8) will be decomposed into the form (Fig. 2):

$$T_1\big|_{x=0} = T_0 + f^{\mathrm{I}}(t), \quad 0 \le t \le t_{cn}$$
$$T_1\big|_{x=0} = T_0 + f^{\mathrm{I}}(t) + f^{\mathrm{II}}(t - t_{cn}), \quad t \ge t_{cn} \tag{16}$$

where:

$$f^{\mathrm{I}}(t) = v_T t, \quad 0 \le t \le t_{cn}$$
$$f^{\mathrm{II}}(t - t_{cn}) = -v_T(t - t_{cn}), \quad t \ge t_{cn} \tag{17}$$

The fluid and the wall temperature are determined from:

$$T_1 = T_0 + T_1^{\mathrm{I}}\left(x,t\right), \quad 0 \le t \le t_{cn} \tag{18}$$

$$T_w = T_0 + T_w^{\mathrm{I}}\left(x,t\right), \quad 0 \le t \le t_{cn} \tag{19}$$

and:

$$T_1 = T_0 + T_1^{\mathrm{I}}\left(x,t\right) + T_1^{\mathrm{II}}\left(x,t-t_{cn}\right), \quad t \ge t_{cn} \tag{20}$$

$$T_w = T_0 + T_w^{\mathrm{I}}\left(x,t\right) + T_w^{\mathrm{II}}\left(x,t-t_{cn}\right), \quad t \ge t_{cn} \tag{21}$$

The temperature $T_1^{\mathrm{I}}(x,t)$ and $T_w^{\mathrm{I}}(x,t)$ are the solutions of the following initial-boundary value problem:

$$\tau_1 \frac{\partial T_1^{\mathrm{I}}}{\partial t} + \frac{1}{N_1}\frac{\partial T_1^{\mathrm{I}}}{\partial x^+} = -\left(T_1^{\mathrm{I}} - T_w^{\mathrm{I}}\right) \tag{22}$$

$$\tau_w \frac{\partial T_w^{\mathrm{I}}}{\partial t} = T_1^{\mathrm{I}} - T_w^{\mathrm{I}} \tag{23}$$

$$T_1^{I}\big|_{x=0} = f^{\mathrm{I}}(t) = v_T\, t, \quad 0 \le t \le t_{cn} \tag{24}$$

$$T_1^{\mathrm{I}}\big|_{t=0} = 0 \tag{25}$$

$$T_w^{\mathrm{I}}\big|_{t=0} = 0 \tag{26}$$

The temperature $T_1^{\mathrm{II}}(x,t)$ and $T_w^{\mathrm{II}}(x,t)$ are the solutions of the follwing initial-boundary value problem:

$$\tau_1 \frac{\partial T_1^{\mathrm{II}}}{\partial t} + \frac{1}{N_1}\frac{\partial T_1^{\mathrm{II}}}{\partial x^+} = -\left(T_1^{\mathrm{II}} - T_w^{\mathrm{II}}\right) \tag{27}$$

$$\tau_w \frac{\partial T_w^{\mathrm{II}}}{\partial t} = T_1^{\mathrm{II}} - T_w^{\mathrm{II}} \tag{28}$$

$$T_1^{\mathrm{II}}\big|_{x=0} = f^{\mathrm{II}}\left(t-t_{cn}\right) = -v_T\left(t-t_{cn}\right), \quad t \ge t_{cn} \tag{29}$$

$$T_1^{\mathrm{II}}\big|_{t=0} = 0 \tag{30}$$

$$T_w^{\mathrm{II}}\big|_{t=0} = 0 \tag{31}$$

The solution of the problem (27)–(31) has the form [3]:

$$T_1^{\mathrm{I}} = v_T\,\tau_w\left\{e^{-(\xi+\eta)}\left[\left(\eta-\xi\right)U + \xi I_0\left(2\sqrt{\xi\eta}\right) + \sqrt{\xi\eta}\, I_1\left(2\sqrt{\xi\eta}\right)\right]\right\}, \quad 0 \le t \le t_{cn} \tag{32}$$

$$T_w^{\mathrm{I}} = T_1^{\mathrm{I}} - v_T\,\tau_w\, e^{-(\xi+\eta)}\left[U - I_0\left(2\sqrt{\xi\eta}\right)\right], \quad 0 \le t \le t_{cn} \tag{33}$$

where the function U is defined as:

$$U = U\left(\xi,\eta\right) = e^{\xi+\eta} - \sum_{n=1}^{\infty}\left(\frac{\xi}{\eta}\right)^{\frac{n}{2}} I_n\left(2\sqrt{\xi\eta}\right) \tag{34}$$

The symbols $I_0(x)$, $I_1(x)$ and $I_n(x)$ denote the modified Bessel functions of the first kind, of order zero, one, and n, respectively. The temperature $T_1^{\mathrm{II}}(x,t)$ and $T_w^{\mathrm{II}}(x,t)$ are the solutions of the initial-boundary value problem (27)–(31):

$$T_1^{\mathrm{II}} = -v_T\,\tau_w\left\{e^{-(\xi+\eta_1)}\left[(\eta_1-\xi)U+\xi I_0\left(2\sqrt{\xi\eta_1}\right)+\sqrt{\xi\eta_1}\,I_1\left(2\sqrt{\xi\eta_1}\right)\right]\right\} \quad t\ge t_{cn} \tag{35}$$

$$T_w^{\mathrm{II}} = T_1^{\mathrm{II}} + v_T\,\tau_w\,e^{-(\xi+\eta_1)}\left[U - I_0\left(2\sqrt{\xi\eta_1}\right)\right], \quad t\ge t_{cn} \tag{36}$$

where:

$$\xi = \frac{x N_1}{L_x},\quad \eta = \frac{t-t_{pr}}{\tau_w},\quad \eta_1 = \frac{t-t_{cn}-t_{pr}}{\tau_w},\quad t_{pr} = x^+ N_1 \tau_1$$

The symbol t_{pr} denotes transit time of the fluid particle from the inlet (x = 0) to the x coordinate given by: $t_{\mathrm{pr}} = x/w_1$.

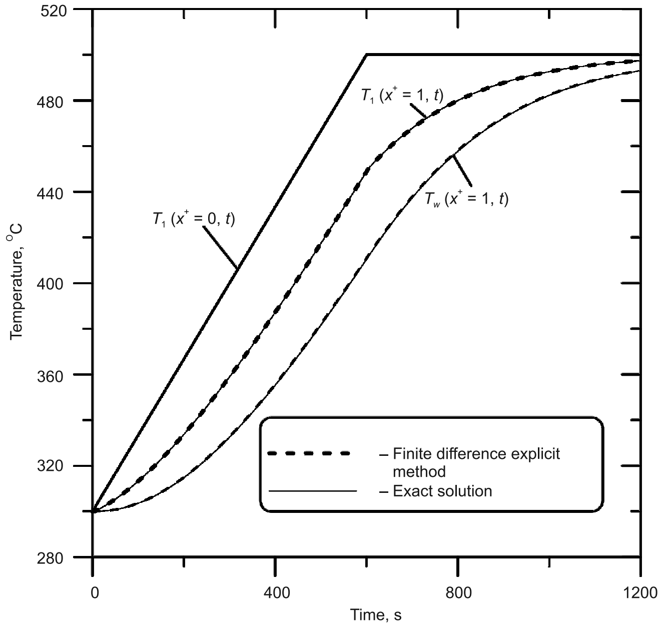

Fig. 3. Comparison of fluid and pipeline wall temperature at the pipeline outlet (x^+ = 1);
- - - - - – finite difference explicit method, —– exact solution

Rys. 3. Porównanie temperatury płynu i ścianki na wylocie z rurociągu (x^+ = 1);
- - - - - – metoda różnic skończonych, —– rozwiązanie ścisłe

where transit time t_{pr} is given by: $t_{\mathrm{pr}} = x\,/\,w_1$.

The comparison of results obtained using the finite difference method (11)–(12) and the analytical exact is presented in Fig. 3. The coincidence between the approximate and exact solution is very satisfactory. The differences between the exact analytical solution and the finite difference solution are almost invisible.

Calculations were performed for the following data: $N_1 = 1.22$, $\tau_1 = 3.94$ s, $\tau_w = 128.89$ s, $T_0 = 300°C$, $T_{nom} = 500°C$, $v_T = 1/3$ K/s, $t_{cn} = 600$ s. The dimensionless spatial and time steps in the finite difference method were: $\Delta x^+ = 1/48$ and $\Delta t = 0.03$ s. The adopted time step satisfies the Courant condition, as the quotient:

$$\frac{\Delta t}{N_1 \; \tau_1 \; \Delta x^+} = \frac{0.03 \cdot 48}{1.22 \cdot 3.94} = 0.3$$

is less than unity.

4. Conclusion

Consistency of the results obtained using the exact analytical method and finite difference method is very good.

It was demonstrated that the accuracy of the explicit finite difference method is very good. The finite difference method can therefore be used to model the pipeline or heat regenerator with time dependent inlet fluid velocity or temperature.

Literatura

[1] Lubecki S., Taler D., Sobota T., *Numerical optimization of steam pipeline heating with respect to thermal stresses*, Archives of Thermodynamics, Vol. 29, No. 4, 2008, 87-96.

[2] Taler J., Lubecki S., *Numerical optimization of steam pipeline heating, EngOpt2010*, 2nd International Conference on Engineering Optimization, 6–9 September 2010 Lisboa, Portugal, Book of Abstracts (p. 308) and CD-ROM Proceedings.

[3] Серов Е.П., Корольков Б.П., *Динамика парогенераторов*, Энергоиздат, Москва 1981.

[4] Taler J., Duda P., *Solving Direct and Inverse Heat Conduction Problems*, Springer, Berlin 2006.